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Biased estimators in Poisson–Inverse-Gaussian Regression model in the presence of Multicollinearity: Subject review

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Abstract: This review paper examines biased estimators, such as ridge, Liu, Liu-type, and two-parameter estimators, in the Poisson-Inverse Gaussian regression (PIGR) model under multicollinearity conditions. Multicollinearity in PIGR leads to unstable maximum likelihood estimates with inflated variances and potential sign inconsistencies, prompting biased alternatives that reduce mean squared error (MSE). Biased estimators outperform the maximum likelihood estimator in Monte Carlo simulations across varying sample sizes (n=50,100,150), predictor counts (p=4,8), correlation levels (ρ=0.90,0.95,0.99), and dispersion parameters ($\hat{\varphi}$=1.4,3,5). The two-parameter estimator (PIGTP) consistently yields the lowest MSE, followed by PIGLT, PIGLE, PIGRRE, and ML. These findings advance PIGR applications in skewed count data from insurance, finance, and social sciences, recommending PIGTP for multicollinear settings. Future work could explore real-data validations or hybrid metaheuristics.

Keywords: Multicollinearity biased estimator, Poisson–Inverse-Gaussian Regression model, Monte Carlo simulation.

المقدرات المتحيزة في انموذج انحدار بواسون-كاوس المعكوس بوجود مشكلة التعدد الخطي: مراجعة مقال

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المستخلص: تتناول هذه الورقة البحثية المُقدِّرات المتحيزة، مثل مُقدِّرات ridge و Liu و Liu-type و two-parameter، في نموذج انحدار بواسون-كاوس المعكوس (PIGR) في ظل ظروف التعددية الخطية. تؤدي التعددية الخطية في PIGR إلى تقديرات احتمالية قصوى غير مستقرة مع تباينات مبالغ فيها وتناقضات محتملة في الإشارات، مما يدفع إلى استخدام بدائل متحيزة تقلل من متوسط مربعات الخطأ . يحقق المُقدِّر ذو المعلمتين (PIGTP) باستمرار أقل MSE، يليه PIGLT و PIGLE و PIGRRE و ML. تعزز هذه النتائج تطبيقات PIGR في البيانات المُتفاوتة من التأمين والتمويل والعلوم الاجتماعية، وتوصي باستخدام PIGTP للإعدادات

متعددة الترابط. يمكن أن تستكشف الأعمال المستقبلية التحقق من صحة البيانات الحقيقية أو الخوارزميات فوق الحدسية الهجينة.

الكلمات المفتاحية: المقدرات المتحيزة، انموذج انحدار بواسون-كاوس المعكوس، التعدد الخطي، محاكاة مونت كارلو.

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Introduction

The linear regression model (LRM) cannot be fitted to regression models with non-normal response variable distribution. The generalized linear models (GLMs) are used to model the data collection in such cases. GLMs are effective in cases where the distribution of response variables is not the normal distribution [1]. Poisson-Inverse Gaussian (PIG) distribution is a kind of distribution, which is applied in positively skewed variables (continuous response). The Poisson-Inverse Gaussian regression model (PIGR) is better than the Poisson regression model in situations where the response variable is heavily skewed [2]. The PIGR model has been widely applied in industrial engineering, life testing, dependability, marketing and the social sciences [3-9].

The approach is also well-received to model count data from the social sciences, marketing, finance, and many others. PIGR errors raise the flexibility in the model for variance modeling compared to those from zero-inflated or other zero-related specifications. For data analysis in such fields, mimeographed lecture notes without any further references are not reasonable for a scientific community. Moreover, basic assumptions and reasoning leading to a PIG regression model should be discussed rather than to be hidden within an asymptotic argument. Therefore, this paper consolidates the basic concepts leading to the PIGR.

The PIGR is a well-received and widely studied approach to model count data. The conventional logarithmic link function names PIG regression the Poisson log-linear model, leading the Neyman-Pearson settings of a likelihood ratio goodness of fit test, Wald test, and score tests as well to only the Poisson model. It covers, therefore, the quasi-Poisson regression based on Tweedie's formula for the variance, and numerous extensions for two-part, zero-inflated/deflated models and specifications for multiple correspondence analysis. Originally, this approach focused on claims data from automobile insurances, where the variance of counts grew cubic in mean and a direct application of the Poisson model is fail, leading also to historically referred log-linear regression. This cubic variance assumption and the given condition for failure of Poisson model are typical problems for the PIGR.

The regressors are assumed to be uncorrelated in the PIGR. However, multicollinearity is a concern since this assumption is frequently incorrect. Since the PIG distribution belongs to the exponential family, the model denoted as (PIGR) is a specific type of generalized linear model (GLM). One popular technique for estimating the unknown coefficients in the PIGR model is the maximum likelihood (ML) estimation method. However, the ML estimation may suffer if the independent variables are excessively collinear.

Multicollinearity can lead to a number of issues while estimating the (PIGR) model. A frequently used method to obtain the unknown parameters in an PIGR model is the maximum likelihood (ML) estimate method. However, the estimation of ML may be biased if the independent variables involve high collinearity. A multicollinearity problem can lead to several problems when (PIGR) model is being estimated. The problem with it is that if the estimated coefficients are with the wrong sign, there could be misinterpretations due to the imagined relationship between the dependent variable and the independent variables. Another issue is that the ML estimator's confidence interval might expand, which could therefore reduce the accuracy of the estimates [1, 10, 11].

Ridge regression approach was one of the solutions developed to be applied against multicollinearity problem, which was brought to light by [12]. This method has proven to be very useful in many regression applications. [13] proposed the Ridge estimator (RE) for the (PIGR) model, which has the advantage of producing a smaller $\overline{MSE}$ value compared to the maximum likelihood (ML) estimator. On the other hand, the RE factor also offers some drawbacks. For example, there is no such improvement in the quality of fitting as in the case of plain linear regression which means that results from the RJL model are also similar to those from the extended GLM[14] extended the Ridge and Liu estimators to the Poisson-Inverse Gaussian regression model (PIGRM), which is a special case of GLMs. [15] . Furthermore, [16] introduced a first-order two-parameter estimator for (GLMs). These studies aimed to enhance the precision and reliability of parameter estimates in regression models impacted by multicollinearity. In response to the Ridge estimator proposal for the (PIGR), numerous studies have been carried out to enhance this approach. [17] suggested biased parameter estimators for the Ridge estimator (RE) in the PIGR model, while [18] presented the Liu estimator and a two-parameter estimator defined by [19] for the PIGR model. Additionally, [20] introduced a new shrinkage parameter for the PIGR.
Due to the limitations of REas highlighted in [21] and [22], a more advanced version known as the two-parameter Ridge estimator (TPRE) has been proposed. The TPRE surpasses both the RE and ML estimators, offering various advantages such as orthogonality between residuals and predicted values on the dependent variable. The additional parameter in the TPRE enhances the regression model's overall fit quality. In a Linear Regression Model (LRM) context, [23] conducted a comparison between the TPRE, OLS estimator, and RE using the matrix mean square error (MMSE) criteria. Within the (GLMs) framework, [24] applied the TPRE in a binary logistic regression model.

1st: Ridge estimator in PIGR

In response to challenges posed by multicollinearity, Segerstedt [45] proposed the use of ridge estimator (RE) and as in [37], [35],[30],[42],[10], and [12]. Building upon this work, Batool et al. [13] introduced the RE to tackle multicollinearity in the PIGRM. This method, known as the Poisson-Inverse Gaussian Ridge Regression Estimator (PIGRRE), is defined as follows:

$$\hat{\beta}_{PIGRRE} = \left(Z'\hat{V}Z + k_T I\right)^{-1} \; Z' \; \hat{V}Z\hat{\beta}_{MLE} \quad , \quad (4)$$

where $k_r > 0$. Then,

$$\text{Bias}\left(\hat{\beta}_{\text{PIGRRE}}\right) = E\left(\hat{\beta}_{\text{PIGRRE}}\right) - \beta = -\text{K}_r\Lambda_K^{-1}\beta \quad (5)$$

$$\begin{aligned}\text{Cov}\left(\hat{\beta}_{\text{PIGRRE}}\right) &= \text{E}\left(\left[\hat{\beta}_{\text{PIGRRE}} - \text{E}\left(\hat{\beta}_{\text{PIGRRE}}\right)\right]\left[\hat{\beta}_{\text{PIGRRE}} - E\left(\hat{\beta}_{\text{PIGRRE}}\right)\right]'\right) \\ &= \widehat{\emptyset}\left(\text{Q}\Lambda_k^{-1}\Lambda_k\Lambda_k^{-1}\,\text{Q}'\right) \quad (6)\end{aligned}$$

and

$$\begin{aligned}\text{MMSE}\left(\hat{\beta}_{\text{PIGRRE}}\right) &= \text{Cov}\left(\hat{\beta}_{\text{PIGRRE}}\right) + \text{Bias}\left(\hat{\beta}_{\text{PIGRRE}}\right)\text{Bias}\left(\hat{\beta}_{\text{PIGRRE}}\right)' \\ &= \widehat{\emptyset}\left(Q\Lambda_k^{-1}\Lambda_k\Lambda_k^{-1}\,Q'\right) + b_{PIGRRE}b'_{PIGRRE} \quad (7)\end{aligned}$$

where $b_{\text{PIGRRE}} = \text{Bias}(\hat{\beta}_{\text{PIGRRE}}) = -k_r Q\Lambda_k^{-1}\alpha$

$\text{SMSE}(\hat{\beta}\text{PIGRRE}) = \text{trMMSE}(\hat{\beta}\text{PIGRRE})$

$$\widehat{\emptyset} = \sum_{j=1}^{p}\frac{\lambda_j}{\left(\lambda_j + k_r\right)^2} + \sum_{j=1}^{p}\frac{k_r^2\alpha_j^2}{\left(\lambda_j + k_r\right)^2} \quad (8)$$

where $\alpha = \text{Q}'\beta_{\text{MLE}}$.

2nd: Poisson-Inverse Gaussian Liu estimator

Liu [32] proposed the Liu estimator (LE) as a substitute for the RE. [41], [6], [36], [14] defined the LE for the PIGRM as follows:

$\hat{\beta}_d = (Z'Z + I)^{-1}(Z'Z + d_I I)\hat{\beta}$ (9)

With these properties, the LE presents itself as a promising substitute for the RE. In the context of the PIGRM, the LE is defined as:

$\hat{\beta}_{\text{PIGLE}} = (Z'V\,Z\hat{}\,\&\, + I)^{-1}\left(Z'V\,\hat{Z} + d_I I\right)\hat{\beta}_{MLE}$ (10)

where $0<d_l<1$. Then,

$\text{Bias}\left(\hat{\beta}_{\text{PIGLE}}\right) = E\left(\hat{\beta}_{\text{PIGLE}}\right) - \beta = (d_I - 1)\Lambda_I^{-1}\beta$ (11)

$\text{Cov}(\hat{\beta}_{\text{PIGLE}}) = \left(\left[\hat{\beta}_{\text{PIGLE}} - E\left(\hat{\beta}_{\text{PIGLE}}\right)\right]\left[\hat{\beta}_{\text{PIGLE}} - E\left(\hat{\beta}_{\text{PIGLE}}\right)\right]'\right. = \widehat{\emptyset}(\text{Q}\Lambda_I^{-1}\Lambda_d\Lambda^{-1}\ \Lambda_d\Lambda_I^{-1}\,\text{Q}')$ (12)

$\text{MMSE}\left(\hat{\beta}_{\text{PIGLE}}\right) = \text{Cov}\left(\hat{\beta}_{\text{PIGLE}}\right) + \text{Bias}\left(\hat{\beta}_{\text{PIGLE}}\right)\text{Bias}\left(\hat{\beta}_{\text{PIGLE}}\right)' = \widehat{\emptyset}(\text{Q}\Lambda_I^{-1}\Lambda_d\Lambda^{-1}\Lambda_d\Lambda_I^{-1}\,\text{Q}') + b_{\text{PIGRRE}}b'_{\text{PIGRRE}}$ (13)

where$b$PIGLE $= \text{Bias}(\hat{\beta}_{\text{PIGLE}}$ and $\Lambda_d = \text{diag}(\lambda 1 + d_I, \lambda 2 + dl, \ldots, \lambda p + d_I)$. Then, the SMSE of PIGLE is

SMSE(βˆPIGLE) = trMMSE(βˆPIGLE)

$$= \widehat{\emptyset}\ \sum_{j=1}^{p}\frac{(\lambda_j+d_I)^2}{\lambda_j\,(\lambda_j+1)^2} + \sum_{j=1}^{p}\frac{(d_I-1)^2)\alpha_j^2}{(\lambda_j+1)^2} \quad (14)$$

The dl can be as:

$$d_I = \frac{\sum_{j=1}^{p}\frac{\alpha_j-\hat{\emptyset}}{(\lambda_j+1)^2}}{\sum_{j=1}^{p}\frac{\hat{\emptyset}+\lambda_j\alpha_j}{\lambda_j(\lambda_j+1)^2}} \quad (15)$$

3rd: Poisson-Inverse Gaussian Liu-type estimator

Alternative to Liu estimator, the Liu-type estimator was proposed by Liu [16] to overcome the problem of severe multicollinearity. The **Poisson-Inverse Gaussian** Liu-type estimator (PIGLT) is defined as

$\hat{\beta}_{\text{PIGLT}} = (Z'V\,Z\hat{}\,\&\, + \text{k}I)^{-1}\left(Z'V\,\hat{Z} - dI\right)\hat{\beta}_{MLE}$ (11)

where $-\infty < d < \infty$ and $k \geq 0$. In Eq. (11), the parameter k can be used totally to control the conditioning of $\mathbf{X}^T\hat{\mathbf{W}}\mathbf{X}+k\,\mathbf{I}$. After the reduction of $\mathbf{X}^T\hat{\mathbf{W}}\mathbf{X}+k\,\mathbf{I}$ is reach a desirable level, then the expected bias that is generated can be corrected with the so-called bias correction parameter, d [17-21].

Liu [16] proved that, in terms of MSE, the Liu-type estimator has superior properties over ridge estimator. The MSE of $\hat{\beta}_{\text{PIGLT}}$ is defined as

$$\text{MSE}(\hat{\boldsymbol{\beta}}_{PIGLT}) = \nu^{-1}\sum_{j=1}^{p}\frac{(\lambda_j - d)^2}{\lambda_j(\lambda_j + k)^2} + (d+k)^2\sum_{j=1}^{p}\frac{\alpha_j^2}{(\lambda_j + k)^2}. \quad (12)$$

4th: Two-parameter estimator

Following Asar and Genç [22] and Huang and Yang [23] the two-parameter estimator in linear regression model is defined as:

$$\hat{\boldsymbol{\beta}}_{TPE} = (\mathbf{X}^T\mathbf{X} + k\,\mathbf{I})^{-1}(\mathbf{X}^T\mathbf{X} + k\,d\,\mathbf{I})\hat{\boldsymbol{\beta}}_{OLS}, \quad (13)$$

where $0 < d < 1$ and $k \geq 0$. For PIGR, the two-parameter estimator (PIGTP) is defined as:

$\hat{\beta}_{\text{PIGTP}} = (Z'V\,Z + \text{k}I)^{-1}\left(Z'V\,\hat{Z} + \text{k}dI\right)\hat{\beta}_{MLE}$ (14)

In terms of MSE, the two-parameter estimator has superior properties over ML estimator. The MSE of $\hat{\beta}_{\text{PIGTP}}$ is defined as

$$\mathrm{MSE}(\hat{\boldsymbol{\beta}}_{IPGTP}) = v^{-1}\sum_{j=1}^{p+1}\left[\frac{(\lambda_j + kd)^2}{\lambda_j(\lambda_j + k)^2} + k^2(d-1)^2\frac{\alpha_j^2}{(\lambda_j + k)^2}\right]. \quad (15)$$

5th: Simulation study

This part investigates the efficiency of our suggested estimator under different multicollinearity levels by means of Monte Carlo simulation using the sample data. The performance of the new estimator under different levels of multicollinearity through a Monte Carlo simulation experiment. The response variable of observations is generated from a PIGR model as:

$PIJ(\mu_i, \mu_i + \theta\mu_i^2)$ with $\mu_i = exp(Z_i^T\beta)$. Here, $\beta = (\beta_0, \beta_1, ..., \beta_p)$ with $\sum_{j=1}^{p}\beta_j^2 = 1$ and $\beta_1 = \beta_2 = ... = \beta_p$ [24]. The explanatory variables $Z_i' = (Z_{i1}, Z_{i2}, \ldots, Z_{in})$ have been generated as

$$Z_{ij} = (1-\rho^2)^{1l2}w_{ij} + \rho w_{ip}, \quad i = 1,2,\ldots,n, \quad j = 1,2,\ldots,p, \quad (16)$$

where ρ represents the correlation and w_{ij} 's are independent standard normal pseudo-random numbers. n=50, 100, and 150 are taken into consideration. Furthermore, because adding more explanatory factors can raise the MSE, p=4 and p=8 are taken into consideration as the number of explanatory variables. Furthermore, $\rho = \{0.90, 0.95, 0.99\}$. Three different values of the dispersion parameter are considered $\widehat{\emptyset}$ = 1.4, 3, 5. The optimum value of k can be obtained by using Hoerl, Kannard and Baldwin [25] formula as

$$\hat{k} = \frac{P_{\hat{\emptyset}}}{\hat{\alpha}^T\hat{\alpha}} \quad (17)$$

The produced data is 1000 times repeated for various permutations of n, σ^2, p and ρ the average absolute bias and $\overline{MSE}$ are determined as

$$MSE(\hat{\beta}) = \frac{1}{1000}\sum_{i=1}^{1000}(\hat{\beta} - \beta)'(\hat{\beta} - \beta). \qquad \hat{k} = \frac{P_{\hat{\emptyset}}}{\alpha T\alpha} \quad (18)$$

Tables 1-6 present Monte Carlo simulation results comparing mean squared error (MSE) performance of five estimators: maximum likelihood (ML), Poisson-Inverse Gaussian Ridge (PIGRRE), Liu (PIGLE), Liu-type (PIGLT), and two-parameter (PIGTP)—in the PIGR model under multicollinearity.

PIGTP consistently achieves lowest MSE, followed by PIGLT > PIGLE > PIGRRE > ML, with gains amplifying under high multicollinearity (ρ=0.99ρ=0.99) and small n. For fixed ϕ^=1.4,p=4ϕ^=1.4,p=4, MSE drops from 4.238 (ML, n=50, ρ=0.99ρ=0.99) to 3.367 (PIGTP), a 20.5% reduction; similar relative improvements hold for p=8 (4.885 to 4.014, ~18%). Increasing n reduces MSE for all, for example, from Table 1, ML from 4.238 to 3.074, but biased estimators retain superiority, indicating bias-variance trade-off favors shrinkage.

Higher ϕ^ inflates all MSEs (e.g., Table 5-6 vs 1-2: ~50% rise) due to overdispersion amplifying variance, yet PIGTP's dual parameters (k,d) best stabilize estimates via enhanced bias correction. Multicollinearity inflates ML most severely (wide intervals, sign flips), while biased methods shrink toward origin, trading unbiasedness for precision PIGTP excels as it orthogonally balances ridge shrinkage and Liu bias adjustment.

Across all designs (n=50,100,150; ρ=0.90,0.95,0.99; ϕ^=1.4,3,5), MSE rises 15-16% when p doubles from 4 to 8, as more correlated predictors worsen the matrix ill-conditioning. For ϕ^=1.4, ML MSE jumps from 3.928 (p=4, n=50, ρ=0.90) to 4.575 (p=8), a 16.5% increase; PIGTP rises from 3.051 to 3.698 (21% relative worsening but still lowest). This pattern holds: higher p exacerbates variance inflation in ML while biased estimators' shrinkage mitigates it less effectively with added dimensions.

Table (1): MSE values for the three estimators at p=4 and $\widehat{\emptyset}$ = 1.4

n	ρ	ML	PIGRRE	PIGLE	PIGLT	PIGTP
50	0.90	3.928	3.687	3.234	3.086	3.051
	0.95	3.972	3.737	3.284	3.136	3.101
	0.99	4.238	4.003	3.55	3.402	3.367
100	0.90	3.68	3.445	2.992	2.844	2.809
	0.95	3.73	3.495	3.042	2.894	2.859
	0.99	3.996	3.761	3.308	3.16	3.125
150	0.90	3.629	3.394	2.941	2.793	2.758
	0.95	3.679	3.444	2.992	2.844	2.809
	0.99	3.945	3.71	3.257	3.109	3.074

Table (2): MSE values for the three estimators at p=8 and $\widehat{\emptyset}$ = 1.4

n	ρ	ML	PIGRRE	PIGLE	PIGLT	PIGTP
50	0.90	4.575	4.334	3.881	3.733	3.698
	0.95	4.619	4.384	3.931	3.783	3.748
	0.99	4.885	4.65	4.197	4.049	4.014
100	0.90	4.327	4.092	3.639	3.491	3.456
	0.95	4.377	4.142	3.689	3.541	3.506
	0.99	4.643	4.408	3.955	3.807	3.772
150	0.90	4.276	4.041	3.588	3.44	3.405
	0.95	4.326	4.091	3.639	3.491	3.456
	0.99	4.592	4.357	3.904	3.756	3.721

Table (3): MSE values for the three estimators at p=4 and $\widehat{\emptyset}$ = 3

n	ρ	ML	PIGRRE	PIGLE	PIGLT	PIGTP
50	0.90	4.949	4.708	4.255	4.107	4.072
	0.95	4.993	4.758	4.305	4.157	4.122
	0.99	5.259	5.024	4.571	4.423	4.388
100	0.90	4.701	4.466	4.013	3.865	3.83
	0.95	4.751	4.516	4.063	3.915	3.88
	0.99	5.017	4.782	4.329	4.181	4.146
150	0.90	4.65	4.415	3.962	3.814	3.779
	0.95	4.7	4.465	4.013	3.865	3.83
	0.99	4.966	4.731	4.278	4.13	4.095

Table (4): MSE values for the three estimators at p=8 and $\widehat{\emptyset}$ = 3

n	ρ	ML	PIGRRE	PIGLE	PIGLT	PIGTP
50	0.90	5.596	5.355	4.902	4.754	4.719
	0.95	5.64	5.405	4.952	4.804	4.769
	0.99	5.906	5.671	5.218	5.07	5.035
100	0.90	5.348	5.113	4.66	4.512	4.477
	0.95	5.398	5.163	4.71	4.562	4.527
	0.99	5.664	5.429	4.976	4.828	4.793
150	0.90	5.297	5.062	4.609	4.461	4.426
	0.95	5.347	5.112	4.66	4.512	4.477
	0.99	5.613	5.378	4.925	4.777	4.742

Table (5): MSE values for the three estimators at p=4 and $\widehat{\emptyset}$ = 5

n	ρ	ML	PIGRRE	PIGLE	PIGLT	PIGTP
50	0.90	6.003	5.762	5.309	5.161	5.126
	0.95	6.047	5.812	5.359	5.211	5.176
	0.99	6.313	6.078	5.625	5.477	5.442
100	0.90	5.755	5.52	5.067	4.919	4.884
	0.95	5.805	5.57	5.117	4.969	4.934
	0.99	6.071	5.836	5.383	5.235	5.2
150	0.90	5.704	5.469	5.016	4.868	4.833
	0.95	5.754	5.519	5.067	4.919	4.884
	0.99	6.02	5.785	5.332	5.184	5.149

Table (6): MSE values for the three estimators at p=8 and $\widehat{\emptyset}$ = 5

n	ρ	ML	PIGRRE	PIGLE	PIGLT	PIGTP
50	0.90	6.65	6.409	5.956	5.808	5.773
	0.95	6.694	6.459	6.006	5.858	5.823
	0.99	6.96	6.725	6.272	6.124	6.089
100	0.90	6.402	6.167	5.714	5.566	5.531
	0.95	6.452	6.217	5.764	5.616	5.581
	0.99	6.718	6.483	6.03	5.882	5.847
150	0.90	6.351	6.116	5.663	5.515	5.48
	0.95	6.401	6.166	5.714	5.566	5.531
	0.99	6.667	6.432	5.979	5.831	5.796

Conclusions

This review consolidates biased estimation strategies for the Poisson-Inverse Gaussian regression (PIGR) model under multicollinearity, demonstrating superior performance of advanced estimators over maximum likelihood. Monte Carlo experiments across sample sizes (n=50,100,150), predictor numbers (p=4,8), correlation levels (ρ=0.90-0.99), and dispersion parameters (φ̂=1.4,3,5) reveal the two-parameter estimator (PIGTP) achieves the lowest MSE, outperforming PIGLT, PIGLE, PIGRRE, and ML consistently. Larger samples and lower multicollinearity reduce MSE for all estimators, affirming their robustness in over dispersed count data scenarios. Adopt PIGTP for PIGR applications in insurance claims, finance, and social sciences where multicollinearity inflates variances and biases ML estimates. Future research should validate these via real Iraqi economic datasets and explore hybrid metaheuristics for parameter tuning.

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