Optimal Reliability Allocation Using the Harmony Search Algorithm (HSA) for Component-Level Cost and Reliability Analysis in Complex Systems

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Abstract:

In this research, the reliability function of a complex system was calculated, identifying the optimal reliability distribution among the system's components. Most recent studies rely on increasing closed-form functions to relate cost to reliability, knowing that the cost of a component increases as its reliability increases. However, these functions can sometimes be ill-defined or difficult to generate. In this research, MATLAB was used to extract the system's reliability function, and the relationship between cost and reliability across separate datasets was discussed. The reliability and cost of each component were calculated individually using the Harmonic Search Algorithm (HSA), in addition to calculating the total cost and reliability of the system. An exponential feasibility factor cost function was applied within this algorithm to improve performance and achieve the optimal reliability distribution.

Keywords: Cost-reliability model, optimization, reliability allocation, harmony search algorithm, and complex systems.

Introduction

In this work, the reliability of a complex, composite system was analyzed using mathematical methods to determine the system's efficiency and stability [2, 3]. The reliability of this system was determined by utilizing short paths through connectivity matrices. Boolean algebra was used to derive all possible paths, then eliminating redundant nodes to find minimal paths that represent the critical paths in the system [1, 4, 5, 11]. To assess the safety of the composite system, its reliability function was determined. Despite its historical background in the study of networks, this research also addresses the mathematical problem of optimal reliability allocation among system components. In this study, the reliability levels of each component were optimized based on the importance of its location in the system, with the goal

of increasing the system's lifetime and improving its reliability while reducing overall cost [6, 7]. Some components, depending on their location in the system architecture, require higher reliability allocations to contribute to increased overall reliability. The optimization of electrical and mechanical systems presents many difficulties for engineers [4, 6, 9]. The reliability of complex systems is the main emphasis of this study, along with the relationship between reliability distribution and system cost—which can be represented by size, weight, or other technical factors. Two primary aspects influence the reliability of components: The model needs to be cost-based before the input items are validated. Depending on the system's characteristics, the suggested cost factor's specifications can be changed. Engineers can use this to plan to attain the minimal level of reliability needed for every system element and to evaluate the cost allocations of components. Additionally, the model needs to consider the mathematical study of the overall reliability of the system, as simple systems can pose significant challenges when applied to larger, more complex systems. The results were obtained using the Harmony Search Algorithm (HSA), a powerful tool for addressing optimization problems in complex systems. An exponential feasibility factor was also used in the cost calculation to ensure a realistic representation of the relationship between cost and reliability.

Allocating dependability and optimizing complicated systems

Consider a complex system with dependability-related components.

[8, 9]. We make use of the notes below:

$C_i(R_i)$ = element i cost;

$0 \leq R_i \leq 1$ = reliability i component;

R_s = reliability of the system;

$[C(R_1, ..., R_n) = \sum_{i=1}^{n} a_i c_i(R_i)]$ is the entire cost of the system.

in which (a_i) is greater than 0;

RG = goal of system dependability.

There are many possible outcomes that may result from the modular design of the system and the responsibilities of each individual component. The same operational capacity can be achieved using a variety of system components, each with varying levels of reliability. The ultimate goal is to enable the system to distribute resources across all components or across specific components in an optimal manner. These problems are fundamental topics in nonlinear programming [9, 10, 11]. Although the system does not follow a linear relationship, its associated constraint and cost functions can be studied and analyzed in depth.

Minimized $C(R_i, \ldots, R_i) = \sum_{i=1}^{n} a_i C_i(R_i)$, $a_i > 0$,

Subject to:
$$R_s \geq R_G$$

$0 \leq R_i < 1$, in which $i = 1, \ldots, n$ \hfill (1)

Let $C_i(R_i)$ satisfy certain constraints [11], and let the partial cost function be reasonable. These are positive, differentiated functions that increase from $\left[\Rightarrow \frac{dC_i}{dR_i} \geq 0 \right]$.

Comparable to the fact that its derivatives $\frac{d^2 C_i}{dR_i^2} \geq 0$ are monotonically raised,

i, e. $\frac{d^2 C_i}{dR_i^2} \geq 0$.

is the part costs function of the Euclidean convexity $[C_i(R_i)]$.

Achieving an all-out framework cost base is the goal of the previous plan, and under R_G, the system reliability restriction is reduced [10].

implementation in a complex system

Similar to how we might create a parallel network out of a group of things, we must convert the complex system into a more manageable network in order to estimate it. The reliability in n-component parallel and series networks is as follows:

$$R_s = \prod_{i=1}^{n} R_i$$
\hfill (2)

$$R_s = 1 - \prod_{i=1}^{n}(1 - R_i)$$ \hfill (3)

In this case, (R_i) is the component i's reliability, and (R_N) is the reliability network[6,8].

Each complex network's reliability with p minimum paths will be compared. provided by equations (1) and (2).

$R_s = 1 - \prod_{z=1}^{p}\left(1 - \prod_{j=\alpha}^{\omega} R_j\right)$ \hfill (4)

The index of the first component of a minimal path z is denoted by α, whereas the index of the last component is denoted by ω.

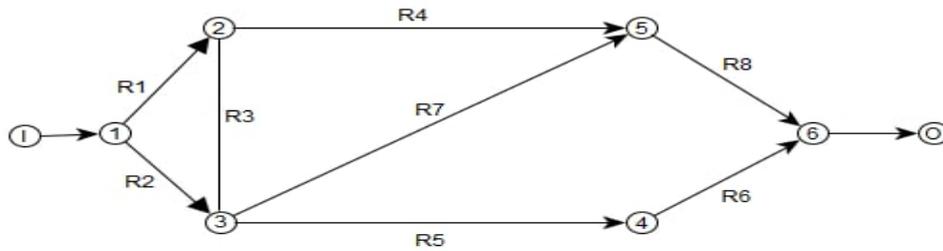The equation can be used to determine the dependability of the complex network in Fig. 1 below (3).

Figure 1. Complex Network

The sets:

$$S = \big\{\{x_1 x_4 x_8\}, \{x_2 x_5 x_6\}, \{x_2 x_7 x_8\}, \{x_1 x_3 x_5 x_6\}, \{x_2 x_3 x_4 x_8\},$$
$$\{x_1 x_3 x_7 x_8\}, \big\}.$$

$$R_s = 1 - [1 - p_r(x_1 x_4 x_8)] \times [1 - p_r(x_2 x_5 x_6)] \times [1 - p_r(x_2 x_7 x_8)]$$
$$\times [1 - p_r(x_1 x_3 x_5 x_6)] \times [1 - p_r(x_2 x_3 x_4 x_8)] \times [1 - p_r(x_1 x_3 x_7 x_8)] \qquad (5)$$

Note: When the $i-\text{th}$ component succeeds, then ($R_i = 1$), and when it fails, then ($R_i = 0$) $\forall\, i = 1, \cdots, 8$, these lead to
$$[\, R_i^n = R_i\,].$$

Equation (5) becomes the following polynomial when the aforementioned note is applied.

$$R_S = R_1 R_4 R_8 + R_2 R_5 R_6 + R_2 R_7 R_8 + R_1 R_3 R_5 R_6 + R_2 R_3 R_4 R_8 + R_1 R_3 R_7 R_8$$
$$- R_1 R_2 R_3 R_5 R_6$$
$$- R_1 R_2 R_3 R_4 R_8 - R_1 R_2 R_3 R_7 R_8 - R_1 R_2 R_4 R_7 R_8 - R_1 R_3 R_4 R_7 R_8$$
$$- R_2 R_3 R_4 R_7 R_8$$
$$- R_2 R_5 R_6 R_7 R_8 + 2 R_1 R_2 R_3 R_4 R_7 R_8 - R_1 R_2 R_4 R_5 R_6 R_8 - R_1 R_3 R_4 R_5 R_6 R_8$$
$$- R_2 R_3 R_4 R_5 R_6 R_8 - R_1 R_3 R_5 R_6 R_7 R_8 + 2 R_1 R_2 R_3 R_4 R_5 R_6 R_8$$
$$+ R_1 R_2 R_3 R_5 R_6 R_7 R_8$$
$$+ R_1 R_2 R_4 R_5 R_6 R_7 + R_1 R_3 R_4 R_5 R_6 R_7 R_8 + R_2 R_3 R_4 R_5 R_6 R_7 R_8 -$$
$$2 R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8$$

Harmony Search Algorithm (HSA)

Optimization algorithms represent a practical alternative for overcoming the challenges associated with complex problems that are difficult to solve using traditional methods. The Harmony Search Algorithm (HSA) is a prominent such approach, inspired by the process of musical improvisation, where musicians strive to achieve the best possible harmony between notes. Similarly, the HSA aims to find optimal values for decision variables to achieve the best value of the objective function, by gradually updating the solutions to reach the global optimal solution. This study examines the reliability analysis of a complex system using connectivity matrices and Boolean algebra to identify critical paths and their minimum number. It also explores improving the reliability distribution among system components according to their relative importance, with the goal of extending the system's lifetime and reducing overall cost. Resource allocation depends on the location of each component and its impact on overall performance, making optimization essential to achieve a balance between cost and reliability.

Figure 2: Gartley Harmonic pattern
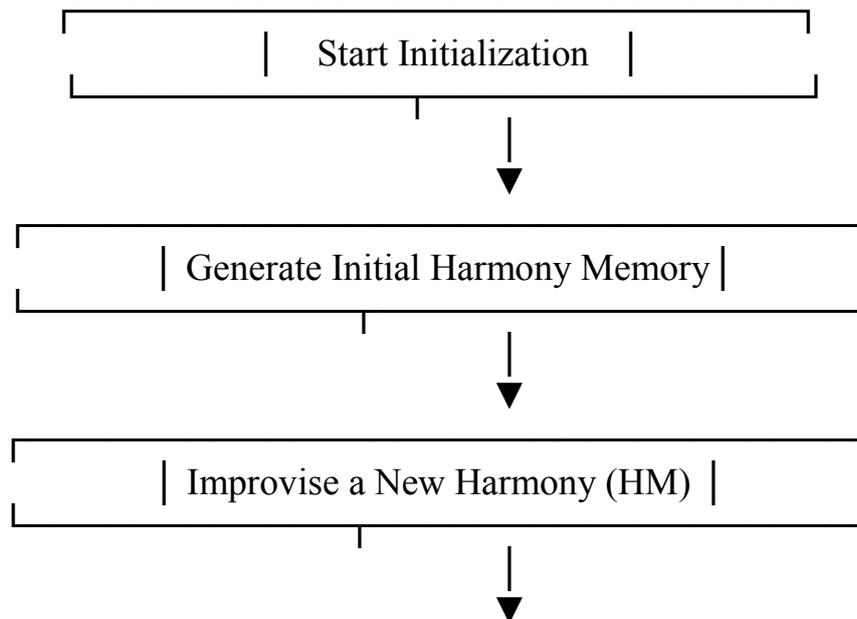
Implementation of HS Algorithem

A viable solution is represented within the Harmony Search (HS) algorithm by the symbol
"harmony," in which each decision variable represents a distinct musical note [1,5]. A set number of harmonies (N) is preserved in a harmonic memory (HM), an element of the HS algorithm. The optimization equation can be described below if the goal is to maximize or minimize a fitness function (f) whilst taking into account d decision elements.

Max. (or Min.) $f(x_1, x_2, ..., x_n)$

In the presented context, f represents the fitness function, $[x_i (i = 1, 2, ..., n)]$ represents the decision variable *i*. *d* signifies the dimension *f* problem. To implement the Harmony Search algorithm for optimization, the subsequent steps are to be followed:

1. Setting up a memory for harmony.
2. Improvisation of new harmony.
3. Adding or removing new harmony from Harmony Memory.
4. Steps 2 and 3 are repeated iteratively until the designated termination condition is   satisfied.
5. Moving on to Step 6 after fulfilling the halting requirement.
6. Return HM as the ideal outcome since it contains the best harmony.

```
┌─────────────────────────────────┐
│        Start Initialization      │
└─────────────────────────────────┘
                 │
                 ▼
┌─────────────────────────────────┐
│   Generate Initial Harmony Memory │
└─────────────────────────────────┘
                 │
                 ▼
┌─────────────────────────────────┐
│     Improvise a New Harmony (HM)  │
└─────────────────────────────────┘
                 │
                 ▼
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```
┌─────────────────────────────────────────────────────┐
│          Evaluate the New Harmony (Fitness)          │
└─────────────────────────────────────────────────────┘
                           │
                           ▼
┌─────────────────────────────────────────────────────┐
│        Is the New Harmony Better than Worst in HM?   │
└─────────────────────────────────────────────────────┘

        Yes              │              No
                         ▼
        ┌────────────────────────────────┐
        │      Replace Worst Harmony      │
        │        with New Harmony         │
        └────────────────────────────────┘
                         │
                         ▼
        ┌────────────────────────────────┐
        │      Check Stopping Criterion?  │
        └────────────────────────────────┘
            No           │           Yes
                         ▼
        ┌────────────────────────────────┐
        │         Return Best HM          │
        │       (Optimal Solution)        │
        └────────────────────────────────┘
```
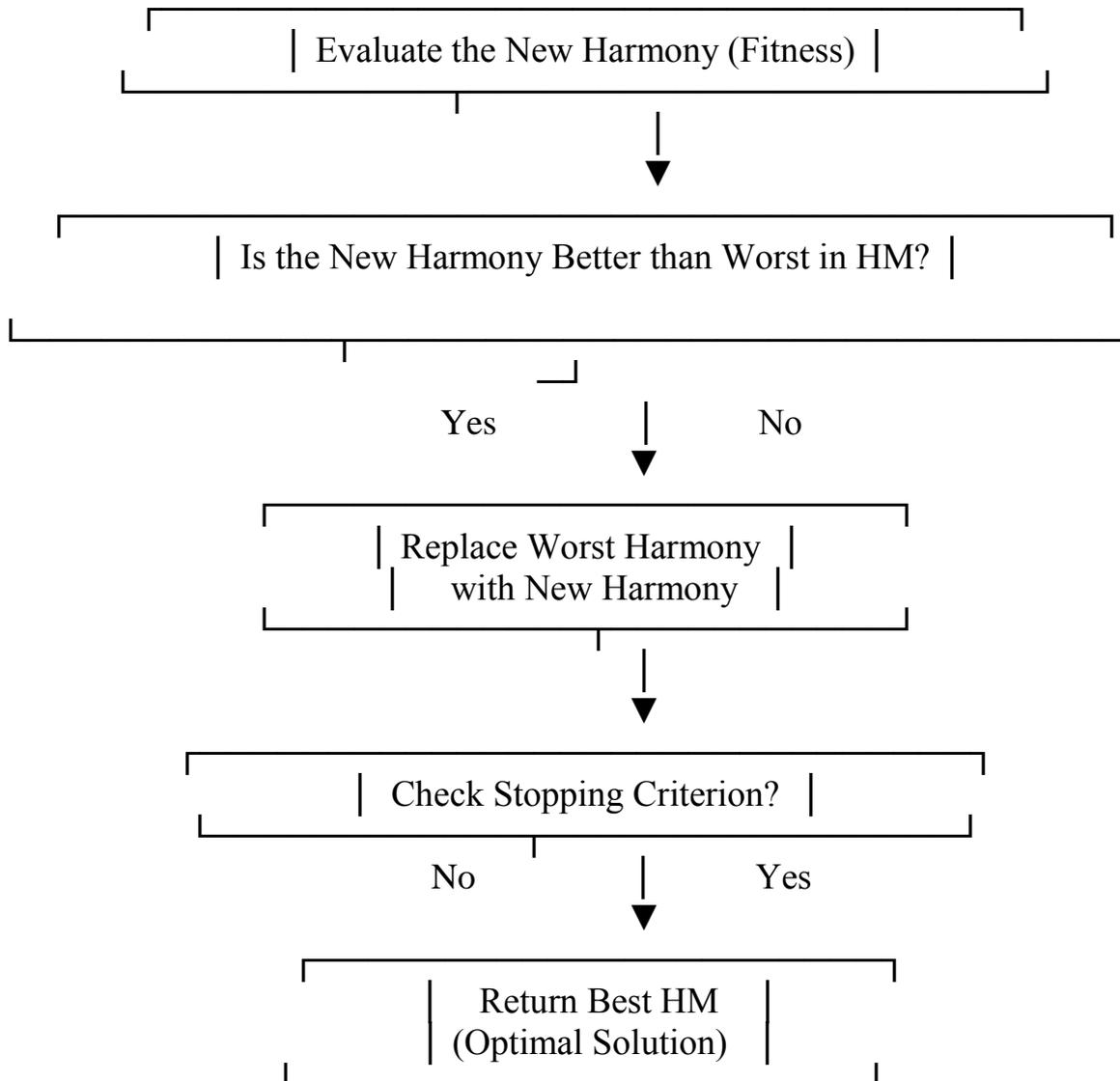
Figure 3. Flow chart of Harmony Search Algorithm (HSA)

Materials and Methods

A. Materials

The following resources and tools were used in this research:

Network reliability theory.

A logarithmic cost function model.

An algorithm known as the combinatorial search algorithm (HSA).

MATLAB R2020a is a simulation and computing software.

Minimum path sets and connectivity matrices were used to simulate complex networks.

B. Methods

For the complex system network, a mathematical model was created to maximize reliability allocation and cost savings.

A nonlinear programming problem with cost and reliability constraints was used to define the optimization problem.

To determine the optimal reliability allocation across system components, the HSA method was used.

Classical formulas for serial and parallel systems were used to evaluate reliability performance.

MATLAB was used to achieve the simulations and final results.

A calculus-based exponential feasibility model

Assume $(0 < f_i < 1)$ is a feasibility factor [8], $[R_{i,max}$ is maximum reliability], and $R_{i,min}$ is minimum reliability.

$$C_i(R_i) = \exp[(1-f_i)\frac{R_i - R_{i,min}}{R_{i,max} - R_i}],$$
$$R_{i,min} \leq R_i \leq R_{i,max}, i = 1,2,\dots,n.$$

The issue with optimization arises

$$MinimizeC(R_i,\dots,R_i)\ \sum_{i=1}^{n} a_i \exp[(1-f_i)\frac{R_i - R_{i,min}}{R_{i,max} - R_i}],$$
$$in\ which\ i = 1,2,\dots,n.$$

Subjected to :

$$R_s \geq R_G$$

$$R_{i,min} \leq R_i \leq R_{i,max}, i1,\dots,n.$$

Table 1: HSA with an applied cost function for the best dependability allocation.

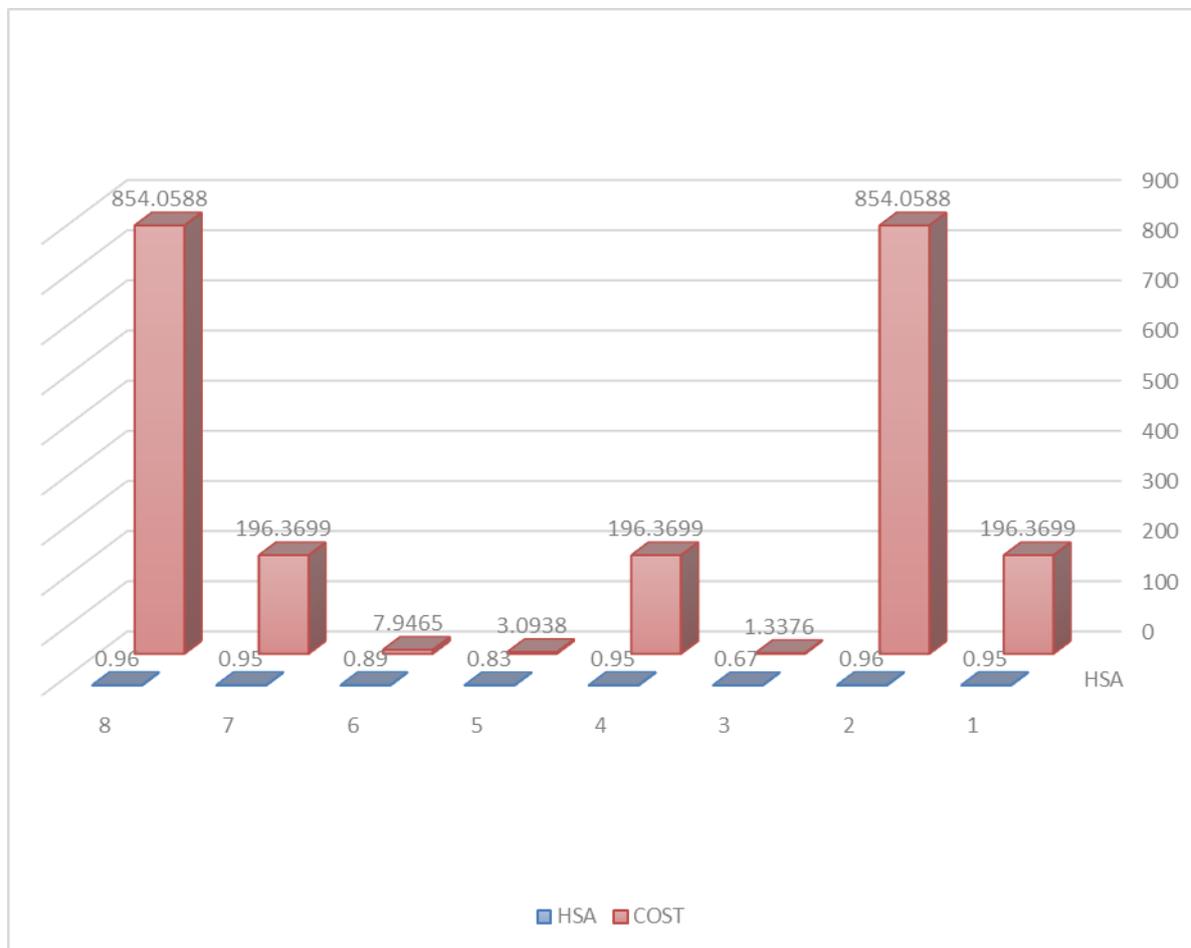| Components	HSA	COST
R_1	0.95	196.3699
R_2	0.96	854.0588
R_3	0.67	1.3376
R_4	0.95	196.3699
R_5	0.83	3.0938
R_6	0.89	7.9465
R_7	0.95	196.3699
R_8	0.96	854.0588
R_{system}	0.99	2.3096e+03

Figure 4: Using the provided feasibility factor model and HSA to distribute reliability.

Conclusion

This study used analytical engineering techniques to design the dependability of each system component in order to improve and increase the reliability of a complicated network. In order to attain the highest levels of efficiency in the reliability of complex systems, the system optimization problem was tackled using a nonlinear programming model that incorporates a cost function and operational restrictions pertaining to labor and resources. The reliability allocation problem was tackled using the Harmony Search Algorithm (HSA). The findings demonstrated that the FFM (Feasibility Factor Model), described in the previous paragraphs, yielded an optimal value for overall reliability of $Rs = 0.99$, which reflects the optimal performance of the system under study. Comparing the system components, it was found that components (2, 8) had the highest allocation ratio and highest cost, while components (3, 5) had the lowest allocation, as shown in

the tables above. The positioning of these elements within the intricate system structure and their direct influence on overall reliability are the reasons for this difference. The importance of the proposed model lies in its ability to handle complex mathematical analyses with flexibility and high efficiency, which enables its application in the design and analysis of large engineering systems that require achieving a balance between cost and reliability.

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التخصيص الأمثل للموثوقية باستخدام خوارزمية بحث التناغم(HSA) لتحليل التكلفة والموثوقية على مستوى المكونات في الأنظمة المعقدة

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مستخلص البحث:

في هذا البحث، تم استخدام دالة الموثوقية لنظام معقد تم حسابها، مع تحديد توزيع الموثوقية الأمثل بين مكونات النظام. تعتمد معظم الدراسات الحديثة على زيادة الدوال ذات الشكل المغلق لربط التكلفة بالموثوقية، مع العلم أن تكلفة المكون تزداد مع زيادة موثوقيته. ومع ذلك، يمكن أن تكون هذه الدوال أحيانًا غير محددة جيدًا أو يصعب إنشاؤها. في هذا البحث، تم استخدام MATLAB لاستخراج دالة موثوقية النظام، ونوقشت العلاقة بين التكلفة والموثوقية عبر مجموعات بيانات منفصلة. تم حساب موثوقية وتكلفة كل مكون على حدة باستخدام خوارزمية البحث التوافقي(HSA) ، بالإضافة إلى حساب التكلفة الإجمالية وموثوقية النظام. تم تطبيق دالة تكلفة عامل الجدوى الأسي ضمن هذه الخوارزمية لتحسين الأداء وتحقيق توزيع الموثوقية الأمثل.

الكلمات الرئيسية: نموذج التكلفة والموثوقية، التحسين، تخصيص الموثوقية، خوارزمية البحث التوافقي، والأنظمة المعقدة.