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Employing the Artificial Cuckoo Swarm Algorithm for Estimating Various Multiple Nonlinear Regression Models

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ABSTRACT

Nonlinear regression models are widely used to model stochastic phenomena, and parameter estimation is essential for interpreting these models. Consequently, this research used the Artificial Cuckoo Swarm Algorithm to improve parameter estimation precision for three nonlinear models: Meyer 1, Meyer 5, and the Nelson Model. We employed two estimation techniques: nonlinear least squares (NLS) and the S estimation approach. To improve the efficacy of these two estimating approaches, they were integrated with the Artificial Cuckoo Swarm Algorithm. This work used simulation to evaluate the effectiveness of these two methodologies before and after the application of the algorithm, using sample sizes (50, 100, 150) and sigma levels (0.1, 0.5, 0.9). The mean squared error (MSE) is used as a benchmark for comparison. Simulation results demonstrate that the s-estimator method, particularly when combined with an AI algorithm, produces the smallest mean squared error (MSE) across all models. Furthermore, this technique achieved the lowest MSE values in almost every instance, even without the algorithm.

In the pollution data for Iraq's Euphrates River, we employed the robust s-estimator method, along with the Cuckoo Swarm Algorithm, identified as optimal in simulation results, to estimate three nonlinear regression models with total dissolved solids as the dependent variable and sulfates and chlorides as the independent variables.

The Cuckoo Swarm Algorithm method significantly improved the three nonlinear regression models included in the study, demonstrating its effectiveness. The findings indicated that chlorine and sulfates directly increase total dissolved solids in the Euphrates River.

1. Introduction:

Regression is a frequently utilised statistical method. Regression analysis clarifies the relationship among causal elements. Research indicates that several relationships between variables are nonlinear. Nonlinear regression models are characterised by the presence of at least one nonlinear coefficient. Nonlinear models are utilised to depict complex interactions among variables. They play an essential role in several scientific and technological disciplines, with typical examples of nonlinear models including growth,

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productivity, density, and dose-response models.

This research will assess several nonlinear regression models utilising the artificial cuckoo method.

This approach identifies appropriate solutions from a collection and optimises them based on existing constraints and variables. Data analysis challenges may be addressed by integrating advanced statistical methodologies with artificial intelligence algorithms to enhance nonlinear regression models.

Various researchers who have explored nonlinear regression models and artificial cuckoo algorithms will be referenced. (Ang et al., 2006) Developed a nonlinear regression model for capacitive accelerometers in microelectromechanical systems using virtual modelling to address scale, bias, and misalignment issues. Tests indicated that the model reduced sensor errors to random noise, thereby enhancing accuracy and efficacy [1]. (Tvrdik, 2007) Employed five adaptive variants of the differential evolution method to estimate nonlinear regression coefficients and conducted an experimental comparison with the controlled random search strategy, which proved to be better [2]. (Srivastava & Tripathi, 2012) The researchers analysed linear regression, nonlinear models, and artificial neural networks. This experiment utilised data on Indian monsoon precipitation. The artificial neural network model surpassed linear regression but did not surpass nonlinear regression [3]. (Pinar Civicioglu & Erkan Besdok, 2013) The cuckoo algorithm's structure and problem-solving ability were examined. Fifty benchmark problems were used to numerically assess CS's algorithmic behaviour. CS was compared with PSO, DE, and ABC for benchmark problem-solving using the Kruskal-Wallis test [4]. (Susanti et al. ,2014) The authors introduced M estimation, S estimation, and MM estimation in robust regression to ascertain the regression model. M estimation extends the maximum likelihood method and serves as a robust estimation technique, while S and MM estimations are extensions of M estimation applied to maize production data [5]. (Xin & Suash, 2014) They examined the core concepts of the cuckoo-hunting algorithm, its recent advancements, and its applications. They examined the algorithm and investigated its search methods to ascertain the factors contributing to its efficiency. They also discussed the nature of algorithms and their connection to self-organising systems [6]. (Gunavathi & Premalatha, 2015) The Cuckoo Search (CS) algorithm improves microarray gene expression data for cancer classification. SNR, F, and T categorise genes. CS identifies pertinent genes from the highest-ranked m. The fitness function for CS is the KNN classification accuracy. The methodology was evaluated using 10 cancer gene expression datasets. Cuckoo search (CS) achieves 100% accuracy on the DLBCL Harvard, Lung Michigan, Ovarian Cancer, AML-ALL, and Lung Harvard2 datasets, surpassing previous approaches on the DLBCL and prostate datasets [7]. (Ozsoy and Orkcu, 2016) Calculated nonlinear regression coefficients employing least squares and particle swarm optimisation. Particle swarm optimisation rapidly converged to the minimum mean-squared error across 28 nonlinear regression models, yielding precise coefficient estimates [8]. (Khan & et al., 2021) Investigated precise estimating techniques to address the influence of outliers and deliver efficient, consistent estimations. They introduced a novel iterative estimator for M, grounded on an innovative loss function. The findings were compared by re-estimating the estimator M using several functions, including Tukey biweight, Huber, Hample, and Andrew-Sign [9]. (Maha & Huda, 2021) They proposed using the Gastwirth-signed estimator as a robust alternative to the arithmetic mean in OLS and the median in other estimation methods. They used simulation to evaluate mean-standard-error-based estimation methods while iteratively applying Huber's M-estimation until convergence [10]. (Madubu & et al., 2023) We employed AIC, BIC, and MSE to simulate M and MM estimators using ordinary least squares to identify the optimal approach for parameter estimation in a nonlinear regression model characterised by extreme values and distinct error distributions. The M estimator performs effectively in exponential and Cauchy distributions, with no extreme values, in small to medium samples, as indicated by simulations. MM and CLS optimally evaluated large samples [11].

The Research Aims are:

1. Estimating nonlinear regression models with the nonlinear least squares method.
2. Estimating nonlinear regression models with the S method.
3. Employing the artificial cuckoo algorithm to estimate nonlinear regression models, subsequently comparing the estimation methodologies for nonlinear models employed in the research before and following implementation, using the MSE comparison criterion.

2. Research Methodology

2.1 Nonlinear Regression:

Nonlinear regression models model the nonlinear relationship between the explanatory variables and the response variable, with unknown parameters. Their typical form is:

$$Y_i = f(x_i;\beta) + \varepsilon_i \quad \text{... (1)}$$

Y_i : It represents the response variable.
x_i : represents a vector of independent variables
β : The vector of parameters requires estimation.
ε_i : The vector of random error is presumed to follow a normal distribution.

Our study included three nonlinear regression models [8].

1. Meyer 1 Model [12]

$$Y_i = \frac{\beta_3\beta_1 x_1}{1+\beta_1 x_1 + \beta_2 x_2} + \varepsilon_i \quad \text{... (2)}$$

2. Meyer 5 Model [12]

$$Y_i = \beta_3(\exp(-\beta_1 x_1) + \exp(-\beta_2 x_2)) + \varepsilon_i \quad \text{... (3)}$$

2. Nelson Model [12]

$$Y_i = \exp(\beta_3 - \beta_1 x_1 \ \exp(-\beta_2 x_2)) + \varepsilon_i \quad \text{... (4)}$$

$\beta_1, \beta_2, \beta_3$: Model parameters

2.2 Methods of Estimation

2.2.1 Nonlinear Least Squares Method (NLS)

A standard method for calculating coefficients in nonlinear models.

$$Q = \sum_{i=1}^{n}\big(Y_i - f(x_i;\beta)\big)^2 \quad \text{.....(5)}$$

Upon determining the partial derivatives of the sum of squares remainder function for each model parameter, we employ numerical methods, namely the Newton-Raphson method, characterised by the following formula [13]:

$$\beta_{(k+1)} = \beta_k - (J_k)^{-1} * f_k \quad \text{....(6)}$$

k: the number of repeats.
$\beta_{(k+1)}$: A vector of dimension (h*1), where h is the number of calculated parameters.
J_k : Matrix of partial derivatives.
f_k : Perpendicular vector derived from the equations.
β_k : Vector of standard coefficients.
k=1 initialises the vector to its default state. The vector incorporates the new default value and the estimated coefficient values from the second iteration at k=2. Nevertheless, if the iteration termination criterion is not met:

$$\left|\beta_{(k+1)} - \beta_k\right| \le 0.01 \quad \text{.....(7)}$$

The values of the coefficient vector fluctuate during iterations until the stopping criterion is met or a predetermined coefficient value is reached, at which point the iteration concludes. The determinant of the calculated coefficient matrix is zero [14].

2.2.1.1 The (NLS) Method for the Meyer1 Model [16]

$$Q = \sum_{i=1}^{n}\left[Y_1 - \frac{\beta_3\beta_1 x_1}{1+\beta_1 x_1+\beta_2 x_2}\right]^2 \quad ...(8)$$

Equation (6) about the second model:

$$\begin{pmatrix}\hat{\beta}_1\\ \hat{\beta}_2\\ \hat{B}_3\end{pmatrix} = \begin{pmatrix}\beta_{10}\\ \beta_{20}\\ \beta_{30}\end{pmatrix} - \begin{bmatrix}\frac{\partial k_1}{\partial\beta_1} & \frac{\partial k_1}{\partial\beta_2} & \frac{\partial k_1}{\partial\beta_3}\\ \frac{\partial k_2}{\partial\beta_1} & \frac{\partial k_2}{\partial\beta_2} & \frac{\partial k_2}{\partial\beta_3}\\ \frac{\partial k_3}{\partial\beta_1} & \frac{\partial k_3}{\partial\beta_2} & \frac{\partial k_3}{\partial\beta_3}\end{bmatrix}^{-1}\begin{bmatrix}\frac{\partial Q}{\partial\beta_1}\\ \frac{\partial Q}{\partial\beta_2}\\ \frac{\partial Q}{\partial\beta_3}\end{bmatrix} \quad ....(9)$$

$$k_1 = \frac{\partial Q}{\partial\beta_1} = -2\left[\sum_{i=1}^{n}\left(\frac{\beta_3 x_1 y_i - \beta_1\beta_3 x_1^2 y_i}{1+\beta_1 x_1+\beta_2 x_2}\right) - \sum_{i=1}^{n}\frac{\beta_1\beta_3^2 x_1^2 - \beta_1^2\beta_3^2 x_1^3}{(1+\beta_1 x_1+\beta_2 x_2)^2}\right] \quad .....(10)$$

$$k_2 = \frac{\partial Q}{\partial\beta_2} = 2\left[\sum_{i=1}^{n}\left(\frac{\beta_1\beta_3 x_1 x_2 y_i}{(1+\beta_1 x_1+\beta_2 x_2)^2}\right) - \sum_{i=1}^{n}\frac{\beta_1^2\beta_3^2 x_1^2 x_2}{(1+\beta_1 x_1+\beta_2 x_2)^3}\right] \quad .....(11)$$

$$k_3 = \frac{\partial Q}{\partial\beta_3} = -2\left[\sum_{i=1}^{n}\left(\frac{\beta_1 x_1 y_i}{1+\beta_1 x_1+\beta_2 x_2}\right) - \sum_{i=1}^{n}\frac{\beta_1^2\beta_3 x_1^2}{(1+\beta_1 x_1+\beta_2 x_2)^2}\right] \quad ......(12)$$

By differentiating Equation (10) with respect to the parameters, we derive

$$\frac{\partial k_1}{\partial\beta_1} = -2\left[-\left(\sum_{i=1}^{n}\frac{\beta_3 x_1^2 y_i}{(1+\beta_1 x_1+\beta_2 x_2)}\right) - \left(\sum_{i=1}^{n}\frac{\beta_3 x_1^2 y_i - \beta_1\beta_3 x_1^2 y_i}{(1+\beta_1 x_1+\beta_2 x_2)}\right) - \left(\sum_{i=1}^{n}\frac{\beta_3^2 x_1^2}{(1+\beta_1 x_1+\beta_2 x_2)^2}\right) + \sum_{i=1}^{n}\frac{4\beta_1\beta_3^2 x_1^4}{(1+\beta_1 x_1+\beta_2 x_2)^3}\right] \quad .....(13)$$

$$\frac{\partial k_1}{\partial\beta_2} = 2\left[\sum_{i=1}^{n}\left(\frac{\beta_3 x_1 x_2 y_i - \beta_1\beta_3 {x_1}^2 x_2 y_i}{(1+\beta_1 x_1+\beta_2 x_2)^2}\right) + \sum_{i=1}^{n}\left(\frac{2(\beta_1{\beta_3}^2{x_1}^2{x_2}^2 - {\beta_1}^2{\beta_3}^2{x_1}^3 x_2}{(1+\beta_1 x_1+\beta_2 x_2)^3}\right)\right] \quad ......(14)$$

$$\frac{\partial k_1}{\partial\beta_3} = -2\left[\sum_{i=1}^{n}\left(\frac{x_1 y_i - \beta_1 x_1^2 y_i}{(1+\beta_1 x_1+\beta_2 x_2)}\right) - \sum_{i=1}^{n}\left(\frac{2\beta_1\beta_3 x_1^2 - 2\beta_1^2\beta_3 x_1^3}{(1+\beta_1 x_1+\beta_2 x_2)^2}\right)\right] \quad ......(15)$$

Taking the derivative of Equation (11) with respect to the parameters results in

$$\frac{\partial k_2}{\partial\beta_1} = 2\sum_{i=1}^{n}\frac{(1+\beta_1 x_1)^2 * \beta_3 x_1 x_2 y_i - \beta_1\beta_3 x_1^2 x_2 y_i * 2(1+\beta_1 x_1+\beta_2 x_2}{(1+\beta_1 x_1+\beta_2 x_2)^4} - \sum_{i=1}^{n}\frac{(1+\beta_1 x_1+\beta_2 x_2)^3 * 2\beta_1\beta_3^2 x_1 x_2 - \beta_1\beta_3^2 x_1 x_2 - \beta_1^2\beta_3^2 x_1^2 x_2 * 3(1+\beta_1 x_1+\beta_2 x_2)^2 * x_1}{(1+\beta_1 x_1+\beta_2 x_2)^6}$$

$$=2\left[\sum_{i=1}^{n}\left(\frac{\beta_3 x_1 x_2 y_i}{(1+\beta_1 x_1+\beta_2 x_2)^2}\right) - \sum_{i=1}^{n}\left(\frac{2\beta_1\beta_3 x_1^2 x_2 y_i}{(1+\beta_1 x_1+\beta_2 x_2)^3}\right) - \sum_{i=1}^{n}\left(\frac{2\beta_1\beta_3^2 x_1 x_2}{(1+\beta_1 x_1+\beta_2 x_2)^3}\right) - \sum_{i=1}^{n}\frac{3\left(\beta_1\beta_3^2 x_1^2 x_2 - \beta_1^2\beta_3^2 x_1^3 x_2\right)}{(1+\beta_1 x_1+\beta_2 x_2)^4}\right] \quad ....(16)$$

$$\frac{\partial k_2}{\beta_2} = 2\left[\frac{-2\beta_1\beta_3 x_1 x_2^2 y_i}{(1+\beta_1 x_1+\beta_2 x_2)^3} - \sum_{i=1}^{n}\frac{3\beta_1^2\beta_3^2 x_1^2 x_2^2}{(1+\beta_1 x_1+\beta_2 x_2)^4}\right] \quad ......(17)$$

$$\frac{\partial k_2}{\partial\beta_3} = 2\left[\sum_{i=1}^{n}\frac{\beta_1 x_1 x_2 y_i}{(1+\beta_1 x_1+\beta_2 x_2)^2} - \sum_{i=1}^{n}\frac{2\beta_1\beta_3 x_1^2 x_2}{(1+\beta_1 x_1+\beta_2 x_2)^3}\right] \quad ....(18)$$

By differentiating Equation (12) with respect to the parameters, one obtains

$$\frac{\partial k_3}{\partial \beta_1} = -2\left[\sum_{i=1}^{n} \frac{x_1 y_i}{(1+\beta_1 x_1+\beta_2 x_2)} - \sum_{i=1}^{n} \frac{\beta_1 x_1^2 y_i}{(1+\beta_1 x_1+\beta_2 x_2)^2} - \sum_{i=1}^{n} \frac{2\beta_1 \beta_3 x_1^2}{(1+\beta_1 x_1+\beta_2 x_2)^3} - \quad \frac{2\beta_1^2 \beta_3 x_1^3}{(1+\beta_1 x_1+\beta_2 x_2)^3}\right] \quad ...(19)$$

$$\frac{\partial k_3}{\partial \beta_2} = -2\left[\sum_{i=1}^{n} \frac{-\beta_1 x_1 x_1 y_i}{(1+\beta_1 x_1+\beta_2 x_2)^2} + \sum_{i=1}^{n} \frac{\beta_1^2 \beta_3 x_1^2 x_2}{(1+\beta_1 x_1+\beta_2 x_2)^3}\right] \quad .....(20)$$

$$\frac{\partial k_3}{\partial \beta_3} = 2\sum_{i=1}^{n}\left(\frac{\beta_1^2 x_1^2}{(1+\beta_1 x_1+\beta_2 x_2)^2}\right) \quad ......(21)$$

2.2.1.2 The (NLS) Method for the Meyer 5 Model [15]

$$Q = \sum_{i=1}^{n}\left[y_i - \beta_3\left(e^{-\beta_1 x_1} + e^{-\beta_2 x_2}\right)\right]^2 \quad .....(22)$$

$$\begin{pmatrix}\hat{\beta}_1\\ \hat{\beta}_2\\ \hat{\beta}_3\end{pmatrix} = \begin{pmatrix}\beta_{10}\\ \beta_{20}\\ \beta_{30}\end{pmatrix} - \begin{bmatrix}\frac{\partial k_1}{\partial \beta_1} & \frac{\partial k_1}{\partial \beta_2} & \frac{\partial k_1}{\partial \beta_3}\\ \frac{\partial k_2}{\partial \beta_1} & \frac{\partial k_2}{\partial \beta_2} & \frac{\partial k_2}{\partial \beta_3}\\ \frac{\partial k_3}{\partial \beta_1} & \frac{\partial k_3}{\partial \beta_2} & \frac{\partial k_3}{\partial \beta_3}\end{bmatrix}^{-1} \begin{bmatrix}\frac{\partial Q}{\partial \beta_1}\\ \frac{\partial Q}{\partial \beta_2}\\ \frac{\partial Q}{\partial \beta_3}\end{bmatrix} \quad .....(23)$$

$$k_1 = \frac{\partial Q}{\partial \beta_1} = 2\left[\sum_{i=1}^{n} y_i \beta_3 x_1 e^{-\beta_1 x_1} - \sum_{i=1}^{n} \beta_3^2 x_1 e^{-2\beta_1 x_1} - \sum_{i=1}^{n} \beta_3^2 x_1 e^{-(\beta_1 x_1+\beta_2 x_2)}\right] \quad ...(24)$$

$$k_2 = \frac{\partial Q}{\partial \beta_2} = 2\left[\sum_{i=1}^{n} y_i \beta_3 x_2 e^{-\beta_2 x_2} - \sum_{i=1}^{n} \beta_3^2 x_2 e^{-(\beta_1 x_1+\beta_2 x_2)} - \sum_{i=1}^{n} \beta_3^2 x_2 e^{-2\beta_2 x_2}\right] \quad .....(25)$$

$$k_3 = \frac{\partial Q}{\partial \beta_3} = -2\left[\sum_{i=1}^{n} y_i\left(e^{-\beta_1 x_1} + e^{-\beta_2 x_2}\right) - \sum_{i=1}^{n} \beta_3\left(e^{-2\beta_1 x_1} + e^{-2\beta_2 x_2}\right) - \sum_{i=1}^{n} 2\beta_3 e^{-(\beta_1 x_1+\beta_2 x_2)}\right] \quad ......(26)$$

By differentiating Equation (24) for the parameters, we derive.

$$\frac{\partial k_1}{\partial \beta_1} = -2\sum_{i=1}^{n}\left[y_i \beta_3 x_1^2 e^{-\beta_1 x_1} - 2\beta_3^2 x_1^2 e^{-2\beta_1 x_1} - \beta_3^2 x_1^2 e^{-(\beta_1 x_1+\beta_2 x_2)}\right] \quad ....(27)$$

$$\frac{\partial k_1}{\partial \beta_2} = 2\sum_{i=1}^{n}\left[\beta_3^2 x_1 x_2 e^{-(\beta_1 x_1+\beta_2 x_2)}\right] \quad .....(28)$$

$$\frac{\partial k_1}{\partial \beta_3} = 2\sum_{i=1}^{n}\left[y_i x_1 e^{-\beta_1 x_1} - 2\beta_3 x_1 e^{-2\beta_1 x_1} - 2\beta_3 x_1 e^{-(\beta_1 x_1+\beta_2 x_2)}\right] \quad .....(29)$$

By differentiating Equation (25) with respect to the parameters, we derive.

$$\frac{\partial k_2}{\partial \beta_1} = 2\sum_{i=1}^{n}\left[\beta_3^2 x_1 x_2 e^{-(\beta_1 x_1+\beta_2 x_2)}\right] \quad .....(30)$$

$$\frac{\partial k_2}{\partial \beta_2} = -2\sum_{i=1}^{n}\left[y_i \beta_3 x_2^2 e^{-\beta_2 x_2} - \beta_3^2 x_2^2 e^{-(\beta_1 x_1)} - 2\beta_3^2 x_2^2 e^{-2\beta_2 x_2}\right] \quad ......(31)$$

$$\frac{\partial k_2}{\partial \beta_3} = 2\sum_{i=1}^{n}\left[y_i x_2 e^{-\beta_2 x_2} - 2\beta_3 x_2 e^{-(\beta_1 x_1+\beta_2 x_2)} - 2\beta_3 x_2 e^{-2\beta_2 x_2}\right] \quad ......(32)$$

By differentiating Equation (26) for the parameters, we derive

$$\frac{\partial k_3}{\partial \beta_1} = 2\sum_{i=1}^{n}\left[y_i x_1 e^{-\beta_1 x_1} - 2\beta_3 x_1 e^{-2\beta_1 x_1} - 2\beta_3 x_1 e^{-(\beta_1 x_1+\beta_2 x_2)}\right] \quad ......(33)$$

$$\frac{\partial k_3}{\partial \beta_2} = 2\sum_{i=1}^{n}\left[y_i x_2 e^{-\beta_2 x_2} - 2\beta_3 x_2 e^{-(\beta_1 x_1+\beta_2 x_2)} - 2\beta_3 x_2 e^{-2\beta_2 x_2}\right] \quad ......(34)$$

$$\frac{\partial k_3}{\partial \beta_3} = 2\sum_{i=1}^{n}\left[e^{-2\beta_1 x_1} + 2e^{-(\beta_1 x_1+\beta_2 x_2)} + e^{-2\beta_2 x_2}\right] \quad .....(35)$$

2.2.1.3 The (NLS) Method for the Nelson Model [15]

$$Q = \sum_{i=1}^{n}\left[y_i - \exp(\beta_3 - \beta_1 x_1 e^{(-\beta_2 x_2)})\right]^2 \qquad ......(36)$$

$$\begin{pmatrix}\hat\beta_1\\ \hat\beta_2\\ \hat\beta_3\end{pmatrix}=\begin{pmatrix}\beta_{10}\\ \beta_{20}\\ \beta_{30}\end{pmatrix}-\begin{bmatrix}\frac{\partial k_1}{\partial\beta_1} & \frac{\partial k_1}{\partial\beta_2} & \frac{\partial k_1}{\partial\beta_3}\\ \frac{\partial k_2}{\partial\beta_1} & \frac{\partial k_2}{\partial\beta_2} & \frac{\partial k_2}{\partial\beta_3}\\ \frac{\partial k_3}{\partial\beta_1} & \frac{\partial k_3}{\partial\beta_2} & \frac{\partial k_3}{\partial\beta_3}\end{bmatrix}^{-1}\begin{bmatrix}\frac{\partial Q}{\partial\beta_1}\\ \frac{\partial Q}{\partial\beta_2}\\ \frac{\partial Q}{\partial\beta_3}\end{bmatrix} \qquad ......(37)$$

$$k_1 = \frac{\partial Q}{\partial \beta_1} = 2\sum_{i=1}^{n}\left[y_i - exp\left(\beta_3 - \beta_1 x_1 e^{(-\beta_2 x_2)}\right) - x_1 \exp(-\beta_2 x_2)\exp\left(\beta_3 - \ \beta_1 x_1 e^{(-\beta_2 x_2)}\right)\right]$$
$$= -2\left[\sum_{i=1}^{n} y_i\, x_1 \exp(-\beta_2 x_2)\exp\left(\beta_3 - \beta_1 x_1 e^{(-\beta_2 x_2)}\right) + \quad \sum_{i=1}^{n} -x_1 \exp(-\beta_2 x_2)\exp 2\left(\beta_3 - \beta_1 x_1 e^{(-\beta_2 x_2)}\right)\right] \qquad .....(38)$$

$$k_2 = \frac{\partial Q}{\partial \beta_2} = -2\sum_{i=1}^{n}\left[y_i - \exp\left(\beta_3 - \beta_1 x_1 e^{(-\beta_2 x_2)}\right) y_i x_1 x_2 \exp\left(\beta_3 - \ \beta_1 x_1 e^{(-\beta_2 x_2)}\right)\right]$$
$$k_2 = -2\left[\sum_{i=1}^{n} y_i x_1 x_2 \beta_1 \ \exp\left(\beta_3 - \beta_1 x_1 e^{(-\beta_2 x_2)}\right) - \sum_{i=1}^{n} x_1 x_2 \beta_1 \exp 2\left(\beta_3 - \quad \beta_1 x_1 e^{(-\beta_2 x_2)}\right)\right] \qquad ....(39)$$

$$k_3 = \frac{\partial Q}{\partial \beta_3} = 2\sum_{i=1}^{n}\left[y_i - \exp\left(\beta_3 - \beta_1 x_1 e^{(-\beta_2 x_2)}\right)\exp\left(\beta_3 - \beta_1 x_1 e^{(-\beta_2 x_2)}\right)\right]$$
$$= 2\left[\sum_{i=1}^{n}\exp\left(\beta_3 - \beta_1 x_1 e^{(-\beta_2 x_2)}\right) - \sum_{i=1}^{n}\exp 2\left(\beta_3 - \beta_1 x_1 e^{(-\beta_2 x_2)}\right)\right] \qquad ......(40)$$

By differentiating Equation (38) with respect to the parameters, we derive

$$\frac{\partial k_1}{\partial \beta_1} = -2\sum_{i=1}^{n} y_i x_1 \exp\left(\beta_3 - \beta_1 x_1 e^{(-\beta_2 x_2)}\right) + 4\sum_{i=1}^{n} x_1 exp2\left(\beta_3 - \ \beta_1 x_1 e^{(-\beta_2 x_2)}\right) \qquad .....(41)$$

$$\frac{\partial k_1}{\partial \beta_2} = 2\sum_{i=1}^{n} y_i x_2 exp\left(\beta_3 - \beta_1 x_1 e^{(-\beta_2 x_2)}\right) + 4\sum_{i=1}^{n} \beta_2 x_1 x_2 exp2\left(\beta_3 - \beta_1 x_1 e^{(-\beta_2 x_2)}\right) \qquad ......(42)$$

$$\frac{\partial k_1}{\partial \beta_3} = 2\sum_{i=1}^{n} y_i \exp\left(\beta_3 - \beta_1 x_1 e^{(-\beta_2 x_2)}\right) - 4\sum_{i=1}^{n}\exp 2\left(\beta_3 - \beta_1 x_1 e^{(-\beta_2 x_2)}\right) \qquad ......(43)$$

By differentiating Equation (39) with respect to the parameters, we derive

$$\frac{\partial k_2}{\partial \beta_1} = 2\sum_{i=1}^{n} y_i x_1^2 \exp(-\beta_2 x_2)\exp\left(\beta_3 - \beta_1 x_1 e^{(-\beta_2 x_2)}\right) - 4\sum_{i=1}^{n} x_1^2 \exp -2(\beta_2 x_2)\exp -2\left(\beta_3 - \beta_1 x_1 e^{(-\beta_2 x_2)}\right) \qquad ......(44)$$

$$\frac{\partial k_2}{\partial \beta_2} =$$
$$-2\left[\sum_{i=1}^{n} y_i x_1 \exp(-\beta_2 x_2) - x_2 x_1 \beta_2 \exp\left(\beta_3 - \beta_1 x_1 e^{(-\beta_2 x_2)}\right)\right] +$$
$$\left[\exp\left(\beta_3 - \beta_1 x_1 e^{(-\beta_2 x_2)}\right) \quad \sum_{i=1}^{n} -x_1 x_2 y_i \exp(-\beta_2 x_2)\right] - \left[x_1 \exp(-\beta_2 x_2) * -x_2 x_1 \beta_2 \exp 2\left(\beta_3 - \beta_1 x_1 e^{(-\beta_2 x_2)}\right)\right] + \quad \left[\exp 2\left(\beta_3 - \beta_1 x_1 e^{(-\beta_2 x_2)}\right) - x_1 x_2 \exp(-\beta_2 x_2)\right]$$
$$= 2\sum_{i=1}^{n} x_1^2\, x_2 \beta_1 y_i \exp(-\beta_2 x_2)\exp\left(\beta_3 - \beta_1 x_1 e^{(-\beta_2 x_2)}\right) - 2\sum_{i=1}^{n} y_i x_1 x_2 \exp(-\beta_2 x_2)\exp\left(\beta_3 - \beta_1 x_1 e^{(-\beta_2 x_2)}\right) + \quad 2\sum_{i=1}^{n} x_1 x_2 \exp(-\beta_2 x_2)\exp 2\left(\beta_3 - \beta_1 x_1 e^{(-\beta_2 x_2)}\right) \qquad ......(45)$$

$$\frac{\partial k_2}{\partial \beta_3} = -2\sum_{i=1}^{n} y_i x_1 \exp(-\beta_2 x_2)\exp\left(\beta_3 - \beta_1 x_1 e^{(-\beta_2 x_2)}\right) - \ 4\sum_{i=1}^{n} x_1 \exp(-\beta_2 x_2)\exp 2\left(\beta_3 - \beta_1 x_1 e^{(-\beta_2 x_2)}\right) \qquad ....(46)$$

By differentiating Equation (40) with respect to the parameters, we derive

$$\frac{\partial k_3}{\partial \beta_1} = -2[\sum_{i=1}^{n} y_i x_1 x_2 \beta_1]\left[-x_1 \exp(-\beta_2 x_2)\exp\left(\beta_3 - \beta_1 x_1 e^{(-\beta_2 x_2)}\right)\right] + \left[\exp\left(\beta_3 - \beta_1 x_1 e^{(-\beta_2 x_2)}\right)\right][x_1 x_2 y_i] - \sum_{i=1}^{n}[x_1 x_2 \beta_1] \quad 2x_1 \exp(-\beta_2 x_2)\exp 2\left(\beta_3 - \beta_1 x_1 e^{(-\beta_2 x_2)}\right) + \exp 2\left(\beta_3 - \beta_1 x_1 e^{(-\beta_2 x_2)}\right) \quad x_1 x_2$$

$$= 2\sum_{i=1}^{n} y_i x_1 x_2 \beta_2 \exp(-\beta_2 x_2)\exp\left(\beta_3 - \beta_1 x_1 e^{(-\beta_2 x_2)}\right) + 2\sum_{i=1}^{n} y_i x_1 x_2 \exp\left(\beta_3 - \beta_1 x_1 e^{(-\beta_2 x_2)}\right) + 4\sum_{i=1}^{n} x_1^2 x_2 y_i \exp(-\beta_2 x_2)\exp 2\left(\beta_3 - \beta_1 x_1 e^{(-\beta_2 x_2)}\right) + 2\sum_{i=1}^{n} x_1 x_2 \exp 2\left(\beta_3 - \beta_1 x_1 e^{(-\beta_2 x_2)}\right) \quad .....(47)$$

$$\frac{\partial k_3}{\partial \beta_2} = 2\sum_{i=1}^{n} x_1^2 x_2^2 y_i \beta_1^2 \exp(-\beta_2 x_2)\exp\left(\beta_3 - \beta_1 x_1 e^{(-\beta_2 x_2)}\right) + 4\sum_{i=1}^{n} x_1^2 x_2^2 \beta_1^2 \exp(-\beta_2 x_2)\exp 2\left(\beta_3 - \beta_1 x_1 e^{(-\beta_2 x_2)}\right) \quad ....(48)$$

$$\frac{\partial k_3}{\partial \beta_3} = -2\sum_{i=1}^{n} y_i x_1 x_2 \beta_1 \exp\left(\beta_3 - \beta_1 x_1 e^{(-\beta_2 x_2)}\right) - 4\sum_{i=1}^{n} x_1 x_2 \beta_1 \exp 2\left(\beta_3 - \beta_1 x_1 e^{(-\beta_2 x_2)}\right) \quad ....(49)$$

2.2.2 S- Estimators

This resilient regression estimator, introduced by Rousseeuw and Yohai in 1984, employs a residual metric derived from a robust M-estimator and is primarily limited by its reliance on the mean as a weighting function, thereby neglecting the data distribution. The residual standard deviation is employed to rectify the deficiencies of the median [16].

The S-estimator is defined by

$$\hat{\beta}_s = min_\beta \hat{\sigma}_s(e_1, e_2, \dots, e_n) \quad ......(50)$$

for the determination of the robust estimator $\hat{\sigma s}$ and its compliance

$$min \sum_{i=1}^{n} p\left(\frac{yi-\hat{y}}{\hat{\sigma}_s}\right) \quad ......(51)$$

$$\hat{\sigma}_s = \begin{cases} \frac{median|e_i - mediane_i|}{0.6745} & iteration = 1 \\ \sqrt{\frac{1}{nK}\sum_{i=1}^{n} w_i e_i^2} & iteration > 1 \end{cases} \quad ....(52)$$

K=0.199

$$w_i = w_\sigma = \frac{p(z)}{z^2} \quad ......(53)$$

By differentiating Equation (51) concerning β and putting the result equal to zero, ψ is identified as the derivative of p.

$$\psi(z_i) = \acute{p}(z_i)\begin{cases} z_i\left[1 - \left(\frac{z_i}{c}\right)^2\right]^2 & ; |z_i| \le c \\ 0 & ; |z_i| > c \end{cases} \quad ......(54)$$

C= 1.547

$z_i = \frac{e_i}{\hat{\sigma}}$ ……(55)

The S-estimator is derived using the Iteratively Reweighted Least Squares (IRLS) approach.

$\hat{\beta}_S = (X^T W X)^{-1} X^T W Y$ …..(56)

Procedures of algorithm S

1. Determining the initial parameters β using the estimated value $\hat{\beta}_M$.

2. Computation of residuals $e_i = y_i - \hat{y}_i$.

3. Determining the value of $\hat{\sigma}$ As per Equation (52).

4. Determining the value of $z_i = \frac{e_i}{\hat{\sigma}}$.

5. Determining weight values via the Tukey function

$$w(z_i) = \begin{cases} \begin{cases} \left[1 - \left(\frac{z_i}{c}\right)^2\right]^2 & ; |z_i| \leq 1.547 \\ 0 & ; |z_i| > 1.547 \end{cases} & iteration = 1 \\ \frac{p(z)}{z^2} & iteration > 1 \end{cases}$$ …..(57)

6. Calculating the value of $\hat{\beta}_s$ In accordance with Equation (56).

7. We redo steps (2) through (7) and persist in solving until we achieve a value approximating $\hat{\beta}_S$ [16].

2.3 Cuckoo Search Algorithm (CS):

The optimisation problems are solved by the 2009 cuckoo search developed by Yang and Deb. Obligatory brood-parasitic cuckoo species inspired this method. Cuckoos deposit their eggs in shared nests but may eliminate other eggs to enhance hatching success. Particular species are obligatory brood parasites, laying their eggs in the nests of host birds [17]. The host bird will abandon or construct a new nest if the eggs are not its own. The parasitic New World cuckoo replicates the colour and pattern of its preferred eggs, potentially depriving other birds of their eggs and enhancing its reproductive success [18].

2.3.1 Artificial Cuckoo Search Algorithm (CSA):

Yang and Depp offered three ideal cuckoo strategies:

1. Cuckoos drop a single egg sequentially into the nest of an arbitrary bird.
2. The subsequent generation inherits the finest nests containing the superior eggs.
3. A specified quantity of host nests enables the host bird to identify the cuckoo egg with a probability of $P \in [0, 1]$. New P nests supplant certain ones.

Each egg in the nest represents a solution, whereas each cuckoo egg signifies a novel solution. The objective function assesses the quality or stability of a solution. Superior solutions (cuckoos) ought to supplant inferior ones in nests [17, 18].

Equation (58) employs a Lévy computation to produce novel cuckoo egg i solutions. $X^{(t+1\}}$.

$X_i^{(t+1)} = X_i^t + \alpha \oplus Lèvy(\lambda)$ ..(58)

The step size must be commensurate with the scales of the relevant problem, where $\alpha > 0$. The product $\oplus$ denotes entrywise multiplications. The Lévy flight produces a stochastic trajectory with random step lengths drawn from a Lévy distribution.

$Lèvy \sim \mu = t^{-\lambda}\ (1 < \lambda < 3)$ ... (59)

It possesses infinite variance and mean. The steps constitute a random walk characterised by a power-law distribution of step lengths and heavy-tailed distributions. Lévy should design innovative ways that enhance the current optimal approach to expedite local search. Nevertheless, far-field randomisation is expected to yield a substantial number of novel solutions that are far from the ideal answer. It will prevent the system from getting stuck in a local optimum [17].

The local search capability of CS may be articulated using Equation (60), which generates new cuckoo solutions based on Equation (58).

$$X_i^{new} = X_i^{old} + 2 * step * \left(X_i^{old} - best\right) \quad (60)$$

Where the step is a random integer produced from a Lévy distribution, the best represents the current optimal solution, X_i^{old} Denotes the previous solution (the old nest is gonna be abandoned) and X_i^{new} Indicates the newly generated solution. A proportion P_α Inadequate responses are discarded, and new solutions are generated using Equation (61), thereby enhancing the CS algorithm's exploratory capacity [19].

$$X_i^{new} = X_i^{old} + rand\ 1 * (rand\ 2 > P_\alpha) * (X_a - X_b) \quad (61)$$

rand1 and rand2 are two random variables drawn from a uniform distribution on the interval [0,1].
P_α Represents the probability of a nest being discovered.
$X_a and\ X_b$ Two are selected at random from existing nests [18, 20].

2.3.2 Variants of the CSA:

Discrete cuckoo search, Binary cuckoo search, Chaotic cuckoo search, Parallel cuckoo search, Cuckoo search for constrained problems, Cuckoo search with adaptive parameters, and Gaussian cuckoo search.
This research will employ the Discrete Cuckoo Search algorithm [18].

2.3.2.1 Discrete Cuckoo Search Algorithm:

The fundamental concept is to reduce the CS step size by employing the sigmoid activation function (62), which serves as an additional threshold to yield 0 or 1, as shown in Equation (63) [22].

$$stp = \frac{1}{1 + exp^{-\alpha}} \quad (62)$$

Where stp is the squashed step size, and α is the CS continuous step size.
The mv is the discrete step size.

$$mv = \begin{cases} 1 & if\ rand < stp \\ 0 & otherwise \end{cases} \quad (63)$$

The newly generated discretised solution is calculated as

$$X_i^{t+1} = mod(X_i^t + mv, m) + 1 \quad \ldots..(64)$$

Where
X_i^{t+1} Is the updated solution or the nest position.
X_i^t Is the old solution or the nest.
m is the range of allowed discrete numbers [21].

Begin

- objective function $f(X), X = (x_1, x_2, \ldots, x_d)^T$
- Generate an initial population of n random solutions (nests).
- while (the ending requirements not satisfied)
  1) Update the cuckoo's position utilizing equation (26).
  2) Choose the best option between Cuckoo's solutions and existing nests.
  3) Discard a portion of the result utilizing equation (27).
  4) Choose the superior option between abandoned nests and newly generated nests.
  5) Update the best solution.
- end while

End

$n \sum_{i=1}$

Where n is the number of a data point, Y_i is the observed value and $\hat{Y}_i$: predicted values.

3. Results and Discussion:

3.1 Experimental Aspect:

We will conduct experiments to evaluate the theoretical framework and demonstrate which of the following estimation methods is superior: **(NLS), S-estimators, and (CS).** The mean squared error criterion will be used to evaluate their results and identify the optimal data estimation method. This will be accomplished via simulation. The outcomes were generated with MATLAB 2024a.

Table 1 Exhibits Mean Squared Error (MSE) results for all methodologies excluding the Cuckoo Swarm Algorithm technique. For varying sample sizes (N = 50, 100, 150) and sigma values (0.1, 0.5, 0.9) in nonlinear regression models with normally distributed errors.

N = 50				
Model	Sigma	NLS	S	Best
Meyer 1	0.1	0.0081252	0.0081292	NLS
Meyer 5		0.7382566	NaN	-
Nelson		0.6000944	NaN	-
Meyer 1	0.5	0.2025297	0.2025773	NLS
Meyer 5		0.9418273	2.8372634	NLS
Nelson		0.6858222	0.3102207	S
Meyer 1	0.9	0.6576309	0.6566275	S
Meyer 5		1.4105958	0.0987646	S
Nelson		1.2046511	1.1499358	S
N = 100				
Meyer 1	0.1	0.0106053	0.0106094	NLS
Meyer 5		0.1096249	0.1039286	S
Nelson		0.0553562	NaN	-
Meyer 1	0.5	0.2645787	0.2645101	S
Meyer 5		0.4113749	0.3764732	S
Nelson		0.3401852	0.2466894	S
Meyer 1	0.9	0.8570885	0.8572420	NLS
Meyer 5		1.0950774	0.4675151	S
Nelson		0.9891456	0.6087309	S
N = 150				
Meyer 1	0.1	0.0117229	0.0117169	S
Meyer 5		0.25495844	0.0303606	S
Nelson		0.1259442	NaN	-
Meyer 1	0.5	0.2930814	0.2903449	S
Meyer 5		0.5696210	0.5337029	S
Nelson		0.4444044	NaN	-
Meyer 1	0.9	0.9692714	0.9644828	S
Meyer 5		1.0315573	1.0252156	S
Nelson		1.1366629	1.1092766	S

Upon examining the MSE values in Table (1), with N = 50

1. sigma = 0.1, it was determined that the NLS approach excelled at predicting the parameters of the Meyer 1 model; however, a comparison between the NLS method and S for the Meyer 5 and Nelson models was not feasible.
2. At sigma = 0.5, the NLS method demonstrated superiority in predicting

the parameters of the Meyer 1 and Meyer 5 models, but the S method excelled in estimating the Nelson model.

3. When sigma equals 0.9, the robust S approach consistently surpassed the NLS method across all three models.

With N=100

1. When sigma equals 0.1, the NLS method demonstrated superiority in predicting Meyer 1 parameters, whereas the S method excelled in estimating Meyer 5 parameters. The two methodologies were incommensurable in evaluating Nelson's model.
2. All three models demonstrated that the robust S-estimator method outperformed the NLS method when sigma was set to 0.5.
3. The S methodology outperformed in the Meyer 5 and Nelson models at sigma = 0.9, although the NLS method excelled at predicting Meyer 1.

When N equals 150, the following observations are made:

1. When sigma equals 0.1, the S approach excelled at estimating the Meyer 1 and Meyer 5 models; however, it was indeterminate whether the technique was best for estimating the Nelson model.
2. Likewise, with sigma = 0.5, the S-Estimators approach demonstrated superiority in estimating the Meyer 1 and Meyer 5 models; however, it was indeterminate whether the technique excelled in predicting the Nelson model.
3. Ultimately, when the sigma value is 0.9, the S-Estimator method emerges as the most effective for estimating the three nonlinear models.

Table (2) Show the (MSE) results for all techniques utilising the synthetic Cuckoo Swarm Algorithm, across varying sample sizes N=(50, 100, 150) and sigma values of (0.1, 0.5, 0.9) for the three nonlinear regression models under the assumption of normally distributed errors.

N = 50				
Model	Sigma	NLS CS	S CS	Best
Meyer 1	0.1	10.1010415e-05	9.2688294e-06	S CS
Meyer 5		2.4131424e-09	1.6213282e-10	S CS
Nelson		0.0025312	0.0127224	NLS CS
Meyer 1	0.5	0.2025297e-03	8.7985026e-06	S CS
Meyer 5		0.9418273e-06	1.1520689e-10	NLS CS
Nelson		0.5858222	0.0130583	S CS
Meyer 1	0.9	7.7404465e-4	8.2358977e-06	S CS
Meyer 5		3.7776716e-06	3.9809590e-08	S CS
Nelson		0.9641051	0.0128333	S CS
N = 100				
Meyer 1	0.1	9.7901411e-04	9.6613134e-06	S CS
Meyer 5		2.8557797e-07	1.1201806e-09	S CS
Nelson		0.0790135	0.0126658	S CS
Meyer 1	0.5	8.5715107e-05	8.1065355e-06	S CS
Meyer 5		3.5406016e-7	5.3165379e-10	S CS
Nelson		0.0578532	0.0126696	S CS
Meyer 1	0.9	7.1798477e-08	9.8637519e-06	NLS CS

Meyer 5		8.0829800e-05	3.5402529e-08	S CS
Nelson		0.0304045	0.0128284	S CS
N = 150				
Meyer 1	0.1	8.5588667e-4	7.9368032e-06	S CS
Meyer 5		7.0882077e-6	2.5496460e-09	S CS
Nelson		0.0680790	0.0126658	S CS
Meyer 1	0.5	9.8000962e-4	7.3579316e-06	S CS
Meyer 5		3.5619849e-7	6.5871653e-10	S CS
Nelson		0.2232150	0.0127167	S CS
Meyer 1	0.9	8.1812220e-3	8.8040194e-06	S CS
Meyer 5		2.2228181e-5	4.5718753e-08	S CS
Nelson		0.0333956	0.0130480	S CS

Where: NLS CS is Nonlinear Least Squares Method with Cuckoo Swarm Algorithm, and S CS is S-Estimator with Cuckoo Swarm Algorithm.

The MSE values in Table (2) indicate that the Cuckoo Swarm Algorithm, when applied to both the NLS and S-Estimator techniques, demonstrated superior performance of the S-CS method relative to the NLS CS method. In the Nelson model, with a sample size of 50 and sigma equal to 0.1, the NLS CS approach proved to be the most effective. This strategy was optimal in the Meyer 5 model with N=50 and sigma=0.5, and it was also superior in the Meyer 1 model with N=100 and sigma=0.9.

Upon comparing the MSE values presented in Tables (1) and (2), it is evident that both the NLS and S techniques yielded superior outcomes when employing the Cuckoo Swarm Algorithm.

3.2 Analyse real data

Data from the Euphrates River were obtained from water-quality monitoring stations operated by the Iraqi Ministry of Environment across all Iraqi governorates through which the river flows. This sample included measurements of Euphrates River water pollution collected from January 10 to October 10, 2025, totalling 108 observations. Four categories of chemical and physical substances that contribute to water contamination were utilised:

Y : Total dissolved solids (TDS) denote the aggregate of organic and inorganic components dissolved in water. Solid salts can leak into water from several sources, including soil, rocks, industrial waste, and sewage.

X_1 Chlorine (Cl) is a chemical element that exists as a positive ion and is present in water from both natural sources and anthropogenic activities, such as sewage or industrial effluents.

X_2 : Sulfates (SO_4) are chemical compounds characterised by a negative ion, formed naturally by the corrosion of sulfur-containing rocks and minerals, or through the application of agricultural fertilisers and the release of industrial waste.

3.2.1 Parameter Estimation:

The S-Estimator methodology and the S CS method, both of which demonstrated exceptional performance in the simulation results, were employed to estimate the parameters of nonlinear regression models (Meyer1, Meyer5, Nelson) using Euphrates River water pollution data.

Table (3) Shows the estimated values of the model parameters

Parameter / Model	β_1	β_2	β_3	MSE
S-Estimator				
Meyer 1	2.8013059	0.7794220	8.1358076	5.7741271
Meyer 5	1.2001181	0.6949775	2.8488735	6.2943622
Nelson	2.1831121	0.9002847	6.0066102	6.1675483
S CS				
Meyer 1	1.7157882	0.9688690	3.5658790	0.4273982
Meyer 5	0.8108610	0.2277280	5.6457549	0.3109859
Nelson	1.0329099	0.5214004	4.1148635	0.5183113

The findings in Table (3) demonstrate that the calculated coefficients β_1 and β_2 were positive throughout all three nonlinear models and both methodologies. This illustrates a distinct positive relationship between chlorine and total dissolved solids, as well as a direct positive relationship between sulfates and total dissolved solids. This affirms the environmental truth that these pollutants significantly contribute to the Euphrates River's elevated salinity. The β_3 is responsible for establishing the model's overall response level or indicating the initial value.

Upon evaluating the MSE values, we noticed that the S CS technique significantly outperformed the s method across all three nonlinear models. This occurred because the MSE values significantly decreased. This indicates that the Cuckoo Swarm Algorithm technique effectively estimates nonlinear models and is appropriate for models grounded on actual data analysis. This serves as crucial evidence of the efficacy of integrating robust estimation methodologies with artificial intelligence algorithms, yielding more reliable and realistic estimates. The significant reduction in MSE serves as a crucial signal of the necessity of its integration.

4. Conclusion

1. A comparison of the two estimation approaches based on MSE values, excluding algorithmic use, indicated that the S method outperformed the NLS method. This occurred because the NLS method had the highest mean squared error (MSE) values.
2. While it did not get the highest evaluation, the performance of the NLS approach was significantly enhanced with its integration with the synthetic Cuckoo Swarm Algorithm. This was demonstrated by a significant decrease in the MSE values, indicating improved estimates from the method.
3. The utilisation of algorithms grounded on artificial intelligence has undeniably improved the performance of the robust technique by reducing the mean squared error (MSE) across all models.
4. Employing robust methodologies alongside artificial intelligence-based algorithms is an effective strategy for acquiring dependable estimations, especially when dealing with complex nonlinear models.
5. This indicates a distinct and direct advantageous effect of Chlorine on total dissolved solids in water, with a clear and direct good influence of Sulfates on total dissolved solids. This is evidenced by the positivity of the parameter values.

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