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توظيف عينات المراقبة من النوع الاول بأسلوب المحاكاة

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المستخلص:

في هذه المقالة لقد قمنا بتعريف توزيع رايلي الاسي المعمم وكذلك دالة الموثوقية تحت ظل عينات المراقبة. تلعب عينات المراقبة دورا مهما في تطبيقات الحياتية، وتنقسم الى ثلاث اقسام: عينات المراقبة من اليسار، عينات المراقبة من اليمين وفترات العينات. في هذه الدراسة كان التركيز على عينات المراقبة من اليمين والتي تعرف عينات المراقبة (النوع الاول) طريقة الامكان الاعظم تحت عينات المراقبة من النوع الاول. تم تقدير واشتقاق ثلاث معلمات لتوزيع رايلي الاسي المعمم باستخدام صيغة الامكان الاعظم للنوع الاول من عينات المراقبة من اليمين. تم ايجاد قيم المعلمات باستخدام خوارزمية نيوتن رافسون. تم اعتماد اسلوب المحاكاة لتقدير الدالة الموثوقية بطريقة مونت كارلو للمحاكاة ذات احجام مختلفة من العينات مع قيم ابتدائية مختلفة لجميع المعلمات لتوزيع رايلي الاسي المعمم باستخدام برنامج ماتلاب .تم اعتماد تقنية متوسط مربعات الخطأ (MSE) للمقارنه بين الدالة الموثوقية قبل التقدير وبعده. وفي النهاية تم ايجاد دالة الاحتمالية والموثوقية.

الكلمات المفتاحية: توزيع رايلي الاسي المعمم، عينات المراقبة، دالة الموثوقية، المحاكاة، عينات مراقبة من النوع الأول.

Employing Type one Censored Sample with Simulation

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1.Abstract

In this article, the generalized exponential Rayleigh distribution and reliability function are defined by using a censored sample. Censored data plays an important role in life over time and is divided into three branches: left-censored data, interval, and right-censored data. In this paper, we study the part of right censoring called (type one) by explaining the maximum likelihood method under type one. We are estimating and deriving the three parameters of the Generalized Exponential Rayleigh distribution under the formula of type one; this method depends on the Newton-Raphson method. to find the estimate of the reliability function using the simulation procedure by the Monte Carlo technique under different sample sizes and various initial values for the parameters for all estimated parameters of the generalized exponential Rayleigh distribution by applying the initial values in the MATLAB program, then comparing the estimated reliability function with the empirical reliability function by utilizing the mean squares error procedure. Finally, finding the probability density function f(t), the reliability function.

Keywords: GERD, Censored sample, Reliability function, Simulation, type three censored (Progressively).

2.Introduction

The reliability function that depends on time is one of the most often utilized functions in lifetime data analysis. When age data are evaluated, it is necessary to find that certain units fail or die while other units do not fail or die, and these units last longer than the experiment's duration without failing or dying. Censored samples are the most common in life. Three branches make up censored data: right, interval, and left censored. Three categories of censorship are included in the right-censored: progressive, type two, and first-class. We show the estimation of the new mixture. Distribution with simulation, using a three-parameter model, Alkinani and Shalan created a novel mixture in 2023 known as the Generalized Exponential Rayleigh. Using a three-parameter model, Alkinani and Shalan presented the mathematical and (Alkanani & Shalan, 2023) statistical properties of the proposed distribution in 2023. This distribution is the generalized exponential Rayleigh distribution. We will also provide some previously explored studies on censored data. In Iden and Qesma submitted Estimating the Survival Function under Type One Censoring Sample for Mixture Distribution (S. Abadi & H. AL-Kanani, 2020). In 2023, Iden and Riham published 'Finding the Exponential-Rayleigh Distribution's Parameters under Type-I Censored Data Abstraction' (Journal et al., 2023). Also, A. M. J. Mohammed and A. T. Mohammed (2021), Parameter Estimation of Inverse Exponential Rayleigh Distribution Based on Classical Methods (M. J. Mohammed & Mohammed, 2021). In 2022, Ali and Mayasa presented The Inverse Exponential Rayleigh Distribution and Related Concepts (A. T. Mohammed et al., n.d.). In 2019, Maysa and Iden estimated the parameters of the new mixture distribution using classical methods. (Jalil & Hussein, 2019)

3. Generalized Exponential Rayleigh Distribution:

Iden and Rehab introduced a new distribution known as the Generalized Exponential–Rayleigh Distribution. With three parameters $\alpha, \theta,$ scale parameters, and γ shape parameter. In this work, we present some properties of this distribution (Rehab Noori Shalan et al., 2023)

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- The Reliability function of this distribution is as follows:

$$R(t_r;\alpha,\theta,\gamma) = \exp\left\{-\left(\alpha t_r^{\frac{1}{\gamma}} + \frac{\theta}{2} t_r^{\frac{2}{\gamma}}\right)\right\} \qquad \alpha, \theta, \gamma > 0,\ t_r > 0$$

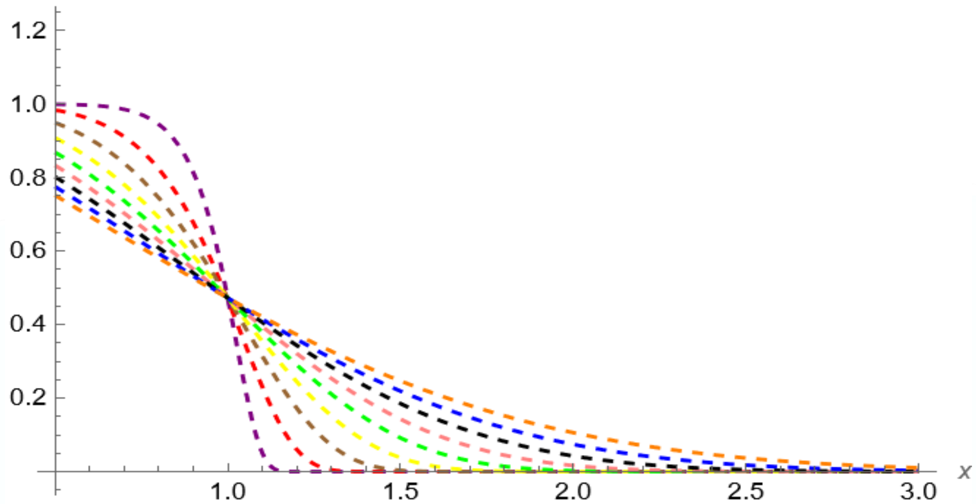

Figure 1 : The reliability function of GER distribution when scale parameter ($\alpha = 0.5, \beta = 0.5$) and different value of shape ($\lambda = 0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9$)

- The cumulative distribution and probability density function are as follows:

$$F(t) = 1 - \exp\left\{-\left(\alpha t_r^{\frac{1}{\gamma}} + \frac{\theta}{2} t_r^{\frac{2}{\gamma}}\right)\right\}$$

$$f(t_i;\alpha,\theta,\gamma) = \left(\frac{\alpha}{\gamma} t_i^{\frac{1}{\gamma}-1} + \frac{\theta}{\gamma} t_i^{\frac{2}{\gamma}-1}\right) e^{-\left(\alpha t_i^{\frac{1}{\gamma}} + \frac{\theta}{2} t_i^{\frac{2}{\gamma}}\right)} \qquad ; t_i \geq 0$$

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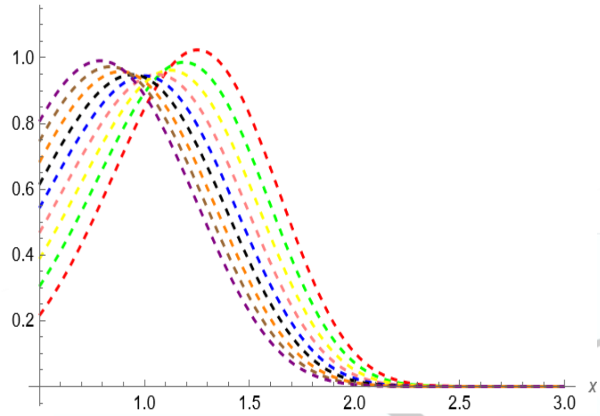

Figure 3 : The density function of generalized exponential –Rayleigh distribution with scale parameter($\beta = 0.5$) ,shape ($\lambda = 0.5$)and different scale($\alpha = 0.1,0.2,0.3,0.4,0.5,0.6,0.7,0,8,0.9$)

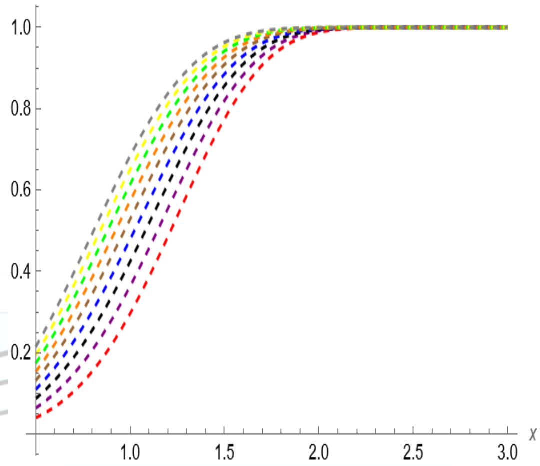

Figure 2: Cumulative function with constant scale ($\beta = 0.5$) ,shape ($\lambda = 0.5$)and different scale($\alpha = 0.1,0.2,0.3,0.4,0.5,0.6,0.7,0,8,0.9$)(shalan, 2023)

- The moment generating for GER distribution formula

$$\mu_x(t) = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{k^m}{m!}\frac{(-\alpha)^n}{n!}\left(\frac{2}{\beta}\right)^{\lambda m+n+\frac{3}{2}}\left(\frac{\alpha+\beta}{2}\right)\left[\Gamma\left(\frac{\lambda m+n+1}{2}\right)+\Gamma\left(\frac{\lambda m+n+2}{2}\right)\right]$$

- The Mean of this distribution is when r=1

$$E(x) = \sum_{n=0}^{\infty}\frac{(-\alpha)^n}{n!}\left[\frac{\alpha}{2}\left(\frac{2}{\beta}\right)^{\frac{n+\lambda+1}{2}}\Gamma\left(\frac{n+\lambda+1}{2}\right)+\left(\frac{2}{\beta}\right)^{\frac{n+\lambda}{2}}\Gamma\left(\frac{n+\lambda+2}{2}\right)\right]$$

4. Censored Data:

Censored samples play an important role in our lives as they help us determine the time of an event or the lifetime of a device, or at least provide partial information about the unit's lifespan to prevent potential future problems. For example, if we take 50 devices and stop the experiment after 20 days, some devices may not have failed by that time. This is called censoring because it does not provide the complete lifetime of the units until failure(Ramos et al., 2020)

Censored data is divided into three main types:

1) **Type one censored data (Ramos et al., 2024)**: Time fixed and any units that do not fail before this time are considered censored, and the MLE formula defines**:**

$$L(t;\alpha,\beta,\lambda)=\frac{n!}{(n-r)!}[R(t_r;\alpha,\theta,\gamma)]^{n-r}\prod_{i=0}^{r}[f(t_i;\alpha,\theta,\gamma)]$$

Where L denotes the likelihood function, t fixed time, r fails units, and n number of units.

2) **Type two censored data (Tolba et al., 2023):** when the experiment was stopped after becoming specifically failed, for example, getting 5 fails from 10.

$$L(t;\alpha,\beta,\lambda)=\frac{n!}{(n-r)!}[R(t_r;\alpha,\theta,\gamma)]^{n-r}\prod_{i=0}^{r}[f(t_i;\alpha,\theta,\gamma)]$$

3) **Type three censored data (progressive) (EL-Sagheer et al., 2025):** Some units are removed during the experiment at different times, while the remaining units continue under observation. This approach provides greater flexibility and reduces costs while retaining sufficient information.

$$L=\frac{n!}{(n-r)!}\prod_{I=1}^{r}\left\{\left(f(t_i;\alpha,\theta,\gamma)\right)^{\delta_i}\left(R(t_r;\alpha,\theta,\gamma)\right)^{1-\delta_i}\right\}$$

$$\delta_i=\begin{cases}1 & is\ fail\\ 0 & is\ not\ fail\end{cases}$$

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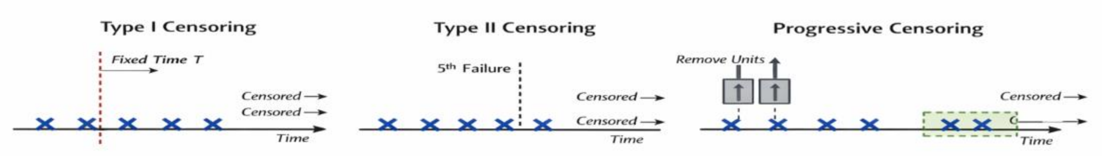

Figure 4 : display the branch of censored sample

5. MLEM for type – Three censoring sample (Liu et al., 2024)

The main advantage of the maximum likelihood properties method is the best affinity. That means when the values of the data increase, the estimate converges will be better and faster with the (parameters) in short; it maximizes the value of the function(Dai et al., 2016)

The general exponential Rayleigh distribution for a random sample of size t_1, t_2,..., t_n is as follows.:

n = The size of every item

r = The quantity of failed attempts

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t= random variable

L= The likelihood function

$$L = \frac{n!}{(n-r)!}\prod_{I=1}^{r}\left\{\left(f(t_i;\alpha,\theta,\gamma)\right)^{\delta_i}\left(R(t_r;\alpha,\theta,\gamma)\right)^{1-\delta_i}\right\}$$

$$\delta_i = \begin{cases} 1 & is\ fail \\ 0 & is\ not\ fail \end{cases}$$

$$L = \frac{n!}{(n-r)!}\prod_{I=1}^{r}\left\{\left(\left(\frac{\alpha}{\gamma}t_i^{\frac{1}{L}-1} + \frac{\theta}{\gamma}t_i^{\frac{2}{\gamma}-1}\right)e^{-\left(\alpha t_i^{\frac{1}{\gamma}}+\frac{\theta}{2}t_i^{\frac{2}{\gamma}}\right)}\right)^{\delta_i}\left(e^{-\left(\alpha t_r^{\frac{1}{\gamma}}+\frac{\theta}{2}t_r^{\frac{2}{\gamma}}\right)}\right)^{1-\delta_i}\right\}$$

Let k = $\frac{n!}{(n-r)!}$ Then we get

$$L = \mathrm{k}\prod_{I=1}^{r}\left\{\left(\left(\frac{\alpha}{\gamma}t_i^{\frac{1}{\gamma}-1} + \frac{\theta}{\gamma}t_i^{\frac{2}{\theta}-1}\right)e^{-\left(\alpha t_i^{\frac{1}{\gamma}}+\frac{\theta}{2}t_i^{\frac{2}{\gamma}}\right)}\right)^{\delta_i}\left(e^{-\left(\alpha t_r^{\frac{1}{\gamma}}+\frac{\theta}{2}t_r^{\frac{2}{\gamma}}\right)}\right)^{1-\delta_i}\right\}$$

$$\ln L = \ln K + \sum_{i=1}^{r}\delta_i \ln\left(\frac{\alpha}{\gamma}t_i^{\frac{1}{\gamma}-1} + \frac{\theta}{\gamma}t_i^{\frac{2}{\gamma}-1}\right) - \sum_{i=1}^{r}\delta_i\alpha\, t_i^{\frac{1}{\gamma}} + \frac{\theta}{2}t_i^{\frac{2}{\gamma}} - \sum_{i=1}^{r}(1-\delta_i)\left(\alpha t_r^{\frac{1}{\gamma}} + \frac{\theta}{2}t_r^{\frac{2}{\gamma}}\right)$$

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$$\frac{\partial \ln L}{\partial \alpha} = \sum_{i=1}^{r} \frac{\delta_i t_i^{\frac{1}{\gamma}-1}}{\alpha t_i^{\frac{1}{\gamma}-1} + \theta t_i^{\frac{2}{\gamma}-1}} - \sum_{i=1}^{r} \delta_i t_i^{\frac{1}{\gamma}} - \sum_{i=1}^{r} (1-\delta_i) t_r^{\frac{1}{\gamma}}$$

$$\frac{\partial \ln L}{\partial \theta} = \sum_{i=1}^{r} \frac{\delta_i t_i^{\frac{2}{\gamma}-1}}{\alpha t_i^{\frac{1}{L}-1} + \theta t_i^{\frac{2}{\gamma}-1}} - \frac{1}{2}\sum_{i=1}^{r} \delta_i t_i^{\frac{2}{\gamma}} - \frac{1}{2}\sum_{i=1}^{r} (1-\delta_i) t_r^{\frac{2}{\gamma}}$$

$$\frac{\partial \ln L}{\partial \gamma} =$$

$$\frac{1}{\gamma^2}\left[\sum_{i=1}^{r} \frac{-\alpha\delta_i t_i^{\frac{1}{\gamma}-1} \ln t_i - \alpha\gamma t_i^{\frac{1}{\gamma}-1} - 2\theta\delta_i t_i^{\frac{2}{\gamma}-1} \ln t_i - \gamma\theta\delta_i t_i^{\frac{2}{\gamma}-1}}{a t_i^{\frac{1}{\lambda}-1} + b t_i^{\frac{2}{\lambda}-1}} + \right.$$

$$\left. \sum_{i=1}^{r} \alpha\delta_i t_i^{\frac{1}{\gamma}} \ln t_i + \theta\delta_i t_i^{\frac{2}{\gamma}} \ln t_i + \sum_{i=1}^{r} (1-\delta_i)\left(\alpha t_r^{\frac{1}{\gamma}} \ln t_r + \theta t_r^{\frac{2}{\gamma}-1} \ln t_r\right)\right]$$

Let $\frac{\partial \ln L}{\partial \alpha} = (\alpha)$, $\frac{\partial \ln L}{\partial \theta} = f(\theta)$, $\frac{\partial \ln L}{\partial \gamma} = f(\gamma)$

The Jacobian matrix J $is\ defined\ as$:

$$J = \begin{bmatrix} \frac{\partial f(\alpha)}{\partial \alpha} & \frac{\partial f(\alpha)}{\partial \theta} & \frac{\partial f(\alpha)}{\partial \gamma} \\ \frac{\partial f(\theta)}{\partial \alpha} & \frac{\partial f(\theta)}{\partial \theta} & \frac{\partial f(\theta)}{\partial \gamma} \\ \frac{\partial f(\gamma)}{\partial \alpha} & \frac{\partial f(\gamma)}{\partial \theta} & \frac{\partial f(\gamma)}{\partial \gamma} \end{bmatrix}$$

$$\frac{\partial f(\alpha)}{\partial \alpha} = \sum_{i=1}^{r} \frac{-\delta_i t_i^{\frac{2}{\gamma}-2}}{\left(\alpha t_i^{\frac{1}{L}-1} + \theta t_i^{\frac{2}{\gamma}-1}\right)^2}$$

$$\frac{\partial f(\alpha)}{\partial \theta} = \sum_{i=1}^{r} \frac{-\delta_i t_i^{\frac{3}{\gamma}-2}}{\left(\alpha t_i^{\frac{1}{\gamma}-1} + \theta t_i^{\frac{2}{\gamma}-1}\right)^2}$$

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$$\frac{\partial f(\alpha)}{\partial \gamma}=\frac{1}{\gamma^2}\left[\sum_{i=1}^{r}\frac{\theta\delta_i t_i^{\frac{3}{\gamma}-2}\ln t_i}{\left(\alpha t_i^{\frac{1}{\gamma}-1}+\theta t_i^{\frac{2}{\gamma}-1}\right)^2}+\sum_{i=1}^{r}\delta_i t_i^{\frac{1}{\gamma}}\ln t_i+\sum_{i=1}^{r}(1-\delta_i)t_r^{\frac{1}{\gamma}}\ln t_r\right]$$

$$\frac{\partial f(\theta)}{\partial \alpha}=\sum_{i=1}^{r}\frac{-\delta_i t_i^{\frac{3}{\gamma}-2}}{\left(\alpha t_i^{\frac{1}{\gamma}-1}+\theta t_i^{\frac{2}{\gamma}-1}\right)^2}$$

$$\frac{\partial f(\theta)}{\partial \theta}=\sum_{i=1}^{r}\frac{-\delta_i t_i^{\frac{4}{\gamma}-2}}{\left(a t_i^{\frac{1}{L}-1}+b t_i^{\frac{2}{L}-1}\right)^2}$$

$$\frac{\partial f(\theta)}{\partial \gamma}=\frac{1}{\gamma^2}\left[\alpha\sum_{i=1}^{r}\frac{-\delta_i t_i^{\frac{3}{\gamma}-2}\ln t_i}{\left(\alpha t_i^{\frac{1}{\gamma}-1}+\theta t_i^{\frac{2}{\gamma}-1}\right)^2}+\sum_{i=1}^{r}\delta_i t_i^{\frac{2}{\gamma}}\ln t_i+\sum_{i=1}^{r}(1-\delta_i)t_r^{\frac{2}{\gamma}}\ln t_r\right]$$

$$\frac{\partial g(\gamma)}{\partial \alpha}=\frac{1}{\gamma^2}\left[\sum_{i=1}^{r}\frac{\theta\delta_i t_i^{\frac{3}{\gamma}-2}\ln t_i}{\left(\alpha t_i^{\frac{1}{\gamma}-1}+\theta t_i^{\frac{2}{\gamma}-1}\right)^2}+\sum_{i=1}^{r}\delta_i t_i^{\frac{1}{\gamma}}\ln t_i+\sum_{i=1}^{r}(1-\delta_i)t_r^{\frac{1}{\gamma}}\ln t_r\right]$$

$$\frac{\partial f(\gamma)}{\partial \theta}=\frac{1}{\gamma^2}\left[\alpha\sum_{i=1}^{r}\frac{-\delta_i t_i^{\frac{3}{\gamma}-2}\ln t_i}{\left(\alpha t_i^{\frac{1}{\gamma}-1}+\theta t_i^{\frac{2}{\gamma}-1}\right)^2}+\sum_{i=1}^{r}\delta_i t_i^{\frac{2}{\gamma}}\ln t_i+\sum_{i=1}^{r}(1-\delta_i)t_r^{\frac{2}{\gamma}}\ln t_r\right]$$

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$$\frac{\partial f(\gamma)}{\partial \gamma}=\sum_{i=1}^{r}\left(\frac{\begin{array}{c}\alpha^2 t_i^{\frac{2}{\gamma}-2}(\ln t_i)^2\delta_i+\alpha^2\gamma t_i^{\frac{2}{\gamma}-2}\ln t_i-\alpha^2\gamma^2 t_i^{\frac{2}{\gamma}-2}+4\theta\alpha\delta_i t_i^{\frac{3}{\gamma}-2}(\ln t_i)^2+2\alpha\theta\gamma\delta_i t_i^{\frac{3}{\gamma}-2}\ln t_i\\ -\alpha\theta\gamma^2\delta_i t_i^{\frac{3}{\gamma}-2}+\alpha\theta\delta_i t_i^{\frac{3}{\gamma}-2}(\ln t_i)^2+\alpha\theta\gamma t_i^{\frac{3}{\gamma}-2}\ln t_i-\gamma^2\alpha\theta t_i^{\frac{3}{\gamma}-2}+4\theta^2\delta_i t_i^{\frac{4}{\gamma}-2}(\ln t_i)^2+2\theta^2\gamma\delta_i t_i^{\frac{4}{\gamma}-2}\ln t_i\\ -\theta^2\gamma^2\delta_i t_i^{\frac{4}{\gamma}-2}-\alpha^2\delta_i t_i^{\frac{2}{\gamma}-2}(\ln t_i)^2+2\alpha^2\gamma\delta_i t_i^{\frac{2}{\gamma}-2}\ln t_i-2\theta\alpha\delta_i t_i^{\frac{3}{\gamma}-2}(\ln t_i)^2\\ +2\theta\gamma\alpha\delta_i t_i^{\frac{3}{\gamma}-2}\ln t_i-\alpha\gamma t_i^{\frac{2}{\gamma}-2}\ln t_i+2\alpha\gamma^2 t_i^{\frac{2}{\gamma}-2}-2\theta\gamma t_i^{\frac{3}{\gamma}-2}\ln t_i+2\theta\gamma^2 t_i^{\frac{3}{\gamma}-2}\\ -2\alpha\theta\delta_i t_i^{\frac{3}{\gamma}-2}(\ln t_i)^2+4\alpha\theta\gamma\delta_i t_i^{\frac{3}{\gamma}-2}\ln t_i-4\theta^2\delta_i t_i^{\frac{4}{\gamma}-2}(\ln t_i)^2-4\theta^2\gamma\delta_i t_i^{\frac{4}{\gamma}-2}\ln t_i\\ -\alpha\theta\gamma\delta_i t_i^{\frac{3}{\gamma}-2}\ln t_i+2\gamma^2\alpha\theta\delta_i t_i^{\frac{3}{\gamma}-2}-2\theta^2\gamma\delta_i t_i^{\frac{4}{\gamma}-2}\ln t_i+2\theta^2\gamma\gamma^2\delta_i t_i^{\frac{4}{\gamma}-2}\end{array}}{\gamma^4\left(\alpha t_i^{\frac{1}{\gamma}-1}+\theta t_i^{\frac{2}{\gamma}-1}\right)^2}\right)$$

$$+\frac{\sum_{i=1}^{r}-\alpha\delta_i t_i^{\frac{1}{\gamma}}(\ln t_i)^2-2\theta\delta_i t_i^{\frac{2}{\gamma}}(\ln t_i)^2-2\gamma\alpha\delta_i t_i^{\frac{1}{\gamma}}\ln t_i-2\gamma\theta\delta_i t_i^{\frac{2}{\gamma}}\ln t_i}{\lambda^4}$$

$$+\sum_{i=1}^{r}\frac{-(1-\delta_i)(\alpha t_r^{\frac{1}{\gamma}}(\ln t_r)^2+2\theta t_r^{\frac{2}{\gamma}}(\ln t_r)^2+2\alpha\gamma t_r^{\frac{1}{\gamma}}\ln t_r+2\theta L t_r^{\frac{2}{\gamma}}\ln t_r)}{\gamma^4}$$

6.Simulation Application:(Rehab Noori Shalan et al., 2024)(Habeeb et al., 2024; Rehab Noori Shalan et al., 2023)

A simulation is characterized as a numerical scientific approach that describes the behavior of a certain system using logical mathematical techniques. Many statistical domains currently use simulation methods. This part has been simulated using a variety of techniques, including Monte Carlo simulation (Rubinstein et al., 2016) (Adnan et al., 2024)To generate simulation data, we start with basic random integers between 0 and 1, modify them to fit the desired distribution, and then repeat the procedure many times to get a sizable dataset that faithfully captures the distribution. Determining the goal, choosing the sample size, creating the random numbers, changing them in accordance with the distribution, and ultimately evaluating the outcomes are the main concerns at each stage (Li, 2025)

U=F(x)

X=$F^{-1}(u)$

$$u = 1 - e^{-\left(\alpha x^{\frac{1}{\theta}} + \frac{\theta}{2} x^{\frac{2}{\gamma}}\right)}$$

$$1 - u = e^{-\left(\alpha x^{\frac{1}{\theta}} + \frac{\theta}{2} x^{\frac{2}{\gamma}}\right)}$$

$$\ln(1-u) = -\left(\alpha x^{\frac{1}{\gamma}} + \frac{\theta}{2} x^{\frac{2}{\gamma}}\right)$$

$$\alpha x^{\frac{1}{\gamma}} + \frac{\theta}{2} x^{\frac{2}{\gamma}} + \ln(1-u) = 0$$

$$x^{\frac{1}{\gamma}}\left(\frac{\theta}{2} x^2 + \alpha x\right) + \ln(1-u) = 0$$

$$x = \left|\frac{-\propto \mp \sqrt{\alpha^2 - 2\ln\theta + 2\ln v\theta}}{\theta}\right|^{\gamma} \qquad \gamma, \theta, \alpha > 0$$

- We select samples (n = 30 , 50, 100)
- We select the initial value (α=0.1 , 0.15) , (θ=0.25, 0.5) , (γ= 1, 1.5)

7. Numerical results: (Alkanani et al., 2023)

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To find the estimation of parameters, R(t), and MSE by using the simulation, we get the results in the table:

Table One: Shows The Parameter Estimate Using M.S.E.

α	θ	γ	n	$\hat{\alpha}$	$\hat{\theta}$	$\hat{\gamma}$
0.1	0.25	1	30	0.100137	0.249981	1.005187
			50	0.100212	0.24986	1.004431
			100	0.100065	0.250111	1.005
	0.25	1.5	30	0.100005	0.249992	1.506709
			50	0.100007	0.249986	1.504273
			100	0.100007	0.250025	1.505692
	0.5	1	30	0.100032	0.499973	1.005523
			50	1.00E-01	0.500017	1.005443
			100	0.100001	0.500125	1.003237
	0.5	1.5	30	1.00E-01	0.499994	1.505426
			50	0.100005	0.49999	1.50593
			100	1.00E-01	0.49999	1.505886
0.15	0.25	1	30	0.149825	0.250146	1.004115
			50	0.149973	0.250223	1.0039
			100	0.149948	0.24986	1.004989
	0.25	1.5	30	0.149997	0.249992	1.504838
			50	0.150011	0.249995	1.505027
			100	0.150009	0.249999	1.505726
0.15	0.5	1	30	0.150005	0.50017	1.004775
			50	0.149892	0.500014	1.005852
			100	0.149878	0.499907	1.003685
	0.5	1.5	30	0.149984	0.500014	1.503445
			50	0.150019	0.500019	1.5043
			100	0.150014	0.500034	1.506292

Table Two: Shows The R(T) Estimation Under Time And MSE

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α	θ	γ	n	t	R	$\hat{R}$
0.1	0.25	1	30	0.1	0.988813	0.988651
0.1	0.25	1	30	0.2	0.97531	0.975039
0.1	0.25	1	30	0.3	0.959589	0.959236
0.1	0.25	1	30	0.4	0.941765	0.941357
0.1	0.25	1	30	0.5	0.921963	0.92153
0.1	0.25	1	50	0.1	0.988813	0.988666
0.1	0.25	1	50	0.2	0.97531	0.975062
0.1	0.25	1	50	0.3	0.959589	0.959265
0.1	0.25	1	50	0.4	0.941765	0.94139
0.1	0.25	1	50	0.5	0.921963	0.921564
0.1	0.25	1	100	0.1	0.988813	0.988663
0.1	0.25	1	100	0.2	0.97531	0.975059
0.1	0.25	1	100	0.3	0.959589	0.959262
0.1	0.25	1	100	0.4	0.941765	0.941387
0.1	0.25	1	100	0.5	0.921963	0.921562
0.1	0.25	1.5	30	0.1	0.973024	0.972802
0.1	0.25	1.5	30	0.2	0.952353	0.952062
0.1	0.25	1.5	30	0.3	0.93247	0.932152
0.1	0.25	1.5	30	0.4	0.9129	0.912581
0.1	0.25	1.5	30	0.5	0.893506	0.893206
0.1	0.25	1.5	50	0.1	0.973024	0.972882
0.1	0.25	1.5	50	0.2	0.952353	0.952167
0.1	0.25	1.5	50	0.3	0.93247	0.932266
0.1	0.25	1.5	50	0.4	0.9129	0.912696
0.1	0.25	1.5	50	0.5	0.893506	0.893314
0.1	0.25	1.5	100	0.1	0.973024	0.972834
0.1	0.25	1.5	100	0.2	0.952353	0.952104
0.1	0.25	1.5	100	0.3	0.93247	0.932196
0.1	0.25	1.5	100	0.4	0.9129	0.912623
0.1	0.25	1.5	100	0.5	0.893506	0.893244
0.1	0.5	0.5	30	0.1	0.998976	0.998944
0.1	0.5	0.5	30	0.2	0.99561	0.995506
0.1	0.5	0.5	30	0.3	0.989036	0.988823
0.1	0.5	0.5	30	0.4	0.977849	0.977494
0.1	0.5	0.5	30	0.5	0.960189	0.959671

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0.1	0.5	0.5	50	0.1	0.998976	0.998938
0.1	0.5	0.5	50	0.2	0.99561	0.995495
0.1	0.5	0.5	50	0.3	0.989036	0.988808
0.1	0.5	0.5	50	0.4	0.977849	0.977477
0.1	0.5	0.5	50	0.5	0.960189	0.959659
0.1	0.5	0.5	100	0.1	0.998976	0.998933
0.1	0.5	0.5	100	0.2	0.99561	0.995458
0.1	0.5	0.5	100	0.3	0.989036	0.988709
0.1	0.5	0.5	100	0.4	0.977849	0.977285
0.1	0.5	0.5	100	0.5	0.960189	0.959334
0.1	0.5	1	30	0.1	0.987578	0.987386
0.1	0.5	1	30	0.2	0.970446	0.970094
0.1	0.5	1	30	0.3	0.948854	0.948373
0.1	0.5	1	30	0.4	0.923116	0.922547
0.1	0.5	1	30	0.5	0.893597	0.892989
0.1	0.5	1	50	0.1	0.987578	0.987392
0.1	0.5	1	50	0.2	0.970446	0.970106
0.1	0.5	1	50	0.3	0.948854	0.94839
0.1	0.5	1	50	0.4	0.923116	0.922566
0.1	0.5	1	50	0.5	0.893597	0.89301
0.1	0.5	1	100	0.1	0.987578	0.987466
0.1	0.5	1	100	0.2	0.970446	0.97024
0.1	0.5	1	100	0.3	0.948854	0.948571
0.1	0.5	1	100	0.4	0.923116	0.922778
0.1	0.5	1	100	0.5	0.893597	0.893233
0.1	0.5	1.5	30	0.1	0.967395	0.967157
0.1	0.5	1.5	30	0.2	0.938531	0.938197
0.1	0.5	1.5	30	0.3	0.909353	0.908976
0.1	0.5	1.5	30	0.4	0.879881	0.879496
0.1	0.5	1.5	30	0.5	0.850264	0.849901
0.1	0.5	1.5	50	0.1	0.967395	0.967131
0.1	0.5	1.5	50	0.2	0.938531	0.938161
0.1	0.5	1.5	50	0.3	0.909353	0.908934
0.1	0.5	1.5	50	0.4	0.879881	0.879452
0.1	0.5	1.5	50	0.5	0.850264	0.849858
0.1	0.5	1.5	100	0.1	0.967395	0.967135
0.1	0.5	1.5	100	0.2	0.938531	0.938167

0.1	0.5	1.5	100	0.3	0.909353	0.908941
0.1	0.5	1.5	100	0.4	0.879881	0.87946
0.1	0.5	1.5	100	0.5	0.850264	0.849866
0.15	0.25	1	30	0.1	0.983881	0.983735
0.15	0.25	1	30	0.2	0.965605	0.965381
0.15	0.25	1	30	0.3	0.945303	0.945031
0.15	0.25	1	30	0.4	0.923116	0.922823
0.15	0.25	1	30	0.5	0.8992	0.898911
0.15	0.25	1	50	0.1	0.983881	0.983728
0.15	0.25	1	50	0.2	0.965605	0.965364
0.15	0.25	1	50	0.3	0.945303	0.945002
0.15	0.25	1	50	0.4	0.923116	0.92278
0.15	0.25	1	50	0.5	0.8992	0.898853
0.15	0.25	1	100	0.1	0.983881	0.983689
0.15	0.25	1	100	0.2	0.965605	0.965308
0.15	0.25	1	100	0.3	0.945303	0.944941
0.15	0.25	1	100	0.4	0.923116	0.922726
0.15	0.25	1	100	0.5	0.8992	0.898814
0.15	0.25	1.5	30	0.1	0.962599	0.96239
0.15	0.25	1.5	30	0.2	0.936206	0.935947
0.15	0.25	1.5	30	0.3	0.911809	0.911534
0.15	0.25	1.5	30	0.4	0.888454	0.888185
0.15	0.25	1.5	30	0.5	0.865801	0.865555
0.15	0.25	1.5	50	0.1	0.962599	0.962379
0.15	0.25	1.5	50	0.2	0.936206	0.935932
0.15	0.25	1.5	50	0.3	0.911809	0.911518
0.15	0.25	1.5	50	0.4	0.888454	0.888168
0.15	0.25	1.5	50	0.5	0.865801	0.865537
0.15	0.25	1.5	100	0.1	0.962599	0.962349
0.15	0.25	1.5	100	0.2	0.936206	0.935895
0.15	0.25	1.5	100	0.3	0.911809	0.911478
0.15	0.25	1.5	100	0.4	0.888454	0.888129
0.15	0.25	1.5	100	0.5	0.865801	0.865501
0.15	0.5	1	30	0.1	0.982652	0.982434
0.15	0.5	1	30	0.2	0.960789	0.960416
0.15	0.5	1	30	0.3	0.934728	0.934236
0.15	0.5	1	30	0.4	0.904837	0.90427

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0.15	0.5	1	30	0.5	0.871534	0.870939
0.15	0.5	1	50	0.1	0.982652	0.982397
0.15	0.5	1	50	0.2	0.960789	0.960358
0.15	0.5	1	50	0.3	0.934728	0.934166
0.15	0.5	1	50	0.4	0.904837	0.904198
0.15	0.5	1	50	0.5	0.871534	0.870876
0.15	0.5	1	100	0.1	0.982652	0.982498
0.15	0.5	1	100	0.2	0.960789	0.96053
0.15	0.5	1	100	0.3	0.934728	0.934393
0.15	0.5	1	100	0.4	0.904837	0.904462
0.15	0.5	1	100	0.5	0.871534	0.871154
0.15	0.5	1.5	30	0.1	0.95703	0.956846
0.15	0.5	1.5	30	0.2	0.922619	0.922373
0.15	0.5	1.5	30	0.3	0.889204	0.888935
0.15	0.5	1.5	30	0.4	0.856318	0.85605
0.15	0.5	1.5	30	0.5	0.8239	0.82365
0.15	0.5	1.5	50	0.1	0.95703	0.956792
0.15	0.5	1.5	50	0.2	0.922619	0.9223
0.15	0.5	1.5	50	0.3	0.889204	0.888852
0.15	0.5	1.5	50	0.4	0.856318	0.855964
0.15	0.5	1.5	50	0.5	0.8239	0.823567
0.15	0.5	1.5	100	0.1	0.95703	0.956684
0.15	0.5	1.5	100	0.2	0.922619	0.922157
0.15	0.5	1.5	100	0.3	0.889204	0.888695
0.15	0.5	1.5	100	0.4	0.856318	0.855807
0.15	0.5	1.5	100	0.5	0.8239	0.82342

- **Effect of time**: $R(t)$, $\hat{R}(t)$ decreasing with increasing t, which reflects the nature decline in the reliability function.
- **Effect size sample:** increasing the sample did not change, except for $R(t)$, but reduced the difference between $R(t)$ and $\hat{R}(t)$ and indicate how to improve the accuracy with big samples.

In overall conclusion, the simulation results confirm the expected behavior of the GERD survival function and demonstrate the effectiveness of the maximum likelihood estimation method for parameter estimation, even with limited sample sizes.

7. Conclusion: (Rehab N Shalan, 2024)

In this article, we estimate the reliability function $\hat{R}(t)$ and three parameters in tables (1, 2), we notice that reliability decreasing under time and becomes very small with respect to the true reliability function *R(t)*,

The estimated values of $\hat{R}(t)$ are decreased when the failure time increases.

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