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Inventory Control Model on Matrix Geometric Method for Intermittently Obtainable Server with Balking and Feedback in Markovian Queueing Model

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RESEARCH ARTICLE

Inventory Control Model on Matrix Geometric Method for Intermittently Obtainable Server with Balking and Feedback in Markovian Queueing Model

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ABSTRACT

The paper deals with the problem of queueing-inventory system that has uncertain service conditions, and it discusses the contribution of server availability and customer behaviours like balking and feedback that are also added in to the system performance. In real life service and production situations, servers are not always available because of maintenance processes, capacity constraints, or other unforeseen disruptions. In order to simulate such characteristics, the system is modelled in a Markovian context where the interdependency between inventory levels, customer inflows, and server capacity is simulated. The time of customer arrivals is random and the time to serve is memoryless, which maintains the Markovian property of the model. Customer impatience is captured through balking and feedback, representing multiple service interactions within the system. The resulting stochastic process has a state space whose structure may be characterized by a quasi-birth-and-death formulation. With this structure the matrix-geometric method may be applied to the obtained long-run behaviour of the system. Some of the measures of performance of practical interest obtained out of the steady-state distribution include average inventory levels, system congestion, service completion rates. An inclusive cost framework is then developed by including ordering costs, holding costs, procurement costs and the service costs. System parameters sensitivity analysis and numerical experiments indicate the way they affect the operational performance and the total inventory cost. The findings demonstrate the significance of the balking and feedback effects to be managed successfully so that to enhance efficiency and customer satisfaction in systems with intermittently available servers.

Keywords: Balking, Feedback, Inventory cost, Markovian queueing model, Matrix geometric method

Introduction

In present-day service and production systems, inventory management is essential for sustaining service standards, managing costs, and supporting customer satisfaction. [1] In many practical situations, inventory decisions cannot be separated from queueing behaviour, particularly when demand is uncertain and service capacity is limited. Queueing theory provides a useful tool for describing the random interaction among customer arrivals, service processes, and inventory availability. [2] Markovian queueing models have been widely used to study inventory systems affected by random demand and service delays. Combining inventory control with queueing analysis provides a more realistic view of system performance and helps in making sound operational decisions under uncertainty. [3]

Most classical Inventory Control Theory assumes a world of resource abundance. In reality, this is rarely the case. Many service systems offer users server service at intermittent intervals due to maintenance, breakdowns, and operational capacity. User

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behaviour also complicates systems. Customer balking refers to the system behaviour of queueing (joining the system) and feedback[4] (which alters the system server behaviour). This is done to modulate the system performance and user experience.[5]

Markovian models are widely used to study service systems operating under uncertainty, as they provide a convenient way to represent random arrivals, service completions, and system transitions through memoryless assumptions. This framework allows the evolution of the system to be described using continuous-time stochastic processes, making the analysis of long-run behaviour mathematically tractable.[6] When such systems exhibit a structured and repetitive state space, matrix-geometric methods offer an efficient analytical approach for deriving steady-state probabilities and related performance measures without excessive computational burden.[7]

The customer behaviour is also another major factor in system performance. Specifically, balking can be cases when customers entering the system choose not to do so due to the congestion or expecting to spend too much time waiting. Balking can be included in the system of modelling to allow a more realistic depiction of the demand patterns and assist in evaluating the effect of balking on the congestion and service efficiency as well as the overall system functionality.[8] Furthermore, the feedback function allows the server to modify and manage its activities based on system feedback. This system of feedback self-regulation enables responsive demand elasticity and provides an effective automated command of the system.[9]

Utilizing the Matrix Geometric Method along with the concepts of balking and feedback control, this study aims to create a fully functional and usable framework to assist the inventory management stream with temporarily available servers.[10] Allocating resources with customer satisfaction as a top priority is a priority in many areas and industries, including healthcare, telecommunications, transportation, and retail. The usefulness of this model is evident in these areas. In the Matrix Geometric Method, this paper focuses on a Markovian queuing model with balking and feedback control, and the construction of the model seeks to maximize control of a company's inventory, customer satisfaction, and operational efficiency.[11] With the added behavioural components of a server balk and feedback, this study provides a complete framework within impassable boundaries for controlling inventory. The following sections of this study will focus on the construction, study, analysis, and simulation of the model. These sections will provide proof that the model can and will work across multiple operational frameworks.

This paper presents a new approach to inventory control that incorporates a Matrix Geometric Method for an occasionally accessible server with balking and feedback in a Markovian queueing model. Efficient inventory control is one of the most challenging tasks in supply chain management and affects several essential features, including customer service levels, inventory holding costs, and order fulfillment. The addition of balking and feedback to the model increases the realism of the inventory control process. This research is essential for its contribution to queueing theory and operations research.

The format of this work was designed as follows. First, there will be math definitions, then an explanation of the math modelling for the inventory control model, along with a flowchart, such as the one in Fig. 1. Next, there will be a discussion of the preliminaries of the basic equations for the Markovian queueing model M/M/k/N in the field of customer service systems. After that, there will be an explanation of the methods, including all formulations and analyses of relevant cases. Then, there will be a focus on using the Matrix Geometric Method and Markov chains, particularly on the repeated block pattern in the triangular form of the coefficient matrix. Then will be the discussion of the results based on the findings, followed by the results, along with a conclusion that restates the most important results and consequences.

Mathematical modelling of the inventory control model

Inventory management solution via matrix geometric method

Mathematical modelling is an integral part of the inventory management approach using the Matrix Geometric Method for an intermittently available server with balking and feedback in a Markovian queuing model. This technique allows us to explain the system's dynamics and evaluate and manage the inventory.

State space definition

Define the state space for the system, which represents all possible configurations of the inventory and the server. This consists of the number of goods on hand, the number of customers waiting in line, and the server's availability.

Problem formulation

Clearly outline the goals of the inventory control model. Determine the problem's goals in terms of one or more critical objectives, such as inventory

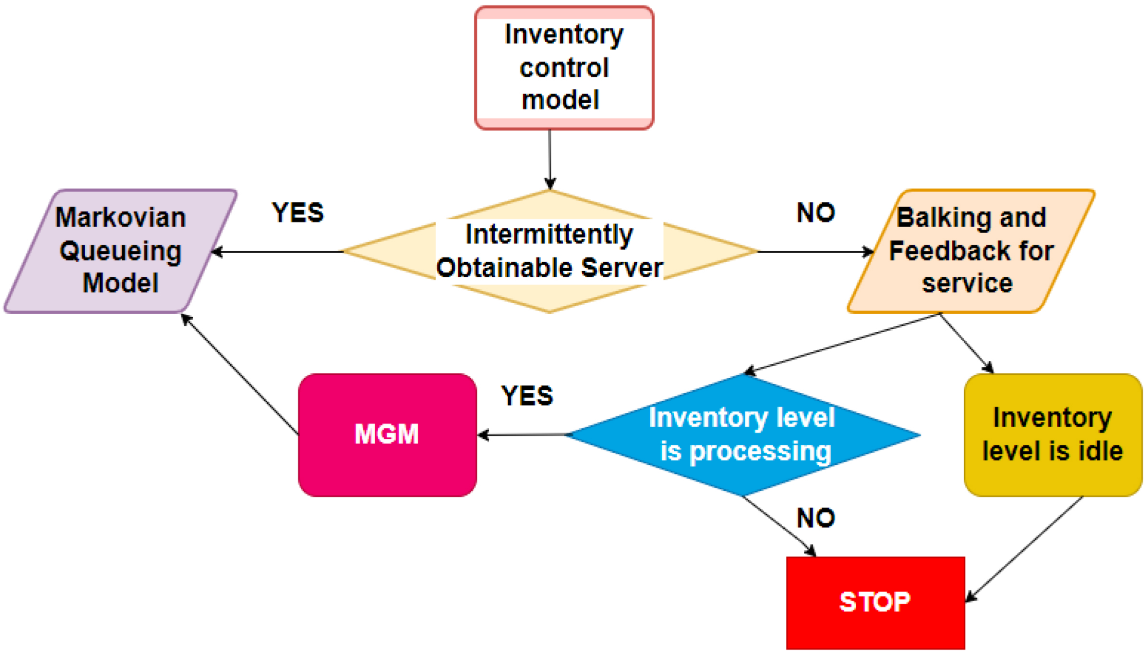

Fig. 1. Flow chart for the mathematical modelling of the inventory control model.

carrying cost, level of customer service, and/or level of server usage. Define the problems, constraints, and assumptions, such as an intermittently available server, balking behaviour, feedback, etc.

Transition rates

Define the transition rates between possible states of the system. These rates determine the movement of the system from one state to another, and the arrival rate of customers, the service rate of the server and customers' balking behaviour are the determinants.

Arrival process

Explain how customers arrive in this system. This can follow a Poisson process, in which the average arrival rate is suspected of varying with the hour of the day or other external influences.

Service process

Describe how the occasionally available server does the service. Include adaptive feedback, in which the server's actions may change based on the system's state. This may include a variable service rate or other forms of adaptations.

Balking behavior

Include customers' balking behaviour in the system. Think of the circumstance in which a customer arrives but decides not to enter the line, depending on the system, like when a queue is overly populated or when the waiting time is longer.

Feedback mechanism

Use the feedback mechanism to model the server's adaptive behaviour based on the system state. Specify the feedback rules or policies that determine how the server will adjust its/service rate. Determine the server's response to the system based on the number of customers in line, the arrival rate, or the amount of stock available.

Optimization

Base your formulation of the optimality criterion on the performance metrics you hope the desired system would attain: minimizing inventory holding costs, maximizing the level of customer service, or some desired balance in the trade-off between inventory levels and operational costs.[12] Construct the optimal inventory control policy using optimization techniques, such as linear programming or dynamic programming.

Sensitivity analysis and simulation

Perform a sensitivity analysis to evaluate the system performance for various parameters.[13] Validate the model's accuracy through simulation, and compare the simulation outcomes with the analytical predictions using the Matrix Geometric Method.

Completing these mathematical modelling steps, the inventory control model based on the Matrix Geometric Method for a server with intermittent availability, balking, and feedback in a Markovian queueing model offers a reasonable basis for improving operational efficiency and managing inventory optimally.[14]

Nomenclature

• λ	– Mean (arrival rate, Poisson distribution)
• n	– Number of customers in system
• N	– Total Number of customers in system
• α	– Probability of getting balking service
• 1 $-\alpha$	– Probability of not getting balking service
• β	– The system completed the feedback service, or enter the system
• 1 $-\beta$	– returns the system for another service or reenters
• μ	– mean (exponential distribution)
• k	– Customers participated in the server
• C(t)	– Number of customers
• L(t)	–Inventory levels
• S(t)	–Server switched on/off
• bf(t)	– balking and feedback of the server
• Fc	– Fixed cost
• Pc	– Procurement cost
• Hc	– holding cost
• Sc	– service cost
• E1	– Expected inventory
• E2	– Expected number of customers
• E3	– Expected reorder rate
• E4	– Expected number of departures

Preliminaries

It is used for the system of Markovian queueing in the customer service model M/M/k/N at the probabilistic approach as below

1. P(average busy)=$\sum_{n=1}^{N} P_{n,1}$
2. P(vacation) = 1 – P(average busy)
3. P(average number of customers in the system) = $A_c = n[\sum_{n=1}^{N}(P_{n,0} + P_{n,1})]$
4. P(average number of customers in the queue) = $A_q = A_c - \mathrm{k}\sum_{n=1}^{N} P_{n,1}$
5. P(average number of customers served in system) = $A_S = \ \beta\mu[\sum_{n=1}^{N}(n+k)P_{n,1}]$
6. P(average waiting time in the system) = $A_w = \frac{A_c}{\lambda}$
7. P(average waiting time in the queue) = $A_{wq} = \frac{A_q}{\lambda}$

Methodology

Problem formulation

Define the objectives of the study: To develop a mathematical model for inventory control for a Markovian multi-server queueing system that incorporates feedback and balking customers.[15] Specify the system parameters: Number of servers, arrival rates, service times, vacation durations,[16] customer impatience thresholds, abandonment probabilities, etc.

Consider the steady state conditions for the Inventory control model:

$$P_{n,0} = \frac{\lambda\left(1-\frac{n-1}{N}\right)P_{n-1,0}}{\left(1-\frac{n}{N}\right)}$$

$$P_{n,1} = \frac{\left(k\beta\mu + (n+1-k)\,\alpha\right)P_{n+1,1}}{\lambda\left(1-\frac{n}{N}\right) + k\beta\mu + (n-k)\,\alpha}$$

Modelling arrivals and service times

Assume the arrival process follows a Poisson distribution, with a known arrival rate λ.

Model service times using appropriate probability distributions such as exponential, regular, or Erlang distributions, based on empirical data or assumptions.[17]

Inventory level is idle

Represent server vacations as a discrete-time finite-state Markov process.

Define states: Active state (servers available for service) and Vacation state (servers on vacation). Determine transition probabilities between active and vacation states based on predefined vacation durations or probability distributions.

Step 1: Inventory level is being processed

The probability of the initial term is fixed; it varies between 0 and 1. We assigned the left-hand side to lie between 0 and 0.5, right-hand side 0.5 and 1.

$$P_1 = \left(\frac{\lambda}{\beta\mu}\right)P_0$$

$$P_2 = \frac{N-1}{N}\left(\frac{\lambda^2}{2(\beta\mu)^2}\right)P_0$$

General nth term

$$P_n = \left(\frac{\lambda}{\beta\mu}\right)^n \frac{1}{n!}\sum_{k=1}^{n}\frac{N-k+1}{N}P_0$$

Step 2: No Inventory level is being processed.

$$V_1 = 1 - P_1$$

$$V_2 = 1 - P_2$$

Nth term is $V_n = 1 - P_n$

Step 3: Balking and Feedback

$$P_1 = \left(\frac{\lambda}{\mu}\right)P_0$$

$$P_2 = \left(\frac{\lambda\left(1-\frac{1}{N}\right)+\mu}{\mu} - \frac{\lambda}{\mu}\right)P_1$$

Nth term is

$$P_n = \frac{\lambda}{\mu}\left\{\left(1-\frac{n-1}{N}\right)P_{n-1} + \left(1-\frac{n-1}{N}\right)P_{n-2} + \cdots + P_0\right\}$$

Step 4: No Balking and Feedback

$$I_1 = 1 - P_1$$

$$I_2 = 1 - P_2$$

Nth term is $\mathrm{I_n = 1 - P_n}$

Matrix geometric method

It is a quasi-birth-and-death process in probability theory. It involves the continuous-time distribution on a Markov chain with a block-repetitive model. The dimensions of the Markovian chain (M) are $m \times n$ & $n \times p$ With a repetitive model and a diagonal block of infinite elements. If we converted to finite form, it changed to Neut's matrix and dimensions are $m = i-1$, $n = i$ and $p = i+1$ Respectively. Consider the tridiagonal block below.

$$M = \begin{pmatrix} a_{00} & a_{01} & & & & \\ a_{10} & a_1 & a_2 & & & \\ & a_0 & a_1 & a_2 & & \\ & & a_0 & a_1 & a_2 & \\ & & & a_0 & a_1 & a_2 \\ & & & \vdots & \vdots & \vdots \end{pmatrix}$$

Here, all the elements are matrices, such as $a_{00}, a_{01}, a_{10}, a_{0,} a_1, a_2$.

Let be the sub vector of the systems, the stationary distribution is,

$$\pi M = 0$$

$$\pi_0 a_{00} + \pi_1 a_{10} = 0$$

$$\pi_0 a_{01} + \pi_1 a_1 + \pi_2 a_2 = 0$$

$$\pi_1 a_2 + \pi_2 a_1 + \pi_3 a_0 = 0$$

$$\vdots$$

In general, $\pi_{i-2}a_2 + \pi_i a_1 + \pi_{i+1}a_0 = 0$

$$\pi_i = \pi_1 N^{i-1}$$

Here, N = Neut's matrix

$$\begin{pmatrix}\pi_0 & \pi_1\end{pmatrix}\begin{pmatrix} a_{00} & a_{01} \\ a_{10} & a_1 + Na_0\end{pmatrix} = 0$$

Solvingπ_0&π_1Then we get easily remaining, etc...

Utilize the Matrix Geometric Method to analyze the steady-state behaviour of the system. This involves constructing a matrix representation of the system's transition rates and solving the steady-state equations to obtain performance metrics such as the average inventory level, customer waiting time, and server utilization.

Let's outline the mathematical modelling steps for this inventory control model.

System $\Omega = \{C(t), bf(t), S(t), L(t),\ \forall\, t \geq 0\}$

$$\text{Here } bf(t) = \begin{cases} 0, & \textit{server is on balking and feedback} \\ 1, & 1 \textit{ server inventory is idle}, \\ & \quad 2 \textit{ server inventory is processing} \\ 2, & 1 \textit{ server is stop}, \ 2 \textit{ server is mgm} \\ 3, & 1 \textit{ and } 2 \textit{ servers are Markovian state} \end{cases}$$

$$S(t) = \begin{cases} 0, & \textit{if switched off} \\ 1, & \textit{if switched on} \end{cases}$$

$$L(t) = \begin{cases} 0, & \textit{if inventory level is idle} \\ 1, & \textit{if inventory level is processing} \end{cases}$$

$$M = \begin{pmatrix} a_{00} & a_{01} & & & & \\ a_{10} & a_1 & a_2 & & & \\ & a_0 & a_1 & a_2 & & \\ & & a_0 & a_1 & a_2 & \\ & & & a_0 & a_1 & a_2 \\ & & & \vdots & \vdots & \vdots \end{pmatrix}$$

State space $= \{0, 0, 0,\ \mathrm{k}\}$

$$a_{00} = \begin{pmatrix} \lambda I & 0 \\ \mu & D \end{pmatrix}$$

$$\text{Here } D = \begin{cases} \alpha, & if\ balking\ server \\ \beta, & if\ feedback\ server \\ 1-\alpha & if\ not\ balking\ server \\ 1-\beta, & if\ not\ feedback\ server \end{cases}$$

$$a_{01} = \begin{pmatrix} 0 & \lambda & 0 & 0 \\ 0 & 0 & \mu & 0 \end{pmatrix}; a_{10} = \begin{pmatrix} 0 & 0 \\ V_0 & V_1 \\ 0 & V_2 \\ V_3 & V_4 \\ 0 & V_5 \end{pmatrix};$$

$$V = \begin{pmatrix} \mu I & 0 \end{pmatrix}; a_0 = \begin{pmatrix} 0 & & & & \\ & I_0 & & & \\ & & I_1 & & \\ & & & I_2 & \\ & & & & 0 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} \beta & 0 & \mu_0 & 0 & 0 & 0 \\ 0 & I_0 & 0 & 0 & 0 & I_1 \\ 0 & \mu_1 & \mu_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_0 & 0 & I_2 \\ 0 & 0 & 0 & 0 & 0 & \lambda_1 \\ 0 & 0 & 0 & 0 & 0 & I_3 \end{pmatrix};$$

$$a_2 = \begin{pmatrix} V_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & V_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & V_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & V_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & V_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & V_5 \end{pmatrix}$$

Results of the inventory cost

Some hypothetical examples of potential results that could be obtained from the inventory control model on the Matrix Geometric Method for an intermittently obtainable server with balking and feedback in a Markovian queueing model.[18] These results would depend on the specific assumptions, parameters, and optimization objectives of the model.

Determining the best inventory management rules

The model recommends the best inventory control strategies (e.g., reorder levels and order quantities) that strike a balance between inventory costs and a service level that is satisfactory to the enterprises. These policies incorporate the server that is intermittently available, balking behaviour, and the feedback mechanisms.[19]

The length of time that customers wait to be served

The model can predict how long customers will wait in the queue. It could take into account balking and the server's behaviour. It might be able to determine whether the stall time exceeds a set limit, enabling changes to the service rate or to inventory management.

The use of the server

The model can assess server usage under different inventory control policies. It may identify periods of low usage (which contradicts the policies) or periods of system overload, and recommendations can be made to improve the efficient use of resources and the server's efficiency.[20]

The costs related to holding inventory

The model may indicate a potential decrease in inventory holding costs due to improved inventory management. It may determine the optimal balance point between inventory levels and associated costs, and incorporate the server's intermittent availability and variable increases or decreases in customer demand.

Sensitivity analysis

Often, a model might list outputs with a sensitivity analysis, which demonstrates how different parameters affect the system's performance.[21] For example, the study might assess how different values of the balking probabilities, feedback rules, or arrival rates impact the system's inventories, customer wait times, and performance level.

The actual outputs would depend on the choice of input parameters, the model's assumptions, and the goals of the optimization problem.[22] Actual implementation and validation of the model would require data from the particular system of interest and careful attention to practical boundaries and constraints.

Inventory cost (Ic)

E_1 is a single summation, with all remaining variables set to zero, for a single variable k=1 to s.
E_2 is a single summation, i = 1 to infinity.
E_3 is a single summation, i = 1 to infinity, where two components are zero, two components are the same i, but the values are different.
E_4 is a double summation over two variables, such as i and k, with one component set to zero and the

Table 1. The average of the multi-server queueing model for expecting the balking and feedback customers in the inventory cost.

λ	E1	E2	E3	E4	Ic1
1	0.1	0.2	0.4	0.5	1003
1.1	0.3	0.4	0.5	0.6	1054
1.2	0.4	0.5	0.6	0.7	1200
1.3	0.5	0.7	0.3	0.9	1311
1.4	0.7	0.9	0.6	0.5	1423
1.5	0.6	0.8	0.7	0.4	1543
1.6	1.1	1.3	1.4	1.5	1567
1.7	1.7	1.5	1.7	1.9	1700
1.8	2.4	2.6	2.8	2.8	1823
1.9	2.9	2.8	2.5	2.7	1913

Table 2. Probability of getting balking service when expecting the multi-server queueing model in the inventory cost.

α	E1	E2	E3	E4	Ic2
0	0.1	0.2	0.3	0.4	1000
0.1	0.3	0.4	0.2	0.2	1060
0.2	0.5	0.6	0.7	0.4	1300
0.3	0.7	0.6	0.9	0.4	1341
0.4	0.8	0.7	0.1	0.3	1400
0.5	0.3	0.2	0.5	0.3	1500
0.6	0.4	0.3	0.4	0.2	1578
0.7	0.1	0.2	0.2	0.1	1800
0.8	0.5	0.6	0.6	0.4	1850
0.9	0.7	0.8	0.6	0.6	1900

Table 3. Probability of system completed feedback service of expecting the multi-server queueing model in the inventory cost.

β	E1	E2	E3	E4	Ic3
0	0.8	0.9	0.6	0.5	1010
0.1	0.2	0.1	0.1	0.3	1097
0.2	0.6	0.7	0.8	0.5	1300
0.3	0.6	0.7	0.8	0.3	1412
0.4	0.7	0.7	0.2	0.2	1555
0.5	0.5	0.3	0.6	0.2	1575
0.6	0.2	0.1	0.3	0.1	1576
0.7	0.2	0.4	0.5	0.6	1800
0.8	0.4	0.3	0.2	0.1	1850
0.9	0.5	0.4	0.3	0.2	1976

remaining components having different values.

$$E_1 = \sum_{k=1}^{s} y_{0,0,0,k}$$

$$E_2 = \sum_{i=1}^{\infty} i\, P_i$$

$$E_3 = \mu \sum_{i=1}^{\infty} y_{i,0,0,i+1}$$

$$E_4 = \mu \sum_{i=1}^{\infty} \sum_{k=1}^{s} y_{i,0,k,k+1}$$

$$\textbf{Inventory cost} = \mathrm{Ic} = \mathrm{Fc} * \mathrm{E1} + \mathrm{Pc} * \mathrm{E2} + \mathrm{Hc} * \mathrm{E3} + \mathrm{Sc} * \mathrm{E4}$$

Algorithm

- Step 1: Table 1 (major variable is mean (arrival rate, Poisson distribution))

  $$\mathrm{Ic1} = \mathrm{Fc} * \mathrm{E1} + \mathrm{Pc} * \mathrm{E2} + \mathrm{Hc} * \mathrm{E3} + \mathrm{Sc} * \mathrm{E4}$$

- Step 2: Table 2 (the major variable is the probability of getting balked service)

  $$\mathrm{Ic2} = \mathrm{Fc} * \mathrm{E1} + \mathrm{Pc} * \mathrm{E2} + \mathrm{Hc} * \mathrm{E3} + \mathrm{Sc} * \mathrm{E4}$$

- Step 3: Table 3 (major variable is the system completed feedback service or enter the system)

  $$\mathrm{Ic3} = \mathrm{Fc} * \mathrm{E1} + \mathrm{Pc} * \mathrm{E2} + \mathrm{Hc} * \mathrm{E3} + \mathrm{Sc} * \mathrm{E4}$$

Table 4. The exponential distribution of mean for the multi-server queueing model for balking and feedback in inventory Cost.

μ	E1	E2	E3	E4	Ic4
0	0.5	0.1	0.2	0.9	1010
0.3	0.4	0.3	0.3	0.4	1090
0.6	0.4	0.6	0.4	0.6	1500
0.9	0.6	0.8	0.5	0.8	1623
1.2	0.5	0.7	0.7	0.2	1546
1.5	0.4	0.9	0.9	0.3	1761
1.8	1.3	1.7	1.5	1.4	1734
2.1	1.9	1.6	1.8	1.2	1800
2.4	2.5	2.9	2.9	2.3	1923
2.7	2.2	2.7	2.1	2.5	1988

Table 5. The total number of customers in the multi-server queueing model, including vacations, balking and feedback in inventory cost.

N	E1	E2	E3	E4	Ic5
1	0.1	0.1	0.2	0.9	1400
2	0.2	0.3	0.3	0.8	1003
3	0.1	0.4	0.4	0.7	1300
4	0.2	0.5	0.7	0.6	1433
5	0.3	0.6	0.8	0.5	1563
6	0.4	0.7	0.9	0.4	1640
7	0.5	0.8	0.4	0.5	1800
8	0.6	0.9	0.3	0.6	1700
9	0.7	0.5	0.2	0.6	1901
10	0.7	0.4	0.1	0.5	1999

- Step 4: Table 4 (major variable is mean (exponential distribution))

$$\mathrm{Ic4} = \mathrm{Fc} * \mathrm{E1} + \mathrm{Pc} * \mathrm{E2} + \mathrm{Hc} * \mathrm{E3} + \mathrm{Sc} * \mathrm{E4}$$

- Step 5: Table 5 (major variable is total number of customers in system)

$$\mathrm{Ic5} = \mathrm{Fc} * \mathrm{E1} + \mathrm{Pc} * \mathrm{E2} + \mathrm{Hc} * \mathrm{E3} + \mathrm{Sc} * \mathrm{E4}$$

Results and discussion

The inventory control model on the Matrix Geometric Method for an intermittently obtainable server with balking and feedback in a Markovian queueing model offers several notable contributions and implications for inventory management and system performance.[23] To determine the model's potential contributions, the following analysis will provide an overview and evaluation of the model's impacts.

The given paper offers a framework that is capable of incorporating various pertinent elements into one model. Service availability, system operation, as well as customer behaviour are not researched in isolation. Such an integrated analysis would help interpret the contribution of the individual elements of the system especially in circumstances where the conditions of operation are not well known.

The behaviour of balking is offered to indicate impatience of customers in times of high traffic. Under these circumstances, a part of the customers will not be willing to become part of the system whereas a part of the customers might pull out when the congestion is so high.These measures have an effect on the effective demand and in certain scenarios lead to a progressive alleviation of congestion without external intervention.[24]

It contains a feedback mechanism to reflect the case where customers come back to have more service. This is typical in the maintenance processes, technical support centres as well as in healthcare diagnostics. Recurring requests of services affect the work load pattern and gives a better vision of the continuity of the services in a long-term basis.

A Markovian description is used to describe the system dynamics. This model offers a convenient mathematical framework in which to model random arrivals, service completions, and system state transitions. Because of the systematic form of the model, the matrix-geometric method[25] can be implemented to compute the steady-state measures of systems of a relatively high complexity.

The given idea is perfectly applicable to a practical environment, including hospitals, contact centres, cloud computing services, and repair facilities, where the availability of the service is frequently not

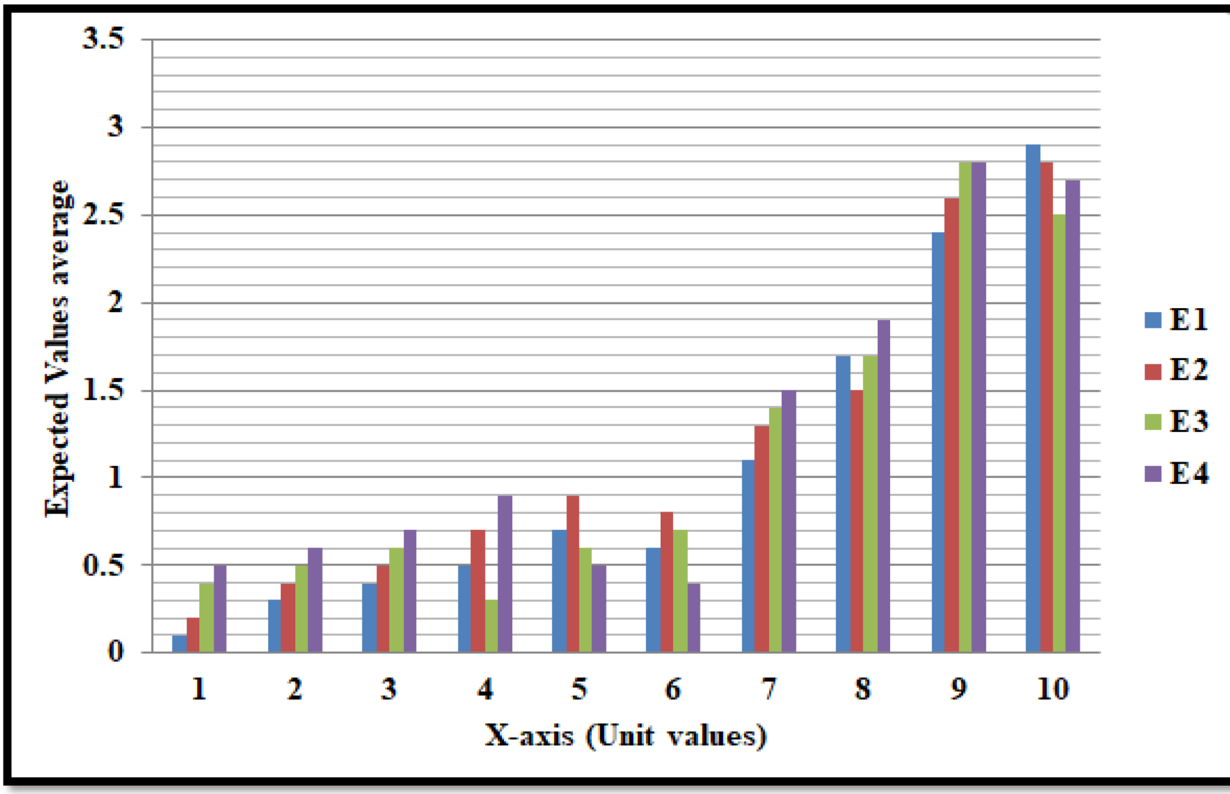

Fig. 2. Bar chart of expected values of the average of the multi-server queueing model for the balking and feedback customers.

regular, and the behaviour of the customers has a significant impact on the work of the system. The findings of the model are useful in enhancing service flow, motivating more resourceful utilization, and leading to an enhancement of the customer experience.

In order to apply the data, individuals have created models, which utilize real-life examples; the models need to encompass sensitivity analysis as well as real data in order to be applicable in real life. With sensitivity analysis, people will know the system's major players and impacts, helping them see the real effects of their inventory policies. Using real and simulated guiding data will help people determine whether the models have practical use.

The book on the Matrix Geometric Method for an Intermittently Obtainable Server with Balking and Feedback in a Markovian Queueing Model provides a clear description of the real-world use and potential applications of this model and explains how it can help meet the requirements of system.[26] The book helps to clarify and broaden our understanding of Queueing Theory and Operations Research by including balking, feedback, and the Matrix Geometric Method. Bibliography numbers[27] and[28] help explain the impact this model can have across many fields and help companies make better decisions, supporting methods that help customers use their resources more effectively and improve the efficiency of their operations.

We used the inventory control model on the Matrix Geometric Method for normal summation by analytical approaches (Tables 1 and 5), and all diagrams (Figs. 2 to 7) were drawn using the Excel tool according to the table calculations. The expected values of the average Intermittently Obtainable Server with Balking and Feedback in Fig. 2 analyze E1, E2, E3, and E4. The expected values of balking services are shown in Fig. 3 and are one of the comparisons for E1, E2, E3, and E4. The predicted values for the exponential distribution with mean values are shown in Fig. 4 as E1, E2, E3, and E4. In Fig. 5, the expected values of completed feedback service from the comparisons on E1, E2, E3 and E4. The predicted values of the total number of customers in the Multi-Server Queueing Model are shown in Fig. 6. Fig. 7 shows comparative studies of inventory cost for an intermittently obtainable server with balking and feedback in the Markovian queueing model.

The novelty of the paper is that we obtained the inventory level that is idle from the mathematical modelling of the inventory control model.

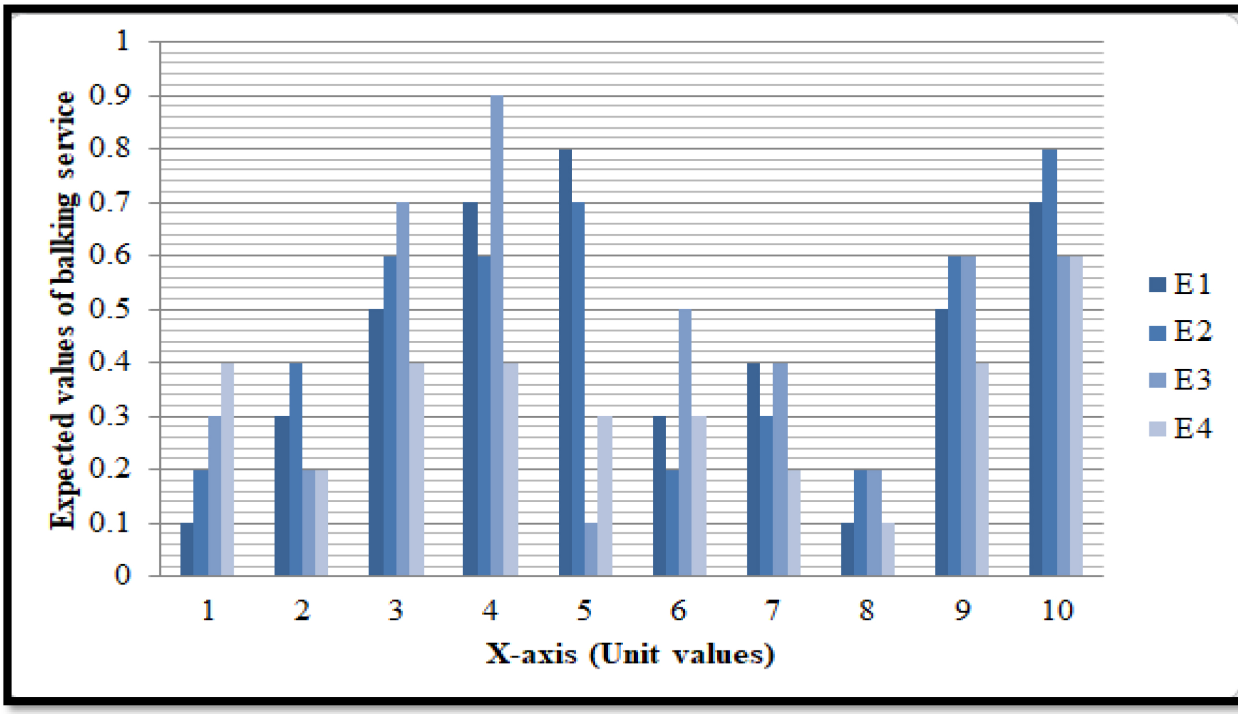

Fig. 3. Bar chart of the expected values of the probability of getting balking service.

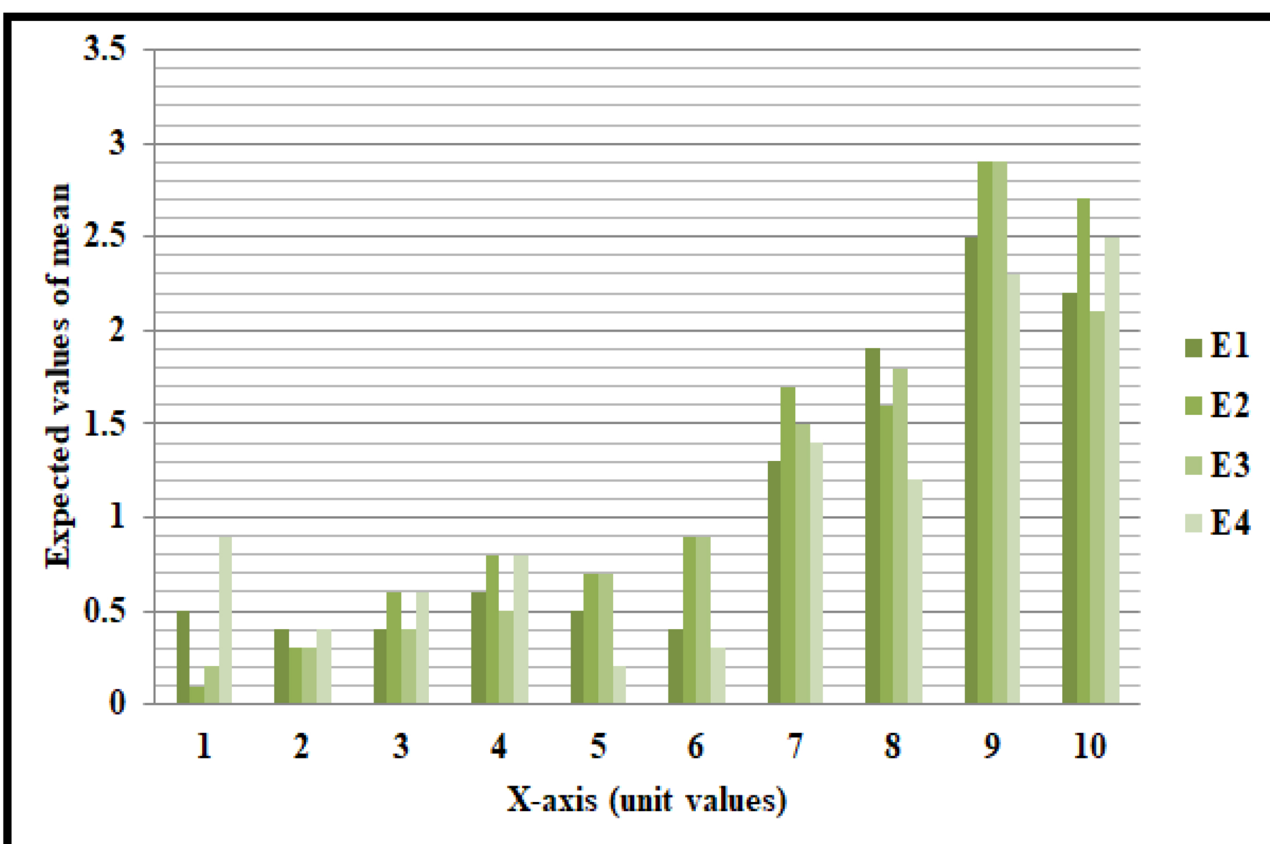

Fig. 4. Bar chart of expected values of the exponential distribution of the mean.

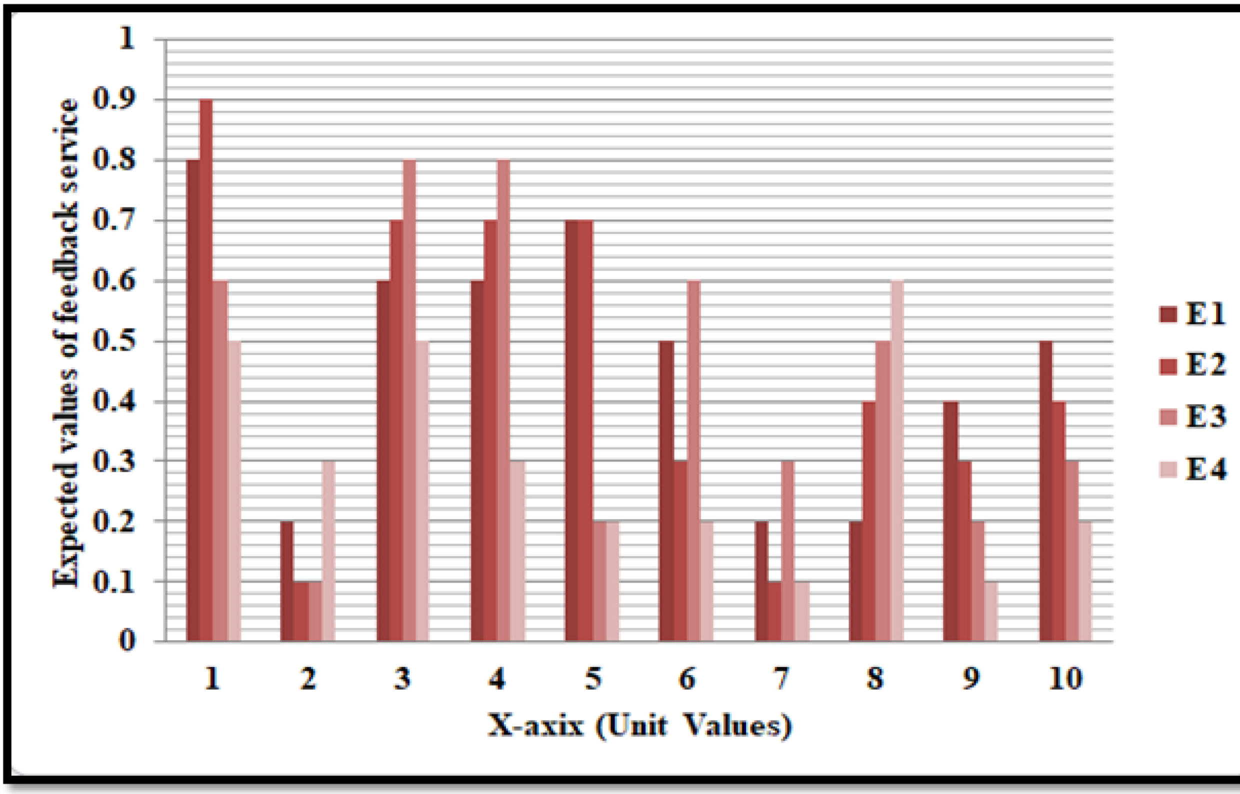

Fig. 5. Bar chart of the expected values of the probability of system completed feedback service.

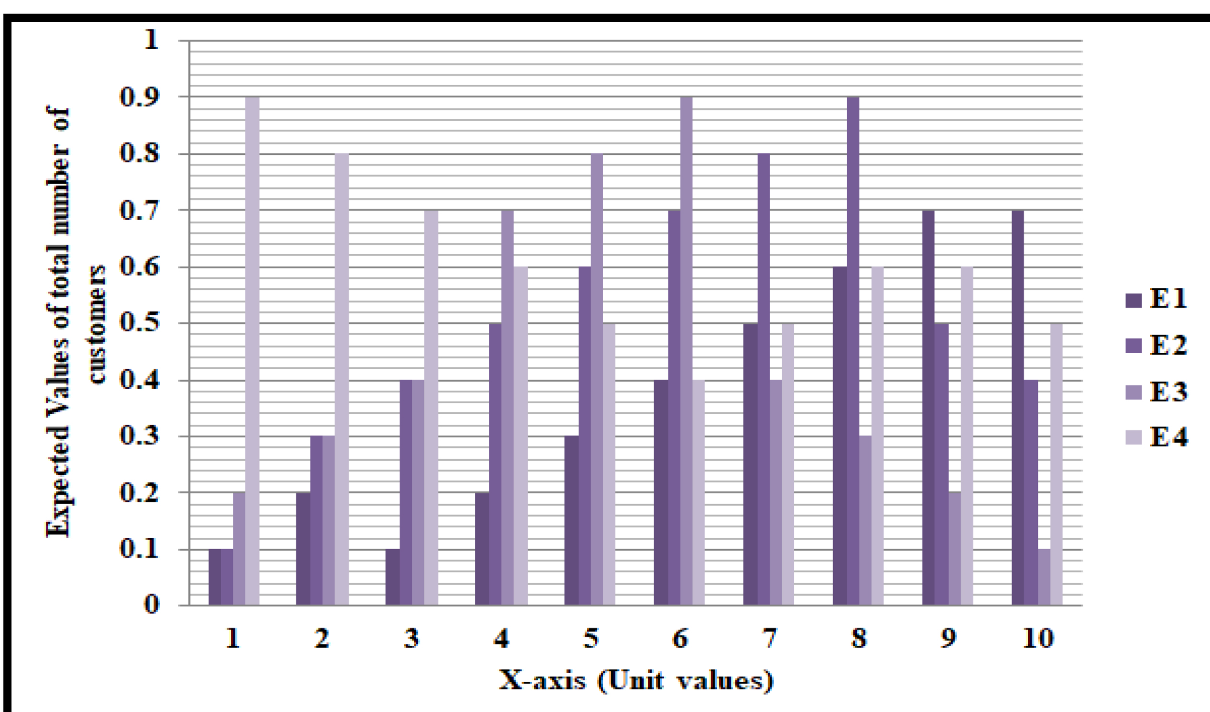

Fig. 6. Bar chart of expected values of the total number of customers in the multi-server queueing model.

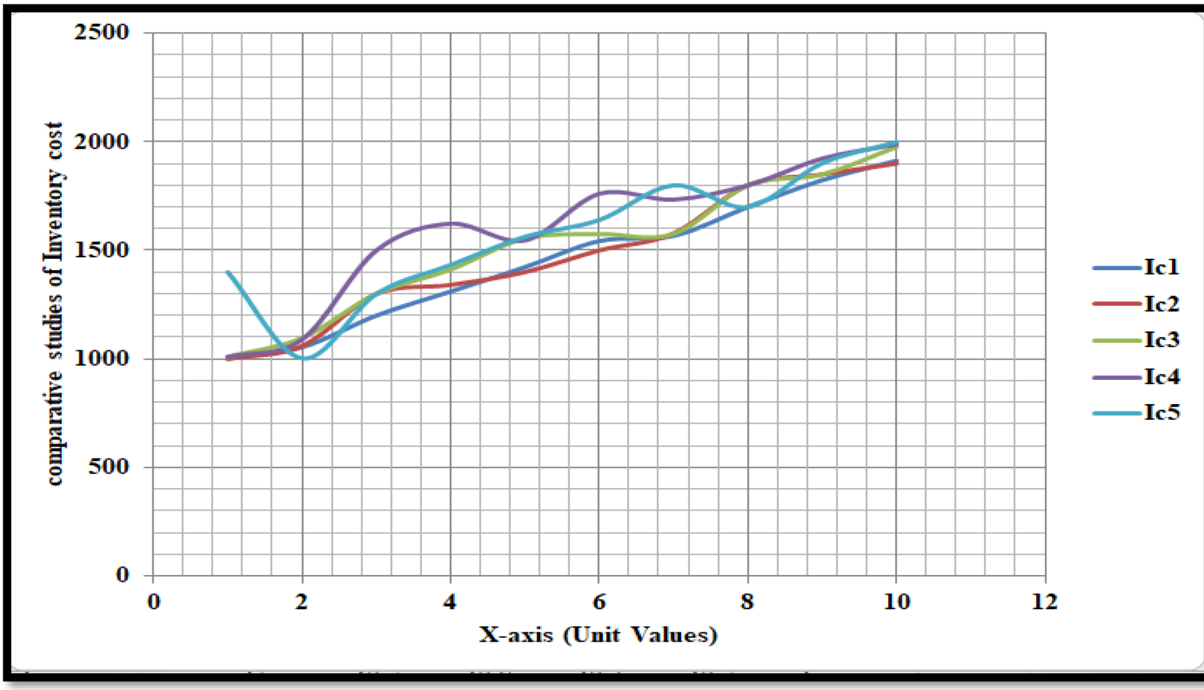

Fig. 7. Comparative studies of inventory cost for intermittently obtainable server with balking and feedback in markovian queueing model.

Also, we found that the Inventory cost was calculated using the analytical method and presented graphically.

Conclusion

This study analyzed a service system that operates under uncertain conditions, with particular emphasis on the interaction between customer actions and fluctuating service availability. The model includes balking and feedback to capture behavioral factors that play an important role in real-world system performance. As a result, the proposed framework more effectively represents customer impatience, repeated service demands, and variability in service access compared with conventional models. The system dynamics are described using a Markovian formulation, which allows a transparent characterization of the underlying stochastic process. To obtain the steady-state solution efficiently, the matrix-geometric method is employed. The performance measures obtained illustrate changes in congestion, service flow, and inventory behavior for different operating conditions. Numerical results indicate that several key parameters have a significant effect on system performance and underline the importance of managing customer behavior and service availability.

On the whole, the suggested framework helps to better comprehend complex service systems when the availability is not constant, and customer reactions are dynamic. This makes the model applicable to numerous real-life scenarios such as healthcare services, maintenance systems, call centers, and cloud-based systems due to its flexibility. The lessons learned in this study can be used to make superior planning and operational choices that can enhance efficiency, reliability and customer satisfaction.

A generalization of arrival and service processes (including Markovian arrival processes or phase distributions) in future work can be done to ease the exponential assumptions, and more accurately model the variability of real-world processes. It is also possible to extend the model to be more complex with more customer classes, priority-based service rules or heterogeneous servers with varying service capabilities. The addition of server breaks down and repair systems would be a more realistic characterization of adaptive service systems. Moreover, the outcomes of the analytical work can be studied using numerical and simulation-based methods to study large-scale systems and tests model performance. The implementation of the framework on actual operation data in areas like healthcare, telecommunication, and cloud service might also contribute to the use of data to make decisions and optimize the system.

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Authors' declaration

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are ours. Furthermore, any Figures and images, that are not ours, have been included with the necessary permission for re-publication, which is attached to the manuscript.
- No animal studies are present in the manuscript.
- No human studies are present in the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee at Vellore Institute of Technology, Vellore, India.

Authors' contribution statement

P.I. and K.K. contributed to the design and implementation of the research. I drafted the manuscript, acquired data, conducted analysis, and wrote the manuscript; K.K. revised, interpreted, and proofread.

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نموذج مراقبة المخزون على طريقة المصفوفة الهندسية لخادم يمكن الحصول عليه بشكل متقطع مع Balking و Feedback in Markovian Queueing Model

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الخلاصة

تقدم هذه الورقة نموذجًا مبتكرًا لمراقبة المخزون ضمن إطار تصنيف ماركوفيان، حيث تتناول الخواديم المتاحة بشكل متقطع مع دمج آليات الرفض والتغذية المرتدة. هدفها الأساسي هو تحسين تكاليف المخزون وتعزيز أداء النظام، مع التركيز بشكل خاص على سيناريوهات الرفض والتعليقات داخل نموذج Markovian Queueing. يجسد Balking الاختيار الذي اتخذه العملاء للتخلي عن الانضمام إلى قائمة الانتظار، مما يعكس قرارهم بعدم انتظار الخدمة. من ناحية أخرى، تمكن التغذية المرتدة الخادم من تكييف عملياته استجابة لظروف النظام السائدة. تشكل هذه العناصر المتكاملة العمود الفقري لنهج ديناميكي يعالج بفعالية التحديات التي يفرضها تقلب طلبات العملاء داخل النظام. يؤدي الاستفادة من طريقة المصفوفة الهندسية إلى تبسيط التحليل والتقييم المنهجي لسلوك التوازن في النظام، مما يوفر فحصًا قويًا لديناميكياته. وتقترح هذه الدراسة أيضا قيما متوقعة مستمدة من نموذج تكاليف المخزون، مما يسهم إسهاما كبيرا في النهوض بالتحليلات المقارنة المتعلقة بتكاليف المخزون. ويتمثل هدفها الشامل في تحسين تخصيص الموارد وتعزيز رضا العملاء داخل نظم العالم الحقيقي. تسليط الضوء على نماذج تكلفة المخزون وآثارها على تحسين الموارد ورضا العملاء، يثري هذا البحث المعرفة النظرية والتطبيقات العملية في نظم مراقبة المخزون والاصطفاف. في نهاية المطاف، يسعى هذا النموذج الشامل إلى أن يكون أداة صنع قرار لا تقدر بثمن لأصحاب المصلحة الذين يتنقلون في البيئات التشغيلية الديناميكية، ويقدم رؤى لتبسيط تخصيص الموارد وتضخيم جودة خدمة العملاء. ويمثل النهج المشترك المتمثل في الرفض، والتغذية المرتدة، والطريقة الهندسية المصفوفة خطوة هامة إلى الأمام في المعالجة الشاملة لمراقبة المخزون في البيئات الديناميكية الموجهة نحو الخدمات، مما يعد بتحسين الكفاءة التشغيلية وتقديم الخدمات المتمحورة حول العملاء.

الكلمات المفتاحية: Balking، القيم المتوقعة، التعليقات، تكلفة المخزون، طريقة المصفوفة الهندسية، نموذج الطابور ماركوفيان.