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An Approach of Employing Some Cosmological Parameters to Describe the Friedmann Micro- Universe

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ABSTRACT

The Friedmann-model was proposed as a solution to Einstein's formulae, so as to get multiple sets of equations specifying cosmological parameters. The time and temperature evolution of a number of cosmological parameters, including scale factor, energy density, pressure, Hubble parameter, and deceleration parameter, which describe the macro world, have been studied in the context of radiation, matter, and dark energy. To describe the micro universe, Fridmann models require to be modified by going back in time to get a new definition for the cosmological parameters. A way that describes Friedmann as a micro universe considered that the strong force (short rang) operates by an equation like that of Einstein equations. The Einstein field equations, which concentrate on the large space-time structure of the universe, will be modified accordingly to take into consideration the small range scale. The cosmological parameter for the macro and micro universes was displayed against temperature and time going back to the end of radiation dominated to matter dominated eras, representing that the universe exhibits a shift from an accelerated expansion of macro universe to a decelerated expansion of micro universe.

Keywords: Friedmann model, early universe, cosmological parameters, hadronic universe

INTRODUCTION

All cosmic models can be categorized into two groups: evolutionary and steady-state models. All evolutionary models are based upon ordinary cosmological principle, while the steady-state models are based principle which states that besides the implication of ordinary cosmological principle, the universe also looks the same to a fundamental observer (Adler, 2022; Ryden, 2016; Beaudu and Lampla, 2022).

In 1922 Friedmann (Gavrikov, *et al*., 2020) derived a set of physical cosmological equations governing the expansion of homogeneous and isotropic space in the framework of general relativity, he demonstrated that there were three evolutionary cosmological models, depending on the universe's curvature parameter k, which is represented by (+1, 0, and -1) for all positive, flat, and negative curving. R is the scale factor (radius of the universe), as in (Fig. 1).

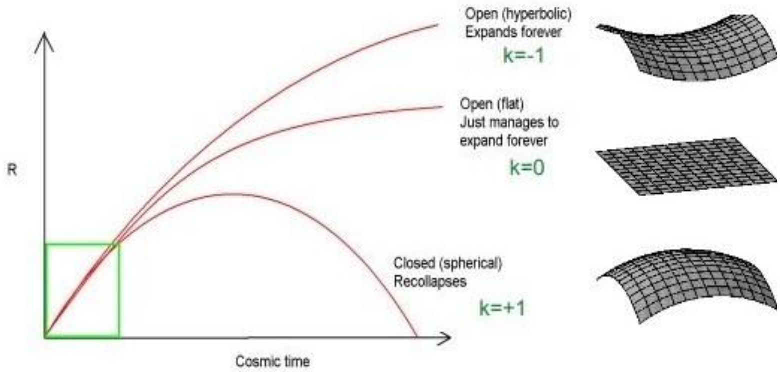

Fig. (1). Evolutionary cosmological models (www.astro.umd.edu)

These set of equations solutions yield a number of parameters, including the scale factor, energy density, Hubble's, deceleration, and the redshift parameters, which describing the expansion of the macro universe compared with an observational data.

At the present time, astronomers have determined that our universe is about $13.8x10^{9}$ y , with a scale factor R(t) = $8.81x10^{26}$ m , present temperature (the cosmic microwave background radiation CMB) is 2.71 K° , energy density ρ=$8.69x10^{-27}$ kg/m^3 in natural units (5 protons per cubic meter), with negative pressure -$4.471x10^{-27}$ pa (for accelerated universe), Hubble's parameter 67.8 km/s/Mpc, deceleration parameter q= -0.27 and zero redshift, with a weak gravitational constant, GN, (Newtonians G=6.6743 $x10^{-11}$ m^3/kg.s^2) governed by a classical mechanics.

In the present work one attempts to give a new picture of how we go back in time to describe these cosmological parameters in the context of Friedmann micro universe (the early universe), this strategy using an Einstein-like equation demonstrating that the strong force also operates but with a very much stronger gravitational constant. Many coworkers (Salam and Strathdee, 1971; Isham *et al*.,1971) have provide sturdy evidence that general relativity may play a decisive rule in the physics of elementary particles with f-meson interpreted as a low range (micro-universe or hadronic universe) gravitational field governed by quantum mechanics. Therefore, according to the corresponding principle (in the limit of large quantum numbers the predictions of quantum physics become identical to the predictions of classical physics). On this basis, it is supposed to understand the association between the gravitational theory with massless graviton (classical physics) and the corresponding finite range theory (quantum physics) similar to the graviton, the f-meson is a spin-2+ particle that mediates the strong force.

RESULTS AND DISCUSSIONS

The research's main goal is to study the cosmic parameters of Friedman's large universe (stars and galaxies) at the present time as a solution to the equations of Einstein, with a large radius and a weak gravitational field, then transform it into a small universe that describes elementary particles with a short range and a strong field and the following theoretical scenario applies to both situations:

1. The Friedmann macro universe's cosmological parameters

In this study, the Friedmann- model was presented as a solution to Einstein's equations, (Adler, 2022).

$$R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu}R + \Lambda\, g_{\mu\nu} = \frac{-8\pi G}{c^4}\, T_{\mu\nu} \quad \ldots\ldots\ldots\ldots \ (1)$$

The left side of equation (1) describes the geometry (curvature) of space-time, with the right side describing matter distribution, where, $R_{\mu\nu}$ is the “Ricci tensor”, $g_{\mu\nu}$ is the “metric tensor”, R is the “scalar curvature”, $T_{\mu\nu}$ is the “energy momentum-tensor”, G is the “Newtonian gravitational” constant, and Λ is the “cosmological parameter”. Then the Friedmann - model solution to equation (1) reads,

$$\left(\frac{\dot{R}}{R}\right)^2 + \frac{kc^2}{R^2} - \frac{\Lambda c^2}{3} = \frac{8\pi G}{3}\rho \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (2A)$$

$$\frac{2\ddot{R}}{R} + \left(\frac{\dot{R}}{R}\right)^2 + \frac{kc^2}{R^2} + c^2\Lambda = \frac{-8\pi G}{3}p \quad \ldots\ldots\ldots\ldots\ldots\ldots (2B)$$

The following is how the cosmological constants are incorporated into these equations: where R is the universe scale factor at present time, ρ is the energy density of the universe, p stands for the universe's pressure, $H = \frac{\dot{R}}{R}$ is Hubble's parameter (a time derivative is represented as an over dot) and q(t) is the deceleration parameter , (Misner, *et al.*, 2017; Vishwakarma and Beesham,1999; Susskind and Cabannes, 2024; Singh, and Kumar, 2014), the following is a description of these parameters.

Using the Freidman as a flat universe (k=0), as a starting point, with the present cosmological constant $\Lambda \sim 10^{-57}$ cm^{-2} ~ 0 (Misner, et al., 2017), the solution to equation (2A) yields the universe's scale factor R(t).

$$R(t) = \frac{R(\mathrm{t_o})}{\mathrm{t_o}^{\alpha}} t^{\alpha} = \mathrm{C}\, t^{\alpha} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Equation (3) represents the scale factor of the universe in its general form, and its evolution through the “eras of radiation” (r), “matter” (m), and “dark energy” (d), are (Susskind and Cabannes, 2024).

$$\mathrm{R_r(t)} = \sqrt[4]{\frac{32\pi Gh}{3C}}\ \mathrm{t}^{1/2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (4A)$$

$$\mathrm{R_m(t)} = \sqrt[3]{6\pi GM}\ \mathrm{t}^{2/3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (4B)$$

$$R_d(\mathrm{t}) = \mathrm{C_1}\ \mathrm{e}^{\mathrm{H(t)t}} \quad ;\ \mathrm{C_1} = R_{\mathrm{o}}\mathrm{e}^{-\mathrm{H_o t_o}} \quad \ldots\ldots\ldots\ldots(4C)$$

The energy density in its general form

$$\rho = \frac{constant}{V^{\omega+1}} \quad ; \quad \omega = \frac{p}{\rho} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Where ω takes the values ($\frac{1}{3}$, 0, -1 for radiation, matter and dark energy eras respectively)

$$\rho_\mathrm{r} = \frac{c_r}{\mathrm{R^4(t)}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (6A)$$

$$\rho_m = \frac{\text{constant}}{R^3(t)} = \frac{M}{R^3(t)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(6B)}$$

$$\rho_d = \frac{\text{constant}}{V^\circ} = \text{constant} = \rho_0 = 9.74 \times 10^{-30}\ gm/cm^3 \ \ldots \quad \text{(6C)}$$

From the definition of the Hubble parameter (Adler, 2022).

$$H = H(t) = \frac{\dot{R}(t)}{R(t)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(7)}$$

Equation (8) can be described at three different eras

$$Hr(t) = \frac{1}{2}\ t^{-1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(8A)}$$

$$Hm(t) = \frac{2}{3}\ t^{-1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(8B)}$$

$$H_d = \sqrt{\frac{8\pi G\rho_o}{3}} = \text{constant} = \alpha \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(8C)}$$

Similarly, the deceleration parameter (Adler, 2022).

$$q_{(t)} = q = -\frac{\ddot{R}(t)}{\dot{R}(t)}\frac{R(t)}{\dot{R}(t)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(9)}$$

Leads to:

$$q_r = +1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(10A)}$$

$$q_m = \frac{1}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(10B)}$$

$$q_d = -1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(10C)}$$

The temperature of the cosmos can finally be calculated by setting k=0 in equation (2) and using the formula σ = T4/c3 for black-body radiation, where σ is the Stevan-Boltzmann constant. Including the relationship between the scale factor R and the temperature T: $\frac{\dot{R}}{R} \propto -\frac{\dot{T}}{T}$, then the temperature of the macro universe is given as (Valev, 2019).

$$T = \left(\frac{3\,c^2}{8\pi G\,\sigma}\right)^{\frac{1}{4}}\ t^{\frac{-1}{2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(11)}$$

Next section gives the illustration of the modified cosmological parameter at micro universe (hadron universe or strong gravity).

2. The Friedmann micro universe's cosmological parameters

A strategy that describes Friedmann as a micro universe contends that the strong force also functions via an Einstein-like equation. The Einstein field equations (1), that focuses on the universe's large space-time structure, will be appropriately changed to account for this short range of interactions. To make this modification, use the following method as simply as possible:

1. Field requires to be mediated by the massive of meson m_f, which makes the interaction range limited and finite (Leibbrandt, *et al.*, 1972).

2. The cosmological constant Λ should be changed to its correspondence (Andersen, 2024; Mishra, 2010).

$$\Lambda_f = \left(\frac{m_f c}{\hbar}\right)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(12)}$$

c is the speed of light and $\hbar = \frac{h}{2\pi}$, h is the Plank's constant.

- Modifying the gravitational constant G_N (or G), by the gravitational constant G_f for f-gravity (Sivaram and Sinha, 1974).

$$G_f = 6.6732x10^{30} = 10^{38}\, G_N \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

- Lastly, the field $f_{\mu\nu}$ took the place of the field $g_{\mu\nu}$. Then the model for the gravitational micro universe that the Einstein equations for strong gravity now provide. In light of the aforementioned,

$$R_{\mu\nu}(f) - \frac{1}{2} f_{\mu\nu} R(f) + \Lambda_f f_{\mu\nu} = -8\pi G_f \; T_{\mu\nu}(f) \quad \ldots (14)$$

Where $R_{\mu\nu}(f)$ is the corresponding Ricci tensor *R(f)* and $T_{\mu\nu}(f)$ is the corresponding energy-momentum tensor $T_{\mu\nu}$ for strong gravity (Andersen, 2024).

equation (14) can be solved and yields a modified Friedman model, which describes the cosmological parameters of the micro-universe (Bolotin, *et al*., 2015) then the scale factor at the Haronic universe (end of radiation matter) reads:

$$R_{Hr}(t) = \sqrt[4]{\frac{32\pi Gh}{3C}}\; t^{1/2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

Similarly, the energy density equation (5), in the macro universe can be modified, since at the hadronic era $\omega = \frac{1}{3}$, and the mass of the macro universe is replaced by the hadronic particle mass (say proton) m_H, and G is replaced by G_f, then equation (5) yields,

$$\rho_{Hr} = \frac{c_r}{R^4(t)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

Then, using the same techniques, the remaining cosmological parameters can be set as follows: Combining equations (7) and (4A), give the Hubble's parameter at the hadronic matter universe:

$$H_{Hr}(t) = \frac{1}{2}\; t^{-1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

It is identical to equation (8A).

The deceleration parameter at the hadronic universe is given by combining equations (9) and (4A):

$$q_{Hr} = +1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (18)$$

It is identical to equation (10A).

If we use G_f instead of G, then equation (11) (Valev, 2019) yields:

$$T = \left(\frac{3c^2}{32\,\pi\, G_f\, \sigma}\right)^{\frac{1}{4}} t^{\frac{-1}{2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

Which represents the beginning temperature of the hadron era.

Graphical Analysis of Theoretical Results

In this section the time and temperature data, were adopted from Crevecoeur 2017, the cosmological parameter was calculated and were plotted versus temperatures and time for the matter era using a MATLAB program, as follows:

In fig. 2, the scale factor shows a raises with time, representing evidently the universe expansion with time, the universe radius (scale factor) at present time (nearly 13.8 Gyrs) has been found to be just an exponential value of (62) at the vertical axis, i.e. R(to) = 8.8E+26 meter, with a temperature at the present universe radius as the exponential value of (0.99) at the horizontal axis, T= 2.69 K°, while at the transition from radiation to matter dominated era $R(-10.44) \sim 2.9E\text{-}5$ meter and temperature $T(10.8) \sim 50000\ K^{o}$.

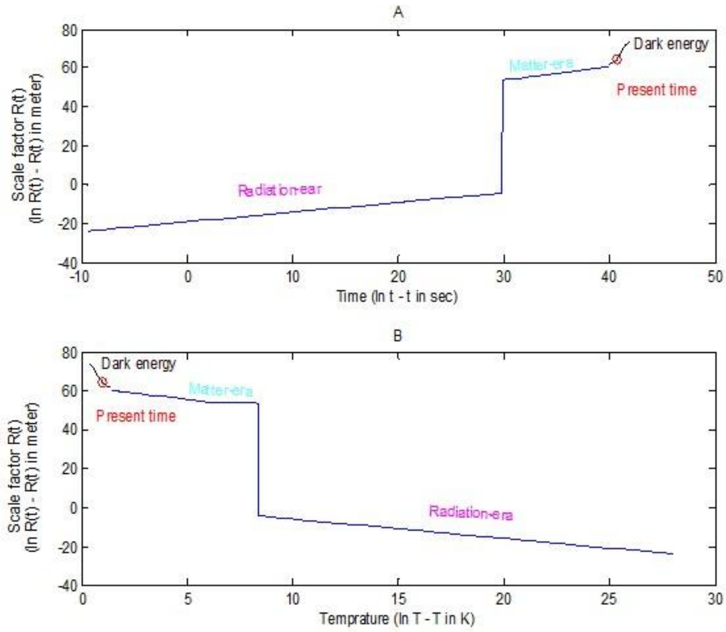

Fig. (2). Scale factor evolution being a function of both time and temperature.

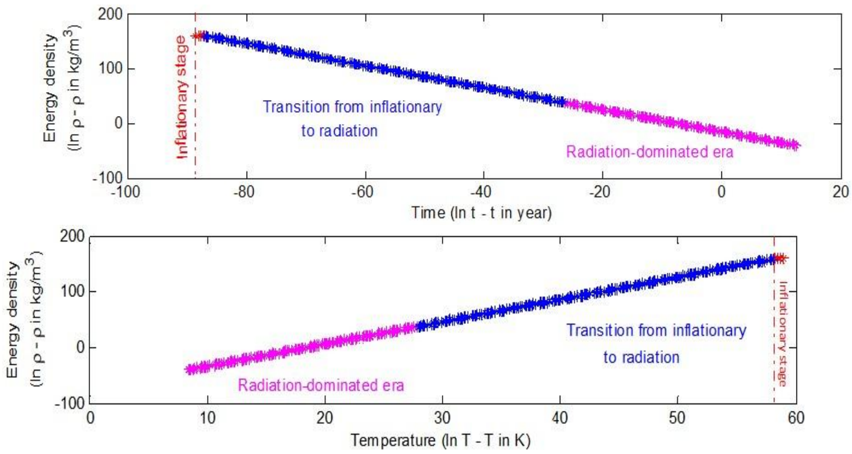

Fig. (3). Evolution of the energy density as a function of time and temperature.

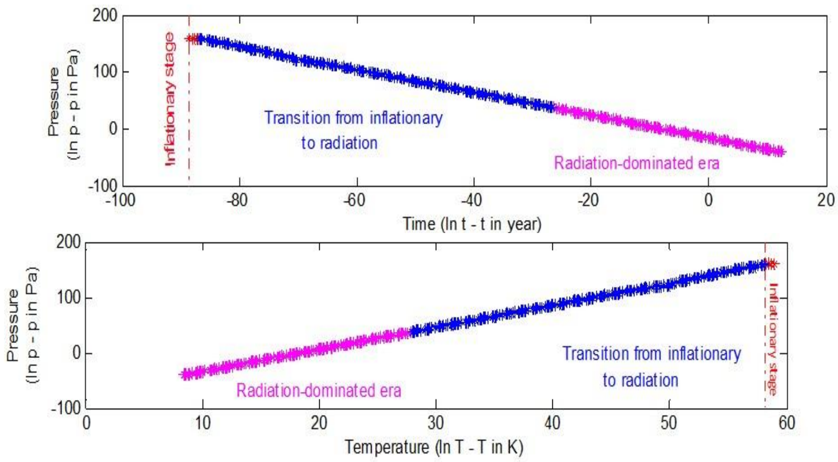

Fig. (4). Pressure evolution being a function of both time and temperature

In (Fig. 3) and (Fig. 4) both the energy density and pressure show a decrease as time goes on, while they show a steady fall as temperature goes on damping through radiation. But in (Fig. 5) the Hubble's parameter decreases with time and temperature with a gradually slower rate of change.

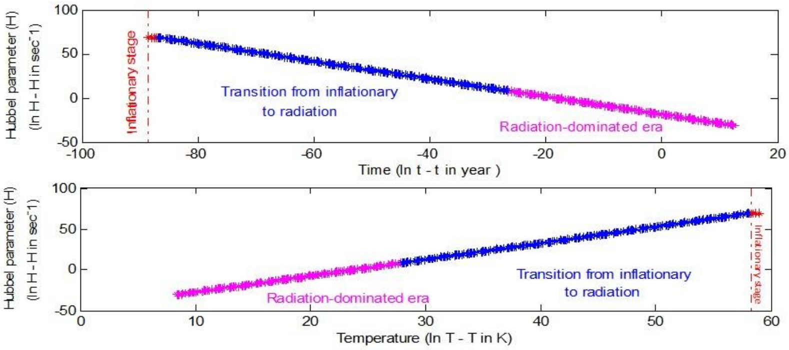

Fig (5): Evolution of the Hubble's parameter being a function of both time and temperature.

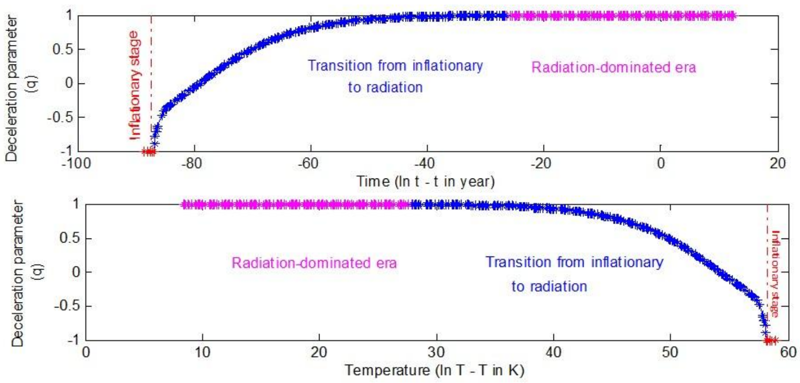

Fig (6): Evaluation of deceleration parameter with time and temperature

In Fig. (6) the deceleration parameter shows a positive sign, which indicates that the universe in a deceleration phase.

From graphical analysis, we can summarize the results of describing the cosmological parameters in the micro universe compared to their values in the macro universe (the present universe), as listed in the (Table 1).

Table 1: A Parametric sample for cosmological evolution [Crevecoeur,2017]

Eras	Radiation	Now
Time (year)	3.17E-12	1.38E+10
Temperature (K°)	1.30E+12	2.690E+00
Scale factor R(t) (meter)	4.59E-11	8.81E+26
Energy Density ρ (kg/m^3) In Natural Units	4.51E+16	8.69E-27
Pressure P (pa)	1.4886316	-4.471E-27
Hubble parameter H (sec^{-1})	5020.503413	2.2035E-18 =67.8 km/s/MPc
Deceleration parameter q	1.00	-0.27

CONCLOSIONS

From (Table 1), going back in time from macro universe to the micro universe, starting from our present time of universe (1.38E+10 year) reaching the time of micro universe (3.17E-12 year), and from the microwave background radiation of the macro universe (T=2.69 K°), back in time to the temperature of the micro hot universe (T=1.30E+12 K°). The scale factor of the present time (R(to) = 8.81E+26 meter), reaching the short rang hadronic universe ($R_{Hadronic}$= 4.59E-11 meter), i.e.

of order 10^{-11}, and from our present energy density (8.69E-27 kg/m^3, i.e. about 4-proton per meter cube) increases to (4.51E+16 kg/m^3), This indicates that the early universe (micro-universe) had an extremely dense matter per unit volume, which is consistent with the Big Bang Theory's prediction that the cosmos began with a very dense matter. Both the pressure and deceleration at the present time show a negative value (-4.471E-27 pa, -0.27), representing that the universe experiences a shifting from a phase of decelerated expansion to an accelerated one, a sign of unknown energy, it has been named dark energy or cosmological constant Λ, while they have positive values (1.48863E+16 pa, 1.0) indicating that the micro universe undergoes deceleration, it indicates that this present-day universe has negative pressure, which results in an expanding cosmos with accelerations, whereas the early universe had positive pressure, which led to a normal expansion with a positive deceleration parameter. Finally, the Hubble's parameter shows an increasing form the present value (67.8 km/s/MPc) to the value (5020.503413 km/s/MPc) in the micro universe.

It should be noted that our findings roughly match the Big Bang theory, which states that the early universe began with a very high temperature of about 10 billion K and an extremely dense matter energy density of ρ = 4.51E+16(kg/m^3), in contrast to the current measured universe density of ρ = 8.5E-27 (kg/m^3) and cosmic microwave background temperature of the universe T = 2.725 K°, our finding of ρ = 8.69E-27 (kg/m^3) and T = 2.69 K° is extremely near (Table 1).

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نهج استخدام بعض المعايير الكونية لوصف الكون الصغير لفريدمان

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الملخص

تم اقتراح نموذج فريدمان كحلٍ لصيغ أينشتاين، وذلك للحصول على مجموعات متعددة من المعادلات التي تُحدد المعاملات الكونية. دُرِسَ تطور الزمن ودرجة الحرارة لعدد من المعاملات الكونية، بما في ذلك عامل المقياس، وكثافة الطاقة، والضغط، ومعامل هابل، ومعامل التباطؤ، التي تصف العالم الكبير، في سياق الإشعاع والمادة والطاقة المظلمة. لوصف الكون الصغير، تتطلب نماذج فريدمان تعديلًا بالعودة إلى الزمن الماضي للحصول على تعريف جديد للمعاملات الكونية. إحدى الطرق التي تصف فريدمان بأنه كون صغير هي أن القوة القوية (قصيرة المدى) تعمل بمعادلة تُشبه معادلة معادلات أينشتاين.

تم تعديل معادلات مجال أينشتاين، التي تُركز على بنية الزمكان الكبيرة للكون، وفقًا لذلك لمراعاة النطاق الصغير. عُرِضت المعلمة الكونية للكونين الكبير والصغير مقابل درجة الحرارة والزمن، بدءًا من نهاية عصور الإشعاع وصولًا إلى عصور المادة، مما يدل على أن الكون يظهر تحولًا من التوسع المتسارع للكون_الكبير إلى التوسع المتباطئ للكون الصغير.

الكلمات المفتاحية: نموذج فريدمان، الكون المبكر، المعلمات الكونية، الكون الهادروني.