

2-23-2026

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### How to Cite this Article

Jayasekaran, C. and Binoja, L. G. (2026) "Relatively Prime Detour Domination Number of Switching Certain Special Graphs," *Baghdad Science Journal*: Vol. 23: Iss. 2, Article 26.

DOI: <https://doi.org/10.21123/2411-7986.5218>

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## RESEARCH ARTICLE

# Relatively Prime Detour Domination Number of Switching Certain Special Graphs

C. Jayasekaran<sup>1,\*</sup>, L. G. Binoja<sup>2</sup><sup>1</sup> Associate Professor, Department of Mathematics, Pioneer Kumaraswamy College, Nagercoil 629003, Tamil Nadu, India<sup>2</sup> Research Scholar, Reg. No: 20213132092002, Department of Mathematics, Pioneer Kumaraswamy College, Nagercoil - 629003, Tamil Nadu, India**ABSTRACT**

A set  $S \subseteq V$  is said to be a relatively prime detour dominating set of a graph  $G$  if it has at least two members is a detour set and a dominating set for any pair of vertices  $b$  and  $w$  such that  $(\deg(b), \deg(w)) = 1$ . The lowest cardinality of a relatively prime detour dominating set is indicated by  $\gamma_{rpdn}(G)$  and defines the lowest cardinality of a relatively prime detour dominating set of order  $\gamma_{rpdn}(G)$  said to as a  $\gamma_{rpdn}$ -set of  $G$ . If there is no relatively prime detour dominating set, the relatively prime detour domination number is zero. In this paper, the idea of switching graphs and providing some conclusions on the relatively prime detour domination number for path graphs, star graphs, bistar graphs, cycle graphs and various standard graphs are probed. This paper could be framed from the terms from the abundant epitome of domination graphs. The speculations are explicated with legitimate illustrations. The proof strategy is given lucidly, and the assessment of finding the detour set, domination set and degree of the vertices are stated plainly. Graph domination is wielded plenty in numerous areas and also we begin research on the concept of relatively prime detour domination number for switching graphs. The difficulty of relatively prime detour domination number is examined for us we starts a study on this idea of relatively prime detour domination number for switching graph.

**Keywords:** Detour number, Detour domination number, Domination number, Relatively prime detour domination number, Relatively prime domination number and switching

**Introduction**

We define a finite undirected graph with many edges and no loops as  $G = (V, E)$  and take into consideration the connected graphs with two or more vertices. One can refer to the book by Chartrand and Lesniak<sup>1</sup> for graph theoretical concepts. Detour distance in graphs was a concept that G. Chartrand et al. first developed in.<sup>2</sup> For vertices  $b$  and  $w$  in a connected graph  $G$ , the detour distance  $D(b, w)$  is the length of the longest  $b - w$  path in  $G$ . An  $b - w$  path of length  $D(b, w)$  is called an  $b - w$  detour. The detour number of a graph was a concept that G. Chartrand et al. first developed in.<sup>3</sup> For  $S \subseteq V$ ,  $I_D[S] = \cup_{b,w \in S} I_D[b, w]$ . A set  $S$  of vertices is a detour set if  $I_D[S] = V$ , and the minimum cardinality of a detour set is the detour number  $dn(G)$ . A detour set of cardinality  $dn(G)$  is called a minimum detour set. A fundamental of domination in graphs was a concept that T. W. Haynes et al. studied in.<sup>4</sup> A set  $S \subseteq V$  in a graph  $G$  is a dominating set of  $G$  if for every vertex  $b$  in  $V - S$ , there exists a vertex  $w \in S$  such that  $b$  is adjacent to  $w$ . The domination number of  $G$ , denoted by  $\gamma(G)$ , is the minimum cardinality of a dominating set of  $G$ . Several kinds of domination parameters are studied in.<sup>5-7</sup> Further studies on domination and its variations can be found in.<sup>8,9</sup> A set  $S \subseteq V(G)$  of vertices in a connected graph  $G$ , is called a detour dominating set of  $G$ , if  $S$  is a detour set and dominating set of  $G$ . The minimum

Received 20 November 2023; revised 3 February 2025; accepted 5 February 2025.  
Available online 24 February 2026

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<https://doi.org/10.21123/2411-7986.5218>

2411-7986/© 2026 The Author(s). Published by College of Science for Women, University of Baghdad. This is an open-access article distributed under the terms of the Creative Commons Attribution 4.0 International License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

cardinality of a detour dominating set of  $G$  is the detour domination number of  $G$  and is denoted by  $\gamma_{dn}(G)$ . The detour domination number of a graph was investigated in.<sup>10</sup> Different types of detour domination parameters are analyzed in<sup>11-13</sup> and other connected detour domination numbers are examined in.<sup>14,15</sup> A set  $S \subseteq V$  is a relatively prime dominating set if  $S$  is a dominating set with at least two elements, and for every pair of vertices  $b$  and  $w$  in  $S$ ,  $(\deg(b), \deg(w)) = 1$ . The smallest cardinality of such a set in a graph  $G$  is the relatively prime domination number  $\gamma_{rpd}(G)$ . Relatively prime dominating sets in graphs was a concept that introduced in.<sup>16</sup> A set  $S \subseteq V$  is said to be a relatively prime detour dominating set of a graph  $G$  if is a detour set and a dominating set with at least two elements, and for every pair of vertices  $b$  and  $w$  such that  $(\deg(b), \deg(w)) = 1$ . The lowest cardinality of a relatively prime detour dominating set is indicated by  $\gamma_{rpdn}(G)$  and defines the lowest cardinality of a relatively prime detour dominating set of order  $\gamma_{rpdn}(G)$  is said to as a  $\gamma_{rpdn}$ -set of  $G$ . If there is no relatively prime detour dominating set, the relatively prime detour domination number is zero. Jayasekaran et al. introduced relatively prime detour domination number of a graph.<sup>17</sup> For a finite undirected graph  $G(V, E)$  and a vertex  $b \in V$ , the switching of  $G$  by  $b$  is defined as the graph  $G^b$  by removing all edges incident to  $b$  and adding edges which are not adjacent to  $b$ . The concept of switching in graphs was studied in.<sup>18,19</sup> In this paper, indicates  $\gamma$ -set is a minimum dominating set,  $\gamma_{dn}$ -set is a minimum detour dominating set and further introduce the idea of relatively prime detour domination number for switching graphs and find the number  $\gamma_{rpdn}(G^b)$ .

## Materials and methods

**Theorem 1:**<sup>12</sup> Each end vertex of a graph  $G$  belongs to every relatively prime detour dominating set of  $G$ .

## Results and discussion

In this section, we find relatively prime detour domination number for switching graph of path graph  $P_g$ , star graph  $K_{1,g}$ , bistar graph  $B_{c,g}$ , cycle graph  $C_g$ , wheel graph  $W_g$  and complete bipartite graph  $K_{c,g}$ .

**Example 1:** Given the graphs  $G$  and  $G^{b_1}$  are depicted in Fig. 1. Here,  $S = \{b_1, b_3, b_4\}$  is a  $\gamma_{rpdn}$ -set of  $G^{b_1}$ . Thus  $\gamma_{rpdn}(G^{b_1}) = 3$ .

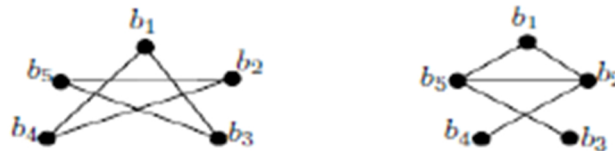


Fig. 1.  $G, G^{b_1}$ .

**Remark 1:** Each end vertex of a graph  $G$  need not belong to every relatively prime detour dominating set of  $G^b$ .

Given the graph  $G = K_{1,3}$  and  $G^{b_1} = C_4$  are depicted in Fig. 2. Since  $G^{b_1} = C_4$ , any minimum dominating set of  $G^{b_1}$  has two vertices with equal degree 2 and so  $G^{b_1}$  does not contains any  $\gamma_{rpdn}$ -set. Thus  $\gamma_{rpdn}(G^{b_1}) = 0$ .



Fig. 2.  $G = K_{1,3}, G^{b_1} = C_4$ .

**Theorem 2:** Each end vertex of a graph  $G^b$  belongs to every  $\gamma_{rpdn}$ -set of  $G^b$ .

**Proof:** The proof follows from Theorem 1.

**Theorem 3:** Let  $G$  be a path graph  $P_g$  and let  $b$  be a vertex of  $P_g$ .

(i) For  $g = 2$ ,  $\gamma_{rpdn}(G^b) = 0$

$$(ii) \text{ For } g \geq 3, \gamma_{rpdn}(G^b) = \begin{cases} 2 & \text{if } b \text{ is an end vertex} \\ 0 & \text{if } b \text{ is a support vertex} \\ 3 & \text{if } b \text{ is neither an end vertex nor a support vertex} \end{cases}.$$

**Proof:** Let  $b_1, b_2, \dots, b_g$  be the path  $P_g$ . Let  $b$  be any vertex of  $G$  and let  $G^b$  be the vertex switching of  $G$  with respect to  $b$ . We consider the following cases.

Case 1.  $g = 2$ .

Clearly,  $G^b = 2K_1$  is a disconnected graph and so there does not exist any  $\gamma_{rpdn}$ -set. Hence  $\gamma_{rpdn}(G^b) = 0$ .

Case 2.  $g \geq 3$

Subcase 1.  $b$  is an end vertex

Here  $b$  is either  $b_1$  or  $b_n$  and  $G^{b_1} \cong G^{b_n}$ . Let  $b$  be  $b_1$ . The graph  $P_g^{b_1}$  is depicted in Fig. 3. Clearly,  $E(G^b) = \{bb_k, b_s b_{s+1} : 3 \leq k \leq g, 2 \leq s \leq g-1\}$  and in  $G^b$ ,  $\deg(b) = g-2, \deg(b_2) = 1, \deg(b_s) = 3$  and  $\deg(b_g) = 2, 3 \leq s \leq g-1$ .

In  $G^b$ ,  $b$  is adjacent to all the vertices other than  $b_s, 3 \leq s \leq g$  and  $b$  is not adjacent to  $b_2$ . Clearly,  $b$  and  $b_2$  dominate every vertices in  $G^b$ . Hence  $S = \{b, b_2\}$  is a  $\gamma$ -set for  $G^b$ . Also  $I_{D[S]} = V(G^b)$ . Hence  $S$  is a  $\gamma_{dn}$ -set and also  $(\deg(b), \deg(b_2)) = (g-2, 1) = 1$ . Thus,  $S$  is a  $\gamma_{rpdn}$ -set of minimum cardinality 2. Hence  $\gamma_{rpdn}(G^b) = 2$ .

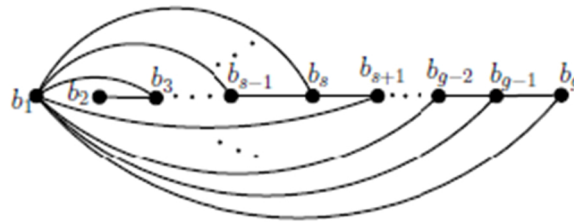


Fig. 3.  $P_g^{b_1}$ .

Subcase 2.  $b$  is a support vertex

Clearly,  $b$  is either  $b_2$  or  $b_{n-1}$  and  $G^{b_2} \cong G^{b_{n-1}}$ . Let  $b$  be  $b_2$ . The graph  $P_g^{b_2}$  is depicted in Fig. 4. Now  $E(G^b) = \{bb_k, b_s b_{s+1} : 4 \leq k \leq g, 3 \leq s \leq g-1\}$  and in  $G^b$ ,  $\deg(b_1) = 0, \deg(b_3) = 1$ . Clearly,  $G^b$  is a disconnected graph and so there does not exist any  $\gamma_{rpdn}$ -set. Hence  $\gamma_{rpdn}(G^b) = 0$ .

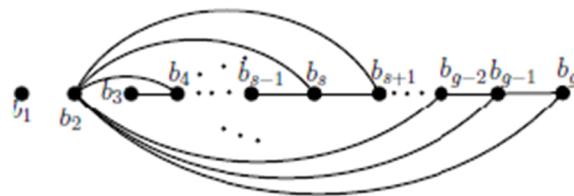


Fig. 4.  $P_g^{b_2}$ .

Subcase 3.  $b$  is neither an end vertex nor a support vertex

Here  $b = b_s, 3 \leq s \leq g-2$ . Let  $b$  be  $b_s$ . The graph  $P_g^{b_s}$  is depicted in Fig. 5. Clearly,  $E(G^b) = \{bb_t, b_r b_{r+1} : 1 \leq t \leq g, 1 \leq r \leq g-1, t \neq s \pm 1, r \neq s-1, s\}$  and in  $G^b$ ,  $\deg(b_{s-1}) = 1, \deg(b) = g-3$  and  $\deg(b_{s+1}) = 1$ .

In  $G^b$ ,  $b$  is adjacent to all the vertices other than  $b_{s-1}$  and  $b_{s+1}$ . Clearly,  $b_{s-1}, b$  and  $b_{s+1}$  dominate every vertices in  $G^b$ . Hence  $S = \{b, b_{s-1}, b_{s+1}\}$  is a  $\gamma$ -set for  $G^b$ . Also  $I_{D[S]} = V(G^b)$ . And so  $S$  is a  $\gamma_{dn}$ -set. Since the degrees of vertices in  $S$  are  $g-3, 1$  and  $1, S$  is a  $\gamma_{rpdn}$ -set of  $G^b$ . Hence  $\gamma_{rpdn}(G^b) = 3$ .

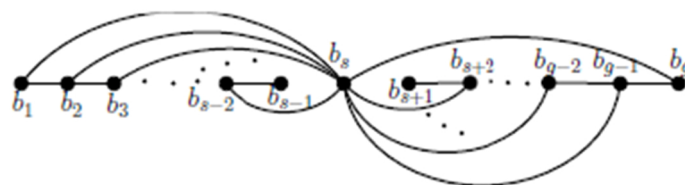


Fig. 5.  $P_g^{b_s}$ .

The following theorem is derived from the preceding three cases.

**Theorem 4:** Let  $b$  a vertex of star  $K_{1,g}$ . Then  $\gamma_{rpdn}(K_{1,g}^b) = 2$  if and only if  $b$  is an end vertex of  $K_{1,g}$  with  $g \geq 2$  is even.

**Proof:** Let  $w$  be the central vertex and  $b_1, b_2, \dots, b_g$  be the vertices of degree one. Clearly,  $K_{1,g}^w = \overline{K_{g+1}}$  and so  $K_{1,g}^w$  has no relatively prime detour dominating set.

Let  $b$  be an end vertex of  $K_{1,g}$ . Then  $b$  is  $b_s$  for some  $s, 1 \leq s \leq g$ . The graph  $K_{1,g}^{b_s}$  is depicted in Fig. 6. Also  $E(G^b) = \{bb_t, wb_t : 1 \leq t \neq s \leq g\}$  and  $\deg(b_s) = g - 1 = \deg(w), \deg(b_t) = 2$ .

In  $K_{1,g}^{b_s}$ ,  $b$  is adjacent to all the vertices except the vertex  $w$ . To dominate the vertex  $w$ , we choose a vertex from  $b_t$  for some  $t, 1 \leq t \neq s \leq g$ . Hence  $S = \{b, b_t\}$  is a  $\gamma$ -set of  $K_{1,g}^{b_s}$ . Also  $I_{D[S]} = V(G^b)$  and  $(\deg(b), \deg(b_t)) = (g - 1, 2) = 1g \geq 2$  is even. Thus  $\gamma_{rpdn}(K_{1,g}^b) = 2$  if and only if  $g$  is even.

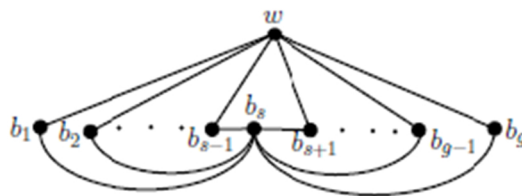


Fig. 6.  $K_{1,g}^{b_s}$ .

**Theorem 5:** Let  $B_{c,g}$  be the bistar graph and  $b$  be a vertex of  $B_{c,g}$ . Then

$$\gamma_{rpdn}(B_{c,g}^b) = \begin{cases} 2 & \text{if either } c + g \text{ is odd or both } c \text{ and } g \text{ are odd and } c \neq g, b \text{ is an end vertex} \\ 0 & \text{otherwise} \end{cases}$$

**Proof:** Let  $G = B_{c,g}$  with  $V(B_{c,g}) = \{w_0, w_s, b_0, b_t : 1 \leq s \leq c, 1 \leq t \leq g\}$  and  $E(B_{c,g}) = \{w_0b_0, w_0w_s, b_0b_t : 1 \leq s \leq c, 1 \leq t \leq g\}$ .

If  $b$  is either  $w_0$  or  $b_0$ , then  $B_{c,g}^b$  is a disconnected graph and hence  $\gamma_{rpdn}(B_{c,g}^b) = 0$ .

Let  $b$  be an end vertex of  $B_{c,g}$ . Then  $b$  is either  $w_s$  or  $b_t, 1 \leq s \leq c, 1 \leq t \leq g$ . Clearly  $G^{w_1} \cong G^{w_2} \cong \dots \cong G^{w_c}$  and  $G^{b_1} \cong G^{b_2} \cong \dots \cong G^{b_g}$ . Let  $b$  be  $w_1$  or  $b_1$ . The graph  $B_{c,g}^{w_1}$  is depicted in Fig. 7.

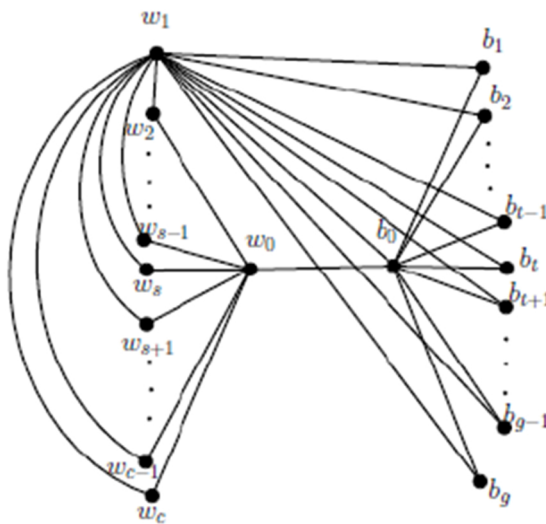


Fig. 7.  $B_{c,g}^{w_1}$ .

Case 1.  $b = w_1$

Clearly,  $E(G^{w_1}) = \{w_0b_0, w_0w_s, b_0b_t, w_1b_0, w_1w_t, w_1b_t : 2 \leq s \leq c, 1 \leq t \leq g\}$  and in  $G^{w_1}, \deg(w_1) = n + m, \deg(w_s) = 2 = \deg(b_t), \deg(w_0) = m$  and  $\deg(b_0) = n + 2, 2 \leq s \leq c, 1 \leq t \leq g$ .

Subcase 1. Either  $c$  or  $g$  is odd but not both

Clearly,  $c$  is odd and  $g$  is even or  $c$  is even and  $g$  is odd. Then  $c + g$  is odd.

In  $G^{w_1}$ ,  $w_1$  is adjacent to all the vertices other than  $w_0$  and  $w_2$  is adjacent to the vertex  $w_0$ . Clearly,  $w_1$  and  $w_2$  dominates every vertices in  $G^{w_1}$ . Hence  $S = \{w_1, w_2\}$  is a  $\gamma$ -set for  $G^{w_1}$ . Also  $I_D[S] = V(G^{w_1})$ . And so  $S$  is a  $\gamma_{dn}$ -set. Now  $(\deg(w_1), \deg(w_2)) = (c + g, 2) = 1$ . Therefore,  $S$  is a  $\gamma_{rpdn}$ -set and so  $\gamma_{rpdn}(B_{c,g}^b) = 2$ .

Subcase 2. both  $c$  and  $g$  are odd and  $c \neq g$

In  $G^{w_1}$ ,  $w_1$  is adjacent to all the vertices other than  $w_0$ . Clearly,  $w_1$  and  $w_0$  dominates every vertices in  $G^{w_1}$  and thereby  $S = \{w_1, w_0\}$  is a  $\gamma$ -set for  $G^{w_1}$ . Also  $I_D[S] = V(G^{w_1})$  and so  $S$  is a  $\gamma_{dn}$ -set. Now  $(\deg(w_1), \deg(w_0)) = (c + g, c) = 1$  if and only if  $c \neq g$ . Therefore,  $S$  is a  $\gamma_{rpdn}$ -set if and only if  $c \neq g$  and so  $\gamma_{rpdn}(B_{c,g}^b) = 2$  if and only if  $c \neq g$ .

Subcase 3. both  $c$  and  $g$  are even

In  $G^{w_1}$ ,  $w_1$  is adjacent to all the vertices other than  $w_0$  and  $w_2; b_0$  is adjacent to the vertex  $w_0$ . Hence  $S = \{w_1, w_0\}$  or  $S = \{w_1, w_2\}$  or  $S = \{w_1, b_0\}$  is a  $\gamma$ -set for  $G^{w_1}$ . Also  $I_D[S] = V(G^{w_1})$  and so  $S$  is a  $\gamma_{dn}$ -set. Now  $(\deg(w_1), \deg(w_0)) = (c + g, c) \neq 1$ ,  $(\deg(w_1), \deg(w_2)) = (c + g, 2) \neq 1$  and  $(\deg(w_1), \deg(b_0)) = (c + g, g + 2) \neq 1$ . Therefore,  $S$  is not a  $\gamma_{rpdn}$ -set and so  $\gamma_{rpdn}(B_{c,g}^b) = 0$ .

Case 2.  $b = b_1$

By a similar argument given in Case 1, we have  $\gamma_{rpdn}(B_{c,g}^b) = 2$  if and only if either  $c + g$  is odd or both  $c$  and  $g$  are odd and  $c \neq g$ .

**Theorem 6:** Let  $b$  a vertex of cycle  $C_g$ . Then  $\gamma_{rpdn}(C_g^b) = 3$  if and only if  $g > 3$ .

**Proof:** Let  $b_1b_2 \dots b_gb_1$  be the cycle  $C_g$ . Clearly,  $C_g^{b_1} \cong C_g^{b_2} \cong \dots \cong C_g^{b_g}$ . Let  $b$  be  $b_1$ . The graph  $C_g^{b_1}$  is depicted in Fig. 8. We have  $E(C_g^{b_1}) = \{b_sb_l, b_tb_{t+1}, b_1b_n : 1 \leq l \leq g, 1 \leq t \leq g - 1, l \neq s \pm 1, j \neq s - 1, s\}$ . In  $C_g^{b_1}$ ,  $\deg(b_2) = \deg(b_n) = 1$ ,  $\deg(b_1) = g - 3$ , and  $\deg(b_t) = 3, 3 \leq t \leq g - 1$ .

If  $g = 3$ , then  $C_3^{b_1} = K_1 \cap K_2$  is a disconnected graph and so  $\gamma_{rpdn}(C_g^b) = 0$ .

For  $g > 3$ , in  $C_g^{b_1}$ ,  $b_1$  is adjacent to all vertices other than the end vertices  $b_2$  and  $b_n$ . Therefore,  $S = \{b_1, b_2, b_n\}$  is a  $\gamma$ -set and also  $I_D[S] = V(C_g^b)$ . This implies that,  $S$  is a  $\gamma_{dn}$ -set. Since the distinct degrees of vertices in  $S$  are  $g - 3, 1$  and  $1$ ,  $S$  is a  $\gamma_{rpdn}$ -set and hence  $\gamma_{rpdn}(C_g^b) = 3$ .

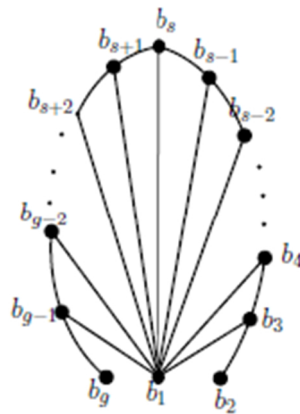


Fig. 8.  $C_g^{b_1}$ .

**Theorem 7:** Let  $b$  a vertex of wheel  $W_g = C_{g-1} + K_1$ . Then

$$\gamma_{rpdn}(W_g^b) = \begin{cases} 2 & \text{if } \deg_{W_g}(b) = 2 \text{ and } g \text{ is odd} \\ 0 & \text{otherwise} \end{cases}$$

**Proof:** Let  $b_1b_2 \dots b_{g-1}b_1$  be the cycle  $C_{g-1}$  and  $w$  be the vertex  $K_1$ . Then join  $C_{g-1} + K_1$  is the wheel graph  $W_g$ .

Case 1.  $b = w$

Clearly,  $W_g^w = C_{n-1} \cup K_1$  is a disconnected graph with an isolated vertex  $w$ , since  $w$  is adjacent to all the vertices in  $W_g$ , there does not exist any  $\gamma_{rpdn}$ -set. Hence,  $\gamma_{rpdn}(W_g^w) = 0$ .

Case 2.  $b \neq w$

Then  $v$  is  $b_s$  for some  $s, 1 \leq s \leq g - 1$ . If  $g = 4$ , then  $W_4^{b_s} = C_3 \cup K_1$  and so  $\gamma_{rpdn}(W_4^{b_s}) = 0$ .

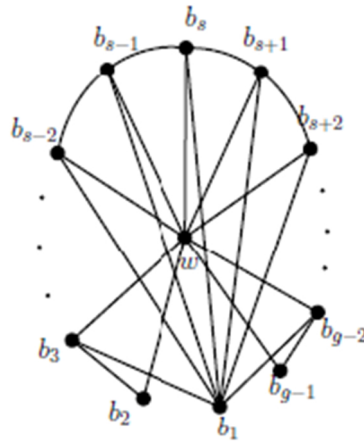


Fig. 9.  $W_g^{b_1}$ .

Let  $g > 4$ . Clearly,  $W_g^{b_1} \cong W_g^{b_2} \cong \dots \cong W_g^{b_{g-1}}$ . Let  $b$  be  $b_1$ . The graph  $W_g^{b_1}$  is depicted in Fig. 9.

Now  $E(W_g^{b_1}) = \{wb_k, b_t b_{t+1}, b_1 b_r : 2 \leq k \leq g-1, 2 \leq t \leq g-2, 3 \leq r \leq g-2\}$ . In  $W_g^{b_1}$ ,  $\deg(w) = g-2$ ,  $\deg(b_1) = g-4$ ,  $\deg(b_2) = \deg(b_{g-1}) = 2$  and  $\deg(b_t) = 4, 2 \leq t \leq g-2$ . In  $W_g^{b_1}$ ,  $w$  is non-adjacent to  $b_1$  and so  $S = \{w, b_1\}$  is a  $\gamma$ -set and also  $I_D[S] = V(W_g^{b_1})$ . Hence,  $S$  is a  $\gamma_{dn}$ -set. Now  $(\deg(w), \deg(b_1)) = (g-2, g-4) = 1$  if and only if  $g$  is odd. Therefore,  $S$  is a  $\gamma_{rpdn}$ -set of  $W_g^{b_1}$  if and only if  $g$  is odd and so  $\gamma_{rpdn}(W_g^{b_1}) = 2$  if and only if  $g$  is odd.

**Theorem 8:** Let  $b$  be any vertex of the complete bipartite graph  $K_{c,g}$ , ( $c, g \geq 2$ ). Then  $\gamma_{rpdn}(K_{c,g}^b) = 2$  if and only if either  $(c-1, g) = 1$  or  $(g-1, c) = 1$ .

**Proof:** Let  $V_1 \cup V_2$  be the bipartition of  $V(K_{c,g})$  where  $V_1 = \{w_s : 1 \leq s \leq c\}$  and  $V_2 = \{b_t : 1 \leq t \leq g\}$ . Let  $b$  be any vertex of  $K_{c,g}$ . Then  $b$  is either  $w_s$  or  $b_t$  for some  $s, t, 1 \leq s \leq c, 1 \leq t \leq g$ .

Case 1.  $b = w_s$ , for some  $s, 1 \leq s \leq c$

Clearly,  $K_{c,g}^{w_1} \cong K_{c,g}^{w_2} \cong \dots \cong K_{c,g}^{w_c}$ . Let  $b$  be  $w_1$ . The graph  $K_{c,g}^{w_1}$  is depicted in Fig. 10. Now  $E(K_{c,g}^{w_1}) = \{w_1 w_l, w_r b_t : 2 \leq l \leq c, 2 \leq r \leq c, 1 \leq t \leq g\}$  and in  $K_{c,g}^{w_1}$ ,  $\deg(w_1) = c-1$ ,  $\deg(w_l) = g$  and  $\deg(b_t) = c-1$ . Hence  $S = \{w_1, w_l\}$ , for some  $l$ , is a  $\gamma$ -set and also  $I_D[S] = V(K_{c,g}^{w_1})$ . Hence  $S$  is a  $\gamma_{dn}$ -set. Thus,  $S$  is  $\gamma_{rpdn}$ -set if and only if  $(c-1, g) = 1$ . Therefore,  $\gamma_{rpdn}(K_{c,g}^b) = 2$  if and only if  $(c-1, g) = 1$ .

Case 2.  $b = b_t, 1 \leq t \leq g$

By a similar argument as given in case 1, we have  $\gamma_{rpdn}(K_{c,g}^b) = 2$  if and only if  $(g-1, c) = 1$ .

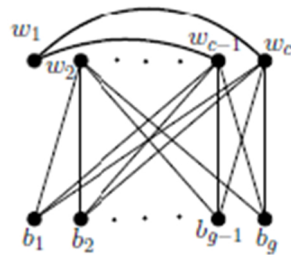


Fig. 10.  $K_{c,g}^{w_1}$ .

### Conclusion

Inspired by the concepts of detour sets and relatively prime dominating sets, we introduce the relatively prime detour dominating set, specifically designed for switching graphs. This paper determines the relatively

prime detour domination number for switching graphs of certain common graphs such as the path, star, bistar, cycle, wheel, and complete bipartite graphs. We concentrate on devising an algorithm capable of determining whether a given graphs have been relatively prime detour domination number. Additionally, we are delving into the challenge of establishing the necessary and sufficient condition for the existence of relatively prime detour dominating set and also investigate the realization problems.

## Acknowledgment

The authors desire to show the thankfulness to the referees for giving their contribution by way of suggestions for improvement.

## Authors' declaration

- Conflicts of Interest: None.
- We hereby confirm that all the Figures in the manuscript are ours. Furthermore, any Figures and images, that are not ours, have been included with the necessary permission for re-publication, which is attached to the manuscript.
- No animal studies are present in the manuscript.
- No human studies are present in the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee at Manonmaniam Sundaranar University's local ethics council in India.

## Authors' contribution statement

CJ contributed for conceptualization and analyzing. LGB contributed for investigation, methodology and writing the draft. CJ reviewed and finalized the draft. All the authors make out and endorsed the finalized the paper.

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# عدد هيمنة الانعطاف الرئيسي نسبياً لتبديل بعض الرسوم البيانية الخاصة

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## المخلص

يقال إن مجموعة  $S \subseteq V$  هي مجموعة منعطفات رئيسية نسبياً مسيطرة على الرسم البياني  $G$  إذا كانت تحتوي على عضوين على الأقل هي مجموعة منعطفات ومجموعة مسيطرة لأي زوج من الرؤوس  $b$  و  $w$  بحيث  $(\deg(w), \deg(b)) = 1$ ، تشير  $(\gamma\_rpdn(G))$  إلى أدنى كاردينية لمجموعة منعطفات رئيسية مسيطرة نسبياً وتحدد أدنى كاردينية لمجموعة منعطفات رئيسية نسبياً مسيطرة على النظام  $(\gamma\_rpdn(G))$  يقال إنها مجموعة  $\gamma\_rpdn$  من  $G$ . إذا لم يكن هناك مجموعة مسيطرة أولية نسبياً، فإن رقم هيمنة الالتفاف الأولي نسبياً هو صفر. في هذه الورقة، يتم فحص فكرة تبديل الرسوم البيانية وتقديم بعض الاستنتاجات حول رقم هيمنة الانعطاف الأولي نسبياً للرسوم البيانية للمسار والرسوم البيانية للنجوم والرسوم البيانية للحزام والرسوم البيانية للدورة والرسوم البيانية القياسية المختلفة. يمكن تأطير هذه الورقة من المصطلحات من المثال الوفير للرسوم البيانية للسيطرة. يتم شرح التكهانات برسوم توضيحية مشروعة. يتم تقديم استراتيجية الإثبات بوضوح، ويتم تحديد تقييم العنور على مجموعة الالتفاف ومجموعة الهيمنة ودرجة الرؤوس بوضوح. يتم استخدام هيمنة الرسم البياني كثيراً في العديد من المجالات، كما نبدأ البحث في مفهوم رقم هيمنة الانعطاف الأولي نسبياً لتبديل الرسوم البيانية. يتم فحص صعوبة رقم هيمنة الانعطاف الأولي نسبياً بالنسبة لنا، حيث نبدأ دراسة حول فكرة رقم هيمنة الانعطاف الأولي نسبياً لتبديل الرسم البياني.

**الكلمات المفتاحية:** رقم هيمنة الالتفاف، رقم الهيمنة، رقم هيمنة الانعطاف الرئيسي نسبياً، رقم الهيمنة الرئيسي نسبياً والتبديل.