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Solving Linear Programming Problems Using Sensitivity Analysis and Heptagonal Ranking Function

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ABSTRACT: This research presents an integrated approach that combines sensitivity analysis with fuzzy logic to address uncertainty in operations research models. The study focuses on the right-hand side (RHS) of constraints as a key parameter subject to variation. To capture this uncertainty, a symmetric heptagonal fuzzy membership function is proposed, where the seven representative values are governed by a single weight parameter (r). A corresponding fuzzy ranking function is developed to transform the fuzzy outcomes into comparable crisp values. The methodology is applied to two illustrative linear programming examples drawn from textbooks, covering both maximization and minimization problems. The results demonstrate that the fuzzy ranking approach, particularly under specific weight configurations, provides more robust and realistic solutions than traditional sensitivity analysis. These findings confirm the effectiveness of integrating fuzzy logic into sensitivity analysis for enhancing decision-making in linear programming under uncertainty.

Keywords: sensitive analysis, linear programming problem, fuzzy numbers, heptagonal function, ranking function

1. INTRODUCTION

The object of linear programming is maximizing or minimizing an objective function while keeping to defined constraints. Sensitivity analysis is an Essential tool used to understand how decisions or outcomes are affected by changes in input parameters or variables, helping to identify the most influential factors to ensure achieving the best possible results Fuzzy linear programming, using a fuzzy ranking function with a heptagonal function to mathematically reflect uncertainty, shows up as a solution in contexts with uncertain or partial data. This method offers solid, flexible strategies for dynamic real-world systems while guaranteeing accurate and flexible analysis of the best solutions [1] Zio, Enrico, and Piero Baraldi (2004). applied various sensitivity analysis techniques to identify the most relevant parameters affecting the performance of an Isolation Condenser system. Fuzzy models were then developed to map system outputs to these inputs, resulting in fast and reliable predictions. [2] Ebrahim Nejad, Ali (2011) extended sensitivity analysisin fuzzy Linear Programming by using fuzzy simplex methods to evaluate how data changes affect the optimal solution.[3] Kumar, Amit, and Neha Bhatia. (2011) addressed additional cases in sensitivity analysis for fuzzy Linear. Programming with trapezoidal fuzzy numbers and provides numerical examples. [4] Kumar, Amit, and N. Bhtie (2012) introduced a simplified sensitivity analysis method for fuzzy LP problems using a fuzzy ranking function, showing that only rank-level accuracy is achievable when both costs and decision variables are fuzzy. [5] Kumar, Amit, and Neha Bhatia (2012) proposed a new sensitivity analysis method for Linear Programming with interval-valued fuzzy numbers and solves an example unsolvable by existing method. [6] Zhong, Yi – Hua, et al (2013)presented a new "revised interior point method" for solving large-scale fuzzy linear programming problems using trapezoidal fuzzy numbers and a linear fuzzy ranking function, including sensitivity analysis for factors like the starting point and safety parameters. [7] Ezzati, Reza, Esmaile Khorram, et al (2014) proposed a new algorithm to solve fuzzy multi – objective linear programming problems using fuzzy operations and ordering, with sensitivity analysis for changing objective priorities. [8] Magni, Carlo Alberto, et al. (2020) introduced a model based on fuzzy logic and an expert system to rate firm default risks, along with a sensitivity analysis to identify the most influential factors and suggest actions to improve the rating. The model simulated expert evaluations automatically and facilitated managerial decision-making.

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[9] Mohan, Suresh, et al (2021) presented sensitivity analysis for intuitionistic fuzzy linear programming using magnitude-based ranking functions, identifying ranges where parameter changes do not affect optimality. [10] Ramezanzade, Mohsen, et al. (2021) applied a hybrid fuzzy multi-criteria decision-making model to rank renewable energy projects based on environmental, economic, technical, and social aspects. Sensitivity analysis was also performed to verify the robustness of the proposed model. [11] Moslem, Sarbast, et al. (2022) proposed a fuzzy ranking method using triangular fuzzy numbers to solve multi − criteria decision − making problems with conflicting criteria, selecting the solution with the least compromise. The method was compared with two other fuzzy approaches, TOPSIS and VIKOR, to demonstrate its effectiveness. [12] Vinogradova-Zinkevič, Irina. (2023) evaluated the stability of various Fuzzy AHP methods, including Arithmetic mean, Geometric mean, Extent analysis, and Lambda Max, by simulating slight changes in expert judgments tens of thousands of times. It analyzed how these changes affected the ranking results, identifying the specific features and advantages of each method. The objective of this study is utilizing traditional linear programming to calculate the objective function. After that changing and replacing value of bi then applying the traditional sensitivity analysis to the objective function. Therefore, using the proposed linear symmetric heptagonal membership function to made the value of bi as fuzzy number and applying the fuzzy sensitive analysis to find the objective function. Section 1: Sensitivity Analysis. Section 2: A simple Overview of fuzzy set theory. Section 3: Proposal of a fuzzy function named the heptagonal function. Section 4: Derivation of the ranking function. Section 5: Presentation of a practical example demonstrating the maximization and minimization functions.

2. SENSITIVITY ANALYSIS

Sensitivity analysis is an essential method for learning how changing inputs affects outputs in different systems in a sensitivity analysis, different inputs are tested to see how they affect the results. For instance, it can determine how profit is affected by changes in the cost of raw materials or how project completion is affected by timetable delays. We use this tool to assess the robustness of a system and examine the relationships between variables. There are several inputs for sensitivity analysis which are as follows:

2.1 Available Resources: In sensitivity analysis, we change the available resources b_i and calculate the sensitivity analysis after the change in linear programming.

2.2 Objective Function: We adjust the coefficients of the objective function c_i and assess the impact of the change on the optimal solution.

2.3 Constraints Coefficients of the Decision Variables: We modify the coefficients of decision variables a_{ij} in the constraints and analyze the effect on the optimal solution**.**

2.4 Adding a Variable: In sensitivity analysis, a variable that was previously omitted can be added, and it is considered one of the essential variables, impacting the optimal solution.

2.5 Adding a Constraint: In sensitivity analysis, a new constraint can be added, modifying the existing constraints, and its impact on the optimal solution is analyzed.

Overview Sensitivity analysis is an essential method for learning how changing inputs affects outputs in different systems. Improving operational efficiency and optimizing management approaches are both possible with the help of components like RHS and LHS, as well as variables and constraints. In order to increase the efficacy and efficiency of goal accomplishment, this analysis gives an opportunity to adjust improvements and decrease risks.

3. THEORY OF FUZZY SETS

In many real-life problems, we cannot deal with satisfactory "Yes" or "No," which means elements belong or elements not belong to the ordinary set theory, meaning it is not an adequate way of dealing with properties of this type. Therefore, Lotfi A.Zadeh (1965) suggested a modified set theoretical procedure in which the element may have a degree of membership value that ranged over a continuum of values ranging between zero and one rather than exactly zero or one then Zadeh is the first one to introduce fuzzy set theory. Fuzzy set theory has been used in many fields, such as operations research. Fuzzy sets are characterized by the degrees of membership function of components within the interval [0,1]. Fuzzy sets provide a strict mathematical framework in which vague conceptual phenomena can be precisely and rigorously studied.

Overview of Fuzzy Set Theory:

This part encompasses fundamental definitions.

3.1 Definition: Let X denote a Set with elements. A fuzzy set Ã is defined by $\mu_{\tilde{A}}(x) : X \longrightarrow [0,1]$, where $\tilde{A}(x) = \{(x, M_{\tilde{A}}(x)) \mid X \rightarrow [0,1]\}$.

3.2 Definition: An α-cut is the precise set $\tilde{A}_\alpha$ of every elements x that fulfill the

$$\tilde{A}_\alpha = \{x \in X | M_{\tilde{A}}(x) \geq \alpha, \alpha \in [0,1]\}.$$

3.3 Definition: A fuzzy set Ã is identified as a fuzzy umber if its membership function meets the following conditions. the following conditions within the set of all real number.

1. There exists at least one x_0 in R such that $M_{\tilde{A}}(x) = 1$ (if $M_{\tilde{A}}(x) = n, 0 < n \leq 1$, we denote Ã as a generalized fuzzy number).
2. $M_{\tilde{A}}(x)$ is Partially continuous.
3. Ã is a normal fuzzy set.

Proposed Linear Symmetric Heptagonal Membership Function

Proposed Linear fuzzy number $\tilde{A}_{LS}(x) = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, r, 1)$ is said to be linear symmetric Heptagonal fuzzy numbers, if its membership function is as follows:

$$\mu_{\tilde{A}LS}(x) = \begin{cases} r(\frac{x - a_1}{a_2 - a_1}) & a_1 \leq x < a_2 \\ r\left(\frac{x - a_2}{a_3 - a_2}\right) & a_2 \leq x < a_3 \\ r + (1 - r)(\frac{x - a_3}{a_4 - a_3}) & a_3 \leq x < a_4 \\ 1 & x = a_4 \\ r + (1 - r)\left(\frac{a_5 - x}{a_5 - a_4}\right) & a_4 < x \leq a_5 \\ r\left(\frac{a_6 - x}{a_6 - a_5}\right) & a_5 < x \leq a_6 \\ r\left(\frac{a_7 - x}{a_7 - a_6}\right) & a_6 < x \leq a_7 \\ 0 \quad x < a_1, x > a_7 \end{cases}$$

Where $a_1, a_2, a_3, a_4, a_5, a_6, a_7$ are real numbers , $a_1 < a_2 < a_3 < a_4 < a_5 < a_6 < a_7$ for maximize objective function and $a_7 < a_6 < a_5 < a_4 < a_3 < a_2 <, a_1$ for minimize objective function ,and $0< r <1$ where conditions $\mu_{\tilde{A}LS}(x)$ are as follows:

$\mu_{\tilde{A}LS}(x)$ is a continuous function in the interval [0 , 1].

1. $\mu_{\tilde{A}LS}(x)$ is nondecreasing and continuous function on $[a_1, a_2]$, $[a_2, a_3]$, $[a_3, a_4]$.
2. $\mu_{\tilde{A}LS}(x)$ is nonincreasing and continuous function on $[a_4, a_5]$, $[a_5, a_6]$, $[a_6, a_7]$.
3. $\mu_{\tilde{A}LS}(x) = 0$ when $x < a_1$ or $x > a_7$.

In this section we draw the figure of heptagonal fuzzy number where $\alpha \epsilon [0,1]$.

$\tilde{A}_{LS}(x)(a_1, a_2, a_3, a_4, a_5, a_6, a_7, r, 1)$ below:

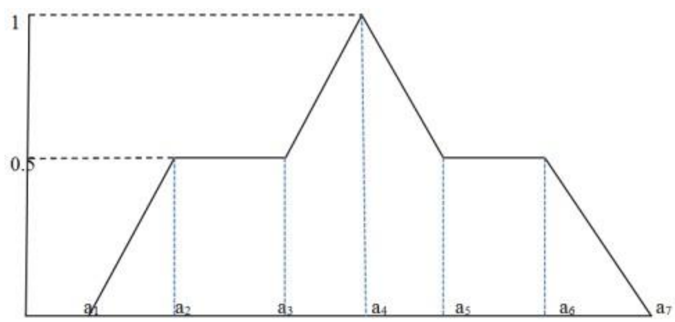

Figure1.: Represent heptagonal fuzzy number

$$\mu_{\tilde{A}LS}(x)_\alpha = \begin{cases} a_1 + \dfrac{\propto (a_2 - a_1)}{r} & \propto\in (0,r] \\ a_2 + \dfrac{\propto (a_3 - a_2)}{r} & \propto\in (0,r] \\ a_3 + \dfrac{\propto (a_4 - a_3) - r(a_3 - a_4)}{1-r} & \propto\in (r,1] \\ a_5 - \dfrac{\propto (a_5 - a_4) - r(a_5 - a_4)}{1-r} & \propto\in (r,1] \\ a_6 - \dfrac{\propto (a_6 - a_5)}{r} & \propto\in (0,r] \\ a_7 - \dfrac{\propto (a_7 - a_6)}{r} & \propto\in (0,r] \end{cases}$$

$$(inf_1\mu_{\tilde{A}LS}(x)_\alpha, sup_3\mu_{\tilde{A}LS}(x)_\alpha) = \left[\left(a_1 + \frac{\propto(a_2 - a_1)}{r}\right), \left(a_7 - \frac{\propto(a_7 - a_6)}{r}\right)\right] \quad \propto\in (0,r]$$

$$(inf_2\mu_{\tilde{A}LS}(x)_\alpha, sup_2\mu_{\tilde{A}LS}(x)_\alpha) = \left[\left(a_2 + \frac{\propto(a_3 - a_2)}{r}\right), \left(a_6 - \frac{\propto(a_6 - a_5)}{r}\right)\right] \quad \propto\in (0,r]$$

$$(inf_3\mu_{\tilde{A}LS}(x)_\alpha, sup_1\mu_{\tilde{A}LS}(x)_\alpha) = \left[\left(a_3 + \frac{\propto(a_4 - a_3) - r(a_3 - a_4)}{1-r}\right), \left(a_5 - \frac{\propto(a_5 - a_4) - r(a_5 - a_4)}{1-r}\right)\right] \propto (r,1]$$

4. RANKING FUNCTION

To find the optimal solution for the fuzzy numbers, the ranking function that was developed by Yager in 1981 is used, which helps evaluate and prioritize possible solutions. The ranking function allows alternatives to be computed according to their level of membership function in a set or category, which is particularly useful in a fuzzy environment when there is uncertainty and imprecision in the data. A heptagonal fuzzy number is represented using seven distinct points. These points describe how much a certain element belongs to a fuzzy set. the membership values start from zero at the first point, gradually increase until they reach the full value of one, and then decrease again back to zero at the last point. The values between the two ends represent varying levels of membership, some weak and some strong. The points where the membership reaches one are considered the most representative values of the fuzzy number. To determine the value that best represents the overall fuzzy number, a weighted average of the seven membership values is calculated, giving each value a weight according to its significance.

$$\mathcal{R}(\tilde{A}_{LS}) = \frac{1}{2}\left[\int_0^1 (inf_m\, \mu_{\tilde{A}LS}(x)_\alpha + sup_n\, \mu_{\tilde{A}LS}(x)_\alpha) d\alpha\right]$$

$$m = 1,2,3 \qquad n = \begin{cases} m+2 & m=1 \\ m & m=2 \\ m-2 & m=3 \end{cases}$$

suppose that:

$$\mathcal{R}(\tilde{A}_{LS}) = F_1 + F_2 + F_3 \quad \textbf{(1)}$$

$$F_1 = \frac{1}{2}\left[\int_0^r \left[a_1 + \frac{\propto(a_2 - a_1)}{r}\right] + \left[a_7 - \frac{\propto(a_7 - a_6)}{r}\right]\right] d\alpha$$

$$F_1 = \frac{r}{2}(a_1 + a_2 + a_6 + a_7) \quad \textbf{(2)}$$

$$F_2 = \frac{1}{2}\left[\int_0^r \left[a_2 + \frac{\propto(a_3 - a_2)}{r}\right] + \left[a_6 - \frac{\propto(a_6 - a_5)}{r}\right]\right] d\alpha$$

$$F_2 = \frac{r}{2}(a_3 + a_2 + a_6 + a_5) \quad \textbf{(3)}$$

$$F_3 = \frac{1}{2}\left[\int_r^1 \left[a_3 + \frac{\propto(a_4 - a_3) - r(a_3 - a_4)}{1-r}\right], \left[a_5 - \frac{\propto(a_5 - a_4) - r(a_5 - a_4)}{1-r}\right]\right] d\alpha$$

$$F_3 = \frac{1-r}{2}(a_3 + 2a_4 + a_5) \quad \textbf{(4)}$$

Now replace equations **(2), (3), (4)** into equation **(1)**

$$\mathcal{R}(\tilde{A}_{LS}) = \frac{1}{4}[\text{r}\,(a_1 + 2a_2 + 2a_6 + a_7 - 2a_4) + (a_3 + 2a_4 + a_5)] \quad \textbf{(5)}$$

4. NUMERICAL RESULTS

Two examples from Hamdi Taha's book were selected to apply fuzzy ordinal constraints in linear programming problems, with the aim of studying the impact of uncertainty in the constraints on the optimal solutions. Two cases of the objective function were considered: maximization and minimization.

In this part discussing the solution for the crisp linear programming depend on the maximize objective function of the following example by using excel programming.

Example (1): $\max_{x_1,x_2,x_3} z = 3x_1 + 2x_2 + 5x_3$

S.to`

$x_1 + 2x_2 + x_3 \leq 430$

$3x_1 + 0 + 2x_3 \leq 460$

$x_1 + 4x_2 + 0 \leq 420$

$x_i \geq 0$, $i \in 1,2,3$

First: Apply the solver in excel to find the solution of crisp linear programming as follows:

Z = 1350 , $x_1 = 0$, $x_2 = 100$, $x_3 = 230$

Second: Now setting up the change or replace the value of (b_2)

Table 1: Sensitivity Report for Constraints (Excel Solver)

Constraint

	Final	Shadow	Constraint	Allowable	Allowable
Name	**Value**	**Price**	**R. H. Side**	**Increase**	**Decrease**
con1	430	1	430	10	200
con2	460	2	460	400	20
con3	400	0	420	1E+30	20

When using Solver in Excel for sensitivity analysis, the program generates a report that includes the Constraints Table. This table presents the final values of the constraints, the shadow prices, the original right-hand side (R.H. Side) values, as well as the allowable increase and decrease before the optimal solution changes, Since the shadow price of the second constraint is 2, which is higher than that of the first constraint 1, increasing the right-hand side of the second constraint generates greater profit. Therefore, prioritizing the second constraint contributes to achieving maximum profitability.

Showing that in constraint b_2 the lower and upper b_2 is as follow:

The change of b_2 = the value of b_2 − decrease allowable = 460 - 20=440

The change of b_2 = the value of b_2 + increase allowable = 460 + 400 = 860

The lower and upper sensitive is: $440 < b_2 < 860$

by using solver, finding the lower right-hand side = 440

Z=1310, $x_1 = 0$, $x_2 = 105$, $x_3 = 220$

by using solver, finding the upper right-hand side = 860

Z= 2150, $x_1 = 0$, $x_2 = 0$, $x_3 = 430$

Third: after that finding the fuzzy number by using proposed linear symmetric heptagonal membership function as follow:

The distance between the first and the last point is $=860 - 440 = 420$

The difference between each pair of points is $= \Delta = \frac{420}{6} = 70$

$a_1 = 440$, $a_2 = 510$, $a_3 = 580$, $a_4 = 650$, $a_5 = 720$, $a_6 = 790$, $a_7 = 860$

Apply the ranking function by using the equation (5)

$$\mathcal{R}(\tilde{A}_{LS}) = \frac{1}{4}[r\,(a_1 + 2a_2 + 2a_6 + a_7 - 2a_4) + (a_3 + 2a_4 + a_5)]$$

When we apply the fuzzy solution for fuzzy linear programming when changing(b_2) and use Solver in Excel the result are as follows :

Table2: represent the fuzzy linear programming when change(b2)

r	b_2_fuzzy	opt. solu.	x_1	x_2	x_3
0.1	715	1860	0	36.25	357.5
0.2	780	1990	0	20	390
0.3	845	2120	0	3.75	422.5
0.4	910	2150	0	0	430
0.5	975	2150	0	0	430
0.6	1040	2150	0	0	430
0.7	1105	2150	0	0	430
0..8	1170	2150	0	0	430
0.9	1235	2150	0	0	430

we can show that the ranking function for the fuzzy linear programming model is best from the crisp linear programming model for all values of the weight function r, but we choose the fuzzy ranking function $r = 0.1,0.2,0.3$ because the values are similar for the crisp linear programming model in variables (x_1, x_2, x_3)

Example(2) : $MinZ = 2x_1 - 4x_2 + 3x_3$
s.t
$5x_1 - 6x_2 + 2x_3 \geq 5$
$-x_1 + 3x_2 + 5x_3 \geq 8$
$2x_1 + 5x_2 - 4x_3 \leq 4$
$x_1, x_2, x_3 \geq 0$
First: apply the solver in excel to find the solution of crisp linear programming as follows :
Z = 3.07407 , $x_1 = 1.81481$, $x_2 = 1.11111$, $x_3 = 1.2963$
second: now setting up the change (b_2) in traditional linear programming by using the solver to find sensitive analysis as follows:

Table 3: Sensitivity Report for Constraints (Excel Solver)

Constraints

	Final	Shadow	Constraint	Allowable	Allowable
Name	**Value**	**Price**	**R.H.Side**	**Increase**	**Decrease**
con1	5	0.53909465	5	45	11.91891892
con2	8	0.176954733	8	1E+30	8.513513514
con3	4	0.259259259	4	35	10

When using Solver in Excel for sensitivity analysis, the program generates a report that includes the Constraints Table. This table presents the final values of the constraints, the shadow prices, the original right-hand side (R.H. Side) values, as well as the allowable increase and decrease before the optimal solution changes, Constraint 2 was selected due to its low shadow price (0.1), indicating that any change in this constraint will minimally affect the objective function, which is reasonable when aiming to minimize cost with minimal impact on the outcome

Showing that in constraint b_2 the lower and and upper b_2 is as follow:
The change of b_2 = the value of b_2 $-$ decrease allowable = $8 - 8.5 = 0$
The change of b_2 = the value of b_2 $+$ increase allowable = $8 + 2 = 10$
The lower and upper sensitive is: $0 < b_2 < 10$
by using solver finding the lower right-hand side = 0
Z=1.65844, x_1 = 1.3539 x_2 =0.3209, x_3 =0.07819
by using solver finding the upper right-hand side =10
Z=3.42798 $x_1 = 1.93004$, $x_2 = 1.30864$, $x_3 = 1.60082$
Third: after that finding the fuzzy number by using proposed linear symmetric heptagonal membership function as follow:
The distance between the first and the last point is $=0 - 7 = -7 = 7$
The difference between each pair of points is $= \Delta = \frac{7}{6} = 1.16666$

$a_7 = 0, a_6 = 1.16666, a_5 = 2.333333, a_4 = 3.5, a_3 = 4.666667, a_2 = 5.833333, a_1 = 7$
Apply the ranking function by using equation (5)

$$\mathcal{R}(\tilde{A}_{LS}) = \frac{1}{4}[r(a_1 + 2a_2 + 2a_6 + a_7 - 2a_4) + (a_3 + 2a_4 + a_5)]$$

When we apply the fuzzy solution for fuzzy linear programming when changing(b_2) and use Solver in Excel the result are as follows:

Table 4: represent the fuzzy linear programming when change(b_2)

r	b_2-fuzzy	Opt. solu.	x_1	x_2	x_3
0.1	3.85	2.33971	1.57572	0.170123	0.6644
0.2	4.2	2.40165	1.59588	0.7358	0.7177
0.3	4.55	2.46358	1.61605	0.77037	0.77099
0.4	4.9	2.52551	1.63621	0.8049	0.82428
0.5	5.25	2.58745	1.65638	0.83951	0.87757
0.6	5.6	2.64938	1.67654	0.87407	0.93086
0.7	5.95	2.71132	1.69671	0.90864	0.98416
0.8	6.3	2.77325	1.71687	0.94321	1.303745
0.9	6.65	2.89712	1.73704	0.97778	1.09074

we can show that the ranking function for the fuzzy linear programming model is best from the crisp linear programming model for all values of the weight function r.

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The main innovation of this research lies in integrating sensitivity analysis with fuzzy logic using a heptagonal membership function and a fuzzy ranking method, along with a practical application that demonstrates how the fuzzy model can improve solutions and adapt to uncertainty. This makes the study novel and valuable for decision-makers dealing with uncertain data.

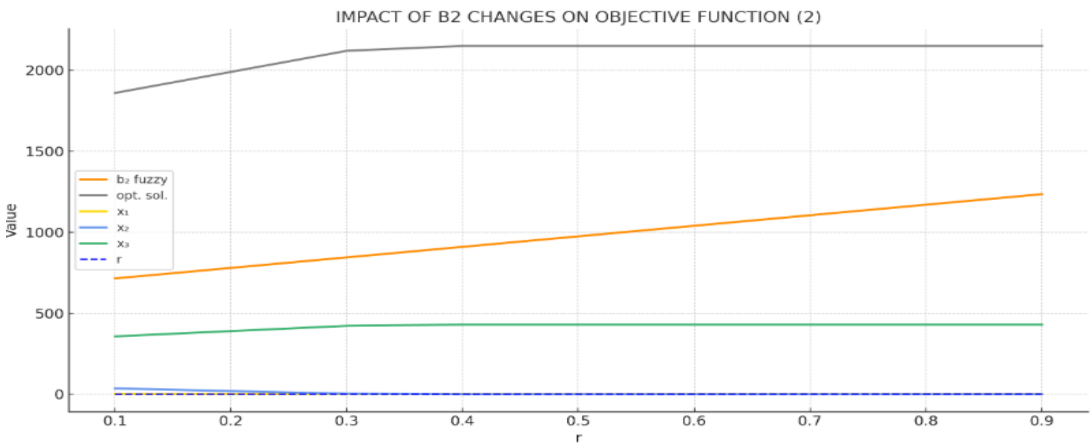

Figure2: impact of change b_2 on objective function

The optimal solution is change by the increase in b_2 until it reaches a state of stability after r=0.4.
The variable x_2 is the most responsive to changes in b_2, while x_1 and x_3 remain almost unaffected.
Introducing fuzziness through r enhances the flexibility of the model and highlights the stability regions of the objective function, which helps decision-makers identify the limits of influence and avoid over-reliance on large changes beyond the stability point.

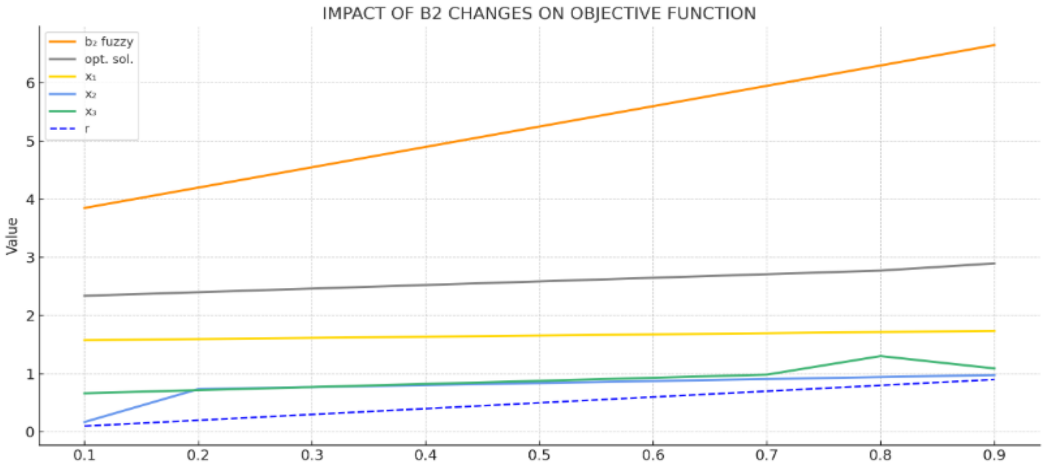

Figure 3: impact of change b_2 on objective function

The optimal solution is gradually affected by the change in b_2 until it reaches higher values at higher levels of r.The variable x_2 shows more fluctuation and sensitivity to changes in b_2, while x_1 remains almost constant and x_3 increases gradually. Introducing fuzziness through r reveals the flexibility of the model and demonstrates how each variable contributes to achieving the objective under different conditions of uncertainty.

5. DISCUSSION

result and discuss the optimal solution of example: maximize and minimize of objective function.

1. We can observe that the crisp linear programming model and fuzzy linear programming model on the application of example when maximize objective function.

LP	r	Optimal solution	x_1	x_2	x_3
crisp	-	1350	0	100	230
$\mathcal{R}_{LS}(\tilde{A})$	0.1 0.2 0.3	1860 1990 2120	0 0 0	36.25 20 3.75	357.5 390 422.5

The results of applying the fuzzy linear programming approach indicate that the optimal solution changes as the fuzziness parameter r varies. Specifically, for r=0.1 to r=0.3, the optimal solution evolves progressively: x_1 remains 0, while x_2 decreases from 36.25 to 3.75, and $x_3$increases from 357.5 to 422.5. This demonstrates that the fuzzy model adapts to variations in the right-hand side b_2, capturing the uncertainty more effectively.

For r=0.4 to r=0.9, the optimal solution remains constant $(x_1 = 0, x_2 = 0, x_3 = 430)$, which shows that beyond a certain level of fuzziness, the solution stabilizes. In these cases, there is no need for further adjustment, as the solution has reached a steady optimal state.

Comparing the fuzzy solution with the crisp (traditional) optimal solution, we observe that the fuzzy approach provides a better outcome when using a maximization objective function, as in the first example. The fuzzy model accommodates the uncertainty in b_2 and allows the decision-maker to achieve higher objective function values than the traditional crisp solution, demonstrating the advantage of using fuzzy linear programming in uncertain environments.

2. we can see that the crisp linear programming model and the fuzzy linear programming model on the application of example when minimize objective function are as follows:

LP	r	Optimal solution	x_1	x_2	x_3
crisp	-	3.07407	1.81481	1.11111	1.2963
$\mathcal{R}_{LS}(\tilde{A})$	0.1	2.33971	1.57572	0.170123	0.6644
	0.2	2.40165	1.59588	0.7358	0.7177
	0.3	2.46358	1.61605	0.77037	0.77099
	0.4	2.52551	1.63621	0.8049	0.82428
	0.5	2.58745	1.65638	0.83951	0.87757
	0.6	2.64938	1.67654	0.87407	0.93086
	0.7	2.71132	1.69671	0.90864	0.98416
	0.8	2.77325	1.71687	0.94321	1.303745
	0.9	2.89712	1.73704	0.97778	1.09074

The results of applying the fuzzy linear programming approach indicate that the optimal solution changes with variations in the fuzziness parameter r. Specifically, for the range r=0.1 to r =0.9, the optimal solution evolves gradually: x_1 remains relatively small and stable, while x_2 and x_3 change progressively to reflect the effects of variations in the right-hand side b_2This demonstrates that the fuzzy model adapts to uncertainty and allows handling parameter variations more effectively.

By the end of the range (r=0.9), the solution reaches a nearly stable state with minimal changes, indicating that the fuzzy model provides stability after a certain level of fuzziness.

Comparing the fuzzy solution with the traditional crisp optimal solution, it is evident that the fuzzy approach yields better outcomes, as it allows dealing with the uncertainty in b_2 and enables the decision-maker to achieve higher objective function values compared to the traditional solution, highlighting the benefit of using fuzzy linear programming in uncertain environments.

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