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A Study on Structural Properties of Operators In Neutrosophic Banach Spaces

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Abstract:
In this paper, I introduce and discuss the notations of linearity, bounded, continuous and invertibility of operators in neuromorphic Banach space. Furthermore, I establish important relationships among linearity, bounded, continuous. Additionally, I extend the study of these properties to the perturbation operator. Moreover, I discuss Lipschitz and contraction mappings, as well as uniqueness results related to fixed points.

Keywords: Neutrosophic Banach Space, neutrosophically continuous, neutrosophically contraction and neutrosophically invertible.

Introduction:

As a treatment for the limitations of classical and fuzzy concepts, the notion of neutrosophy was first introduced by Florentine Smarandache [1]. Since then, interest in this generalization known as the neutrosophic space—has grown, and it has begun to be applied in various fields such as decision-making, artificial intelligence, image processing, data analysis, and mathematical modeling. [2] proposed a new generalization of the fuzzy normed space by employing the concept of neutrosophic sets. In their work, they investigated several fundamental properties of this space and named it the neutrosophic normed space.

[3] presented chain rule and some algebraic properties of Fréchet differentiation of operators between neutrosophic normed spaces. [4] studied the analysis of sequences in terms of ordinary and Cauchy convergence under certain conditions in neutrosophic normed spaces. They also defined the forms of operators on these spaces and investigated some equivalent relationships among them. [5] proposesed a novel approach to address the analysis of stability in functional equations within neutrosophic normed spaces, offering a comprehensive framework for investigating stability properties in such contexts. [6] defined and discussed the concepts of continuous and bounded of operators in neuromorphic normed space.

In this paper, I introduce and discuss a new extended of concepts linearity, continuity and boundedness, invertibility of operators in neutrosophic Banach spaces. Consequently, I reveal new properties and relationships between the bounded linear operators and perturbation operators as well as their connections with the invertibility in neutrosophic Banach spaces. also, we obtain a generalized version of boundedness and continuity of intuitionistic fuzzy norms, while will play an important role in study neutrosophic analysis. Finally, I presented the new concepts of neutrosophic Lipschitz and contraction function in neutrosophic Banach space.

Previous Studies:

Definition 1. [7] A continuous $t-norm$ is defined as a mapping $\oplus:[0,1]\times[0,1]\to[0,1]$ that satisfies the following conditions, for all $g,\hbar,\ell,m\in[0,1]$

a) $\oplus$ is an associative, commutative and continuous,

b) $g\oplus 1=g$,

c) $g\oplus\hbar\le\ell\oplus m$ implies that $g\le\ell$ and $\hbar\le m$

Definition 2. [2] A continuous $t-co\_norm$ is defined as a mapping $\odot:[0,1]\times[0,1]\to[0,1]$ that satisfies the following conditions, for all $g,\hbar,\ell,m\in[0,1]$

a) $\odot$ is an associative, commutative and continuous,

b) $g\odot 0=g$,

c) $g\odot\hbar\le\ell\odot m$ implies that $g\le\ell$ and $\hbar\le m$

Definition 3.[8] An NS $\check{N}$ over $\mathbb{N}$ is described as follows

$\{\langle m,\check{N}_1(m),\check{N}_2(m),\check{N}_3(m)\rangle; m\in\mathbb{N}\}$

where, $\check{N}_i:\mathbb{N}\to[0,1], i=1,2,3$

Definition 4.[9] **A function** NS $\check{N}$ defined on a linear space $\mathcal{M}$ over $\mathbb{R}$ is referred to as a neutrosophic normed linear space NSŇLS $(\mathcal{M},\mathbb{R},\oplus,\odot)$, if it satisfies the following axioms

1. $0\le\check{N}_1(m,\tau),\check{N}_2(m,\tau),\check{N}_3(m,\tau)\le 1$
2. $0\le\check{N}_1(m,\tau)+\check{N}_2(m,\tau)+\check{N}_3(m,\tau)\le 3$
3. $\check{N}_1(m,\tau)=0,\check{N}_2(m,\tau)=1,\check{N}_3(m,\tau)=1, \tau\le 0$
4. $\check{N}_1(m,\tau)=1,\check{N}_2(m,\tau)=0,\check{N}_3(m,\tau)=0, \tau>0$ *if and only if* $m=0$
5. $\check{N}_1(\alpha m,\tau)=\check{N}_1\left(m,\frac{\tau}{|\alpha|}\right),\check{N}_2(\alpha m,\tau)=\check{N}_2\left(m,\frac{\tau}{|\alpha|}\right),\check{N}_3(\alpha m,\tau)=\check{N}_3\left(m,\frac{\tau}{|\alpha|}\right)$, for each $\alpha\ne 0$ and $\tau>0$
6. $\check{N}_1(m,\tau)\oplus\check{N}_1(\ell,\delta)\le\check{N}_1(m+\ell,\tau+\delta)$, , for each $\tau,\delta\in\mathbb{R}$
7. $\check{N}_1(m,\tau)$ approaches 1 as τ approaches infinity such that $\check{N}_1$ is a non-decreasing and continuous function for $\tau>0$
8. $\check{N}_2(m,\tau)\oplus\check{N}_2(\ell,\delta)\ge\check{N}_2(m+\ell,\tau+\delta)$, , for each $\tau,\delta\in\mathbb{R}$
9. $\check{N}_2(m,\tau)$ approaches 0 as τ approaches infinity such that $\check{N}_2$ is a non-

increasing and continuous function for $\tau > 0$

10. $\check{N}_3(m,\tau)\oplus\check{N}_3(\ell,\delta) \geq \check{N}_3(m+\ell,\tau+\delta)$, , for each $\tau,\delta \in \mathbb{R}$

11. $\check{N}_3(m,\tau)$ approaches 0 as τ approaches infinity such that $\check{N}_2$ is a non-increasing and continuous function for $\tau > 0$

Definition 5. [10]Let $\{m_n\}$ be a sequence in NSŇLS $(\mathcal{M},\mathbb{R},\oplus,\odot)$, then the sequence $\{m_n\}$ converges to $m \in \mathcal{M}$, if and only if for each $0 < \epsilon < 1, \tau > 0$, there exists $n_0 \in \mathbb{N}$ satisfies

$$\check{N}_1(m_n - m,\tau) > 1-\epsilon$$
$$\check{N}_2(m_n - m,\tau) < \epsilon$$
$$\check{N}_3(m_n - m,\tau) < \epsilon \text{ ,for all } n \geq n_0$$
$$\lim_{n\to\infty} \check{N}_1(m_n - m,\tau) = 1$$
$$\lim_{n\to\infty} \check{N}_2(m_n - m,\tau) = 0$$
$$\lim_{n\to\infty} \check{N}_3(m_n - m,\tau) = 0$$

In this case the sequence $\{m_n\}$ is said to be convergent in the space NSŇLS $(\mathcal{M},\mathbb{R},\oplus,\odot)$.

Definition 5. [10]Let $\{m_n\}$ be a sequence in NSŇLS $(\mathcal{M},\mathbb{R},\oplus,\odot)$, then the sequence $\{m_n\}$ is called a Cauchy, if and only if for each $0 < \epsilon < 1, \tau > 0$, there exists $n_0 \in \mathbb{N}$ satisfies

$$\check{N}_1(m_n - m_k,\tau) > 1-\epsilon$$
$$\check{N}_2(m_n - m_k,\tau) < \epsilon$$
$$\check{N}_3(m_n - m_k,\tau) < \epsilon \text{ ,for all } n,k \geq n_0$$
$$\lim_{n\to\infty} \check{N}_1(m_n - m_k,\tau) = 1$$
$$\lim_{n\to\infty} \check{N}_2(m_n - m_k,\tau) = 0$$
$$\lim_{n\to\infty} \check{N}_3(m_n - m_k,\tau) = 0$$

In this case the sequence $\{m_n\}$ is said to be Cauchy in the space NSŇLS $(\mathcal{M},\mathbb{R},\oplus,\odot)$.

Definition 6.[11] A Neutrosophic Banach Space is a neutrosophic normed linear space NSŇLS $(\mathcal{M},\mathbb{R},\oplus,\odot)$ that is complete with respect to the neutrosophic norm that is, every neutrosophic Cauchy sequence in $\mathcal{M}$ converges neutrosophically to an element of $\mathcal{M}$.

Main Results

Definition 7. Let $(\mathcal{M},\mathbb{R},\oplus,\odot)$ and $(\mathcal{N},\mathbb{R},\oplus,\odot)$ be two neutrosophic Banach linear spaces over the same field $\mathbb{R}$. A mapping $\Upsilon:(\mathcal{M},\mathbb{R},\oplus,\odot)\to(\mathcal{N},\mathbb{R},\oplus,\odot)$ is said to be linear if and only if it satisfies

$$\Upsilon(\alpha m_1+\beta m_2)=\alpha\Upsilon(m_1)+\beta\Upsilon(m_2)\ ,for\ all\ m_1,m_2\in\mathcal{M}\ \textbf{and}\ \alpha,\beta\in\mathbb{R}$$

Definition 8. Let $(\mathcal{M},\mathbb{R},\oplus,\odot)$ and $(\mathcal{N},\mathbb{R},\oplus,\odot)$ be two neutrosophic Banach linear spaces over the same field $\mathbb{R}$. A mapping $\Upsilon:(\mathcal{M},\mathbb{R},\oplus,\odot)\to(\mathcal{N},\mathbb{R},\oplus,\odot)$ is called neutrosophically bounded if and only if there exist $0\neq € \in\mathbb{R}$, for each $m\in\mathcal{M}$ **and** $\tau>0$

$$\check{N}_1(\Upsilon(m),\tau)\geq\check{N}_1(€m,\tau)$$
$$\check{N}_2(\Upsilon(m),\tau)\leq\check{N}_2(€m,\tau)$$
$$\check{N}_3(\Upsilon(m),\tau)\leq\check{N}_3(€m,\tau)$$

Definition 9. Let $(\mathcal{M},\mathbb{R},\oplus,\odot)$ and $(\mathcal{N},\mathbb{R},\oplus,\odot)$ be two neutrosophic Banach linear spaces over the same field $\mathbb{R}$. A mapping $\Upsilon:(\mathcal{M},\mathbb{R},\oplus,\odot)\to(\mathcal{N},\mathbb{R},\oplus,\odot)$ is called neutrosophically continuous at $m_0\in\mathcal{M}$ if and only if for all $m\in\mathcal{M}$, $\mathbf{0<\varepsilon<1}$ **and** $\tau>0$, there exists $0<\delta<1$ and $t>0$, such that

$$\check{N}_1(\Upsilon(m)-\Upsilon(m_0),\tau)>1-\varepsilon$$
$$\check{N}_2(\Upsilon(m)-\Upsilon(m_0),\tau)<\varepsilon$$
$$\check{N}_3(\Upsilon(m)-\Upsilon(m_0),\tau)<\varepsilon$$

whenever,

$$\check{N}_1(m-m_0,\tau)>1-\delta$$
$$\check{N}_2(m-m_0,\tau)<\delta$$
$$\check{N}_3(m-m_0,\tau)<\delta$$

Υ is continuous on $\mathcal{M}$ if it is continuous at every point in $\mathcal{M}$.

Theorem 1. Let $(\mathcal{M},\mathbb{R},\oplus,\odot)$ and $(\mathcal{N},\mathbb{R},\oplus,\odot)$ be two neutrosophic Banach linear spaces over the same field $\mathbb{R}$ and $\Upsilon:(\mathcal{M},\mathbb{R},\oplus,\odot)\to(\mathcal{N},\mathbb{R},\oplus,\odot)$ be a neutrosophically linear operator. Then, the following statements are equivalent:

1. Υ is neutrosophically continuous on $\mathcal{M}$.
2. Υ is neutrosophically continuous at the origin $m=0$.
3. Υ is neutrosophically bounded.

Proof: $\mathbf{1.\Rightarrow 2.}$ Suppose that Υ is neutrosophically continuous on $\mathcal{M}$, that it must be continous at every point, including $m=0$.

$\mathbf{2.\Rightarrow 3.}$ Suppose that Υ is neutrosophically continuous at the origin

$m = 0$, then for every neutrosophic number $\mathbf{0} < \varepsilon < \mathbf{1}$ **and** $\tau > 0$, **there exists** $0 < \delta < 1$ and $t > 0$ such that

$$\check{N}_1(\Upsilon(m) - \Upsilon(0), \tau) > 1 - \varepsilon$$
$$\check{N}_2(\Upsilon(m) - \Upsilon(0), \tau) < \varepsilon$$
$$\check{N}_3(\Upsilon(m) - \Upsilon(0), \tau) < \varepsilon$$

whenever,

$$\check{N}_1(m, \tau) > 1 - \delta$$
$$\check{N}_2(m, \tau) < \delta$$
$$\check{N}_3(m, \tau) < \delta$$

But, Υ is neutrosophically linear operator this leads

$$\check{N}_1(\Upsilon(m), \tau) > 1 - \varepsilon$$
$$\check{N}_2(\Upsilon(m), \tau) < \varepsilon$$
$$\check{N}_3(\Upsilon(m), \tau) < \varepsilon$$

This gives

$$\check{N}_1\left(\Upsilon\left(\frac{m}{\check{N}_1(m,\tau)}\right), \tau\right) > 1 - \varepsilon$$
$$\check{N}_2\left(\Upsilon\left(\frac{m}{\check{N}_2(m,\tau)}\right), \tau\right) < \varepsilon$$
$$\check{N}_3\left(\Upsilon\left(\frac{m}{\check{N}_2(m,\tau)}\right), \tau\right) < \varepsilon$$

Consequently,

$$\check{N}_1(\Upsilon(m), \tau) > \frac{2\varepsilon}{\delta}$$
$$\check{N}_2(\Upsilon(m), \tau) < \frac{2\varepsilon}{\delta}$$
$$\check{N}_3(\Upsilon(m), \tau) < \frac{2\varepsilon}{\delta}$$

Therefore, there exists $0 \neq € \in \mathbb{R}$, for each $m \in \mathcal{M}$ **and** $\tau > 0$

$$\check{N}_1(\Upsilon(m), \tau) \geq \check{N}_1(€m, \tau)$$
$$\check{N}_2(\Upsilon(m), \tau) \leq \check{N}_2(€m, \tau)$$
$$\check{N}_3(\Upsilon(m), \tau) \leq \check{N}_3(€m, \tau)$$

Theorem 2. Let $\Upsilon: (\mathcal{M}, \mathbb{R}, \oplus, \odot) \to (\mathcal{N}, \mathbb{R}, \oplus, \odot)$ be a neutrosophically continuous on a Banach space and let $0 < \beta < 1$. Then, $\beta I + \Upsilon$ is a continuous on $\mathcal{M}$ if and only if Υ is neutrosophically continuous on $\mathcal{M}$.

Proof: Since Υ is neutrosophically continuous on $\mathcal{M}$, then for all $m \in \mathcal{M}$, $\mathbf{0} < \varepsilon < \mathbf{1}$ **and** $\tau > 0$, there exists $0 < \delta < 1$ and $t > 0$, such that

$\check{N}_1(\Upsilon(m)-\Upsilon(m_0),\tau)>1-\varepsilon$
$\check{N}_2(\Upsilon(m)-\Upsilon(m_0),\tau)<\varepsilon$
$\check{N}_3(\Upsilon(m)-\Upsilon(m_0),\tau)<\varepsilon$
whenever,
$\check{N}_1(m-m_0,\tau)>1-\delta$
$\check{N}_2(m-m_0,\tau)<\delta$
$\check{N}_3(m-m_0,\tau)<\delta$
The above inequalities can be written as

$\check{N}_1\big((\beta I+\Upsilon)(m)-(\beta I+\Upsilon)(m_0),\tau\big)>1-\varepsilon$
$\check{N}_2\big((\beta I+\Upsilon)(m)-(\beta I+\Upsilon)(m_0),\tau\big)<\varepsilon$
$\check{N}_3\big((\beta I+\Upsilon)(m)-(\beta I+\Upsilon)(m_0),\tau\big)<\varepsilon$
whenever,
$\check{N}_1(m-m_0,\tau)>1-\delta$
$\check{N}_2(m-m_0,\tau)<\delta$
$\check{N}_3(m-m_0,\tau)<\delta$
This means $\beta I+\Upsilon$ is a continuous on $\mathcal{M}$.

Definition 10. Let $\Upsilon:(\mathcal{M},\mathbb{R},\oplus,\odot)\rightarrow(\mathcal{N},\mathbb{R},\oplus,\odot)$ be a neutrosophically bounded linear operator on two neutrosophic Banach linear spaces $(\mathcal{M},\mathbb{R},\oplus,\odot)$ *and* $(\mathcal{N},\mathbb{R},\oplus,\odot)$. Then, Υ is neutrosophically invertible with inverse $\Upsilon^{-1}:(\mathcal{N},\mathbb{R},\oplus,\odot)\rightarrow(\mathcal{M},\mathbb{R},\oplus,\odot)$ if Υ^{-1} is neutrosophically bounded linear operator and it satisfies the following

$\check{N}_1(\Upsilon^{-1}\Upsilon(m),\tau)=\check{N}_1(\Upsilon\Upsilon^{-1}(m),\tau)=\check{N}_1(m,\tau)$
$\check{N}_2(\Upsilon^{-1}\Upsilon(m),\tau)=\check{N}_2(\Upsilon\Upsilon^{-1}(m),\tau)=\check{N}_2(m,\tau)$
$\check{N}_3(\Upsilon^{-1}\Upsilon(m),\tau)=\check{N}_3(\Upsilon\Upsilon^{-1}(m),\tau)=\check{N}_3(m,\tau)$
, for each $m\in\mathcal{M}$ **and** $\tau>0$.

Theorem 3. Let $\Upsilon:(\mathcal{M},\mathbb{R},\oplus,\odot)\rightarrow(\mathcal{N},\mathbb{R},\oplus,\odot)$ be a neutrosophically bounded linear operator on two neutrosophic Banach linear spaces $(\mathcal{M},\mathbb{R},\oplus,\odot)$ *and* $(\mathcal{N},\mathbb{R},\oplus,\odot)$ and let $0<\beta<1$. Then, *consider* $S=\beta I+\Upsilon$. If the neutrosophically bounded linear operator Υ satisfies

$\check{N}_1(\Upsilon(m),\tau)>\beta$
$\check{N}_2(\Upsilon(m),\tau)<\beta$
$\check{N}_3(\Upsilon(m),\tau)<\beta$

Then S is invertible, and its inverse $S^{-1}:(\mathcal{N},\mathbb{R},\oplus,\odot)\rightarrow(\mathcal{M},\mathbb{R},\oplus,\odot)$ is bounded and linear in the neutrosophic sense. Moreover, the inverse is given by the Neutrosophic

Neumann series:

$$S^{-1} = \frac{1}{\beta}\sum_{n=0}^{\infty}\left(-\frac{\Upsilon}{\beta}\right)^{n}$$

with convergence component-wise on $(\check{N}_1, \check{N}_2, \check{N}_3)$.

Proof:

Boundedness and linearity of S: Since Υ is linear, then

$$S(\alpha_1 m_1 + \alpha_2 m_2) = (\beta I + \Upsilon)(\alpha_1 m_1 + \alpha_2 m_2)$$
$$= \beta I(\alpha_1 m_1 + \alpha_2 m_2) + \Upsilon(\alpha_1 m_1 + \alpha_2 m_2) \text{ ,for all } m_1, m_2 \in \mathcal{M} \text{ and } \alpha_1\alpha_2 \in \mathbb{R}$$
$$= \beta\alpha_1 m_1 + \beta\alpha_2 m_2 + \alpha_1\Upsilon(m_1) + \alpha_2\Upsilon(m_2)$$
$$= \alpha_1(\beta I + \Upsilon)(m_1) + \alpha_2(\beta I + \Upsilon)(m_2)$$

Since Υ is bounded, there exists a neutrosophic constant $0 \neq € \in \mathbb{R}$, for each $m \in \mathcal{M}$ and $\tau > 0$

$$\check{N}_1(\Upsilon(m), \tau) \geq \check{N}_1(€m, \tau)$$
$$\check{N}_2(\Upsilon(m), \tau) \leq \check{N}_2(€m, \tau)$$
$$\check{N}_3(\Upsilon(m), \tau) \leq \check{N}_3(€m, \tau)$$

And by hypotheses of this theorem, I have

$$\check{N}_1(\Upsilon(m), \tau) > \beta$$
$$\check{N}_2(\Upsilon(m), \tau) < \beta$$
$$\check{N}_3(\Upsilon(m), \tau) < \beta$$

Therefore, S is bounded.

Neutrosophic Neumann series: Consider the series

$$\sum_{n=0}^{\infty}\left(-\frac{\Upsilon}{\beta}\right)^{n}.$$

Convergence component-wise is guaranteed because the norm of each term decreases geometrically for $\check{N}_1, \check{N}_2, \check{N}_3$. Thus, this series defines a bounded linear operator on $\mathcal{M}$.

Verification of the inverse: Define

$$S^{-1} := \frac{1}{\beta}\sum_{n=0}^{\infty}\left(-\frac{\Upsilon}{\beta}\right)^{n}.$$

Then

$$S^{-1}S = \frac{1}{\beta}\sum_{n=0}^{\infty}\left(-\frac{\Upsilon}{\beta}\right)^{n}(\beta I + \Upsilon) = I,$$

$$SS^{-1} = (\beta I + \Upsilon)\frac{1}{\beta}\sum_{n=0}^{\infty}\left(-\frac{\Upsilon}{\beta}\right)^{n} = I.$$

Boundedness of S^{-1}: Component-wise, I have

$$\check{N}_1(S^{-1}(m),\tau) \geq \check{N}_1\left(\frac{1}{\beta}\sum n\, m,\tau\right)$$
$$\check{N}_2(\Upsilon(m),\tau) \leq \check{N}_2\left(\frac{1}{\beta}\sum n\, m,\tau\right)$$
$$\check{N}_3(\Upsilon(m),\tau) \leq \check{N}_3\left(\frac{1}{\beta}\sum n\, m,\tau\right)$$

Thus, S^{-1} is bounded component-wise for $(\check{N}_1,\check{N}_2,\check{N}_3)$.
Hence, $S=\beta I+T$ is invertible, and S^{-1} is linear and bounded in the neutrosophic sense.
The theorem ensures that the inverse preserves the neutrosophic structure: each of the truth, indeterminacy, and falsity components is controlled, guaranteeing both algebraic invertibility and component-wise boundedness.

Definition 11.[6] A mapping $\Upsilon:(\mathcal{M},\mathbb{R},\oplus,\odot)\to(\mathcal{N},\mathbb{R},\oplus,\odot)$ is called neutrosophically Lipschitz on $\mathcal{M}$, if there exists $\mathcal{C}>0$ satisfies the following

$$\check{N}_1(\Upsilon(m_1)-\Upsilon(m_2),\tau) \geq \check{N}_1\left(m_1-m_2,\frac{\tau}{\mathcal{C}}\right)$$
$$\check{N}_2(\Upsilon(m_1)-\Upsilon(m_2),\tau) \leq \check{N}_2\left(m_1-m_2,\frac{\tau}{\mathcal{C}}\right)$$
$$\check{N}_3\left(\Upsilon(m_1)-\Upsilon(m_2)\right) \leq \check{N}_3\left(m_1-m_2,\frac{\tau}{\mathcal{C}}\right)$$

, for each $m_1,m_2\in\mathcal{M}$ **and** $\tau>0$. Υ is said to be neutrosophically contraction, if $\mathcal{C}<1$.

Theorem 4. [6] **A** neutrosophically operator Υ which defined on Banach space $(\mathcal{M},\mathbb{R},\oplus,\odot)$ has a unique fixed point if, Υ is neutrosophically contraction.

Theorem 5. Let Υ be a neutrosophically operator which defined on Banach space $(\mathcal{M},\mathbb{R},\oplus,\odot)$ and let $0<\beta<1$. Then, $\beta I+\Upsilon$ has a unique fixed point if, Υ is neutrosophically contraction.

Proof: Since Υ is neutrosophically contraction, then there exists $\mathcal{C}<1$ such that

$$\check{N}_1(\Upsilon(m_1)-\Upsilon(m_2),\tau) \geq \check{N}_1\left(m_1-m_2,\frac{\tau}{\mathcal{C}}\right)$$
$$\check{N}_2(\Upsilon(m_1)-\Upsilon(m_2),\tau) \leq \check{N}_2\left(m_1-m_2,\frac{\tau}{\mathcal{C}}\right)$$

$$\check{N}_3\left(\Upsilon(m_1)-\Upsilon(m_2)\right)\le \check{N}_3\left(m_1-m_2,\frac{\tau}{\mathcal{C}}\right)$$

, for each $m_1, m_2 \in \mathcal{M}$ and $\tau > 0$.

This gives

$$\check{N}_1\left((\beta I+\Upsilon)(m_1)-(\beta I+\Upsilon)(m_2),\tau\right)\ge \check{N}_1\left(m_1-m_2,\frac{\tau}{\mathcal{C}}\right)$$

$$\check{N}_2\left((\beta I+\Upsilon)(m_1)-(\beta I+\Upsilon)(m_2),\tau\right)\le \check{N}_2\left(m_1-m_2,\frac{\tau}{\mathcal{C}}\right)$$

$$\check{N}_3\left((\beta I+\Upsilon)(m_1)-(\beta I+\Upsilon)(m_2),\tau\right)\le \check{N}_3\left(m_1-m_2,\frac{\tau}{\mathcal{C}}\right)$$

Therefore, $\beta I+\Upsilon$ is a neutrosophically contraction which defined on Banach space $(\mathcal{M},\mathbb{R},\oplus,\odot)$, using Theorem 4. $\beta I+\Upsilon$ has a unique fixed point.

Conclusion:

In this paper, I have extended the definitions of linearity, boundedness, continuity and invertibility of bounded operators within neutrosophic Banach spaces. accordingly, I have introduced a new class of bounded linear operators in this context. Several noteworthy relationships among these operators have been rigorously investigated. Finally, the necessary and sufficient conditions ensuring the uniqueness of the fixed point, derived through the concept of neutrosophic contractions, have been rigorously determined.

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دراسة حول الخصائص البنيوية للمشغلات في فضاءات بناخ

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مستخلص البحث:

في هذه الورقة، قدمت وناقشت التدوينات الخطية، والمحدودة، والمتصلة، وقابلية عكس العوامل في فضاء باناخ النيوترسوفي. علاوةً على ذلك، نُرسي علاقاتٍ مهمة بين الخطية، والمحدودة، والمتصلة. كما نُوسّع دراسة هذه الخصائص لتشمل عامل الاضطراب. علاوةً على ذلك، نناقش تعيينات Lipschitz والانكماش، بالإضافة إلى نتائج التفرد المتعلقة بالنقاط الثابتة.

الكلمات المفتاحية : فضاء باناخ النيتروسوفي، متصل نيوتروسوفيًا، متقلص نيوتروسوفيًا وقابل للانعكاس نيوتروسوفيًا.