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العدد السابع
والثلاثون

((النموذج الرياضي للتحكم في الصيد التجاري من خلال التحسين التفاضلي العددي))

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(الرياضيات التطبيقية)

المستخلص:

في الصيد التجاري، يسعى الصيادون إلى الاعتماد على دالة ربحية متكاملة للتحكم الأمثل في تكلفة الصيد وعائدات بيع الأسماك. ويتأثر الصيد بكمية الأسماك التي يتم صيدها سنوياً. والهدف هو تعظيم عائدات الصيد على مدى خمس سنوات. في حال الصيد الجائر (ارتفاع معدل الصيد)، تنخفض عائدات السنوات اللاحقة ولا يتعافى مخزون الأسماك. وتتخلص هذه المسألة في إيجاد الشكل الأمثل للاستخراج من أجل تعظيم ربح الصيد التجاري. لذا، قمنا بإنشاء نموذج رياضي لحل مشكلة التحكم الاقتصادي الأمثل في الصيد التجاري. استخدمنا برنامجاً لتحسين النموذج وعرض النتائج.

الكلمات المفتاحية: التحكم الأمثل في الصيد، استراتيجية الصيد، هيكلية مصايد الأسماك، المنهجية.

**A mathematical control model for commercial fishing through
Numerical Differentiation Optimization**

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Abstract

Commercial fishing in which fishermen seek to rely on an integrated profit function to optimally control the cost of fishing and revenues from fish sales. Fishing is influenced by the number of fish caught each year. The goal is to maximize fishing returns over 5 years. If there is overfishing (high v), subsequent years' revenues decline, and the fish population does not recover.



This control problem boils down to finding the optimal form of extraction to maximize the profit of commercial fishing. Create a mathematical model to solve the problem of economic control of optimal commercial fishing. So we used software to optimize and display the results.

Keywords: Optimal Control for Fishing, Fishing Strategy, The Structure of Fisheries, Methodology.

INTRODUCTION

Even while overfishing is the fisheries environmental concern that receives the greatest attention, numerous other challenges could also have an impact on the global seafood trade (Yogatama & Wibisono, 2025). To put it simply, unchecked global overfishing will have an impact on the amount of seafood available for trade internationally. For example, it is undeniable that a large number of fish stocks worldwide are damaged (Kim et al., 2021).

In addition, overfishing affects ecological equilibrium and jeopardizes biodiversity by depleting fish stocks to unnecessarily low levels. However, a wide range of environmental concerns about fisheries and fish production, some of which cross national boundaries, others do not, as well as domestic fisheries management and international agreements, may interact to produce much more subtle effects on the global trade in seafood products. WTO regulations about anti-dumping, countervailing, and technical trade barriers, as well as subsidies (Asani et al., 2024). May clash with these interactions.

On the other hand, concerns about seafood safety are more likely to surface in relation to sanitary and phytosanitary regulations (Rokhani et al., 2021). There is growing concern that many countries lack the infrastructure needed to manage fish stocks sustainably, guard against food-borne illnesses, and lessen the environmental effects of fishing and fish farming (Ramezani et al., 2024). This is because globalization has significantly increased the amount of fish traded internationally (Almufti et al., 2021).

With a greater fish stock than under open access, fisheries will normally function on the traditional section of the supply curve, which is not backward-bending (Alkhalifa et al., 2025). Several theoretical findings regarding trade and renewable resources are based on the supply of fish,



which bends backwards. As a result, depending on how successful resource management institutions are, trade liberalization may reduce welfare.

New Mathematics Model: (Maximize)

Many systems where the accumulation of a quantity is maximized or decreased naturally represent themselves as integrals. The profit function over the course of five years is the integral (Xiao et al., 2024).

$$\max_{v(t)} \int_0^5 (L - \frac{c}{z}) v Vmax dt$$

Subject to

$$\frac{dz}{dt} = r z(t) (1 - \frac{z(t)}{k}) - v Vmax$$

$$Z(0) = 60$$

$$0 \leq v(t) \leq 1$$

$$L = 1, c = 17.5, r = 0.71, k = 80.5, Vmax = 20$$

A new variable is defined to rewrite integral expressions in differential and algebraic form.

$$g = \max_{v(t)} \int_0^5 (L - \frac{c}{z}) v Vmax dt$$

There is an additional equation that includes the differentiation of the new variable g and an integral. At the last moment, the issue shifts to maximizing the new variable g .

$$\max_{v(t)} g(t_f)$$

Subject to

$$\frac{dz}{dt} = r z(t) (1 - \frac{z(t)}{k}) - v Vmax$$

$$\frac{dg}{dt} = (L - \frac{c}{z}) v Vmax$$

$$Z(0) = 60$$



$$g(0) = 0$$

$$0 \leq v(t) \leq 1$$

$$L=1, c=17.5, r=0.17, k= 80.5, Vmax = 20$$

In the time interval 0 to 5, the integral g 's initial condition, which begins at zero, becomes the integral (Zheng et al., 2021). In the end, the optimum control problem's end value is maximized within the given time horizon. Multiplying the objective by a negative number transforms a maximizing problem into a minimization challenge (Drugo\vna et al., 2020).

$$\max_{v(t)} g(t_f) = - \min_{v(t)} g(t_f)$$

Optimal Control for Fishing

An optimal control problem can be used to determine the best course of action for a fishery (Achema et al., 2022). We take the variable $z(t) \in R^n$ that fulfills the equation for an ordinary differential A single fishery is first taken into consideration while developing a fishery resource management model. The canonical fishing model gives rise to the fundamental bio-economic model (Pironneau, 2023). The state equation indicates that the fish stock is changing :

$$\dot{z}(t) = \frac{dz}{dt} = F(t) - d(t)$$

where $z(t)$ is the fish biomass, $d(t)$ is the harvest rate, and $F(x)$ is the natural growth function. The Shaefer model expresses this as follows: $F(x) = r x(1-x/K)$, where K is the maximum biomass, or $\lim_{t \rightarrow \infty} z(t) = K$. The planner uses the formula $h(t) = gL(t)x(t)$ to determine the harvest, where $L(t)$ is the rate of fishing effort at time t and g is the "catchability" coefficient (Humphrey et al., 2023).

For commercial fishermen, choosing a place at the trip level is a short-term, dynamic spatial decision (Hammarlund et al., 2021). Selecting fishing locations to optimize the total trip earnings is the difficulty that each fisher faces. Discontinuities in the spatial structure are caused by the spatial patchiness of marine resources that have varying population densities and economic attributes (Aljehani et al., 2023). In commercial fisheries, spatial



decisions are therefore modeled as discrete choices among a finite collection of fishing sites(Ghambari et al., 2024).

Fishing Strategy

According to conventional economic theory, producers will choose the production level at which the marginal cost and the output price are equal (Hammarlund et al., 2021; Zekavat et al., 2021). This also happens when the output price is equal to the average cost and the resulting biomass stock and effort are, respectively(Jabari et al., 2024), smaller and higher than in the ideal fishery in a competitive fishery with free entry and exit (open access).

The harvest level will typically be lower as well; this is not guaranteed(Jabari et al., 2024). This suggests that management is necessary if society is concerned with efficiency, namely, to coordinate fishermen's actions to create an ideal fishery. As a result, one additional research task in policy analysis is to forecast how fishermen will respond to changes in fishery management. For example, if marine reserves are implemented as a management tool or if inputs or outputs are restricted, the primary concern is forecasting how fishermen will modify their fishing practices.

Using this data(Mustafa & Abboud, 2025), the general normative problem, that is, whether the policy achieves the desired levels of effort, biomass, harvest, and predicted economic surplus, can be addressed. The fundamental externality problem, or the stock externality(Ghouali et al., 2022), is at the heart of fishery management. It refers to the fact that individual fishermen fail to consider the impact their fishing has on other fishermen's unit costs due to changes in the stock(Guo et al., 2025).

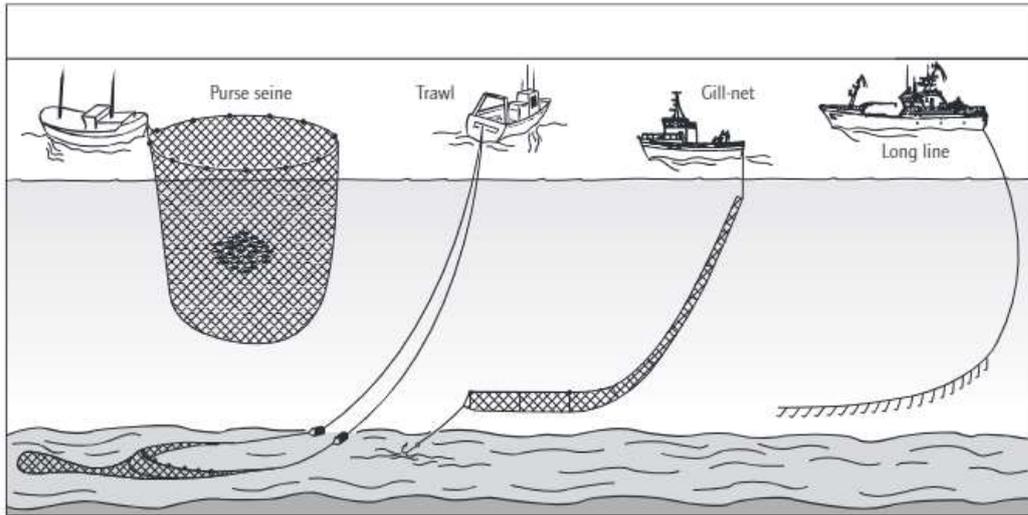


FIGURE 1. - Fishing Strategy

The Structure of Fisheries

Fisheries can be divided into subsectors based on the fish production chain (Pierallini et al., 2025), which includes fishing as well as fishing support, fish processing, and fish marketing. The fishery production chain and its connections to social and ecological systems are depicted in Figure 1, along with the structural ties between fishing and other pertinent sectors. Initially, the main factor influencing fishing activity is the variable of fishing effort (Liu et al., 2023).

The fishing industry drives the fishing support sector from the back, providing services like maintenance and production of gear. Fish processing is a forward-thinking industry that uses specific methods, such as freezing, to prepare raw fish. brining, preserving, seasoning, etc (Skinderowicz, 2022) (Parker et al., 2015; Teh et al., 2017). Propelled forward by the fishing and processing industries, the fish marketing industry offers both raw and processed fishery goods for sale. All the non-fisheries sectors of the economy are dragged backward by the fisheries sectors collectively. Métiers are a group of diverse micro fishers that fall under the fishing industry category (Song et al., 2025). A fleet that is outfitted with specific gear and targets a specific species as its primary catch, with other potential species



being caught as bycatch, is referred to as a fishing métier. More than one target species is typically associated with a métier (Xu & Zhang, 2024). Still, the idea that a single producer type can only generate a single product type is one of the basic tenets and constraints of input-output models.

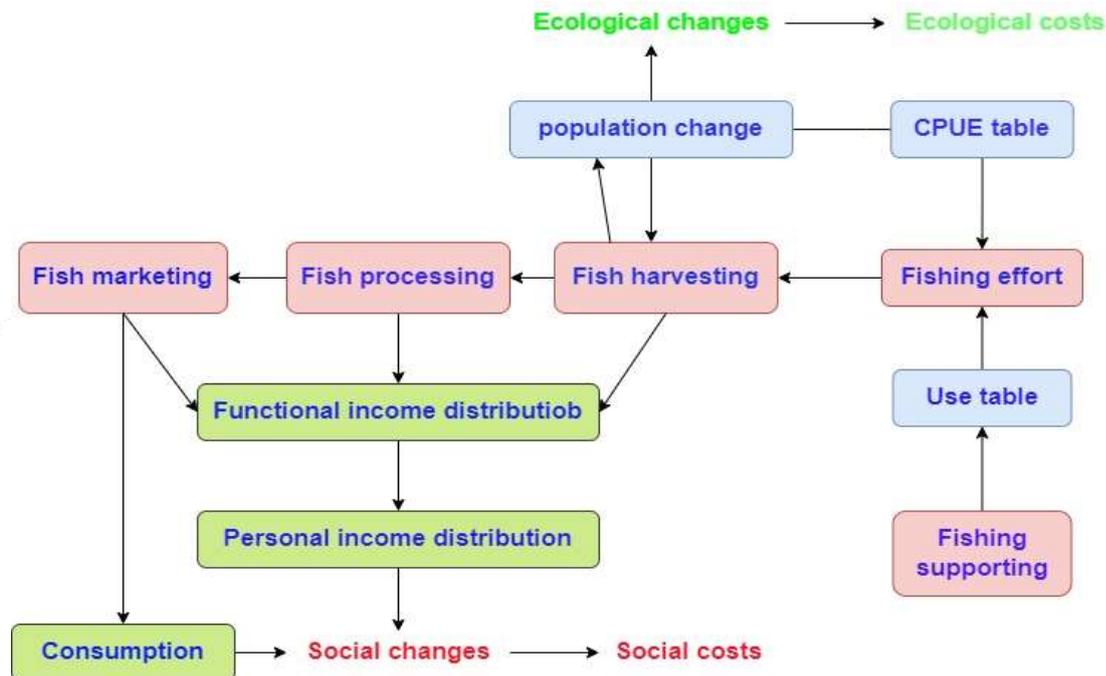


FIGURE 2. - The Structure of Fisheries

Methodology

We will create a model using Python Gekko. First, we will write some special Gecko packages. Next (Hansen et al., 2022), we have some time points we want to solve from 0 to 5 years using 501 time points, and we also have the seconds that were in the originally defined problem (Domínguez-May et al., 2024). We also write the constants L , c , r , and V_{max} . In the next step, we place our catch rate within the time period from 0 to 5 years, where we start from a cost optimization of D equal to 0 at no cost. After that we have an initial state of the fishing population equal to 60 and then we perform the equilibrium equation for the fish using the differential equation and at a later stage we have our target which is initially 0 and then we define the final target which we will know as FB (Sahoo et al., 2023), which will be



just one value over the entire time horizon, The state will be equal to one meaning, which is our goal (Nataraj & Subramanian, 2025).

After that we have a connection with gekko and we will link it permanently to the topic g to optimize g where the differential equation for profit and we will maximize the profit by minimizing the negative profit which is the optimal control solution, so it will be optimized over a dynamic horizon of nodes (Smith-Miles et al., 2025), where the greater the number of nodes, the greater the accuracy of the solution for How to solve Ipop.

Numerical Results

We use the new fishing model to determine the best time and lowest cost to solve the problem, as shown in Figure 4 and Table 1,2.

Table 1 - Mathematics Model

Iter	Objective	Inf-pr	Inf-du	Alpha-du	Alpha-pr
0	0.0000000e+00	1.40e+01	1.00e+00	0.00e+00	0.00e+00
1	5.4482962e+04	2.75e+01	9.53e+01	4.02e-14	1.44e-12f
2	-2.1792602e+04	3.03e+01	2.67e+04	1.01e-12	3.95e-13f
3	-1.3650910e+04	3.04e+01	2.25e+05	1.05e-03	4.53e-04f
4	1.1528297e+04	3.11e+01	1.58e+05	1.26e-09	2.71e-09f
5	1.1248938e+04	3.11e+01	1.44e+05	3.58e-08	3.00e-08f
6	1.1568454e+004	3.11e+001	4.02e+005	4.80e-007	4.55e-007f
7	1.1296565e+004	3.11e+001	5.94e+004	7.04e-006	7.26e-006f
8	1.1559240e+004	3.11e+001	2.11e+006	1.11e-004	1.06e-004f
9	9 1.1214841e+00	3.10e+001	1.25e+007	1.77e-003	1.64e-003f
10	1.0223370e+004	3.02e+001	1.30e+008	3.28e-002	2.59e-002f
11	-1.1463049e+004	1.80e+001	4.41e+008	5.60e-001	4.53e-001f



12	-3.6345800e+004	3.72e+000	4.41e+006	9.90e-001	9.90e-001f
13	-3.5905538e+004	2.43e-001	6.46e+003	9.90e-001	9.90e-001h
Iter	Objective	Inf-pr	Inf-du	Alpha-du	Alpha-pr
14	-3.5879210e+004	4.13e-004	2.05e+006	9.90e-001	1.00e+000h
15	-3.5778107e+004	4.67e-003	1.08e+012	9.93e-001	4.02e-002f
16	-3.5779985e+004	1.60e-005	1.74e-001	1.00e+000	1.00e+000f
17	-3.5782991e+004	4.07e-008	1.08e+010	9.94e-001	1.00e+000f
18	-3.6263906e+004	1.04e-003	1.05e-003	1.00e+000	1.00e+000f
19	-5.0205804e+004	9.21e-001	1.02e+000	1.00e+000	1.00e+000f
20	-5.8478879e+004	1.31e+000	2.63e+000	1.00e+000	1.00e+000f
21	-5.7896547e+004	2.90e-001	1.20e+000	1.00e+000	1.00e+000h
22	-5.7812571e+004	2.98e-001	3.97e+003	1.00e+000	9.82e-001h
23	-5.7816639e+004	2.38e-002	5.71e-003	1.00e+000	1.00e+000h
24	-5.7852926e+004	1.08e-002	2.79e-003	1.00e+000	1.00e+000f
25	-5.7871316e+004	4.29e-003	7.26e-003	1.00e+000	1.00e+000f
26	-5.7873416e+004	1.78e-003	4.29e-003	1.00e+000	1.00e+000h
27	-5.7873813e+004	7.52e-004	2.17e+001	9.94e-001	1.00e+000h
28	-5.7873912e+004	1.85e-004	3.84e-004	1.00e+000	1.00e+000h
29	-5.7873935e+004	2.38e-005	9.43e-001	9.96e-001	1.00e+000h
30	-5.7873936e+004	2.84e-006	6.93e-006	1.00e+000	1.00e+000h
31	-5.7873936e+004	2.42e-007	2.70e-003	1.00e+000	9.94e-001h



32	-5.7873936e+04	1.29e-08	9.13e-06	1.00e+00	1.00e+00h
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INFO(1) = -9 was returned by MUMPS, and additional memory needs to be reallocated. First Strike.

Moving icntl[13] up from 1000 to 2000. MUMPS needs to reallocate more memory and returned an error with the value INFO(1) = -9. Second Try

An increase in icntl [13] between 2000 and 4000. MUMPS needs to reallocate more memory and returned an error with the value INFO(1) = -9.

Strike Three

Increasing icntl [13] from 4000 to 8000.

33 -5.7873936e+004 1.53e-014 6.55e-009 -11.0 6.55e-005 -4.0 1.00e+000
1.00e+000h

Number of Iterations.....: 33

Table 1. - Mathematics Model

	scaled	unsealed
Objective	-5.7873936428602829e+03	-5.7873936428602829e+04
Dual infeasibility	6.5511615743667048e-09	6.5511615743667046e-08
Constraint violation	1.4210854715202004e-14	1.4210854715202004e-14
Complementarity	1.0288582522487802e-11	1.0288582522487801e-10
Overall NLP error	6.5511615743667048e-09	6.5511615743667046e-08

Quantity of evaluations for objective functions = 37

Evaluations of objective gradients = 34

Count of evaluations for equality constraints = 37

Quantity of evaluations for inequality constraint = 37

As many equality constraints as possible, Jacobian assessments = 34

Number of constraints on inequality Jacobian assessments = 34

Evaluations of the Lagrangian Hessian = 33

Total CPU secs in IPOPT (w/o function evaluations) = 42.510



Total CPU secs in NLP function evaluations = 2.379

EXIT: Optimal Solution Found.

The solution was found.

The final value of the objective function is -57873.93642860283

Solution time: 32.7573000000011 sec

Objective : -57873.9364286028

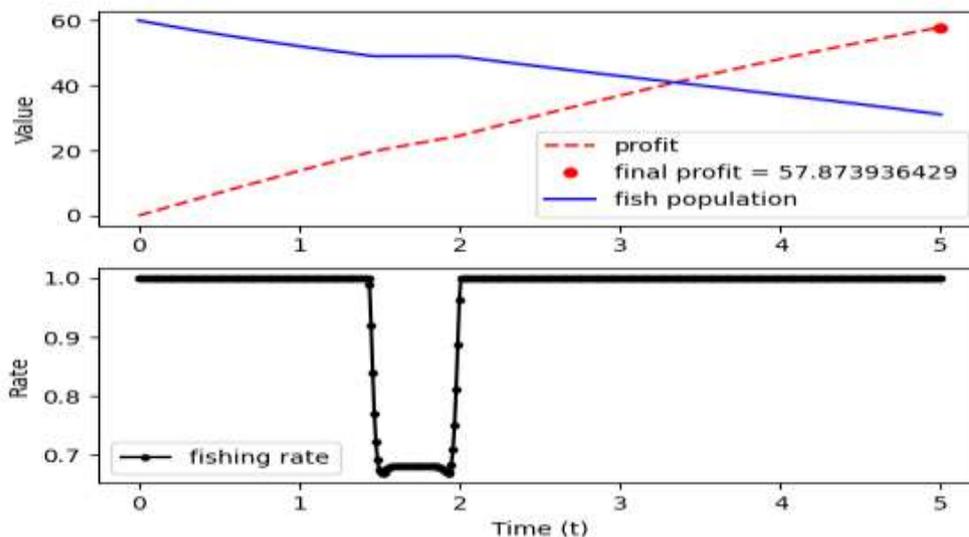


FIGURE 3. - Math Model (Maximize)

The Description

The problem of finding the ideal amount of fish that we can catch by maximizing profit is addressed through a simple differential equation that relates to the number of fish, where we have an initial number of fish, which we assume to be equal to 60 during a time period of 32.7573000000011 seconds, which is a small fraction of the amount of time that we cover. In fishing during the year, we also have some constants $L=1$, $c=17.5$, $r=0.71$, $k=80.5$, $V_{max}=20$, which are the constant factors.

These parameters enable us to update the system through the equation found here for the number of fish. We have maximized that function set by using optimization tools, as we are working to reduce it, and this is not a problem, as we can multiply by negative one, and this turns the maximization problem into a minimization of 57.873936429 - which is the



objective function. We notice from the drawing that the blue line will represent the fish population, where we observe the fish populations and what they do over time. The catch rate will be in black with some points of increase, which we know is 501 points in time, and the ideal profit was 57.873936429.

Conclusion

Balancing the economics of fishing costs with the reproduction of fish each year gives us very good fishing the following year. As a result, our average fishing intensity drops to just below 0.7, and then at the end of the last three years, we return to the maximum again. This means that the work will stop after five years, as the interest will now decrease in the number of fish, so we will abandon it until it does not reach zero, but rather decreases. This is a type of risk in commercial fishing, because it may compromise the sustainability of the resource. An economy that basically tries to maximize profit by controlling the amount that we catch, which is related to the amount of fish that we will withdraw and be able to sell, and we must maintain them at the previously described levels for five years, and then maintain the fish population to maximize profit revenues and have sustainable fish populations in the long term.

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