# On (T, L)-α- Homotopy and (T, L)-α-Isotopy

**Shahba'a M. Yousif**
Education College - Al-Mustansirh University

**Abstract:**

In this paper we defines the (T, L)-α- Homotopy, strongly (T, L)-α-Isotopy and we study the relation among then and (T, L)- Homotopy, as well as we present the definitions of (T, L)-α-Isotopy, strongly (T, L)-α-Isotopy and we study the relation among them and (T, L)-Isotopy .

**المستخلص :**

في البحث الحالي ، عرفنا هوموتوبي -α -(T, L) وهوموتوبي -α -(T, L) بقوة ، ودرسنا العلاقة بينهم وبين هوموتوبي (T, L) ، بالإضافة إلى ذلك قدمنا تعاريف ايزوتوبي -α -(T, L) وايزوتوبي -α -(T, L) بقوة، ودرسنا العلاقة بينهم وبين ايزوتوبي .(T, L)

## Introduction:

In this work we introduce the definition of (T, L)-α- Homotopy and (T, L)-α-Isotopy and we study the properties of them. In section two we present the definition of (T, L)-α- Homotopy and strongly (T, L)-α-Homotopy and we study the relation among then and (T, L)-Homotopy. In section three we define (T, L)-α- deformation and strongly (T, L)-α- deformation also we study the relation between them and deformation. In section four we give the deformation of (T, L)-α-construction and strongly (T, L)-α- construction and we stud some characters of tem. In section five we introduce the definition (T, L)-α-imbedding and strongly (T, L)-α- imbedding into the section six.

Where we present the definitions of (T,L)-α-Isotopy and strongly (T,L)-α-Isotopy also we study the relation among them and (T,L)-Isotopy.

*In [3] we get the proprieties of a-space.*

We need the following definitions.

**1.1 Definition**: Let (X, Γ) be a topological space, B a subset of x and T be an operator from Γ into P(X) i.e $T:\Gamma\rightarrow P(X)$. We say that T is an operator associated with Γ if the following conditions holds.

(O) $U\subseteq T(u)$ for all $U\in \Gamma$. [4]

(S) T induces an operator $T_B:\Gamma_B\rightarrow P(B)$.

such that. $T_B (U\cap B)= T(u) \cap B$ for every $u\in\Gamma$ where $\Gamma_B$ is the relative topology on B. [5] [6]

**1.2 Definition**: Let (X, Γ) be a topological space and T be an operator on Γ. A subset A of x is said to be T-open set every $x\in A$. There exists an open set U containing x such that. A subset B is said its complement is *T-open* [2].

**1.3 Definition**: Let (X, Γ) be a topological space and T be a mapping from $\Gamma^\alpha$ into P(X) i.e $T:\Gamma^{\alpha} \rightarrow P(X)$. then T is said to be an α-operator associated with. if the following conditions hold $U\subseteq T(u)$ for all $U\in \Gamma$ every, and we say that the α-operator T is stable with respect to $B\in x$ if T induce and α- operator $T_\beta:\Gamma^\alpha\rightarrow P$ such that. $T_\beta$ $(U\cap B)$ such that. $T_\beta$ $(U\cap B)= T(u) \cap B$ for every $u\in\Gamma^{\alpha}$ where $\Gamma_\beta{}^{\alpha}$ is the relative topology on β. [2]

**1.4 Definition**: Let (X, Γ) be a topological space and T be an α-operator Γ A subset A of x is said to be T α-open set if for every x∈A, there exists an α-open set U containing x such that. T(u)⊆A. [1]

**2)** (T, L)-α- Homotopy: in this section we define (T, L)-α- Homotopy and strongly (T, L)-α- Homotopy.

**2.1 Definition**: Let (X, $\Gamma_x$) be T- topological space and (y, $\Gamma_y$) be L-topological space. The map B:x×I→Y, I=[0,1] is said to be a (T, L)-α-Homotopy between the maps f,g: I→Y iff B(X, i), ∀x∈X, i∈I is a (T, L)-a- construction such that B(X, 0)=f(X), B(X, 1)=g(X), ∀x∈X and f,g are said to be (T, L)-α- Homotopic.

**Theorem (1):** Every (T, L)- Homotopy is (T, L)-α- Homotopy.

***Proof***: Let (X, $\Gamma_x$) be T- topological space and (y, $\Gamma_y$) be L- topological space.

Let B:x×I→Y be (T, L)- Homotopy map between the maps f,g: X→Y that means B(X, i), ∀x∈X, i∈I, is (T, L)-a- continuous map

Since [every (T, L)- continuous map (T, L)-α- continuous map] [2]

Therefore B(X, i), ∀x∈X, i∈I is a (T, L)-α- construction such that B(X, 0)=f(X), B(X, 1)=g(X), ∀x∈X and f,g are said to be (T, L)-α-Homotopy.

**Theorem (2):** The (T, L)-α- Homotopy relation between maps of x into Y is an equivalence relation.

***Proof***: we must prove the (T, L)-α- Homotopy is reflexive, symmetric and transitive.

1- To show (T, L)-α- Homotopy is reflexive let f:x→Y be map and define a (T, L)-α- Homotopy B:x×I→Y, between the (T, L)-α-continuous maps f,g: $X \to Y$ such that means B(X, i)=f(X), $\forall x \in X$, $i \in I$ we get B(X, 0)=f(X) and B(X, 1)=f(X)

Thus (T, L)-a- Homotopy reflexive.

2- To show (T, L)-α- Homotopy is symmetric let f,g:$x \to Y$ such that f is (T, L)-α- Homotopic to g.

Then $\exists$ B:$x \times I \to Y$ *such that* B(X, 0)=f(X), B(X, 1)=g(X).

We define (T, L)-α- Homotopy k:$x \times I \to Y$

k(X, i)=B(X, 1-i) $\forall x \in X$, $i \in I$

we get k(X, 0)=B(X, 1)=g(X)

k(X, 1)=B(X, 0)=f(X)

then g is (T, L)-α- Homotopic to f

thus (T, L)-α- Homotopic is symmetric

3- To show (T, L)-α- Homotopy is transitive.

Suppose f is (T, L)-α- Homotopic to g, and g is (T, L)-α- Homotopic to h then there exists (T, L)-α- Homotopy B:$x \times I \to Y$, K:$x \times I \to Y$ such that

B(X, 0)= f(X), B(X, 1)=g(X), k(X, 0)=g(X), k(X, 1) = h(X).

Now we define H: $x \times I \to Y$ *such that:*

$$H(x,i) = \begin{cases} B(x,2i), & 0 \le i \le 1/2 \\ k(x,2i-1), & 1/2 \le i \le 1 \end{cases}$$

we get H(X, 0) = B (X, 0) = f (X)

and H(X, t) = K (X, 1) = h (X)

Therefore f is (T, L)-α- Homotopic to h

Hence (T, L)-α- Homotopy is transitive

Thus (T, L)-α- Homotopy is an equivalence relation.

**2.2 Definition**: Let (X, $\Gamma_x$) be T- topological space and (Y, $\Gamma_y$) be L- α-topological space. The map B:x×I→Y, I=[0,1] is said to be strongly (T, L)-α- Homotopy between f,g: X→Y iff B(X, i), ∀x∈X, i∈I is strongly (T, L)-α- construction such that B(X, 0)=f(X) and B(X, 1)=g(X), ∀x∈X.

**Theorem (3):** *Every strongly (T, L)- Homotopy between maps is* (T, L)-α- Homotopy.

***Proof***: Let (X, $\Gamma_x$) be T-α- topological space and (Y, $\Gamma_y$) be L- α- topological space.

Let B:x×I→Y be strongly (T, L)- α- Homotopy, we get B(X, i), ∀x∈X and i∈I is strongly (T, L)-α- continuous.

Since [every strongly (T, L)- α-continuous is (T, L)-α- continuous] [2]

Therefore B(X, *i*), ∀x∈X, *i*∈*I is a* (T, L)-*α- construction such that* B(X, 0)=f(X) and B(X, 1)=g(X), ∀x∈X and i∈I.

**Theorem (4):** Every strongly (T, L)- Homotopy relation between maps of x into Y is an equivalence relation.

***Proof***: Let B:x×I→Y be strongly (T, L)- α- Homotopy, then by theorem (3) B is (T, L)-α- Homotopy and by theorem (4) we get the strongly (T, L)-α- Homotopy is an equivalence relation.

**3) (T, L)-α- deformation**: in this section we present the definition of (T, L)-α- deformation and strongly (T, L)-α- deformation.

**3.1) Definition:** the (T, L)- α- Homotopy B:x×I→Y is said be (T, L)- α- deformation, iff B(X, 0) is inclusion map.

**Theorem (5):** Every (T, L)- deformation is (T, L)-α- deformation.

***Proof***: suppose that B:x×I→Y is Homotopy and B(X, 0) is inclusion map. Then by theorem (1) we get (T, L)- α- Homotopy B(X, 0) is inclusion map, then B is (T, L)-α- Homotopy and by theorem (4) we get the strongly (T, L)-α- deformation.

**3.2 Definition**: Let strongly (T, L)-α- Homotopy B:x×I→Y is called strongly (T, L)-α- deformation iff B(X, 1) is the inclusion map.

**Theorem (6):** Every strongly (T, L)- α-deformation is (T, L)-α- deformation.

***Proof***: suppose that B:x×I→Y is strongly (T, L)- α-deformation.

That is B strongly (T, L)-α- Homotopy and B(X, 0) is inclusion map, and Then by theorem (3) and B(X, 0) is inclusion map, therefore B is semi α- deformation.

**4) (T, L)-α-construction**: In this section we use section 2 to give the definition of (T, L)-α- construction and strongly (T, L)-α- construction we begin by the following definition.

**4.1) Definition:** The (T, L)- α- deformation B:x×I→Y is called (T, L)- α- construction iff B(X, 0) is the constant map.

**Theorem (7):** Every (T, L)-a- construction is (T, L)-α- construction.

***Proof***: suppose that B:x×I→Y is (T, L)- construction .

That mean B is deformation and B(X, 1) is constant map,

Then by theorem (5) we get B(X, i) ∀x∈X and i∈I is (T, L)-α- deformation and B(X, 1) is the constant map.

**4.2) Definition:** The strongly (T, L)- α- deformation B:x×I→Y is called strongly (T, L)-α- construction iff B(X, 1) is the constant map.

**Theorem (8):** Every strongly (T,L)-α-contraction is (T,L)-α- contraction.

***Proof***: Suppose that B:x×I→Y is strongly (T, L)-α-contraction

That mean B is strongly (T, L)-α- deformation and B(X, 1) is constant map, and by theorem (6) we get B(X, i) ∀x∈X and i∈I is (T, L)-α-deformation and B(X, 1) is the constant map, therefore B is (T, L)-α-construction.

**5) (T, L)-α-Imbedding**: We present this section in order to present the definition of (T, L)-a-Isotopy.

**5.1 Definition**: Let (X, $\Gamma_x$) be T-topological space and (Y, $\Gamma_y$) be L- *topological space. A one to one map f:x×I→Y is called* (T, L)-α-Imbedding iff ∃ a (T, L)-α- Homotopy from x onto f(X).

**Theorem (9):** Every (T, L)-Imbedding is (T, L)-α-Imbedding.

***Proof***: Let (X, $\Gamma_x$) be T- topological space and (Y, $\Gamma_y$) be L- topological space. And f:x×I→Y be (T, L)-α-Imbedding map.

That mean there exists a (T, L)- Homeomorphism g:x→ f(X) since [every a (T, L)- Homeomorphism is a (T, L)- α-Homeomorphism] [2]

Then g is a (T, L)- α-Homeomorphism

Hence f is (T, L)-α-Imbedding.

**5.2 Definition**: Let (X, $\Gamma_x$) be T- topological space and (Y, $\Gamma_y$) be L- topological space. A one to one map f:x×I→Y is called strongly (T, L)-α-Imbedding iff there exists a strongly (T, L)- α-Homeomorphism from x onto f(X).

**Theorem (10):** Every strongly (T, L)-α-Imbedding is (T, L)-α-Imbedding.

***Proof***: Let (X, $\Gamma_x$) be T- topological space and (Y, $\Gamma_y$) be L- topological space respectively, and let f:x→Y be strongly (T, L)-α-Imbedding.

That is there exists strongly (T,L)- Homeomorphism g:x→f(X) since [every strongly (T,L)-α-Homeomorphism is a (T,L)- α-Homeomorphism] [1], [2]

Thus g is (T, L)- α-Homeomorphism

Hence f is (T, L)-α-Imbedding.

**6) (T, L)-α-Isotopy**: In this section we define of (T, L)-α-Isotopy and strongly (T, L)-a-Isotopy also we study the relation between them.

**6.1 Definition**: Let $(X, \Gamma_x)$ and $(Y, \Gamma_y)$ be two T- topological and L- topological spaces respectively.

A *map* f:x×I→Y, I=[0,1], *is said to be* (T, L)-a-*Isotopy* between f:g×I→Y for and and i∈I is (T, L)-α-Imbedding such that B(X, 0)=f(X) and B(X, 1)= g(X).

**Theorem (11):** Every (T, L)-Isotopy is (T, L)-a-Isotopy.

***Proof***: Let $(X, \Gamma_x)$ and $(Y, \Gamma_y)$ be two T- topological and L- topological spaces respectively, and let B:x×I→Y be (T, L)-Isotopy map between f,g:x→Y.

That mean B(X, i) for x∈X and i∈I is Imbedding map, such that B(X,0)=f(X) and B(X,1)=g(X).

Since by theorem (9) B(X, i) for x∈X and i∈I is (T, L)-α-Imbedding such that

B(X,0)=f(X) and B(X,1)=g(X)

Thus B(X,i) is semi a-Isotopy.

**Theorem (12):** The (T, L)-α- Isotopy relation between maps of x into Y is an equivalence relation.

***Proof***: We must to prove the (T, L)-α- Isotopy is reflexive, symmetric and transitive.

first to show (T, L)-a- Isotopy is reflexive let f:x→Y be map and define a B:x×I→Y by B(X, i)=f(X), ∀x∈X, i∈I we get (X, 0)=f(X) and B(X, 1)=g(X).

therefore (T, L)-a-Isotopy is reflexive.

Neaxt to show (T, L)-a-Isotopy is symmetric

Let B:x→Y such that f is (T, L)-a-Isotopy to g.

Then there exist B:x×I→Y such that B(X, 0)=f(X) and B(X, 1)=g(X), we define (T, L)-a-Isotopy K:x→Y by k(X, i)= B(X, i-1) ∀x∈X and i∈I we get K(X, 0)=B(X, 1) = g(X) and

Then g is (T, L)-a-Isotopy to f.

Thus (T, L)-a-Isotopy *is symmetric*

Finally, suppose f is (T, L)-a-Isotopy to g and g is (T, L)-a-Isotopy to h, then there exist (T, L)-a-Isotopy B:x×I→Y and K:x→Y such that. B(X, 0)=f(X), B(X, 1) = g(X), k(X, 0)=g(X) and k(X, 1)=h(X), we define (T, L)-α-Isotopy K:x→Y by:

$$H(x,i)=\begin{cases} B(x,2i), & 0\le i\le 1/2 \\ k(x,2i-1), & 1/2\le i\le 1 \end{cases}$$

we get H(X, 0) = B (X, 0) = f (X)

and H(X, 1) = K (X, 1) = h (X)

Therefore f is (T, L)-α- Isotopy to h

Thus (T, L)-α- *Isotopy is an equivalence relation.*

**6.2 Definition**: Let (X, $\Gamma_x$) and (Y, $\Gamma_y$) T- topological and L-topological spaces respectively, a map B:x×I→Y is said to be strongly (T, L)-α- Isotopy between f,g: X→Y iff B(X, i), ∀x∈X, i∈I is strongly (T, L)-α-Imbedding such that B(X, 0)=f(X) and B(X, 1)=g(X).

**Theorem (13):** Every strongly (T, L)-α- Isotopy is (T, L)-a-Isotopy.

***Proof***: Suppose that (X,$\Gamma_x$) and (Y,$\Gamma_y$) be T-topological and L-topological spaces.

Let B:x×I→Y be strongly (T, L)-α- Isotopy between f,g: X→Y

That means B(X, i), $\forall x \in X$, $i \in I$ is strongly (T, L)-$\alpha$-Imbedding map such that B(X, 0)=f(X) and B(X, 1)=g(X)

Since by theorem (10) we get B is (T, L)-$\alpha$-Imbedding such that B(X, 0)=f(X) and B(X, 1)=g(X).

Thus B is (T, L)-$\alpha$-Isotopy.

**Theorem (14):** *the strongly (T, L)-$\alpha$- Isotopy relation between maps of* x into Y is an equivalence relation.

***Proof***: Suppose that (X, $\Gamma_x$) and (Y, $\Gamma_y$) be T- topological and L-topological respectively, and $B: x \times I \rightarrow Y$ be strongly (T, L)-$\alpha$- Isotopy since by theorem (12) we get B is (T, L)-$\alpha$-Imbedding and by theorem (13) we get require result.

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