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Biased estimators in gamma regression model in the presence of Multicollinearity: Subject review

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Abstract: The presence of the high correlation among predictors in regression modeling has undesirable effects on the regression estimating. There are several available biased methods to overcome this issue. The gamma regression model (GRM) is a special model from the generalized linear models. The GRM is a well-known model in research application when the response variable under the study is skewed data. Numerous biased estimators for overcoming the multicollinearity in GRM have been proposed in the literature using different theories. An overview of recent biased methods for GRM is provided. A comparison among these biased estimators allows us to gain an insight into their performance.

Keywords: Multicollinearity. biased estimator, gamma regression model, Monte Carlo simulation.

المقدرات المتحيزة في انموذج انحدار كاما بوجود مطسلة التعدد الخطي-مراجعة مقال

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المستخلص: إن وجود ارتباط عالٍ بين المتغيرات التنبؤية في نمذجة الانحدار له آثار غير مرغوب فيها على تقدير الانحدار. هناك عدة طرائق متاحة للتغلب على هذه المشكلة. نموذج الانحدار كاما (GRM) هو نموذج خاص من النماذج الخطية المعممة. نموذج GRM هو نموذج معروف في التطبيقات البحثية عندما تكون المتغيرات المستجيبة قيد الدراسة بيانات منحرفة. تم اقتراح العديد من المُقدِّرات المتحيزة للتغلب على التعددية الخطية في نموذج GRM في الأدبيات باستخدام نظريات مختلفة. يتم تقديم نظرة عامة على الطرق المتحيزة الحديثة لنموذج GRM. تتيح لنا المقارنة بين هذه المُقدِّرات المتحيزة الحصول على فكرة عن أدائها.

الكلمات المفتاحية: المقدرات المتحيزة، انموذج انحدار كاما، التعدد الخطي، محاكاة مونت كارلو.

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Introduction

Gamma regression model is widely applied for studying several real data problems, such as automobile insurance claims, healthcare economics, and medical science [1-3]. Specifically, “gamma regression model is used when the response variable under the study is not distributed as normal distribution or the response variable is positively skewed. Consequently, the gamma regression assumes that the response variable has a gamma distribution [4, 5].

In dealing with the GRM, it is assumed that there is no correlation among the explanatory variables [6-26]. In practice, however, this assumption often not holds, which leads to the problem of multicollinearity. In the presence of multicollinearity, when estimating the regression coefficients for GRM using the maximum likelihood (ML) method, the estimated coefficients are usually become unstable with a high variance, and therefore low statistical significance [27]. Numerous remedial methods have been proposed to overcome the problem of multicollinearity [28-32]. The ridge regression method [33] has been consistently demonstrated to be an attractive and alternative to the ML estimation method.

Ridge regression is a biased method that shrinks all regression coefficients toward zero to reduce the large variance [34]. This done by adding a positive amount to the diagonal of $\mathbf{X}^T\mathbf{X}$. As a result, the ridge estimator is biased but it guaranties a smaller mean squared error than the ML estimator.

In linear regression, the ridge estimator is defined as

$$\hat{\boldsymbol{\beta}}_{Ridge} = (\mathbf{X}^T\mathbf{X} + k\,\mathbf{I})^{-1}\mathbf{X}^T\mathbf{y}, \tag{1}$$

where $\mathbf{y}$ is an $n \times 1$ vector of observations of the response variable, $\mathbf{X} = (\mathbf{x}_1, \ldots, \mathbf{x}_p)$ is an $n \times p$ known design matrix of explanatory variables, $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_p)$ is a $p \times 1$ vector of unknown regression coefficients, $\mathbf{I}$ is the identity matrix with dimension $p \times p$, and $k \geq 0$ represents the ridge parameter (shrinkage parameter). The ridge parameter, k, controls the shrinkage of $\boldsymbol{\beta}$ toward zero. The OLS estimator can be considered as a special estimator from Eq. (1) with $k = 0$. For larger value of k, the $\hat{\boldsymbol{\beta}}_{Ridge}$ estimator yields greater shrinkage approaching zero [33, 35].

1st: Gamma regression model

Positively skewed data often arise in epidemiology, social, and economic studies. This type of data consists of nonnegative values. Gamma distribution is a well-known distribution that fits to such type of data. Gamma regression model (GRM) is used to model the relationship between the positively skewed response variable and potentially regressors [36].

Let y_i be the response variable and follows a gamma distribution with nonnegative shape parameter ν and nonnegative scale parameter τ, i.e. $y_i \sim Gamma(\nu, \tau)$, then the probability density function is defined as

$$f(y_i) = \frac{\tau}{\Gamma(\nu)}(\tau y_i)^{\nu-1}e^{-\tau y_i}, \quad y_i \geq 0, \tag{2}$$

with $E(y) = \nu/\tau = \theta$ and $\mathrm{var}(y) = \nu/\tau^2 = \theta^2/\nu$. It is observed that the variance of the response variable is proportional to the square of its mean when ν parameter is known.

In a GRM, $\theta_i = \exp(\mathbf{x}_i^T\boldsymbol{\beta})$ is expressed as a linear combination of regressors $\mathbf{x}_i = (x_{i1}, \ldots, x_{ip})^T$. The θ_i is called the log link function which making the relationship between regressors and response variable linear. This log like function is alternatively used rather than the canonical link function (reciprocal link function, $\theta_i = -1/\mathbf{x}_i^T\boldsymbol{\beta}$) because it ensures that $\theta_i > 0$.

The most common method of estimating the coefficients of GRM is to use the maximum likelihood method. Given the assumption that the observations are independent and $\tau = \nu / \theta$, the log-likelihood function is given by

$$\ell(\boldsymbol{\beta}) = \sum_{i=1}^{n} \left\{ \nu \left[-\frac{y_i}{\mathbf{x}_i^T \boldsymbol{\beta}} - \mathbf{x}_i^T \boldsymbol{\beta} \right] - \ln \Gamma \nu + \nu \ln(\nu y_i) - \ln(y_i) \right\}. \quad (3)$$

The ML estimator is then obtained by computing the first derivative of the Eq. (4) and setting it equal to zero, as

$$\frac{\partial \ell(\boldsymbol{\beta})}{\partial \boldsymbol{\beta}} = \sum_{i=1}^{n} \nu \left[\frac{y_i}{(\mathbf{x}_i^T \boldsymbol{\beta})^2} - 1 \right] \mathbf{x}_i = 0. \quad (4)$$

Unfortunately, the first derivative cannot be solved analytically because Eq. (4) is nonlinear in $\boldsymbol{\beta}$. The iteratively weighted least squares (IWLS) algorithm or Newton–Raphson algorithm can be used to obtain the ML estimators of the gamma regression parameters. In each iteration, the parameters are updated by

$$\boldsymbol{\beta}^{(r+1)} = \boldsymbol{\beta}^{(r)} + I^{-1}(\boldsymbol{\beta}^{(r)}) S(\boldsymbol{\beta}^{(r)}), \quad (5)$$

where $S(\boldsymbol{\beta}) = \partial \ell(\boldsymbol{\beta}) / \partial \boldsymbol{\beta}$ and $I^{-1}(\boldsymbol{\beta}) = \left(-E\left(\partial^2 \ell(\boldsymbol{\beta}) / \partial \boldsymbol{\beta} \partial \boldsymbol{\beta}^T \right) \right)^{-1}$. The final step of the estimated coefficients is defined as

$$\hat{\boldsymbol{\beta}}_{GRM} = (\mathbf{X}^T \hat{\mathbf{W}} \mathbf{X})^{-1} \mathbf{X}^T \hat{\mathbf{W}} \hat{\mathbf{u}}, \quad (6)$$

where $\hat{\mathbf{W}} = \mathrm{diag}(\hat{\theta}_i^2)$ and $\hat{\mathbf{u}}$ is a vector where i$^{\text{th}}$ element equals to $\hat{u}_i = \hat{\theta}_i + ((y_i - \hat{\theta}_i) / \hat{\theta}_i^2)$. The ML estimator is asymptotically normally distributed with a covariance matrix that corresponds to the inverse of the Hessian matrix

$$\mathrm{cov}(\hat{\boldsymbol{\beta}}_{GRM}) = \left[-E\left(\frac{\partial^2 \ell(\boldsymbol{\beta})}{\partial \beta_i \, \partial \beta_k} \right) \right]^{-1} = (\mathbf{X}^T \hat{\mathbf{W}} \mathbf{X})^{-1}. \quad (7)$$

The mean squared error (MSE) of Eq. (7) can be obtained as

$$\begin{aligned} \mathrm{MSE}(\hat{\boldsymbol{\beta}}_{GRM}) &= E(\hat{\boldsymbol{\beta}}_{GRM} - \hat{\boldsymbol{\beta}})^T (\hat{\boldsymbol{\beta}}_{GRM} - \hat{\boldsymbol{\beta}}) \\ &= tr\, \nu^{-1} [(\mathbf{X}^T \hat{\mathbf{W}} \mathbf{X})^{-1}] \\ &= \nu^{-1} \sum_{j=1}^{p} \frac{1}{\lambda_j}, \end{aligned} \quad (8)$$

where λ_j is the eigenvalue of the $\mathbf{X}^T \hat{\mathbf{W}} \mathbf{X}$ matrix. In the presence of multicollinearity, the matrix $\mathbf{X}^T \hat{\mathbf{W}} \mathbf{X}$ becomes ill-conditioned leading to high variance and instability of the ML estimator of the gamma regression parameters.

2nd: Ridge estimator

In the presence of multicollinearity, the matrix $\mathbf{X}^T \hat{\mathbf{W}} \mathbf{X}$ becomes ill-conditioned leading to high variance and instability of the ML estimator of the GRM parameters. As a remedy, Månsson and Shukur [37] proposed the GR ridge estimator (GRR) as

$$\begin{aligned} \hat{\boldsymbol{\beta}}_{GRR} &= (\mathbf{X}^T \hat{\mathbf{W}} \mathbf{X} + k\mathbf{I})^{-1} \mathbf{X}^T \hat{\mathbf{W}} \mathbf{X} \hat{\boldsymbol{\beta}}_{GRM} \\ &= (\mathbf{X}^T \hat{\mathbf{W}} \mathbf{X} + k\mathbf{I})^{-1} \mathbf{X}^T \hat{\mathbf{W}} \hat{\mathbf{v}}, \end{aligned} \quad (9)$$

where $k \geq 0$. The ML estimator can be considered as a special estimator from Eq. (10) with $k = 0$. Regardless of k value, the MSE of the $\hat{\boldsymbol{\beta}}_{GRR}$ is smaller than that of $\hat{\boldsymbol{\beta}}_{GRM}$ because the MSE of $\hat{\boldsymbol{\beta}}_{GRR}$ is equal to [27]

$$\mathrm{MSE}(\hat{\boldsymbol{\beta}}_{GRR}) = \nu^{-1}\sum_{j=1}^{p}\frac{\lambda_j}{(\lambda_j+k)^2} + k^2\sum_{j=1}^{p}\frac{\alpha_j}{(\lambda_j+k)^2}, \tag{10}$$

where α_j is defined as the j^th^ element of $\gamma\hat{\boldsymbol{\beta}}_{GRM}$ and γ is the eigenvector of the $\mathbf{X}^T\hat{\mathbf{W}}\mathbf{X}$ matrix. Comparing with the MSE of Eq. (11), $\mathrm{MSE}(\hat{\boldsymbol{\beta}}_{GRR})$ is always small for $k>0$.

3^rd^: Liu estimator

Another popular biased estimator which is known as Liu estimator has been adopted in Poisson regression model. The gamma Liu estimator (GLE) is defined as

$$\hat{\boldsymbol{\beta}}_{GLE} = (\mathbf{X}^T\hat{\mathbf{W}}\mathbf{X}+\mathbf{I})^{-1}(\mathbf{X}^T\hat{\mathbf{W}}\mathbf{X}+d\,\mathbf{I})\hat{\boldsymbol{\beta}}_{GRM}, \tag{11}$$

where $0<d<1$. Regardless of d value, the MSE of the $\hat{\boldsymbol{\beta}}_{GLE}$ is smaller than that of $\hat{\boldsymbol{\beta}}_{GRM}$ because the MSE of $\hat{\boldsymbol{\beta}}_{GLE}$ is equal to [27]

$$\mathrm{MSE}(\hat{\boldsymbol{\beta}}_{GLE}) = \nu^{-1}\sum_{j=1}^{p}\frac{(\lambda_j+d)^2}{\lambda_j(\lambda_j+1)^2} + (d-1)^2\sum_{j=1}^{p}\frac{\alpha_j^2}{(\lambda_j+1)^2}. \tag{12}$$

4^th^: Liu-type estimator

Alternative to Liu estimator, the Liu-type estimator was proposed by Liu [38] to overcome the problem of severe multicollinearity. The gamma Liu-type estimator (GLT) is defined as

$$\hat{\boldsymbol{\beta}}_{GLT} = (\mathbf{X}^T\hat{\mathbf{W}}\mathbf{X}+k\,\mathbf{I})^{-1}(\mathbf{X}^T\hat{\mathbf{W}}\mathbf{X}-d\,\mathbf{I})\hat{\boldsymbol{\beta}}_{GRM}, \tag{13}$$

where $-\infty<d<\infty$ and $k\ge 0$. In Eq. (14), the parameter k can be used totally to control the conditioning of $\mathbf{X}^T\hat{\mathbf{W}}\mathbf{X}+k\,\mathbf{I}$. After the reduction of $\mathbf{X}^T\hat{\mathbf{W}}\mathbf{X}+k\,\mathbf{I}$ is reach a desirable level, then the expected bias that is generated can be corrected with the so-called bias correction parameter, d [39-43].

Liu [38] proved that, in terms of MSE, the Liu-type estimator has superior properties over ridge estimator. The MSE of $\hat{\boldsymbol{\beta}}_{IGLT}$ is defined as

$$\mathrm{MSE}(\hat{\boldsymbol{\beta}}_{GLT}) = \nu^{-1}\sum_{j=1}^{p}\frac{(\lambda_j-d)^2}{\lambda_j(\lambda_j+k)^2} + (d+k)^2\sum_{j=1}^{p}\frac{\alpha_j^2}{(\lambda_j+k)^2}. \tag{14}$$

5^th^: Two-parameter estimator

Following Asar and Genç [44] and Huang and Yang [45] the two-parameter estimator in linear regression model is defined as:

$$\hat{\boldsymbol{\beta}}_{TPE} = (\mathbf{X}^T\mathbf{X}+k\,\mathbf{I})^{-1}(\mathbf{X}^T\mathbf{X}+k\,d\,\mathbf{I})\hat{\boldsymbol{\beta}}_{OLS}, \tag{15}$$

where $0<d<1$ and $k\ge 0$. For GRM, the two-parameter estimator (GTP) is defined as:

$$\hat{\boldsymbol{\beta}}_{GTP} = (\mathbf{X}^T\hat{\mathbf{W}}\mathbf{X}+k\,\mathbf{I})^{-1}(\mathbf{X}^T\hat{\mathbf{W}}\mathbf{X}+k\,d\,\mathbf{I})\hat{\boldsymbol{\beta}}_{GRM}. \tag{16}$$

It is obviously noted that the $\hat{\boldsymbol{\beta}}_{GTP}$ is a combination of two different estimators GRR and GLE. Furthermore, if $k=1$, Eq. (18) will be the $\hat{\boldsymbol{\beta}}_{GLE}$ while if $k=0$, Eq. (17) will be the $\hat{\boldsymbol{\beta}}_{GRM}$. Besides, when $d=0$, then Eq. (17) will equal $\hat{\boldsymbol{\beta}}_{GRR}$.

In terms of MSE, the two-parameter estimator has superior properties over ML estimator. The MSE of $\hat{\boldsymbol{\beta}}_{GTP}$ is defined as

$$\mathrm{MSE}(\hat{\boldsymbol{\beta}}_{GTP}) = \nu^{-1}\sum_{j=1}^{p+1}\left[\frac{(\lambda_j+kd)^2}{\lambda_j(\lambda_j+k)^2} + k^2(d-1)^2\frac{\alpha_j^2}{(\lambda_j+k)^2}\right]. \tag{17}$$

6th: Real application

To demonstrate the usefulness of the shrinkage estimators in real application, we present here a chemistry dataset with $(n, p) = (212,10)$, where n represents the number of antifungal agents. The antimicrobial activities were measured as pMIC (the logarithm of reciprocal of MIC, where MIC is minimum inhibitory concentration against C. albicans in mM/L). While p denotes the number of molecular descriptors, which are treated as explanatory variables [46, 47]. Quantitative structure-activity relationship (QSAR) study has become a great deal of importance in chemometrics. The principle of QSAR is to model several biological activities over a collection of chemical compounds in terms of their structural properties. Consequently, using of regression model is one of the most important tools for constructing the QSAR model. A description of the used explanatory variables is provided in Table 1. All the variables are numerical.
First, to check whether the response variable belongs to the gamma distribution, Chi-square test is used. The result of the test equals to 10.0286 with p-value equals to 0.9117. It is indicated form this result that the gamma distribution fits very well to this response variable. The estimated dispersion parameter is 0.0153.
Second, to test the existence of multicollinearity after fitting the gamma regression model using log link function and the estimated dispersion parameter is 0.0153, the eigenvalues of the matrix $\mathbf{X}^T \hat{\mathbf{W}} \mathbf{X}$ are obtained as $1.97\times10^9, 3.74\times10^6$, 1.21×10^4, 1.34×10^3, $1.22\times10^3, 1.07\times10^3$, 4.63×10^2, 2.08×10^1, 10.68, and 1.57.
The determined condition number $\text{CN} = \sqrt{\lambda_{\max} / \lambda_{\min}}$ of the data is 35422.83 indicating that the severe multicollinearity issue is exist.
The estimated MSE values for the GRM, GRR, GLE, GLT, and GTP estimators are listed in Table 2. According to Table 2, it is clearly seen that the GLE estimator shrinkages the value of the estimated coefficients efficiently". Additionally, in terms of the MSE, there is an important reduction in favor of the GLE estimator. Specifically, it can be seen that the MSE of the GLE method was about 47.615%, 40.241%, 21.062%, and 21.602% lower than that of GRM, GRR, GLT, and GTP estimators, respectively.

Table (1): Description of the used explanatory variables

Variable name's	description
SpMax3_Bh(s)	largest eigenvalue n. 3 of Burden matrix weighted by I-state
P_VSA_e_3	P_VSA-like on Sanderson electronegativity, bin 3
IC3	Information Content index (neighborhood symmetry of 3-order)
Mor21e	signal 21 / weighted by Sanderson electronegativity
MATS2s	Moran autocorrelation of lag 2 weighted by I-state
GATS4p	Geary autocorrelation of lag 4 weighted by polarizability
SpMax8_Bh(p)	largest eigenvalue n. 8 of Burden matrix weighted by polarizability
ATS8v	Broto-Moreau autocorrelation of lag 8 (log function) weighted by van der Waals volume
MATS7v	Moran autocorrelation of lag 7 weighted by van der Waals volume
TDB08m	3D Topological distance-based descriptors - lag 8 weighted by mass

Table (2): The estimated MSE values for the real data application

Methods	MSE
GRM	4.3291
GRR	3.8507
GLE	**2.3008**
GLT	2.9147
GTP	2.9348

Conclusions

In this paper, we presented a thorough review of literature regarding the biased estimators in gamma regression model when the multicollinearity is existing. According to real data application, the Liu estimator has better performance than GRM, GRR, GLT, and GTP, in terms of MSE. In conclusion, the use of the Liu estimator is recommended when multicollinearity is present in the gamma regression model.

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