Wasit Journal for Pure Science

Journal Homepage: https://wjps.uowasit.edu.iq/index.php/wjps/indexe-
ISSN: 2790-5241 p-ISSN: 2790-5233

On Nano g- Connectedness via Graph with Medical Application

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DOI: https://doi.org/10.31185/wjps.856
Received 11 March 2025; Accepted 15 July 2025; Available online 30 March 2026

ABSTRACT: The importance of nano-topology via graphs lies in its ability to represent data within a small yet meaningful topological framework, which enhances the accuracy of classification and decision-making, and supports comparison and categorization. This makes any model applicable in various fields such as healthcare and data analysis. The main objective of this paper is to introduce a new concept of nano-connectivity, called nano-g connectedness via graph. Which defined as the disjoint union of any pair of open sets. We also present a new application to identify the main factors contributing to atherosclerosis.

Keywords: Nano-Topological Space Via Graph, Nano g-connectedness via graph, Nano g-disconnectedness via graph, Ng-Os and Ng-Cs

1. INTRODUCTION

The importance of nano topology derived from graph theory lies in fact, it helps us deal with imprecise and ambiguous data using theories such as the integer set and approximation space [1] [2], and thus extract underlying patterns. It also enables us to understand fine-scale structures by analyzing relationships between small components (nodes) in a system and studying the structures and connections between them using graphs. The notion of generalized closed sets introduced by N. Levine [3] in 1970. Arhangel'ski and Richard Wiegandt [4] introduced in 1975 the notion of connectedness and disconnection in topological space. Lellis Thivagar [5] [6] introduced the concept of nano topology in 2013. S Krishnaprakash, R Ramesh and R Suresh in 2018 [7] introduced the notion of nano connectedness in nano topological space. K Bhuvaneswari and K Mythili Gnanapriya [8] in 2014 introduced the notion of generalized closed nanostructures. In 2016, M. Lellis Thivagar [9] introduced the concept of graph-generated nano topology. In this paper a new generalization of nanostructures derived from graph theory is studied, and its properties are discussed. Several researchers have studied and identified the main factors that lead to atherosclerosis. Atherosclerosis is a gradual condition that occurs in the coronary arteries, where substances containing cholesterol and fats accumulate on the artery walls, causing them to narrow. At some critical stages, the arteries may become completely blocked, which can cause numerous health problems for the patient, including angina, heart attacks, heart failure, or even chronic kidney failure. Studies have shown that lifestyle changes, such as quitting smoking, exercising regularly and consistently, and following a healthy diet to maintain weight, are among the methods that significantly reduce the risk of atherosclerosis. The research sheds light on the main factors and one of the causes that lead to atherosclerosis using graph-induced

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Nano topology, where we wrote a table of analyzing the influencing factors in the form of factors, and in addition to that, we wrote the distance matrix of the table through the distance function to provide a deeper understanding of the links and relationships in various scientific applications.

2. PRELIMINARIES

In this section, we present the key concepts that will serve as the foundation for the remainder of the research.

Definition 1. [10] [11] A graph $G(P, L)$ consists of the following three paragraphs:

1. Points set $P(G)$.
2. Lines set $L(G)$.
3. A relation between a line and a pair of points.

Definition 2. [10] [11] Let $G(P, L)$ be a graph, $p \in P(G)$.Then ,the neighborhood of p define as:$N_p = \{\{p\} \cup \{q \in P(G): pq \text{ or } qp \in L(G)\}$ which N_p is the union of p with all adjacent points of it.

Definition 3. [1] Let W be a finite set of items called universe and E be an equivalence relation on W named as the indiscernibility classes elements belonging to the same equivalence are considered to be approximation space. Let $M \subseteq W$:-

1. the set of every element that can be for sure classified as M with respect to E, called the lower approximation space of M respect to E and it is denotes by $L_E(M)$ that is:$L_E(\delta) = \cup_{\delta\in W}\{E(\delta); E(\delta) \subseteq \delta \neq \phi\}$. Where $E(\delta)$ denotes the equivalence class determined by δ
2. The set of every element that can possibly be defined as M with respect to E, called the upper approximation space of M respect to E and it is denotes by $U_E(M)$ that is: $U_E(\delta) = \cup_{\delta\in W}\{E(\delta); E(\delta)\cap\delta \neq \phi\}$.
3. The set of every element that can be classified neither as M nor as not M with respect to E, called the boundary region of M whith respect to E and is denotes by $B_E(M)$ that is: $B_E(\delta) = U_E(\delta)\setminus L_E(\delta)$.

Definition 4 [5] [6]. Let W the universe set & E be the equivalence relation on W and $\tau_E(M)$= $\{W,\phi,L_E(M),U_E(M),B_E(M)\}$ where $M \subseteq W$ satisfies the axioms of topology, then, $\tau_E(M)$ called the nano topology W with respect to M,(W,$\tau_E(M)$) called nano topological spaces. The elements of $\tau_E(M)$ are known as nano open sets ((briefly N-Os) and the complement of it called a nano closed set ((briefly N-Cs).

Definition 5. [8] Let $(W, \tau_E(M))$ be a nano topological space. N subset of $(W, \tau_E(M))$ said to be nano generalized closed set (briefly Ng-closed set) if $Ncl(N) \subseteq U$ where $N \subseteq U$ and U is nano-open.

Definition 6. [8] If $(W, \tau_E(M))$ is a nano topological space with respect to M where $M \subseteq W$ and if, $N \subseteq W$ then:

1. The nano generalized interior of the set N is denoted as $N_g int(N)$ and defined by:
$$N_g int(N) = \cup\{H \subseteq W; H \text{ is nano g open set and } H \subseteq N\}$$
2. The nano generalized closure of the set N is denoted as $N_g cl(N)$ and defined by
$$N_g cl(N) = \cap\{F \subseteq W; F \text{ is nano g closed and } N \subseteq F\}$$

Definition 7. [9] Let $G(P, L)$ be a graph, N_p is the neighborhood of a point $p \in P(G)$ then, $\tau_G(M)$= $\{P(G),\phi,L_G(M),U_G(M),B_G(M)\}$ where the approximation space of every subset M of a graph $P(G)$ are defined by:

1. $L_G(M)=\{p \in P(G); N_G(p) \subseteq M\}$
2. $U_G(M)=\{ p \in P(G); N_G(p) \cap M \neq \phi \}$
3. $B_G(M)=U_G(M)\setminus L_E(M)$.
That is, $\tau_G(M)$ known as the nano topology induced by graph (briefly NTS_G).

Definition 8. [12] [13] Let $(P(G), \tau_G(M))$ be a NTS_G then, two non-empty subgraphs S and F of P(G) are said to be N- separated via graph if $S \cap Ncl(F) = \phi$ and $F \cap Ncl(S) = \phi$.

Definition 9. [12] [13] We said that the NTS_G is N − connected if it cannot be defined as a union of two non-empty disjoint N-Os. A subset M of NTS_G is N − connected as a subspace of NTS_G.

Definition 10. [14] Let $(P(G), \tau_G(M))$be a NTS_G , if $p \in P(G)$, N^{ng} is said to be N-neighborhood for a point p if there exist an N-Open set S such that $p \in S \subseteq N^{ng}$.

Definition 11. [14] A point $p \in P(G)$ is said to be a N-limit point of S if and only if for each $P \in Ng - Os$

We have $P\backslash\{p\} \cap A \neq \phi$.

3. NANO g-CONNECTEDNESS VIA GRAPH

In this part, we investigate the concept of Nano g-connectedness via graph (briefly Ng-connectedness via graph), a new generalization of the concept of nano-connectedness in nano topological via graph. The purpose of this work is to elucidate the importance of this concept in developing nano topological investigations based on graph theory.

Definition 12. Let $(P(G), \tau_G(\delta))$ be a NTS_G then, two non-empty subgraphs S and F of P(G) are said to be Ng separated via graph if $S \cap N_g cl(F) = \phi$ and $F \cap N_g cl(S) = \phi$.

Example 1. Consider the graph shown in **Figure 1** then, $N(p_1) = \{p_2, p_1\}, N(p_2) = \{p_4, p_2, p_3, p_1\}$,$N(p_3) = \{p_4, p_3, p_2\}, N(p_4) = \{p_5, p_4, p_2, p_3\}$ and $N(p_5) = \{p_5, p_4\}$. Let $\delta = \{p_4, p_5\}$ then,$L_G(\delta) = \{p_5\}$,$U_G(\delta) = \{p_5, p_4, p_3, p_2\}$ then $B_G(\delta) = \{p_2, p_4, p_3\}$ that is the NTS_G:-$\tau_G(\delta) = \{P(G), \phi, \{p_5\}, \{p_2, p_4, p_3\}, \{p_5, p_4, p_3, p_2\}\}$, $\tau_G^c(\delta) = \{P(G), \phi, \{p_1\}, \{p_1, p_5\}, \{p_4, p_3, p_2, p_1\}\}$ and Ng-Cs=$\{P(G), \phi, \{p_1\}, \{p_2, p_1\}, \{p_1, p_3\}, \{p_4, p_1\}, \{p_1, p_5\}, \{p_3, p_2, p_1\}, \{p_1, p_4, p_2\}, \{p_5, p_2, p_1\}, \{p_1, p_4, p_3\}, \{p_3, p_1, p_5\}, \{p_5, p_4, p_1\}, \{p_5, p_2, p_3, p_1\}, \{p_5, p_1, p_2, p_3\}, \{p_4, p_3, p_2, p_1\}$ .Now, let $S = \{p_3\}$ and $F = \{p_4\}$ be a subgraph from graph G then, $N_g cl(S) = \{p_1, p_3\}$,$N_g cl(F) = \{p_1, p_4\}$ such that $S \cap N_g cl(F) = \{p_3\} \cap \{p_1, p_4\} = \phi$ and $F \cap N_g cl(S) = \{p_4\} \cap \{p_1, p_3\} = \phi$. That is, S and F are Ng − separated subgraph.

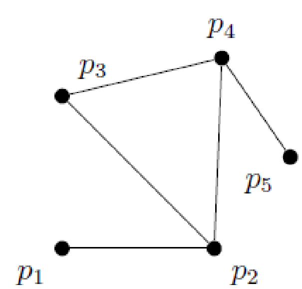

FIGURE 1: A Connected Cyclic Graph

Remark 1. Each pair of Ng-separated sets is disjoint, but the converse does not necessarily hold.

Example 2. From Example 1, we take two subgraphs of graph: $S = \{p_1\}$ and $F = \{p_2\}$, which are disjoint then, $N_g cl(S) = \{p_1\}$ and $N_g cl(F) = \{p_2, p_1\}$ such that $S \cap N_g cl(F) = S \neq \phi$ .Hence, S and F are not Ng-separated.

Theorem 1. Any two open or closed subsets of NTS_G that don't intersect are called disjoint.

Proof. Suppose S, F be two subgraphs from graph G are not intersect that is, $S \cap F = \phi$:-

1. If S and F are Ng-open then, $(S\backslash F) \cap N_g cl((F\backslash S)) \subseteq S \cap N_g cl(F\backslash S) \subseteq N_g cl(S \cap (F\backslash S)) = \phi$ and $N_g cl((S\backslash F)) \cap ((F\backslash S)) \subseteq (S\backslash F) \subseteq N_g cl((S\backslash F) \cap S) = \phi$. That is, $S\backslash F$, $F\backslash S$ are separated.

2. In a similar way, we can prove if S and F are closed then, $S\backslash F$, $F\backslash S$ are separated. ■

Proposition 1. If S and F are Ng-separated subgraph in NTS_G such that $S_1 \subseteq S, F_1 \subseteq F$ then, S_1 and F_1 are Ng-separated subgraph.

Proof. Since S and F are Ng-separated that means $S \cap F = \phi$ consequently, $S_1 \cap F_1 = \phi$ and: -

1. $S_1 \cap N_g cl(F_1) \subseteq S \cap N_g cl(F) = \phi$
2. $F_1 \cap N_g cl(S_1) \subseteq F \cap N_g cl(S) = \phi$ as a result, S_1 and F_1 are Ng-separated subgraph. ■

Theorem 2. Two Ng-OS subgraph of a NTS_G are Ng − separated iff they are disjoint.

Proof. Let S and F be a Ng-Os of a NTS_G and suppose S, F are disjoint. To prove that S, F are Ng-separated. Now, suppose for contradiction, that $N_g cl(S) \cap F \neq \phi$ then, $\forall k \in N_g cl(S) \cap F$ we have $k \in N_g cl(S)$ and $k \in F$ that is, k is limit point of S hence, S is a Ng-nbhd of each it is point that is give us $S \cap F \neq \phi$ C!. So that $N_g cl(S) \cap F = \phi$, in the same way get $N_g cl(F) \cap S = \phi$ .Hence S, F are Ng-separated. **Conversely**: Because S, F are Ng-separated then, in the same way, we have $N_g cl(S) \cap F = \phi$ and $N_g cl(F) \cap S = \phi$. Let $S \subset N_g cl(S), F \subset F \Longrightarrow S \cap F \subset N_g cl \cap F \Longrightarrow S \cap F \subset \phi$ that gives us $S \cap F = \phi$ ■

Definition 13. In NTS_G, suppose there are two Ng-open subgraphs S and F in P(G), such that for any subgraph H from P(G) is:

1. $H \cap S \neq \phi, H \cap F \neq \phi$.
2. $(H \cap S) \cap (H \cap F) = \phi$.
3. $(H \cap S) \cup (H \cap F) = H$

Then, the set $S \cup F$ said to be Nano generalized-disconnection(briefly Ng-disconnection) for subgraph H and from that definition, we have: $(H \cap S) \cap (H \cap F) = H \cap (S \cap F) = \phi \Leftrightarrow S \cap F \subseteq H^c$ and $(H \cap S) \cup (H \cap F) = H \cap (S \cup F) = H \Leftrightarrow H \subseteq S \cup F$, so we can redefined it as follows: The set $S \cup F$ called Ng-disconnection for $H \Leftrightarrow H \cap S \neq \phi, H \cap F \neq \phi, H \subseteq S \cup F, S \cap F \subseteq H^c$.

Definition 14. A NTS_G is said to be Ng − connected if NTS_G is not defined as a disjoint union of two non-empty Ng-Os. A subset M of NTS_G is Ng − connected as a subspace of NTS_G.

Example 3. From Example 1. If we take complement of Ng-Cs we get Ng-Os=$\{P(G), \phi, \{p_5\}, \{p_4\}, \{p_3\}, \{p_2, p_5\}, \{p_2, p_4\}, \{p_4, p_5\}, \{p_5, p_3\}, \{p_3, p_4\}, \{p_2, p_3\}, \{p_3, p_2, p_4\}, \{p_2, p_5, p_3\}, \{p_2, p_5, p_4\}, \{p_3, p_5, p_4\}, \{p_2, p_5, p_4, p_3\}$ Here, $(P(G), \tau_G(\delta))$ cannot be expressed as a disjoint union of Ng − Os then, $(P(G), \tau_G(\delta))$ is Ng − connectedness

Example 4. Consider the graph shown in **Figure 2** then, $N_{p_1} = \{p_1, p_2, p_3\} = N_{p_2}, N_{p_3} = P(G), N_{p_4} = \{p_3, p_4\}, N_{p_5} = \{p_3, p_5\}$:If $\delta = \{p_4\}$ then, $L_G(\delta) = \phi, U_G(\delta) = \{p_3, p_4\} = B_G(\delta)$ such that $\tau_G(\delta) = \{P(G), \phi, \{p_3, p_4\}\}, \tau_G^c(\delta) = \{P(G), \phi, \{p_1, p_2, p_5\}\}$ and Ng-Os=$\{\{P(G), \phi, \{p_5\}, \{p_4\}, \{p_3\}, \{p_2\}, \{p_1\}, \{p_5, p_1\}, \{p_2, p_5\}, \{p_2, p_4\}, \{p_4, p_5\}, \{p_5, p_3\}, \{p_2, p_3\}, \{p_1, p_3\}, \{p_2, p_1\}, \{p_4, p_3\}, \{p_3, p_2, p_1\}, \{p_4, p_1, p_2\}, \{p_3, p_4, p_5\}\{p_3, p_2, p_4\}, \{p_3, p_1, p_4\}, \{p_5, p_3, p_1\}, \{p_5, p_4, p_1\}, \{p_2, p_5, p_3\}, \{p_2, p_5, p_4\}, \{p_2, p_5, p_4, p_3\} \{p_5, p_4, p_3, p_1\}, \{p_4, p_2, p_1, p_3\}\}$ then, there exist non-empty $\{p_4, p_5\}$and $\{p_1, p_2, p_3\}$ are disjoint sets and it union give the point graph such that:$\{p_4, p_5\} \cap \{p_1, p_2, p_3\} = \phi$,$\{p_4, p_5\} \cup \{p_1, p_2, p_3\} = P(G)$
then NTS_G is Ng-disconnected.

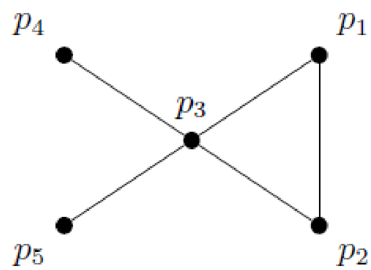

FIGURE 2: **A Connected Cyclic Graph**

Theorem 3. The subgraph S from NTS_G $(P(G), \tau_G(\delta))$ be Ng-disconnected in total space iff S is Ng-disconnected in Subgraph space $(P(G), \tau_G(S))$.

Proof. Let S be a Ng-disconnected subgraph in a total space NTS_G then, we have F and H be a non-empty, Ng-open disjoint of subgraph S in P(G) such that $F \cup H$. Now because F, H Ng-open then, $S \cap F, S \cap H \in \tau_G(S)$, and from this both $S \cap F, S \cap H$ be disjoint subgraph for S in subgraph space $(P(G), \tau_G(S))$. Hence, S be subgraph Ng-disconnected in $(P(G), \tau_G(S))$. **Conversely**: Let S be a Ng-disconnected subgraph in$(P(G), \tau_G(S))$. Let $F^*, H^* \in \tau_G(S)$ be non-empty, Ng-open disjoint of subgraph S therefore, there exist $F, H \in \tau_G(\delta)$ such that $F^* = S \cap F, H^* = S \cap H$. But , $S \cap F^* = S \cap (S \cap F) = S \cap F$ and $S \cap H^* = S \cap (S \cap H) = S \cap H$ then, $F \cup H$ be a subgraph disjoint for S in a total space $(P(G), \tau_G(\delta))$ so we get S be a subgraph Ng-disconnected in P(G) ■

Theorem 4. The Ng-continuous image of a Ng-connected set is Ng-connected.

Proof. Let $f:(P(G_1), \tau_{G_1}(\delta)) \rightarrow (P(G_2), \tau_{G_2}(f(\delta)))$ be a Ng-continuous function from a Ng-connected $(P(G_1), \tau_{G_1}(\delta))$ to a NTS_G $(P(G_2), \tau_{G_2}(f(\delta)))$,to prove that $f(P(G_1))$ is a Ng-connected subgraph. So, suppose that $f(P(G_1))$ is a Ng-disconnected subgraph. S, F be a subgraph Ng-disconnected for subgraph $f(P(G_1))$ such that: $f(P(G_1)) \subseteq S \cup F$, $S \cap F = \phi$ and from this we get: $P(G_1) \subseteq f^{-1}(S) \cup f^{-1}(F), f^{-1}(S) \cap f^{-1}(F) = \phi$. And since $f^{-1}(S), f^1(F)$ be a non-empty disjoint subgraph then $f^{-1}(S), f^1(F)$ be a subgraph Ng-disconnected for $P(G_1)$ and this is contradiction because it is a Ng-connected subgraph, and thus $f(P(G_1))$ is a Ng-connected graph. ■

Theorem 5. Let $(P(G), \tau_G(\delta))$ be a NTS_G.If there exists a Ng-Open and Ng-closed non-empty proper subgraph of P(G) then, P(G) is Ng-disconnected.

Proof. Let $S \neq \phi$ be a proper subgraph of P(G) , and let S be both Ng-open and Ng-closed. Let $F = S^c$. Then, $F \neq \phi$ since S is proper subgraph of P(G). Also, $S \cup F = P(G)$ and $S \cap F = \phi$. Since S is both Ng-open and Ng-closed then ,F also $Ng-open$ and $Ng-closed$. Hence, $N_g cl(S) = S$ and $N_g cl(F) = F$. we have $S \cap N_g cl(F) = \phi$ and $F \cap N_g cl(S) = \phi$, which means P(G) has been expresses as union of two Ng-separated sets. Therefore, P(G) is Ng-disconnected ■

Proposition 2. Let $(P(G), \tau_G(\delta))$ be a NTS_G then, the following are equivalence: -

1. $(P(G), \tau_G(\delta))$ be a Ng-disconnected.
2. Cannot be expressed as the union of two non-empty disjoint closed set.
3. The two set $P(G), \phi$ are only two Ng-clopen set in $(P(G))$.
4. For any subgraph $S \neq \phi$ in P(G) , $b_G(S) \neq \phi$.

Proof. (i) $\Longrightarrow$ (ii): Suppose the opposite. That is, there exist two Ng-closed S, F such that: $P(G) = S \cup F, S \cap F = \phi, S \neq \phi, F \neq \phi$. Then, S^c, F^c be two Ng-open such that $S^c \cup F^c = P(G), S^c \cap F^c = \phi$.
(ii) $\Longrightarrow$ (iii): Suppose the opposite. That is, there exist subgraph S from $P(G)$ be Ng-clopen such that $S \neq \phi, S \neq P(G)$,then we get $S \neq \phi, S^c \neq \phi$ and since $S \cap S^c = \phi, P(G) = S \cup S^c$ then, S and S^c non-empty disjoint Ng-Cs and there union equals $P(G)$ this contradicts assumption(ii),thus, they are the only two Ng-C and Ng-O sets in $P(G)$ are $\{P(G), \phi\}$.
(iii) $\Longrightarrow$ (iv): Suppose that S be a proper non-empty subgraph from $P(G)$ such that $b_g(S) \neq \phi$ .But, $b_g(S) = N_g cl(S) \cap N_g cl(S^c) = N_g cl(S) \cap \left(N_g int(S)\right)^c = \phi$,that is $S \subseteq N_g cl(S) \subseteq N_g int(S) \subseteq S$ it follows that $S = N_g cl(S), S = N_g int(S)$. Hence S be a Ng-clopen set in the same time, and this contradicts the assumption(iii). ■

Theorem 6. Let $\left(P(G), \tau_G(\delta)\right)$ be a NTS_G and S, F be non-empty disjoint subgraph in $P(G)$,if there is H be a Ng-connected in $P(G)$ such that $H \subseteq S \cup F$ then, $H \subseteq S$ or $H \subseteq F$.

Proof. Consider H is not contained in S, F then: $H \cap S \neq \phi, H \cap F \neq \phi$. and that give a subgraph H be Ng-disconnected to prove this: Since S, F be non-empty disjoint subgraph in $P(G)$ then: $Ncl_{g_H}(H \cap S) \cap (H \cap F) = \left(H \cap Ncl_{g_H}(S)\right) \cap (H \cap F) = H \cap \left(Ncl_g(S) \cap F\right) = H \cap \phi$ and: $Ncl_{g_H}(H \cap F) \cap (H \cap S) = \left(H \cap Ncl_{g_H}(F)\right) \cap (H \cap S) = H \cap \left(Ncl_{g_H}(F) \cap S\right) = H \cap \phi$ .Hence, $H \cap S$ and $H \cap F$ be subgraph disjoint in a subgraph space $\left(P(G), \tau_G(H)\right)$ and since $H \subseteq S \cap F$ then: $(H \cap S) \cup (H \cap F) = H \cap (S \cup F) = H$ .That is, H be Ng-disconnected subgraph C! ■

Theorem 7. If S, F be a non-disjoint, Ng-connected subgraph in $NTS_G \left(P(G), \tau_G((\delta))\right)$ then, $S \cup F$ be a Ng-connected subgraph.

Proof. Suppose $S \cup F$ be Ng-disconnected subgraph and $H \cup K$ be a Ng-separated subgraph in $S \cup F$, since $S \subseteq S \cup F$, S be Ng-connected then, from above theorem either $S \subseteq H$ or $S \subseteq K$. In a similar way, we can obtain either $F \subseteq H$ or $F \subseteq K$. Now if $S \subseteq H$ and $F \subseteq K$ or ($S \subseteq K$ and $F \subseteq H$) then, $(S \cup F) \cap H = S, (S \cup F) \cap K = F$ be a disjoint subgraph C! then, either $S \cup F \subseteq H$ or $S \cup F \subseteq K$. Which means $H \cup K$ be non-Ng-separated subgraph for $S \cup F$. Hence $S \cup F$ is Ng-connected. ■

Theorem 8. Let $\{S_i\}_{i \in I}$ be a family of subgraph Ng-connected in NTS_G such that: $\forall i, j; S_i \cap S_j \neq \phi$ then, $S = \cup_{i \in I} S_i$ is Ng-connected.

Proof. Suppose S_1 be a Ng-clopen subgraph in S. And $s \in S$ then, $s \in S_{i_o}, i_o \in I$. Now since $S \cap S_i$ be a Ng-clopen subgraph in S_{i_o} gives S_{i_o} be a Ng-connected subgraph therefore, $S \cap S_{i_o} = S_{i_o}$. And for any $j \in I$ then, $S_i \cap S_j$ be a Ng-clopen subgraph in S_j .But $S_{i_o} \subseteq S$ then, $\phi \neq S_{i_o} \cap S_j \subseteq S_1 \cap S_j = S_j$. Such that S_j be a Ng-connected subgraph therefore, $S_j \subseteq S_1$; $\forall j$ then, $S = \cup S_j \subseteq S_1$.So $S = S_1$ ■

4. APPLICATION

In this part ,We are going to apply the nano topological space via graph to identify the primary keys and causes that lead to exposure to atherosclerosis, as we will rely on an algorithm that we invented to convert patient information into: firstly, factors to analyze data and train algorithms secondly, to distance matrix to measure the extent of similarity or difference between patients and finally, graphs are used in order to facilitate the visual understanding of complex models, which speeds up the analysis and decision-making process.
Algorithm :
CASE I : patients with Atherosclerosis
Step 1 :We convert patients into factors.
Step 2 : We find the distance between the factors.
Step 3 :We generate a distance matrix with a threshold value:distances > 0 are marked as 0, while distances = 0 are marked as 1.
Step 4 : We convert the distance matrix to a graph, adding edges for each 1 and leaving zeros unconnected.
Step 5 :Convert the graph to nano topology.
CASE II : patients without Atherosclerosis We proceed with the same steps as before.

Example 5. Atherosclerosis is a condition in which fat, cholesterol & different materials assemble on the walls of your arteries, causing them to narrow and harden. Over time, your arteries become less elastic, restricting blood flow to your organs and tissues, and if left untreated, it can lead to serious problems such as heart attacks or strokes. We will use the new algorithm to detect and predict the causes of atherosclerosis. Similarity and disease identification are performed using a new technique generated by the distance matrix, which constructs the relative topological graph. We proceed with the same steps as before. The following table contains information on 15 patients.

TABLE 1. Patient Survey and data collection.

Patients	High blood pressure	Diabetes	Smoking	Obesity	Family history	High cholesterol	Iron Deficiency Anemia	Result
U_1	✓			✓		✓	✓	✓
U_2	✓	✓			✓	✓		✓
U_3	✓	✓	✓	✓		✓	✓	✓
U_4	✓		✓			✓		✓
U_5	✓		✓					✓
U_6	✓	✓			✓	✓		
U_7	✓	✓			✓	✓		
U_8	✓		✓			✓		
U_9	✓		✓					
U_{10}	✓	✓	✓	✓		✓	✓	✓
U_{11}		✓	✓		✓			
U_{12}		✓	✓		✓			
U_{13}	✓	✓	✓					
U_{14}		✓			✓	✓		
U_{15}	✓			✓		✓	✓	✓

Here $P(G) = \bigcup_{i=1}^{15} U_i$ is a list of patients with qualities A equal to HBP=High blood pressure, DB=Diabetes, SMK=Smoking, OB= Obesity, FH=Family history, HC=High cholesterol and IDA= Iron Deficiency Anemia. Here C.C represents Condition characteristics ={HBP,DB,SMK,OB,FH,HC,IDA} and D represents Decision attributes (Atherosclerosis).

Step 1 :Convert the previous table into a factor: $U_1 = [1\ \ 0\ \ 0\ \ 1\ \ 0\ \ 1\ \ 1]$, $U_2 = [1\ \ 1\ \ 0\ \ 0\ \ 1\ \ 1\ \ 0]$, $U_3 = [1\ \ 1\ \ 1\ \ 1\ \ 0\ \ 1\ \ 1]$,$U_4 = [1\ \ 0\ \ 1\ \ 0\ \ 0\ \ 1\ \ 0]$, $U_5 = [1\ \ 0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0]$, $U_6 = [1\ \ 1\ \ 0\ \ 0\ \ 1\ \ 1\ \ 0]$, $U_7 = [1\ \ 1\ \ 0\ \ 0\ \ 1\ \ 1\ \ 0]$, $U_8 = [1\ \ 0\ \ 1\ \ 0\ \ 0\ \ 1\ \ 0]$ $U_9 = [1\ \ 0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0]$, $U_{10} = [1\ \ 1\ \ 1\ \ 1\ \ 0\ \ 1\ \ 1]$, $U_{11} = [0\ \ 1\ \ 1\ \ 0\ \ 1\ \ 0\ \ 0]$, $U_{12} = [0\ \ 1\ \ 1\ \ 0\ \ 1\ \ 0\ \ 0]$, $U_{13} = [1\ \ 0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0]$, $U_{14} = [0\ \ 1\ \ 0\ \ 0\ \ 1\ \ 1\ \ 0]$ and $U_{15} = [1\ \ 0\ \ 0\ \ 1\ \ 0\ \ 1\ \ 1]$

Step 2 : We use the following mathematical relationship to get the distance between the quantities of the factors:

$d(U_i, U_j) = \begin{cases} \sum_{n=1}^{15} |U_{in} - U_{jn}|, & \text{if } U_i \neq U_j, \\ 0, & \text{if } U_i = U_j. \end{cases}$ then,: $d(U_1, U_2) = 4$, $d(U_1, U_3) = 2$, $d(U_1, U_4) = 3$... in the same approach, we calculate the distance between all factors.

Step 3: Now, we will express the distance between each factors as a distance matrix and we will take a threshold value < 1 (which means that if the distance between the factors is equal to or more than one, it will be written in the distance matrix as zero, and if it is equal to zero, it will be written in the distance matrix as one):

$$U_{ij} = \begin{array}{c|ccccccccccccccc} & U_1 & U_2 & U_3 & U_4 & U_5 & U_6 & U_7 & U_8 & U_9 & U_{10} & U_{11} & U_{12} & U_{13} & U_{14} & U_{15} \\ U_1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ U_2 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ U_3 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ U_4 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ U_5 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ U_6 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ U_7 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ U_8 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ U_9 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ U_{10} & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ U_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ U_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ U_{13} & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ U_{14} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ U_{15} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}$$

Step 4: Then, we will transform this matrix into a graph that depicts the relationship and total similarity of the patient's attributes:

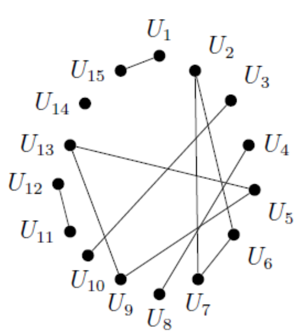

FIGURE 3: **Graph illustrating the complete network of relationships among atherosclerosis patients**

Step 5 : Now, we transform the graph to nano topology, when the nbhd of the graph is defined as $N_{U_1} = N_{U_{15}} = \{U_1, U_{15}\}$,$N_{U_2} = N_{U_6} = N_{U_7} = \{U_2, U_6, U_7\}$,$N_{U_3} = N_{U_{10}} = \{U_3, U_{10}\}$,$N_{U_4} = N_{U_8} = \{U_4, U_8\}$,$N_{U_5} = N_{U_9} = N_{U_{13}} = \{U_5, U_9, U_{13}\}$,$N_{U_{11}} = N_{U_{12}} = \{U_{11}, U_{12}\}$ and $N_{U_{14}} = \{U_{14}\}$

Case 1 : patients that have atherosclerosis:$\delta = \{U_1, U_2, U_3, U_4, U_{10}, U_{15}\}$ then, $L_{C.C}(\delta) = \{U_1, U_3, U_{10}, U_{15}\}$,$U_{C.C}(\delta) = \{U_{15}, U_6, U_7, U_2, U_8, U_3, U_4, U_{10}, U_1\}$, $B_{C.C}(\delta) = \{U_8, U_6, U_4, U_7, U_2\}$. Hence, the nano topology induced by graph(NTS_G) is $\tau_{C.C}(\delta) = \{P(G), \phi, \{U_1, U_3, U_{10}, U_{15}\}$ $, \{U_8, U_6, U_4, U_7, U_2\}, \{U_{15}, U_6, U_7, U_2, U_8, U_3, U_4, U_{10}, U_1\}\}$.

1. If the attribute "High blood pressure" is deleted from C.C we apply the same previous steps and we will get the following graph:

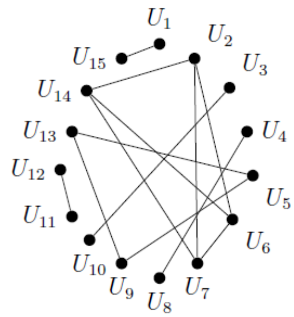

FIGURE 4: **The relationship between patients after deleting High blood pressure**

Then, NTS_G is $\tau_{C.C\backslash HBP}(\delta)\left\{\begin{matrix} P(G), \phi, \{U_1, U_3, U_{10}, U_{15}\}, \{U_8, U_6, U_4, U_7, U_2, U_{14}\}, \\ \{U_{15}, U_6, U_7, U_2, U_8, U_3, U_4, U_{10}, U_{14}, U_1\} \end{matrix}\right\} \neq \tau_{C.C}(\delta)$.Thus, the High blood pressure is one of the main keys to the disease. Inasmuch we note that there is a difference between the nano topological structure when the Atherosclerosis is deleted and the nano topological structure of the disease.

2. If the attribute "Diabetes" is deleted from C.C, we obtain the result shown in **Figure 3**. Clearly, $\tau_{C.C\backslash DB}(\delta) = \tau_{C.C}(\delta)$.Deleting the Diabetes does not lead to a change in the nano topological structure and therefore, does not have that strong effect on the occurrence of the disease.

3. If the attribute "Smoking" is deleted from C, we obtain the result shown in **Figure 3** .Clearly, $\tau_{C.C\backslash SMK}(\delta) = \tau_{C.C}(\delta)$.

4. If the attribute "Obesity" is deleted from C.C, we obtain the result shown in **Figure 3** .Clearly,$\tau_{C.C\backslash OB}(\delta) = \tau_{C.C}(\delta)$.

5. If the attribute "Family history" is deleted from C.C, we obtain the result shown in **Figure 3** .Clearly, $\tau_{C.C\backslash FH}(\delta) = \tau_{C.C}(\delta)$.

6. If the attribute "High cholesterol" is deleted from C.C then, $U_1 = [1 \quad 0 \quad 0 \quad 1 \quad 0 \quad 1]$, $U_2 = [1 \quad 1 \quad 0 \quad 0 \quad 1 \quad 0]$, $U_3 = [1 \quad 1 \quad 1 \quad 1 \quad 0 \quad 1]$, $U_4 = [1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0]$, $U_5 = [1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0]$, $U_6 = [1 \quad 1 \quad 0 \quad 0 \quad 1 \quad 0]$, $U_7 = [1 \quad 1 \quad 0 \quad 0 \quad 1 \quad 0]$, $U_8 = [1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0]$ $U_9 = [1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0]$, $U_{10} = [1 \quad 1 \quad 1 \quad 1 \quad 0 \quad 1]$,$U_{11} = [0 \quad 1 \quad 1 \quad 0 \quad 1 \quad 0]$, $U_{12} = [0 \quad 1 \quad 1 \quad 0 \quad 1 \quad 0]$, $U_{13} = [1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0]$, $U_{14} = [0 \quad 1 \quad 0 \quad 0 \quad 1 \quad 0]$ and $U_{15} = [1 \quad 0 \quad 0 \quad 1 \quad 0 \quad 1]$.The distance matrix is obtained by calculating the distance between factors and applying the threshold value < 1 :

7.

$U_{ij}=$

	U_1	U_2	U_3	U_4	U_5	U_6	U_7	U_8	U_9	U_{10}	U_{11}	U_{12}	U_{13}	U_{14}	U_{15}
U_1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1
U_2	0	1	0	0	0	1	1	0	0	0	0	0	0	0	0
U_3	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0
U_4	0	0	0	1	1	0	0	1	1	0	0	0	1	0	0
U_5	0	0	0	1	1	0	0	1	1	0	0	0	1	0	0
U_6	0	1	0	0	0	1	1	0	0	0	0	0	0	0	0
U_7	0	1	0	0	0	1	1	0	0	0	0	0	0	0	0
U_8	0	0	0	1	1	0	0	1	1	0	0	0	1	0	0
U_9	0	0	0	1	1	0	0	1	1	0	0	0	1	0	0
U_{10}	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0
U_{11}	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0
U_{12}	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0
U_{13}	0	0	0	1	1	0	0	1	1	0	0	0	1	0	0
U_{14}	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
U_{15}	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1

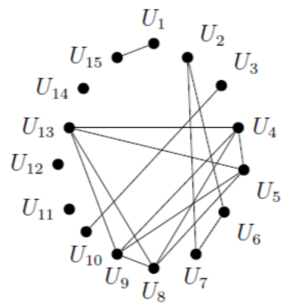

FIGURE 5: **The relationship between patients after deleting high cholesterol**

Now, we transform the graph to nano topology, when the nbhd of the graph is $N_{U_1} = N_{U_{15}} = \{U_1, U_{15}\}$,$N_{U_2} = N_{U_6} = N_{U_7} = \{U_2, U_6, U_7\}$,$N_{U_3} = N_{U_{10}} = \{U_3, U_{10}\}$,$N_{U_4} = N_{U_5} = N_{U_8} = N_{U_9} = N_{U_{13}} = \{U_4, U_5, U_8, U_9, U_{13}\}$,$N_{U_{11}} = N_{U_{12}} = \{U_{11}, U_{12}\}$ and $N_{U_{14}} = \{U_{14}\}$ then, $U_{C.C}(\delta) = \{U_{15}, U_6, U_7, U_2, , U_8, U_3, U_4, U_9, U_{13}, U_5, U_{10}, U_1\}$,$L_{C.C}(\delta) = \{U_1, U_3, U_{10}, U_{15}\}$ *and* $B_{C.C}(\delta) = \{U_9, U_8, U_6, U_{13}, U_4, U_7, U_2\}$. Hence NTS_G is$\{P(G), \phi, \{U_{15}, U_6, U_7, U_2, U_8, U_3, U_4, U_9, U_{13}, U_5, U_{10}, U_1\}\{U_9, U_8, U_6, U_{13}, U_4, U_7, U_2\}, \{U_1, U_3, U_{10}, U_{15}\}\} = \tau_{C.C\setminus HC}(\delta) \neq \tau_{C.C}(\delta)$. That is the High cholesterol is one of the main keys to the disease. Inasmuch we note that there is a difference between the nano topological structure when the Atherosclerosis is deleted and nano topological structure of the disease.

8. If the attribute "Iron Deficiency Anemia" is deleted from C.C, we obtain the result shown in **Figure 3** .Clearly, $\tau_{C.C\setminus IDA}(\delta) = \tau_{C.C}(\delta)$.Hence,

$$\textbf{CORE} = \{\textbf{High blood pressure}, \textbf{High cholesterol}\}$$

***Case* 2**: *Patients without Atherosclerosis*:In this situation, we will receive the same distance between the factors and thus the identical graph; the only difference is that $\delta_1 = \{U_5, U_6, U_7, U_8, U_9, U_{11}, U_{12}, U_{13}, U_{14}\}$ then, $L_{C.C}(\delta_1) = \{U_1, U_3, U_{10}, U_{15}\}$,$U_{C.C}(\delta_1) = \{U_8, U_3, U_{15}, U_2, U_6, U_4, U_1, U_7, U_{10}\}$ and $B_{C.C}(\delta_1) = \{U_2, U_6, U_8, U_7, U_4\}$. Hence, the nano topology induced by graph(NTS_G) is $\{\{U_8, U_3, U_{15}, U_2, U_6, U_4, U_1, U_7, U_{10}\}$,$\{U_2, U_6, U_8, U_7, U_4\}, \{U_1, U_3, U_{10}, U_{15}\}\}$= $\tau_{C.C}(\delta_1)$

1. If the attribute " High blood pressure" is deleted from C.C then, by **Figure 4**, we obtain $U_{C.C}(\delta_1) = \{U_5, U_8, U_3, U_{13}, U_2, U_6, U_{14}, U_4, U_{12}, U_7, U_{11}\}$, $L_{C.C\setminus SMK}(\delta_1) = \{U_5, U_9, U_{11}, U_{12}, U_{13}\}$, $B_{C.C\setminus SMK}(\delta_1) = \{U_2, U_4, U_6, U_8, U_{14}\}$ that is $\tau_{C.C\setminus HBP}(\delta) = \begin{Bmatrix} P(G), \phi, \{U_5, U_9, U_{11}, U_{12}, U_{13}\}, \{U_2, U_4, U_6, U_8, U_{14}\}, \\ \{U_5, U_8, U_3, U_{13}, U_2, U_6, U_{14}, U_4, U_{12}, U_7, U_{11}\} \end{Bmatrix} \neq \tau_{C.C}(\delta_1)$
2. If the attribute "Diabetes" is deleted from C.C then, by **Figure 3**, we obtain $L_{C.C\setminus DB}(\delta_1) = \{U_1, U_3, U_{10}, U_{15}\}$, $U_{C.C\setminus DB}(\delta_1) = \{U_8, U_3, U_{15}, U_2, U_6, U_4, U_1, U_7, U_{10}\}$, $B_{C.C\setminus DB}(\delta_1) = \{U_2, U_4, U_6, U_7, U_8\}$ that is$\{\{U_8, U_3, U_{15}, U_2, U_6, U_4, U_1, U_7, U_{10}\}, \{U_2, U_6, U_8, U_7, U_4\}, \{U_1, U_3, U_{10}, U_{15}\}\}$= $\tau_{C.C\setminus DB}(\delta_1)$
3. If the attribute "Smoking" is deleted from C.C then, by **Figure 3**, we obtain $L_{C.C\setminus HBP}(\delta_1) = \{U_1, U_3, U_{10}, U_{15}\}$, $U_{C.C\setminus HBP}(\delta_1) = \{U_8, U_3, U_{15}, U_2, U_6, U_4, U_1, U_7, U_{10}\}$, $B_{C.C\setminus HBP}(\delta_1) = \{U_2, U_4, U_6, U_7, U_8\}$ that is $\{\{U_8, U_3, U_{15}, U_2, U_6, U_4, U_1, U_7, U_{10}\}, \{U_2, U_6, U_8, U_7, U_4\}, \{U_1, U_3, U_{10}, U_{15}\}\}$= $\tau_{C.C\setminus SMK}(\delta_1)$=$\tau_{C.C}(\delta_1)$
4. If the attribute " Obesity" is deleted from C.C then, by **Figure 3**, we obtain $L_{C.C\setminus OB}(\delta_1) = \{U_1, U_3, U_{10}, U_{15}\}$, $U_{C.C\setminus OB}(\delta_1) = \{U_8, U_3, U_{15}, U_2, U_6, U_4, U_1, U_7, U_{10}\}$, $B_{C.C\setminus OB}(\delta_1) =$

$\{U_2, U_4, U_6, U_7, U_8\}$ that is $\{\{U_8, U_3, U_{15}, U_2, U_6, U_4, U_1, U_7, U_{10}\}$, $\{U_2, U_6, U_8, U_7, U_4\}, \{U_1, U_3, U_{10}, U_{15}\}\} = \tau_{C.C\setminus OB}(\delta_1) = \tau_{C.C}(\delta_1)$

5. If the attribute " Family history" is deleted from C.C then, by **Figure 3**, we obtain $L_{C.C\setminus FH}(\delta_1) = \{U_1, U_3, U_{10}, U_{15}\}$, $U_{C.C\setminus FH}(\delta_1) = \{U_8, U_3, U_{15}, U_2, U_6, U_4, U_1, U_7, U_{10}\}$, $B_{C.C\setminus FH}(\delta_1) = \{U_2, U_4, U_6, U_7, U_8\}$ that is $\{\{U_8, U_3, U_{15}, U_2, U_6, U_4, U_1, U_7, U_{10}\}$, $\{U_2, U_6, U_8, U_7, U_4\}, \{U_1, U_3, U_{10}, U_{15}\}\} = \tau_{C.C\setminus FH}(\delta_1)) = \tau_{C.C}(\delta_1)$
6. If the attribute " High cholesterol" is deleted from C.C then, by **Figure 5**, we obtain $U_{C.C\setminus HC}(\delta_1) = \{U_2, U_4, U_5, U_6, U_7, U_8, U_9, U_{11}, U_{12}, U_{13}, U_{14}\}$, $B_{C.C\setminus HC}(\delta_1) = \{U_2, U_4, U_6, U_7, U_8, U_9, U_{13}\}$, $L_{C.C\setminus HC}(\delta_1) = \{U_{11}, U_{12}, U_{14}\}$that is $\{\{U_2, U_4, U_5, U_6, U_7, U_8, U_9, U_{11}, U_{12}, U_{13}, U_{14}\}, \{U_2, U_4, U_6, U_7, U_8, U_9, U_{13}\}, \{U_{11}, U_{12}, U_{14}\}\} = \tau_{C.C\setminus HC}(\delta_1)) \neq \tau_{C.C}(\delta_1)$
7. If the attribute " Iron Deficiency Anemia" is deleted from C.C then, by **Figure 3**, we obtain $L_{C.C\setminus IDA}(\delta_1) = \{U_1, U_3, U_{10}, U_{15}\}$, $U_{C.C\setminus IDA}(\delta_1) = \{U_8, U_3, U_{15}, U_2, U_6, U_4, U_1, U_7, U_{10}\}$, $B_{C.C\setminus IDA}(\delta_1) = \{U_2, U_4, U_6, U_7, U_8\}$ that is $\{\{U_8, U_3, U_{15}, U_2, U_6, U_4, U_1, U_7, U_{10}\}$, $\{U_2, U_6, U_8, U_7, U_4\}, \{U_1, U_3, U_{10}, U_{15}\}\} = \tau_{C.C\setminus IDA}(\delta_1)) = \tau_{C.C}(\delta_1)$

$$\boldsymbol{Hence\ CORE = \{High\ blood\ pressure, High\ cholesterol\}}$$

Observation : From the above two cases we see that the "High blood pressure" and "high cholesterol" are primary causes of atherosclerosis. High blood pressure stresses and damages artery walls, leading to enlargement and hardening. Elevated cholesterol increases inflammation, promoting harmful cholesterol accumulation in artery walls. Together, they intensify pressure on blood vessels, accelerating atherosclerosis and raising the risk of heart disease and strokes. Preventing this requires a healthy diet, regular exercise, and maintaining a healthy weight. The proposed nano-topological methodology, derived from graph theory, may assist physicians in rapidly identifying individuals at high risk of developing atherosclerosis by analyzing their medical data through the presented approach. Moreover, it enables the diagnosis of patients with the highest likelihood of contracting the disease, thereby supporting timely and accurate clinical decisions.

5. CONCLUSION

In this paper, a new concept called “nano generalization connectedness via graph” is introduced, and its various properties are studied. We are also developing and testing a new algorithm that can be applied using [factors, distance matrix, graphs] to analyze data in the context of medical applications, such as detecting the causes of atherosclerosis. The results show that “High blood pressure” and “high cholesterol” are the main factors that contribute to the development of atherosclerosis. This finding highlights the importance of monitoring health factors related to diet, physical activity, and healthy weight. It also demonstrates the importance of using nano topology to analyze factors affecting medical conditions in an innovative way. This study suggests that the use of nano topology provides a powerful mathematical framework to study the relationships between different variables in medical data, opening up new avenues for its application in other fields.

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