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Review Paper

Structural synthesis and classification of planetary gear-cam mechanisms using graph theory

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ABSTRACT

This paper presents a comprehensive study on the structural synthesis and classification of planetary gear mechanisms (PGCMs), which combine the advantages of planetary gear trains (PGTs) and cam mechanisms to produce precise, intermittent, or variable-speed motions. These mechanisms are particularly suitable for high-performance industrial applications, such as indexing tables, path generation systems, and rigid body guidance. The work begins by detailing the kinematic and structural characteristics of PGCMs and identifying possible configurations based on degrees of freedom, link types, and joint arrangements. Using graph theory, rooted graph representations are developed to systematically enumerate two-degree-of-freedom PGCMs, up to seven links. A spanning-tree-based synthesis method is used to generate candidate structures, followed by genetic compatibility analysis to identify viable PGCM configurations. The proposed method produces a set of functionally and structurally distinct PGCMs, including a novel five-link mechanism that achieves precise path generation with minimal complexity. This study not only enhances the theoretical framework for PGCM design, but also provides a practical basis for its application in modern mechanical systems.

1. Introduction

To facilitate the cyclic motion of the output link of input mechanisms in numerous industrial machines, such as those employing packaging, printing, textile, and paper machinery, it is necessary to accomplish a controlled speed over the course of an operation cycle. Processing industries must improve their productivity and efficiency. As a result, high-productivity and high-reliability machines are in high demand. As a result, design trends favour high-speed machines with high accuracy, low noise, and low energy consumption. For example, Fig. 1 depicts the rotating table of an indexing machine [1]. The process is carried out in a series of operations that take place at various stations. At each station, the relative velocity between the tools and workpieces must be zero. As a result, the table will perform an indexed movement. The indexing machine transforms the constant speed of the input motor into the intermittent motion of the work table. This situation has a lot of power and load, and the best way to meet it is often to combine cams and gears that are all driven by a motor or engine at a steady speed. Accuracy, dependability, minimal vibration, and low noise may be obtained even when large masses are moved at rapid speeds. Cams provide all of the benefits of controlled design by enabling diversity in choosing the type of follower action, resulting in the optimum dynamic features of the mechanism. If a mechanism has gear pairs, it is termed a geared mechanism; if it contains cam pairs, it is called a cam mechanism; and if it contains both gear and cam pairs, it is called a gear-cam mechanism [2]. Three lower pairs and two upper pairs are typically employed in gear-cam mechanisms. Revolute (R), prismatic (P), and cylindrical (C) pairs are lower pairs, while higher pairs are cam and gear pairs. There are two types of PGCMs: those with a fixed link and those with all links moving. Despite the great number of PGCM structural variations, there are very few mechanisms that are actually useful. Mechanisms with few links are preferred in practical designs for their simplicity. Therefore, PGCMs with six or more links have little use in the real world. PGCMs with a fixed link are simple, have few links, and are widely used. PGCMs appear to be a suitable alternative for the creation of indexed motion. Their applications are not limited to indexing mechanisms; they can be used for exact path generation mechanisms [3], rigid body guidance mechanisms [4], and infinitely variable transmissions [5].

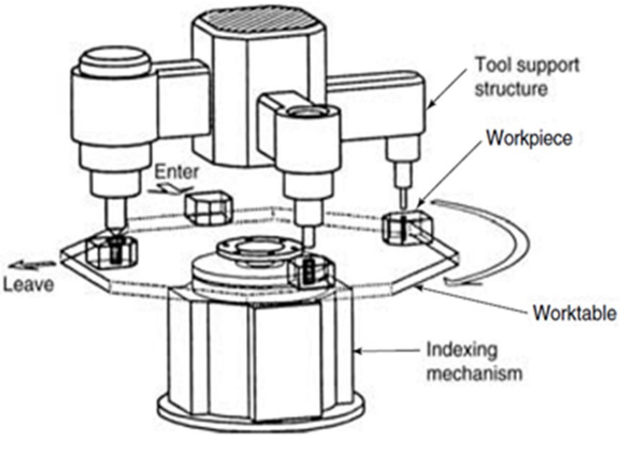

Figure 1. Indexed movement of the work table [1].

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Nomenclature

DOF	Degrees of freedom	v	Number of vertices in the graph
$PGCM$	Planetary gear-cam mechanism	e	Number of edges
PGT	Planetary gear train	F	Number of degrees of Freedom
R	Revolute joint (lower pair)	Gp	gear pair
P	Prismatic joint (lower pair)	J	The number of joints
LAA	Link Assortment Array	L	Number of independent loops
VDA	Vertex Degree Array	k	The maximal degree of the vertex
RR	The Revolute-Revolute	N	Number of Links
PR	The Prismatic –Revolute	j_1	Number of 1-DOF joints (R and P)
F	The number of degrees of freedom	j_2	Number of 2-DOF joints (gear pair, cam pair)
D	The vertex degree array $=\left[d_0, d_1, \ldots, d_{(v-1)}\right]$	J	Total number of joints
IVT	Infinitely variable transmission		

1.1 Scope and contribution

This research is primarily concerned with the structural synthesis and systematic classification of planar PGCMs), with a focus on 2-DOF systems. The scope encompasses a comprehensive exploration of both the kinematic architecture and graph-theoretic representation of PGCMs, particularly those that integrate both gear and cam elements to produce non-uniform motion suitable for high-performance applications such as indexing, path generation, and rigid body guidance. This study contributes in four aspects:

- *Systematic classification*:
  The study presents a new classification system for PGCMs with up to seven links, based on degrees of freedom and joint types. Applying the Grobler-Kutzbach criterion, along with graph enumeration techniques, this classification identifies the minimum requirements for links and connections necessary to achieve the desired kinematic function.
- *Graph-Based Synthesis*:
  The spanning tree approach and the parent graph are used to generate all valid topological structures for PGCMs. The study introduces the concept of genetic matching between spanning trees and parent graphs to identify appropriate gear-cam topologies. This enables the automated synthesis of mechanism structures with the desired kinematic properties.
- *Functional and Structural Validation*:
  The proposed structural graphs are translated into functional mechanisms. Several representative PGCM configurations are modeled and analyzed to verify their kinematic feasibility and suitability for industrial applications requiring intermittent or static motion.
- *Design Insights for Application Areas*:
  The results demonstrate the suitability of PGCM models for use in mechanical systems that require compactness, precision, and mechanical advantage, such as packaging, indexing tables, or cam-based variable transmissions. The simplicity of the five-link configurations and the versatility of planetary gear integration offer tangible benefits in real-world machine design.

By linking theoretical graph synthesis with practical mechanical design, this work advances the state of knowledge in the field of mechanism design and potentially opens new possibilities for developing efficient PGCM-based systems.

1.2 Paper structure

This paper is structured as follows: A general overview of indexing mechanisms in industrial applications and discusses the limitations of traditional systems is provided in the (Introduction) section 1. It introduces gear-cam mechanisms as efficient alternatives. The historical development and mechanical advantages of planetary gear trains and cam mechanisms are explored in the (Background and Literature Review) section 2. It highlights recent advancements and notable applications in various engineering fields. Section 2 defines key concepts including the degrees of freedom (DOF), link types, and joint classifications in PGCMs. It also outlines the fundamental structural and kinematic criteria used for mechanism synthesis. The methodology for generating and classifying PGCMs using graph theory and spanning trees are described in Section 3. It introduces link assortment arrays and vertex degree arrays, and explains the synthesis process in detail. Section 4 presents classifications of two-DOF PGCMs with up to seven links and outlines the structural properties of viable mechanisms. Tables and figures are used to categorize and compare mechanism types. It provides graphical representations of PGCMs, identifies cam and gear edges, and explains the process of generating potential PGCM graphs through genetic compatibility analysis. A novel five-link PGCM for continuous path generation, supported by simulated results and functional diagrams is introduced in this section. The mechanism's compact design and performance characteristics are evaluated. The significance of the synthesized PGCMs, their design feasibility, and potential applications in industry are discussed in section 5. Key performance factors such as simplicity, efficiency, and motion accuracy are considered. Section 6 summarizes the contributions and findings of the study, emphasizing the novelty and practical relevance of the proposed PGCM designs.

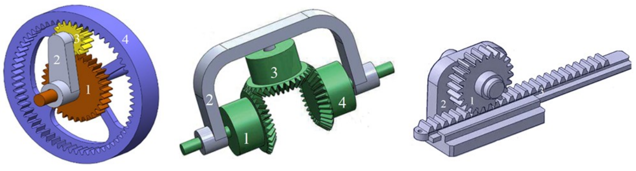
2. Background and literature review

2.1 Indexing mechanisms in industry

To achieve high-speed movements, irregular drive mechanisms such as the Geneva mechanism, star wheel, or ratchet mechanism are avoided due to the jerk that occurs when these mechanisms are used in high-speed machinery that has massive moving masses.

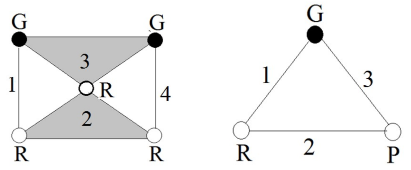
2.2 Planar geared mechanisms

A planar geared mechanism is a geared kinematic chain that contains only prismatic, P, revolute, R, and gear joints, G. A planetary gear train (PGT) is a gear train that includes a supporting link between each pair of meshing gears to maintain the distance between them constant. This supporting link is known as a carrier. The most basic types of gear trains with supporting links are shown in Fig. 2. A polygon with vertices that represent the kinematic pairings serves as the structural representation of each link in a mechanism. Figure 3 illustrates the structural representation of the PGTs illustrated in Fig. 2.



(a) Spur gear (b) Bevel gear (c) Rack and pinion

Figure 2. Basic types of gear trains with a supporting link.



(a) Spur or Bevel gears (b) Rack and pinion

Figure 3. Structural representation of the PGTs shown in Fig. 2.

We observe that the type of gear mesh cannot be determined at that level of abstraction. In this respect, the functional representation shown in Fig. 3a represents the sketch of more than one kinematic structure, Fig. 2a and Fig. 2b. Either gear pair can assume either spur or bevel gears, external or internal gear mesh, or rack and pinion. Consequently, there is NO exact correspondence between both structural and functional representations. To identify the differences, an additional level of clarity is required. Kinematic inversion allows for the derivation of a wide variety of gear mechanisms. If, for instance, the carrier of a PGT is designated as the stationary link, the resulting mechanism is a conventional gear train. Ordinary gear trains can be considered special cases of PGTs.

2.3 Planetary gear trains

A PGT is a geared kinematic chain that contains only revolute and geared joints and conforms to the rules below [2]:

- R1: The mechanism must follow the general equation of degrees of freedom.
- R2: All links must be rotatable unlimitedly.
- R3: Each gear needs to have a revolute joint on its axis so as to keep a constant center spacing between gear pairs.
- R4: Every link must include at least one revolute joint in order to function properly.

PGTs are not only small and lightweight, but they are also capable of creating a large mechanical advantage as well as a high-speed single-stage reduction. PGTs are frequently used in robotic manipulators [6], automatic transmissions [7–10], and aerospace drives such as helicopter transmissions [11–14] or wind turbine reduction gears [15]. The transmission depicted in Fig. 4 is infinitely variable and capable of achieving any transmission ratio. This transmission has five primary components in its most basic form. A three-dimensional cam is in the center. A few followers are mounted on the carrier around the cam. The rotation of the carrier causes the followers to revolve about the axis of the cam. An indexing clutch, shown in Fig. 5, connects the output shaft to all of the sun gears.

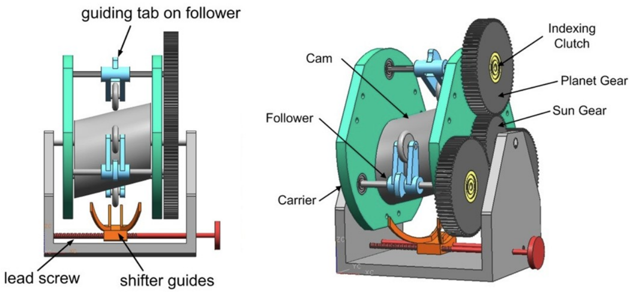


Figure 4. The front and isometric view of the cam-based infinitely variable transmission [12].

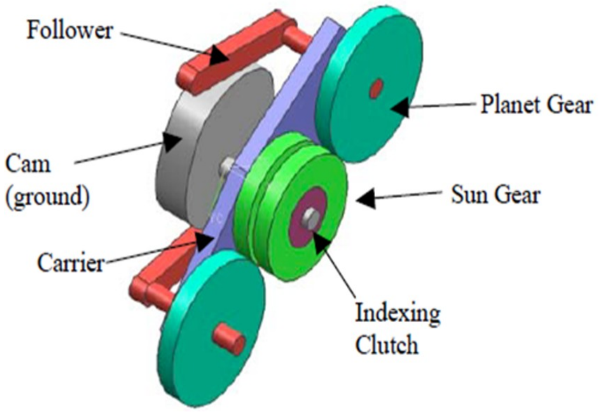


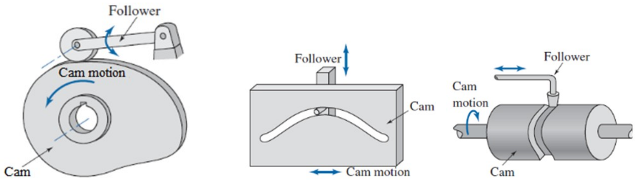
Figure 5. Simplified depiction of the IVT [5].

2.4 Cam mechanisms

The simplest cam mechanism consists of three components: the cam, the cam-follower, and the housing. Prismatic, revolute, cylindrical, or screw pairs can be used to connect the frame to the cam or follower [1]. Three features distinguish cams from other mechanisms:

- Greater velocity capability.
- Capability to impart larger torque at higher momentum.
- Reliable performance that is characterized by great repeatability and accuracy [2].

The notations P, R, and C_p are used to refer to prismatic joint, revolute joint, and cam pair, respectively. Thus, the RRC_p, PPC_p and RPC_p are refer to Fig. 6a, b, and Fig. 6c, respectively. As well as, the Computer Aided Design (CAD) is used to generate highly accurate cam profiles. Furthermore, the Computer-Aided Manufacturing (CAM) has significantly improved the accuracy of cam machining, where the cam mechanisms are utilized for a variety of purposes, including speed reduction.



(a) RR Cam pair (b) PP Cam pair (c) RP Cam pair

Figure 6. Three types of cam mechanisms joints. shown in Fig. 7.

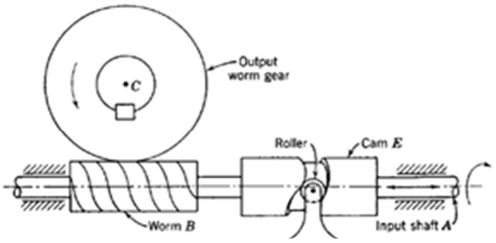


Figure 7. Gear-Cam mechanism with worm gear drive [1].

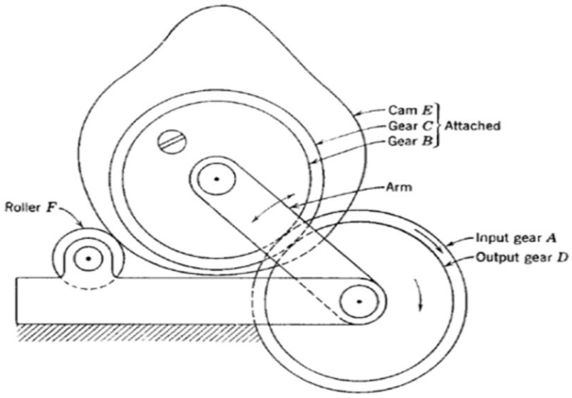


Figure 8. PGCM with fixed follower [1].

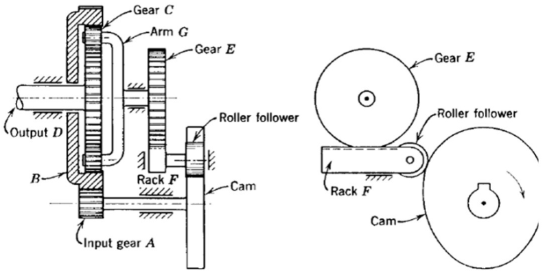


Figure 9. Planetary Gear-Cam mechanism [1].

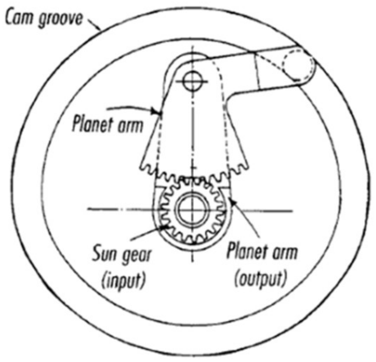


Figure 10**.** Planetary Gear-Cam mechanism with fixed cam [16].

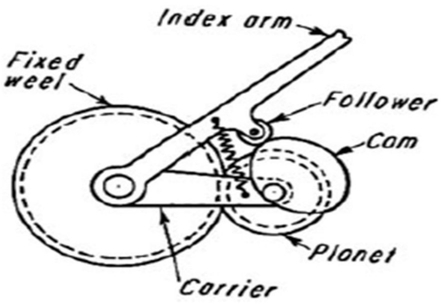


Figure 11**.** Planetary Gear-Cam mechanism with fixed sun gear.

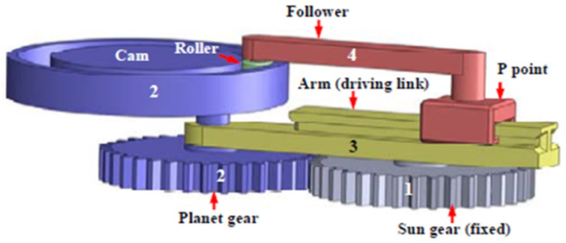


Figure 12**.** Exact path generation mechanism [17].

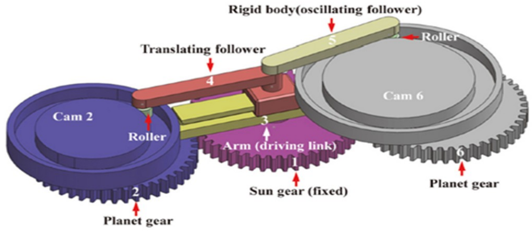


Figure 13**.** A rigid body guidance planetary gear-cam mechanism [18].

2.5 Combining gears and cams

PGCMs leverage the strengths of both elements, producing efficient and accurate motion with minimal components. In the worm and worm gear system shown in Fig. 7, shaft A propels worm gear C through worm B. An immobile roller in the rotating cam E causes the reciprocating and rotating motion of shaft A. As a result, the output of the worm gear consists of the rotation of the input shaft in addition to its axial movement. Figure 8 depicts a compound PGT with gear A as the driving gear. The output of the planet gear as well as the movement of the carrier drive Gear D. Figure 9 depicts a kinematically identical design to the previous one. As stated previously, the output speed is the sum of two motions: the inputs of both gear A and planet gear carrier G, where G is driven by the cam. Gear A drives gear B, which turns the planetary gear C, which meshes with gear D on the output shaft. Gear C can freely rotate on its shaft. Gear E is driven by rack F, which is attached to the cam follower. As a result, the output is comprised of the cam and gear A inputs. Figure 10 shows a PGCM with a fixed cam. The planet carrier as well as the planet gear are both driven by the sun gear. The output of the carrier is propelled by the rotation of the planet gear in conjunction with the rotation of the follower in its fixed cam groove. Figure 11 depicts an indexing mechanism made up of a planetary gear and a cam. A planet gear and a cam are fixed in relation to one another, while the carrier rotates at a constant speed around the fixed sun gear. The index arm moves non-uniformly and has dwell periods. Path generation is the process of producing a trajectory for a given point along a predetermined path [16]. Soong [17] created a PGCM for accurate path generation by combining a cam-follower with a PGT. The different links of the PGCM are differentiated by number and color in Fig. 12. If the ratio between the planet and sun gears is one, the coupler point p can create a specified path while the input link makes a single cycle. Motion generation can be defined as the control of a planar link to move through a specified set of successive locations. Soong [18] proposed a rigid body guidance PGCM consisting of two cams with two followers, translating and oscillating. A rigid-body guided PGCM operated by a rotating input link is shown in Fig. 13.

2.6 Graph theory in mechanism synthesis

The graph representation of a mechanism is created by representing each link as a vertex and each joint as an edge [19]. Each edge connection between two vertices corresponds to a link-to-link joint connection. The graph completely ignores the dimensions of the mechanism, preserving only information about the number of links and the type of joints connecting them. A graph is said to be rooted if one of its vertices stands out from the others [20]. In the mechanism, the root represents a unique link, which is usually the ground link (or the frame). Because it is easier to enumerate rooted graphs, one should always attempt to employ the ground link when generating a class of mechanisms. For convenience, we utilize a thick edge to indicate a gear joint, a thin edge to indicate a revolute or prismatic joint, and a thick red edge to indicate a cam joint. Figure 14 is a graphical representation of PGTs, cams, and followers, as well as PGCM. GCMs should be able to produce non-uniform motion in order to be a better candidate for driving variable-speed input mechanisms. Gear cam mechanisms can be classified into two groups, each with its own set of structural characteristics depending on their gear train structures. The first group contains ordinary gear trains, while the second group contains only planetary gear trains [21–27]. The structural synthesis of PGCMs has only been the focus of little research in the past [24]. Because planetary gear-cam mechanisms are of particular relevance to this work, the section that follows will concentrate on their structural characteristics. Furthermore, four- and seven-link gear-cam mechanisms like those shown in Fig. 7 and 9 that have in their structures conventional gear trains should be excluded from further consideration. Based on R2 and R3 above, the gear-cam mechanism with worm gear drive discussed before is an ordinary gear-cam mechanism. When the planet gear in a PGCM is fixed, the planetary gear-cam mechanism transforms into a conventional gear-cam mechanism. For example, the three-jaw self-centering chuck, shown in Fig. 15a, is a type of PGCM with the planet carrier relatively fixed. Fig. 15b illustrates the graph representation of the three-jaw chuck and the input and output links. For the links of the PGT part of the PGCM to possess unlimited rotation (Rule 2), the joints will all be revolute. Consequently, only gear and revolute joints are permitted for the structure synthesis of the PGT part of the mechanism. For the cam-follower mechanism, the prismatic joint is included.

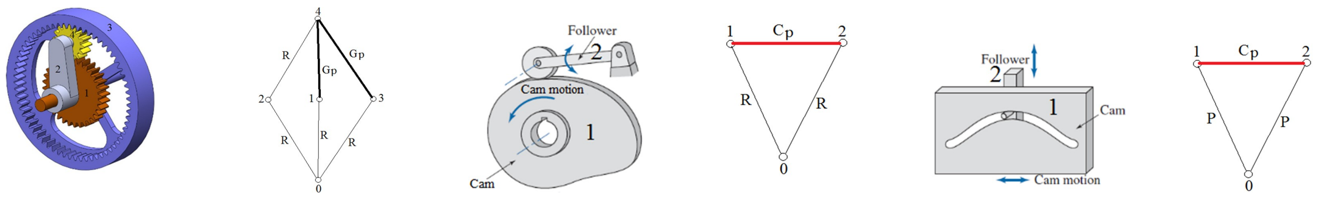
3. Methodology

The methodology aims to systematically enumerate and classify 2-DOF PGCMs using graph theory and structural constraints.

3.1 Overview of approach

The proposed synthesis process follows the following main steps, where Figure 16 summarizes this sequence.

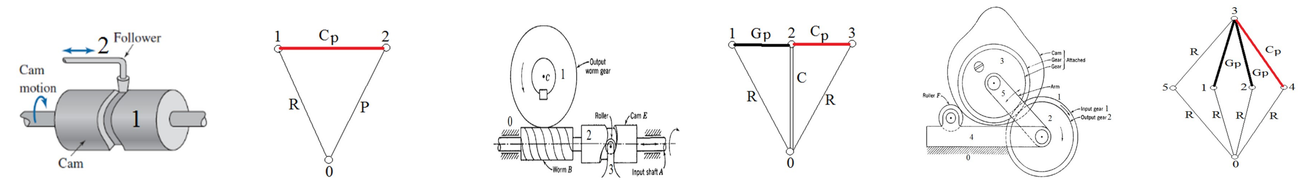
- Define kinematic constraints for 2-DOF PGCMs based on degrees of freedom, joint types, and joint arrangements.
- Represent mechanisms as graphs, with links as vertices and joints as edges.
- Use spanning trees to enumerate all possible basic topologies.
- Add gear and cam joints to construct the parent graphs.
- Verify the genetic compatibility between the spanning trees and the original graphs to identify valid PGCMs.
- Annotate joints as revolute, prismatic, gears, or cams based on structural rules.
- Verify functional feasibility through graphical representation and simulation.



(a) See Fig 2a for details.

(b) See Fig 6a for details.

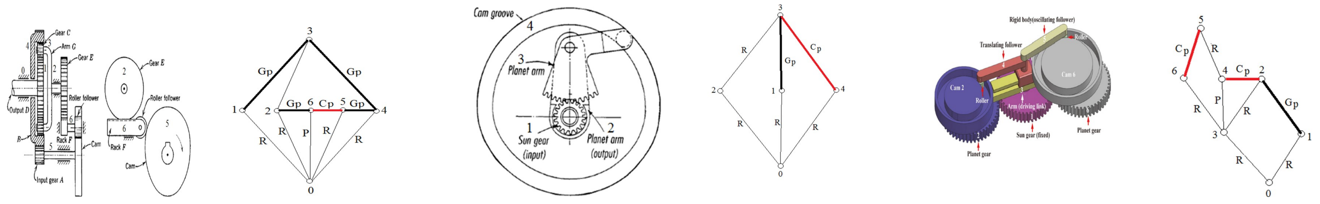
(c) See Fig 6b for details.



(d) See Fig 6c for details.

(e) See Fig 7 for details.

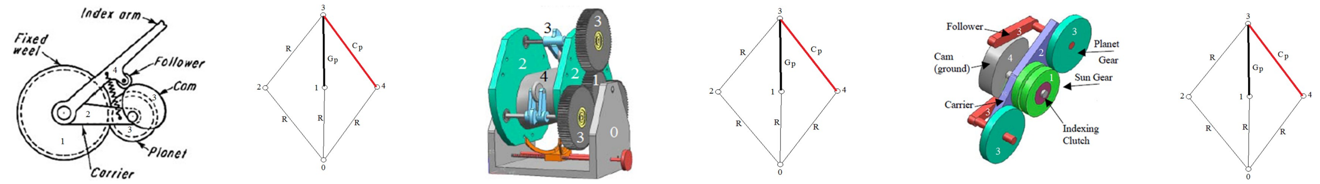
(f) See Fig 8 for details.



(g) See Fig 9 for details.

(h) See Fig 10 for details.

(i) See Fig 13 for details.

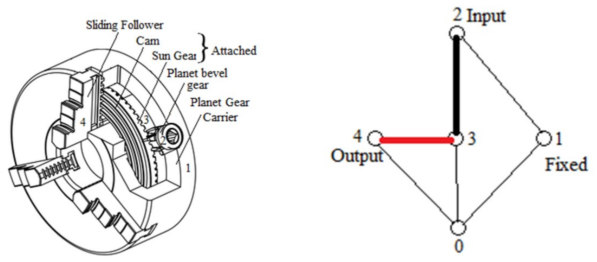


(j) See Fig 11 for details.

(k) See Fig 4 for details.

(l) See Fig 5 for details.

Figure 14. Rooted graphs for different GCMs, (The mechanism and graph representation).



(a) The three-jaw self-centring chuck

(b) Its graph representation.

Figure 15. Three-jaw self-centering chuck.

3.2 Classifying 2DOF planetary gear-cam mechanisms

Let the number of $i-DOFs$ be represented by j_i, therefore Eq. 1.

$$j = j_1 + j_2 \tag{1}$$

Since the prismatic and revolute joints have one degree of freedom and the cam and geared joints have two degrees of freedom, the total DOF in all joints may be stated as Eq. 2.

$$\sum_{i=1}^{j} f_i = j_1 + 2j_2 \tag{2}$$

The DOF of a mechanism can be written according to the Grübler or Kutzbach criteria for n-link, j-joint PGCM as in Eq. 3.

$$F = \lambda(n-j-1) + \sum_{i=1}^{j} f_i \tag{3}$$

where λ is the motion parameter and is equal to 3 for planar and spherical mechanisms. Substituting Eq. 1 and $\lambda = 3$ into Eq. 3 yields to Eq. 4.

$$F = 3(n-1) - 2j_1 - j_2 \tag{4}$$

According to R4, the number of lower pair joints is always one less than the number of links.

$$j_1 = n-1 \tag{5}$$

Substituting Eq. 5 into Eq. 4 yields to Eq. 6.

$$j_2 = n-1-F \tag{6}$$

For two-DOF PGCMs, see Eq. 7.

$$j_2 = n-3 \tag{7}$$

As previously explained, a PGCM combines gear and cam components. Due to the fact that both components have joints with two degrees of freedom, each PGCM will have a minimum of two joints with two degrees of freedom. The number of two-DOF joints is present in Eq. 8.

$$j_2 \geq 2 \tag{8}$$

By substituting Eq. 7 into Eq. 6 and simplifying, we get,Eq. 9

$$n \geq 5 \tag{9}$$

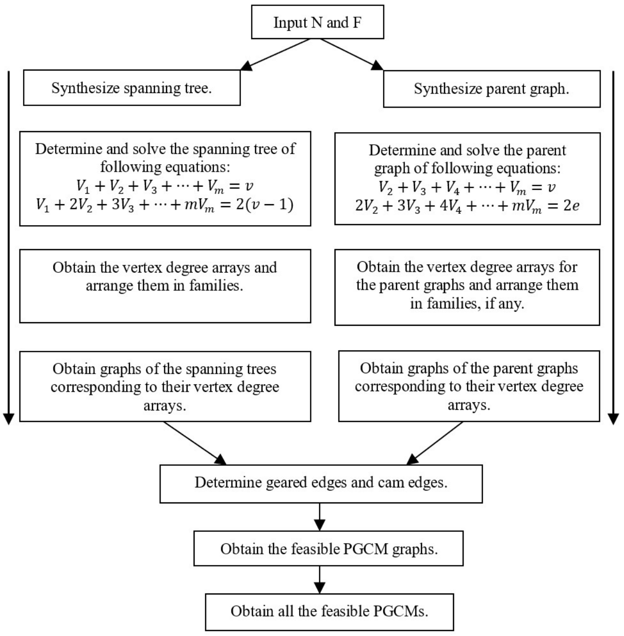


Figure 16. Flow chart for the structure synthesis of PGCMs.

3.3 Structure synthesis of planetary gear-cam mechanisms

To construct the whole set of gear-cam graphs, PGCMs are synthesized using the spanning tree-based method. Figure 16 illustrates the major steps of the synthesis process. Assume that the number of vertices of degree i is denoted by v_i, then Eq. 10.

$$\sum_i^k v_i = v = n \tag{10}$$

Using the fact that k (the maximal degree of a vertex) is greater than the j_2 by one, we have Eq. 11.

$$k = j_2 + 1 \tag{11}$$

Because each edge has two end vertices with which it is incident, it adds two to the total sum of the degrees of the vertices,Eq. 12.

$$\sum_i^k i v_i = 2j \tag{12}$$

The lower bound is $i = 1$ for spanning trees and $i = 2$ for parent graphs.

Table 1. Classification of two-DOF PGCMs with up to 7 links.

Class	No. of links n	No. of DOF F	2-DOF Joints		1-DOF Joints	
			j_2,	Eq. 6	j_1,	Eq. 4
1	5	2	2	—	4	—
			Gear	Cam	R	P
			1	1	4	0
			1	1	3	1
2	6	2	3	—	5	—
			Gear	Cam	R	P
			2	1	5	0
			2	1	4	1
			1	2	5	0
			1	2	4	1
			1	2	3	2
3	7	2	4	—	6	—
			Gear	Cam	R	P
			3	1	6	0
			3	1	5	1
			2	2	6	0
			2	2	5	1
			2	2	4	2
			1	3	6	0
			1	3	5	1
			1	3	4	2
			1	3	3	3

4. Results

4.1 Classification of PGCMs

Any two-DOF PGCM must have at least five links. For $n = 5$ and $F = 2$, Eq. 4 and Eq. 6 result in $J_1 = 4$ and $J_2 = 2$. In light of this, planar two-DOF PGCMs with five links have four one-DOF joints, one cam joint, and one geared joint. As previously discussed, the joints of the PGT part of the PGCM are all revolute joints, thus, the three one-DOF joints associated with the geared joint are revolute joints. The fourth one-DOF joint might be revolute or prismatic. Table 1 presents the classification of planar two-DOF PGCMs for mechanisms composed of 5 to 7 links. These configurations follow the constraints derived from Grübler's criterion and joint-degree relationships. Each configuration ensures F = 2 and includes both gear and cam joints.

4.2 Structural properties

Key characteristics of valid PGCMs include the presence of revolute joints in the PGT component, a prismatic or rotating joint in the cam follower component, and a fixed ground connection as the root of the graph. Table 2 summarizes these characteristics.

Table 2. The structural characteristics for PGCMs.

Property	Structural characteristics
Mechanism type	Planar planetary gear-cam mechanism
Degree of freedom	Planar PGCMs are two-DOF geared mechanisms
Link types	Gear, carrier, cam, follower, and ground link (0).
Joint types	Revolute (R), and prismatic (P).
Ground link	Binary or ternary link.
No redundant connections or partially rigid sub-chains shall be permitted.	

4.3 Enumeration using spanning trees

Below is a case study that illustrates how to systematically construct a graphical representation of a PGCM starting from a LAA, using the graph theory-based synthesis method proposed in this paper.

4.3.1 Spanning trees

The link assortment array (LAA) for a spanning tree is designated as $[[v_1, v_2, v_3, v_4, \ldots, v_k]$, it encodes the count of links in the mechanism, grouped by their degrees. Each LAA satisfies the structural constraints derived from degrees of freedom equations and rules (e.g., Eq. 9 and Eq. 11). All feasible LAAs can be derived for a given n and F. For instance, two LAAs can be obtained for 2-DOF 5-link PGTs are: $[2,3,0]$ and $[3,1,1]$. The vertex degree array (VDA) $[d_1, d_2, d_3 \ldots\ldots d_N]$ is obtained by ordering the degrees of vertices in a descending manner. For instance, the LAA $[2,3,0]$ means that there are two pendant vertices and three binary vertices, so the corresponding degree array is $[2,2,2,1,1]$. By considering all the LAAs in turn, all possible candidate spanning trees can be synthesized. For each VDA, all possible spanning trees (basic, acyclic graphs connecting all links) are enumerated. four spanning trees are listed, two from each family of VDAs. These trees capture the link connectivity but not yet the type of joints (gear, cam, revolute, prismatic). The four spanning trees that correspond to the two VDAs $[2,2,2,1,1]$ and $[3,2,1,1,1]$ are shown in Table 3.

Table 3. The four spanning trees.

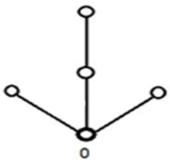
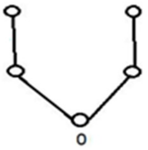
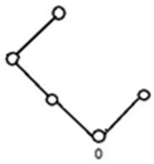
Family 1	Family 2
$[3,2,1,1,1]$	$[2,2,2,1,1]$
 (a)	 (b)
 (c)	 (d)

Table 4**.** The four rooted parent graphs.

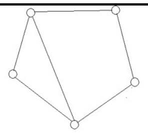
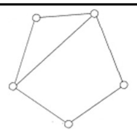
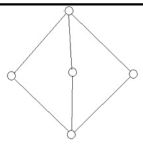
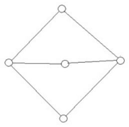
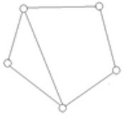
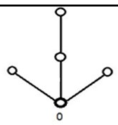
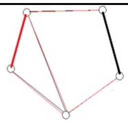
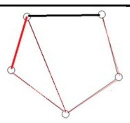
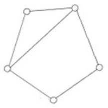
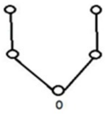
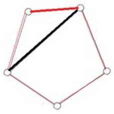
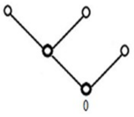
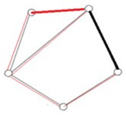
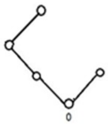
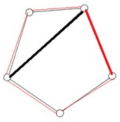
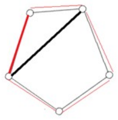
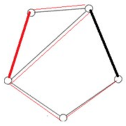
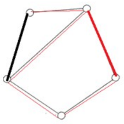
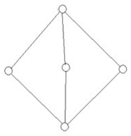
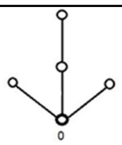
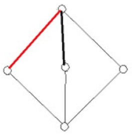
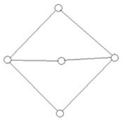
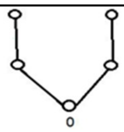
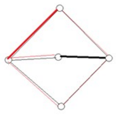
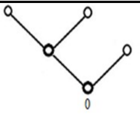
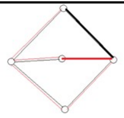
Parent graph 1	Parent graph 2
	
Parent graph 3	**Parent graph 4**
	

Table 5**.** The potential PGCMs graphs.

Family 1			
			
[3,3,2,2,2]	[3,2,1,1,1]		
Family 2			
			
[3,3,2,2,2]	[2,2,2,1,1]		
			
	[3,2,1,1,1]		
			
	[2,2,2,1,1]		
			
Family 3			
			
[3,3,2,2,2]	[3,2,1,1,1]		
Family 4			
			
[3,3,2,2,2]	[2,2,2,1,1]		
			
	[3,2,1,1,1]		

4.3.2 Parent graphs and genetic compatibility

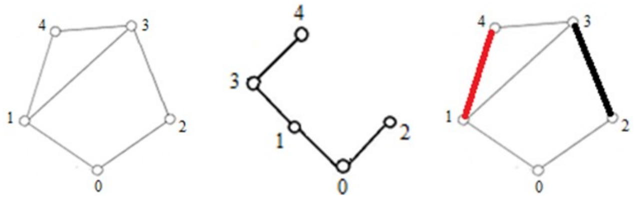
Parent graphs include the remaining connections (i.e., loops) to account for the additional DOF beyond the tree. The link assortment array (LAA) for a parent graph is designated as $[v_2, v_3, v_4, \ldots, v_k]$. For F = 2 and v = 5, only one LAA can be derived from Eq. 9 and Eq. 11, namely, $[3,2]$ and the corresponding vertex-degree array is $[3,3,2,2,2]$. Table 4 shows the four parent graphs for this vertex degree array up to the second level only.

4.3.3 Enumeration of geared-cam graphs

Not all combinations of spanning trees and parent graphs produce valid mechanisms. The concept of genetic compatibility is used: only those spanning tree and parent graph pairs that can coexist structurally and kinematically form feasible PGCMs [28–30]. Table 5 shows how different families of spanning trees and parent graphs match to produce valid graphs representing PGCMs. Once a compatible graph is identified, edges are annotated: thick edges for gear joints, thin edges for revolute/prismatic joints, and a thick red edge for cam joints. This annotation is determined by comparing spanning trees and parent graphs and assigning the additional connections as gear or cam joints based on structural rules.

4.4 Functional designs and simulations

An example of this is a five-link PGCM model, which combines a ring gear, a planet gear, a carrier, a cam, and a follower in a compact configuration capable of path generation. In the graphs in Fig. 17, vertex (1) is the carrier of planet gear (3), vertex (2) is the coaxial gear, and vertex (4) is cam follower (1). Figure 18 shows a functional diagram that corresponds to Fig. 17c. This new path-generation mechanism is a 5-link PGCM with 3R, G_p, and C_p. The PGT consists of ring gear (2), planet gear (3), and planet carrier (1). Planet carrier (1) is coupled to the cam and acts as an input link, while ring gear (2) is fixed. The path of the coupler point is determined by the motion of the roller in the moving-cam groove and the motion of the follower around the axis of the planet gear. The new mechanism is simple and compact, and it can generate continuous and symmetric curve paths. This practical example demonstrates how the proposed methodology combines abstract structural formulation (via graphs and constructs), kinematic feasibility (via degree-of-freedom equations and constraints), and practical application (via the identification of real joints and motion simulation). It demonstrates, step by step, how to move from a purely mathematical LAA model via VDA, spanning trees, and parent graphs to a realistic and efficient PGCM design.



(a) Rooted parent graph. (b) Spanning tree. (c) PGCM graph.

Figure 17**.** The cam and geared edges are detected using the variation between the spanning tree and the parent graph.

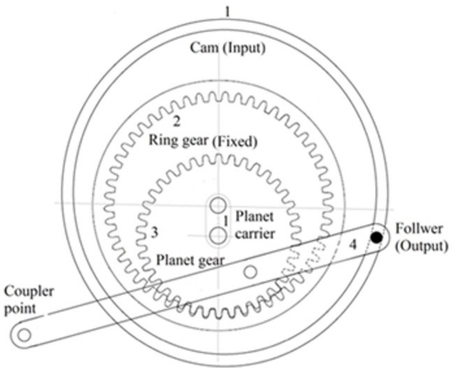


Figure 18**.** Functional diagram of the selected PGCM showing the mechanical layout.

5. Discussion

The structural synthesis of PGCMs based on graph theory and spanning trees has led to a comprehensive classification framework that enables a systematic enumeration of feasible 2-DOF mechanisms. The methodology focuses on mechanisms with five to seven links, identifying configurations with practicality and minimal complexity. Using the VDA LAA, a complete set of candidate mechanisms was generated. For the five-link PGCMs, the analysis revealed that only two LAAs $[2, 3, 0]$ and $[3, 1, 1]$ satisfied the required structural constraints. These models were used to generate four distinct spanning trees, as summarized in Table 3. Similarly, the VDA-derived parent-root graphs $[3, 3, 2, 2, 2]$ were matched with the generated spanning trees to identify compatible PGCM structures. Four parent graphs were generated and evaluated using genetic compatibility tests. Intersecting the valid spanning trees and parent graphs yielded potential PGCM models, as shown in Table 5 and Fig. 17. The selected PGCMs were analyzed for compliance with the constraints imposed by degrees of freedom, joint types, and link arrangements. All synthesized mechanisms included one cam pair and one gear pair, as required for 2-DOF systems. The classification confirmed that PGCMs with fewer than five links were structurally infeasible, consistent with the degrees-of-freedom equation derivations. Common structural features observed in valid PGCMs include:

- Revolute joints are used in all components of a planetary gear train,
- A single prismatic joint is typically associated with the cam-follower pair,
- A fixed ground link serves as the root in the graphs.

Table 2 summarizes these consistent features and establishes the theoretical basis for the synthesis. Fig. 18 shows a functional model of a selected five-link PGCM, combining cam and planetary gear systems into a compact unit. Simulation results Fig. 19, indicate that this structure can achieve smooth, continuous, and symmetrical path generation, with potential applications in path control systems. This simulation demonstrates the effectiveness of the proposed synthesis method, not only in structural design, but also in providing practical and controllable motion patterns, overcoming the limitations of conventional designs that rely solely on cams or gears.

The use of graph theory greatly simplifies the structural synthesis process. By encoding joints and links as vertices and edges, a precise and automated framework is developed for identifying structurally valid PGCMs. The spanning tree approach ensures the comprehensiveness of the enumeration process, while compatibility checks with the parent graphs provide a logical mechanism for design validation. The study also demonstrates the scalability of this method to more complex structures and its potential integration with computer-aided design/computer-aided manufacturing (CAD/CAM) systems for design automation.

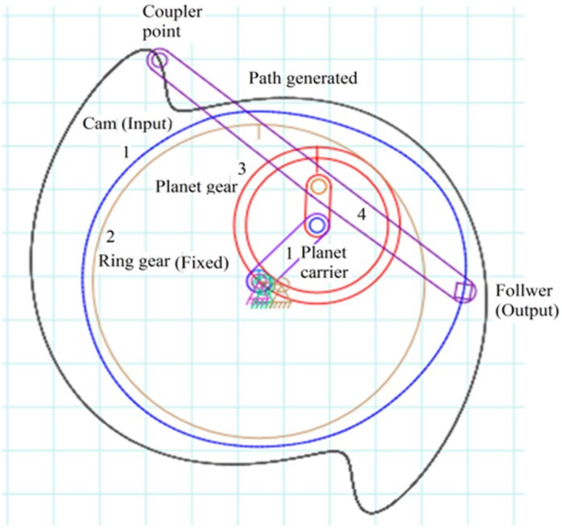


Figure 19**.** Simulated path generated by the mechanism.

6. Conclusions

This study presents a comprehensive framework for the structural synthesis and classification of PGCMs, which are essential for the design of high-performance motion systems, such as index tables, robotic drives, and variable transmissions. By integrating the functions of both PGTs and cam mechanisms, PGCMs provide an ideal solution for generating complex and irregular motion with high accuracy and reliability. The research demonstrates how to leverage spanning trees, graph theory, and kinematic rules to enumerate and classify 2-DOF PGCM models of up to seven joints. This process, supported by a systematic flowchart and graphical representation, enables the identification of possible configurations based on structural constraints, such as joint types, link connectivity, and degrees of freedom. Furthermore, the concept of genetic compatibility between spanning trees and the parent graphs proves effective in revealing the positions of geared and cam joints, facilitating the construction of new, functionally sound PGCM models. Several practical PGCMs are proposed and analyzed in this work, including a newly designed five-link mechanism capable of generating a continuous trajectory. The simplicity, small size, and versatility of this configuration point to promising applications in areas requiring symmetric motion generation and space-saving design. Overall, the structural classification and enumeration approach developed in this study enhances the understanding of the PGCM design space and provides a valuable foundation for future developments in robotic mechanism synthesis, especially for applications in high-speed and precision mechanical systems.

Authors' contribution

All authors contributed equally to the preparation of this article.

Declaration of competing interest

The authors declare no conflicts of interest.

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Data availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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