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Driving a new Quasi-Newton method for solving Mult objective optimization problems

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Abstract:
One of the more restrictive methods of improvement is quasi –Newton, in this paper, we drive a new matrix which is considered a development of BFGS quasi –Newton (QN)matrix by equating the direction of 3-term conjugate gradient method and the direction of modified QN method by based on the second –order Taylor series and by benefit from the properties of the conjugate gradient method (orthogonal and descent). Theoretically, it was proven that the new matrix is positively defined and satisfies QN condition, Finally, numerical results are provided to illustrate the robustness and the activity of new QN method when we compete with BFGS QN method by Dom Mor`s performance profile (is a program used to illustrate the effective of numerical results by figures which is written by MATLAB language) based on number of function , number of iteration and measurement of the length of time that data is being worked on by the processor

Keywords-Quasi –Newton, unconstrained optimization, , three term, QN condition, positive define.

تطوير طريقة شبه نيوتن جديدة لحل مسائل التحسين متعددة الأهداف

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الملخص:
تُعدّ طريقة شبه نيوتن من أكثر طرق التحسين انتشارا. في هذه الورقة، قمنا بتطوير مصفوفة جديدة تُعتبر تطويرًا لمصفوفة شبه نيوتن BFGS (QN) من خلال مساواة اتجاه طريقة التدرج المترافق ثلاثي الحدود مع اتجاه طريقة شبه نيوتن المعدلة. بالاعتماد على متسلسلة تايلور من الرتبة الثانية، والاستفادة من خصائص طريقة التدرج المترافق (التعامد والانحدار). نظريًا، ثبت أن المصفوفة الجديدة مُوجبة تعريف وتُحقق شرط شبه نيوتن. أخيرًا، تم تقديم نتائج عددية لتوضيح متانة وفعالية طريقة شبه نيوتن الجديدة عند مقارنتها بطريقة BFGS QN باستخدام ملف تعريف أداء dole more (وهو برنامج مكتوب بلغة MATLAB يُستخدم لتوضيح فعالية النتائج العددية بيانيًا) بناءً على عدد الدوال، وعدد التكرارات، وقياس مدة معالجة البيانات بواسطة المعالج.
الكلمات المفتاحية: شبه نيوتن، التحسين غير المقيد، ثلاثة حدود، شرط شبه نيوتن، مُوجبة التعريف

1-Introduction

Quasi-Newton methods are iterative methods for solving an unconstrained minimization problem [1]

$$\min f(x); \qquad x \in R^n$$

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where $f: R^n \to R$; Represent an n-dimensional Euclidean space in which the function is continuously differentiable, the search direction for the minimizer of [1] carried out by using:

$$x_{k+1} = x_k + \alpha_k d_k \qquad (2)$$

When α_k could be a step length to be computed by a line search procedure [1] satisfyStrong Wolfe Conditions consist

$$f(x_k + \alpha_k d_k) - f(x_k) \le \delta \alpha_k \nabla f(x_k)^T d \qquad (3)$$

$$|g(x_k + \alpha_k d_k)^T d_k| \le -\sigma\, g_k^T d_k \qquad (4)$$

for more details can found in [2].

The search directions in quasi-Newton methods are computed as follows:

$$d_k = -H_k g_k \qquad (5)$$

where $H_k \in R^{n\times n}$ is an approximation to the inverse Hessian. At the iteration k, the approximation H_k to the inverse Hessian is updated to achieve H_{k+1} such that H_{k+1} satisfies a Quasi-Newton condition ,

$$H_{k+1} y_k = s_k$$

Where $s_k = x_{k+1} - x_k$ and $y_k = g_{k+1} - g_k$

The Broydon-Fletcher-Goldfarb-Shanno method, also known as the BFGS method,which is define [3].

$$H_k^{BFGS} = H_k - \frac{s_k y_k^T H_k + H_k y_k s_k^T}{y_k^T s_k} + \left(1 + \frac{y_k^T H_k y_k}{y_k^T s_k}\right)\frac{s_k s_k^T}{y_k^T s_k} \qquad (6)$$

$$H_k^{DFP} = H_k - \frac{H_k y_k y_k^T H_k}{y_k^T H_k y_k} + \frac{s_k s_k^T}{y_k^T s_k} \qquad (7)$$

Many modified Quasi-Newton equations have been proposed in [4-8].

2- The New QN method

To introduce our new algorithm, let us define the three term CG method which is define in [9] as:

$$d_{k+1} = -g_{k+1} - \frac{\|g_{k+1}\|^2}{s_k^T g_k} s_k - \frac{g_{k+1}^T s_k}{s_k^T g_k} y_k \;.............(8)$$

And define the direction of modified QN method which is define in [10] as:

$$d_{k+1} = -H_{k+1} g_{k+1} + \lambda_k s_k \;..............(9)$$

By equality eq. (8) and eq. (9), we get

$$)10.............(-H_{k+1} g_{k+1} + \lambda_k s_k = -g_{k+1} - \frac{\|g_{k+1}\|^2}{s_k^T g_k} s_k - \frac{g_{k+1}^T s_k}{s_k^T g_k} y$$

Multiplying both side of eq.(10) by s_k^T ,we obtain:

$$-s_k^T H_{k+1} g_{k+1} + \lambda_k s_k^T s_k = -s_k^T g_{k+1} - \frac{\|g_{k+1}\|^2}{s_k^T g_k} s_k^T s_k - \frac{g_{k+1}^T s_k}{s_k^T g_k} s_k^T y \;..........(11)$$

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Since,

$$g_{k+1}^T g_{k+1} = g_{k+1}^T g_{k+1} - g_{k+1}^T g_k + g_{k+1}^T g_k$$

$$g_{k+1}^T g_{k+1} = g_{k+1}^T (g_{k+1} - g_k) + g_{k+1}^T g_{k+1} \text{.............}(12)$$

From orthogonal property,eq.(12) can be written

$$g_{k+1}^T g_{k+1} = g_{k+1}^T (g_{k+1} - g_k)$$

$$g_{k+1}^T g_{k+1} = g_{k+1}^T (g_{k+1} - g_k) + g_k^T y_k - g_k^T y_k$$

$$g_{k+1}^T g_{k+1} = g_{k+1}^T y_k + g_k^T y_k - g_k^T y_k$$

$$g_{k+1}^T g_{k+1} = y_k (g_{k+1} - g_k) + g_k (g_{k+1} - g_k) \text{......................}(13)$$

Since the eq. (13) obtain from the direction of conjugate gradient, therefore the direction satisfied orthogonal $(g_{k+1}^T \; g_k = 0)$ and descent condition $(d_k = -g_k)$ and $s_k = -\alpha_k d_k$,then :

$$g_{k+1}^T g_{k+1} = \|y_k\|^2 - \frac{\|s_k\|^2}{\alpha_k^2} \text{..................}(14)$$

After we submit eq. (14) in eq. (11), we get

$$-s_k^T H_{k+1} g_{k+1} + \lambda_k s_k^T s_k = -s_k^T g_{k+1} - (\|y_k\|^2 - \frac{\|s_k\|^2}{\alpha_k^2}) \frac{s_k^T s_k}{s_k^T g_k} - \frac{g_{k+1}^T s_k}{s_k^T g_k} s_k^T y \text{ .... } (12)$$

since $s_k^T g_k = s_k^T g_k + s_k^T g_{k+1} - s_k^T g_{k+1}$

$$s_k^T g_k = -s_k^T (g_{k+1} - g_k) + s_k^T g_{k+1} \text{.....................}(13)$$

$$s_k^T g_k = -s_k^T y_k + s_k^T g_{k+1}$$

Since $(d_k = -g_k)$ and from orthogonal property we obtain:

$$s_k^T g_k = -s_k^T y_k \text{.............................}(14)$$

After we submit eq. (14) in eq. (12), we get

$$-s_k^T H_{k+1} g_{k+1} + \lambda_k s_k^T s_k = -s_k^T g_{k+1} + (\|y_k\|^2 - \frac{\|s_k\|^2}{\alpha_k^2}) \frac{s_k^T s_k}{s_k^T y_k} + \frac{g_{k+1}^T s_k}{s_k^T y_k} s_k^T y \qquad (15)$$

Using the Taylor expansion to second-order terms with convex quadratic function, f and $g_{k+1}^T s_k$ can be written as:

$$f_k = f_{k+1} - g_{k+1}^T s_k + \frac{1}{2!} s_k^T y_k$$

$$2 f_k = 2 f_{k+1} - 2 g_{k+1}^T s_k + s_k^T y_k$$

then we obtain

$$g_{k+1}^T s_k = (f_{k+1} - f_k) + \frac{s_k^T y_k}{2} \text{ ....}(16)$$

After some algebra operation, we obtain the new matrix as:

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$$H_{k+1}^{new} = I + \left(\frac{\frac{\|s_k\|^2}{\alpha_k^2} - \|y_k\|^2}{s_k^T y_k (f_{k+1} - f_k) + \frac{s_k^T y_k}{2})} + \frac{\lambda_k}{(f_{k+1} - f_k) + \frac{s_k^T y_k}{2})} \right) s_k s_k^T - \frac{s_k y_k^T + y s_k^T}{s_k^T y_k} \quad ...(17)$$

eq.(16) can be written as:

$$H_{k+1}^{new} = I + \left(\frac{\frac{\|s_k\|^2}{\alpha_k^2} - \|y_k\|^2}{((f_{k+1} - f_k) + \frac{s_k^T y_k}{2})} - \frac{\lambda_k}{(\frac{(f_{k+1} - f_k)}{s_k^T y_k} + 0.5)} \right) \frac{s_k s_k^T}{s_k^T y_k} - \frac{s_k y_k^T + y s_k^T}{s_k^T y_k} \quad ......(18)$$

This new matrix formula (H_{k+1}^{new} is similar to the memoryless BFGS matrix; only the difference is in the two terms.

3- the **algorithm of new QN method**

Step (1): Choose an initial point x_0 and ε sufficiently small. Set $H_0 = I$ and $k = 0$.
Step (2): Calculate the gradient g_k
Step (3): cheek if $\|g_k\| < \varepsilon$,then stop
Step (4): *Compute* $d_k = -H_k g_k$

Step (5): Compute $x_{k+1} = x_k + \alpha_k d_k$, $y_k = g_{k+1} - g_k$
Step (6): Compute the new matrix, which is define in aq. (18)
Step (7): Set $k = k + 1$ and go to step (2).

Theorem (1) The new matrix H_{k+1} , which is defined by (17), satisfy QN- like condition (i.e.$H_{k+1} y_k = \rho_k s_k$,).
Proof:
By multiplying eq. (17) by y_k ,we obtain

$$H_{k+1}^{new} y_k = \left[I + \left(\frac{\frac{\|s_k\|^2}{\alpha_k^2} - \|y_k\|^2}{((f_{k+1} - f_k) + \frac{s_k^T y_k}{2})} - \frac{\lambda_k}{(\frac{(f_{k+1} - f_k)}{s_k^T y_k} + 0.5)} \right) \frac{s_k s_k^T}{s_k^T y_k} - \frac{s_k y_k^T + y s_k^T}{s_k^T y_k} \right] y_k$$

$$H_{k+1}^{new} y_k = \left[y_k + \left(\frac{\frac{\|s_k\|^2}{\alpha_k^2} - \|y_k\|^2}{((f_{k+1} - f_k) + \frac{s_k^T y_k}{2})} - \frac{\lambda_k}{(\frac{(f_{k+1} - f_k)}{s_k^T y_k} + 0.5)} \right) \frac{s_k s_k^T y_k}{s_k^T y_k} - \frac{s_k y_k^T}{s_k y_k^T} y_k - \frac{\|y_k\|^2}{s_k^T y_k} s_k \right]$$

$$H_{k+1}^{new} y_k = \left[\left(\frac{\frac{\|s_k\|^2}{\alpha_k^2} - \|y_k\|^2}{((f_{k+1} - f_k) + \frac{s_k^T y_k}{2})} - \frac{\lambda_k}{(\frac{(f_{k+1} - f_k)}{s_k^T y_k} + 0.5)} - \frac{\|y_k\|^2}{s_k^T y_k} \right)_k s_k \right]$$

Let $\rho_k = \frac{\frac{\|s_k\|^2}{\alpha_k^2} - \|y_k\|^2}{((f_{k+1} - f_k) + \frac{s_k^T y_k}{2})} - \frac{\lambda_k}{(\frac{(f_{k+1} - f_k)}{s_k^T y_k} + 0.5)} - \frac{\|y_k\|^2}{s_k^T y_k}$

$H_{k+1}^{new} y_k = \rho_k s_k$, where ρ_k is positive constant

Now, we will prove ρ_k is positive constant

Form (wolf)

$f_{k+1} - f_k \le c\alpha g_k d_k \le -c\alpha \|g_k\|^2$

$$\rho_k = \frac{\frac{\|s_k\|^2}{\alpha_k^2} - \|y_k\|^2}{(-c\alpha \|g_k\|^2 + \frac{s_k^T y_k}{2})} - \frac{\lambda_k}{(\frac{-c\alpha \|g_k\|^2}{s_k^T y_k} + 0.5)} - \frac{\|y_k\|^2}{s_k^T y_k}$$

Let $-c\alpha \|g_k\|^2 < \frac{s_k^T y_k}{2}$ and $\frac{-c\alpha \|g_k\|^2}{s_k^T y_k} > 0.5$, and suppose sum of first and second term is great them third term

$\therefore \rho_k > 0$

The prove is complete.

Lemma: If H_1 is a positive definite matrix, then all matrix H_{k+1}^{new} which is generated by (17) is also a positive definite, i.e., $z_k^T H_{k+1}^{new} z_k > 0$, for any vector $z_k \neq 0$.

Proof:

$$z_k^T H_{k+1}^{new} \; z_k =$$

$$z_k^T \left[I + \left(\frac{\frac{\|s_k\|^2}{\alpha_k^2} - \|y_k\|^2}{((f_{k+1} - f_k) + \frac{s_k^T y_k}{2})} - \frac{\lambda_k}{(\frac{(f_{k+1} - f_k)}{s_k^T y_k} + 0.5)} \right) \frac{s_k s_k^T}{s_k^T y_k} - \frac{s_k y_k^T + y s_k^T}{s_k^T y_k} . \right] z_k \quad (5.8)$$

$$= z_k^T z_k - \frac{(z_k^T s_k)(y_k^T z_k)}{s_k^T y_k} - \frac{(z_k^T y_k)(s_k^T z_k)}{s_k^T y_k} +$$

$$\left(\frac{\frac{\|s_k\|^2}{\alpha_k^2} - \|y_k\|^2}{((f_{k+1} - f_k) + \frac{s_k^T y_k}{2})} - \frac{\lambda_k}{(\frac{(f_{k+1} - f_k)}{s_k^T y_k} + 0.5)} \right) \frac{(s_k^T z_k)^2}{s_k^T y_k}$$

$$= z_k^T z_k - \frac{2(z_k^T s_k)(y_k^T z_k)}{s_k^T y_k} + \left(\frac{\frac{\|s_k\|^2}{\alpha_k^2} - \|y_k\|^2}{((f_{k+1} - f_k) + \frac{s_k^T y_k}{2})} - \frac{\lambda_k}{(\frac{(f_{k+1} - f_k)}{s_k^T y_k} + 0.5)} \right) \frac{(s_k^T z_k)^2}{s_k^T y_k}$$

Since $z_k^T z_k, \|y_k\|,$ |

Case 1:
If $(z_k^T s_k)(y_k^T z_k)$ have the different sign, then the second term is positive

$$z_k^T H_{k+1}^{new} z_k = z_k^T z_k - \frac{2(z_k^T s_k)(y_k^T z_k)}{s_k^T y_k} + \left(\frac{\frac{\|s_k\|^2}{\alpha_k^2} - \|y_k\|^2}{((f_{k+1} - f_k) + \frac{s_k^T y_k}{2})} - \frac{\lambda_k}{(\frac{(f_{k+1} - f_k)}{s_k^T y_k} + 0.5)} \right) \frac{(s_k^T z_k)^2}{s_k^T y_k} \quad (5.11)$$

Moreover, prove it is complete.
Case (2):
If $(z_k^T s_k)(y_k^T z_k)$ have the same sign, then:

$$z_k^T H_{k+1}^{new} z_k = z_k^T z_k - \frac{2(z_k^T s_k)(y_k^T z_k)}{s_k^T y_k} + \left(\frac{\frac{\|s_k\|^2}{\alpha_k^2} - \|y_k\|^2}{((f_{k+1} - f_k) + \frac{s_k^T y_k}{2})} - \frac{\lambda_k}{(\frac{(f_{k+1} - f_k)}{s_k^T y_k} + 0.5)} \right) \frac{(s_k^T z_k)^2}{s_k^T y_k}$$

Let $\rho = \frac{2(z_k^T s_k)(y_k^T z_k)}{s_k^T y_k}$ and let $\rho < z_k^T z_k$, then:
$z_k^T H_k z_k \geq z_k^T z_k > 0$
The proof is complete.

3-Numerical Result

In this study, we concentrate on the new QN's numerical performance, which provides the best performance when compared to the BFGS QN in Equation (20). To solve 50 well-known test functions [11] with various dimensions, we wrote a MATLAB method. The Dolan-Mor'e performance profile [12], which is dependent on the method's (NOF), NOI, and CPU time, serves as the termination criteria in our approach. Focusing on the number of function evaluations (NOI), number of iterations, and processing time, particularly for the three problem dimensions (50, 100, and 500), Figures (1, 2) in this study demonstrate the strength of the proposed technique utilizing a Dolan-Mor'e graph. Figures (3,4) show a similar trend, demonstrating the effectiveness of our approach.

Table 1 compares the BFGS QN approach with the new QN method (n=50).

NO.	Test Fun.	New QN Method			NH1 QN Method			BFSG QN Method		
	N	CPU	NOI	NOF	CPU	NOI	NOF	CPU	NOI	NOF
1	"ARGLINA"	1	42	126	108	63	189	18	22	24
2	"BEALE"	1	1	2	7	1	2	1	1	2
3	"BIGGS6"	1	2	8	17	2	8	6	13	15
4	"BOX3"	3	1	2	7	1	2	3	1	2
5	"BRKMEC"	5	1	2	14	1	2	5	1	2
6	"BROWNAL"	161	28	84	215	43	129	135	21	23
7	"BROWNBS"	40	7	21	21	2	6	217	40	42
8	"BROWNDEN"	22	5	15	27	5	15	9506	2500	2502
9	"CLIFF"	27	5	15	32	5	15	179	37	39
10	"DENSCHNA"	1	1	2	2	1	2	1	1	2
11	"DENSCHNC"	31	5	15	29	5	15	294	59	61
12	"DENSCHND"	206	3	11	3256	58	174	10132	188	190
13	"DENSCHNE"	29	5	15	34	6	18	11996	2500	2502
14	"DENSCHNF"	30	5	15	32	5	15	530	106	108
15	"DIXON3DQ"	1	2	8	6	75	225	4	27	29
16	"DJTL"	1	1	2	1	1	2	1	1	2
17	"EIGENALS"	1	2	8	15	76	228	2	27	29
18	"EIGENBLS"	2482	298	894	652	80	240	2837	341	343
19	"ERRIMROS"	77	8	24	70	7	21	14860	1842	1844
20	"EXPFIT"	62	33	99	93	44	132	318	149	151
21	"FLETCHER"	382	34	102	396	41	123	27776	2500	2502
22	"GENHUMPS"	530	1840	5520	551	1846	5538	990	2070	2072
23	"GROWTHLS"	70	15	47	74	16	50	60	13	15
24	"HAIRY"	51	9	27	35	6	18	1767	367	369
25	"HEART6LS"	37	28	84	60	47	141	373	250	252
26	"HELIX"	9	2	8	8	2	8	963	461	463
27	"HILBERTA"	39	56	164	1616	2500	2514	55	74	76
28	"HILBERTB"	22	32	96	37	38	114	113	135	137
29	"HIMMELBB"	1	1	2	1	1	2	1	1	2
30	"HIMMELBF"	55	11	33	11798	2500	7502	10995	2500	2502
31	"HIMMELBG"	80	17	51	11463	2500	7502	11075	2500	2502
32	"HIMMELBH"	112	22	66	12350	2500	7502	11104	2500	2502
33	"HUMPS"	194	38	114	277	56	168	634	134	136
34	"JENSMP"	28	5	15	29	5	15	563	122	124
35	"KOWOSB"	155	30	90	232	49	147	108	23	25

36	"LOGHAIRY"	72	12	36	20	3	9	1119	232	234
37	"MANCINO"	3940	7	21	3800	7	21	3897	2500	2502
38	"MARATOSB"	209	40	120	3111	226	678	53997	2500	2502
39	"MEXHAT"	11510	2500	2822	14618	2500	2822	48821	2500	2502
40	"OSBORNEB"	44	7	21	46	7	21	121	4	6
41	"PALMER1C"	62	32	96	90	45	135	1877	224	226
42	"PALMER2C"	60	30	90	96	47	141	1636	191	193
43	"PALMER7C"	328	166	498	120	55	165	2631	296	298
44	"PALMER8C"	467	228	684	113	55	165	2292	276	278
45	"POWELLSQ"	54	20	60	30	10	30	28056	2500	2502
46	"SINEVAL"	0	0	2	2	1	2	1	1	2
47	"SISSER"	128	25	75	218	43	129	1510	64	66
48	"TOINTQOR"	1	12	36	2	6	18	19	33	35
49	"VARDIM"	1	5	15	2	5	15	1044	2152	2154
50	"YFITU"	19	9	29	9	3	11	21770	2500	2502

Table 2 compares the BFGS QN approach with the new QN method (n=100).

NO.	Test Fun.	New QN Method			NH1 QN Method			BFSG QN Method		
	N	CPU	NOI	NOF	CPU	NOI	NOF	CPU	NOI	NOF
1	"ARGLINA"	1	37	111	13	60	180	14	22	24
2	"BEALE"	1	1	2	1	1	2	1	1	2
3	"BIGGS6"	2	2	8	2	2	8	20	13	15
4	"BOX3"	6	1	2	6	1	2	24	1	2
5	"BRKMEC"	17	1	2	18	1	2	79	1	2
6	"BROWNAL"	507	26	78	832	41	123	1715	22	24
7	"BROWNBS"	177	8	24	55	2	6	4731	49	51
8	"BROWNDEN"	45	5	15	44	5	15	93275	2500	2502
9	"CLIFF"	105	5	15	102	5	15	4987	57	59
10	"DENSCHNA"	1	1	2	1	1	2	1	1	2
11	"DENSCHNC"	104	5	15	103	5	15	4694	56	58
12	"DENSCHND"	6180	29	87	11361	54	162	279599	284	286
13	"DENSCHNE"	103	5	15	125	6	18	208527	2500	2502
14	"DENSCHNF"	118	5	15	114	5	15	8835	101	103
15	"DIXON3DQ"	1	2	8	6	70	210	12	28	30
16	"DJTL"	1	1	2	1	1	2	1	1	2
17	"EIGENALS"	1	2	8	30	72	216	15	28	30
18	"EIGENBLS"	1313	40	120	3066	94	282	88580	629	631
19	"ERRIMROS"	302	8	24	247	7	21	32633	232	234
20	"EXPFIT"	125	31	93	157	40	120	2701	149	151
21	"FLETCHER"	168	3	11	178	3	11	10015	56	58
22	"GENHUMPS"	162	23	69	2122	1844	5530	6403	2054	2056
23	"GROWTHLS"	141	15	47	141	16	50	527	13	15
24	"HAIRY"	221	10	30	139	6	18	22933	269	271
25	"HEART6LS"	73	26	78	123	42	126	2686	250	252
26	"HELIX"	14	2	8	19	2	8	9093	461	463
27	"HILBERTA"	217	38	114	15561	2500	2508	677	49	51
28	"HILBERTB"	167	30	90	225	35	105	6542	431	433
29	"HIMMELBB"	1	1	2	1	1	2	1	1	2
30	"HIMMELBF"	432	22	66	45617	2500	7502	191740	2500	2502
31	"HIMMELBG"	140	6	18	45582	2500	7502	200970	2500	2502
32	"HIMMELBH"	119	6	18	47691	2500	7502	191530	2500	2502
33	"HUMPS"	695	38	114	1058	52	156	19124	238	240

34	"JENSMP"	718	37	111	109	5	15	208791	2500	2502
35	"KOWOSB"	620	34	102	937	51	153	2214	27	29
36	"LOGHAIRY"	200	10	30	76	3	9	20431	233	235
37	"MANCINO"	25186	6	18	25162	6	18	528206	34	36
38	"MARATOSB"	646	32	96	2288	118	354	198845	2500	2502
39	"MEXHAT"	45081	2500	7436	48276	2500	7436	217190	2500	2502
40	"OSBORNEB"	86	7	21	95	7	21	235	4	6
41	"PALMER1C"	138	30	90	184	40	120	3689	224	226
42	"PALMER2C"	124	29	87	166	42	126	3137	191	193
43	"PALMER7C"	1521	342	1026	210	49	147	4886	296	298
44	"PALMER8C"	747	169	507	204	50	150	4555	275	277
45	"POWELLSQ"	209	20	60	89	8	24	95862	2500	2502
46	"SINEVAL"	2	2	3	1	1	2	2	2	3
47	"SISSER"	1148	60	180	918	43	129	4975	68	70
48	"TOINTQOR"	3	13	39	2	6	18	219	281	283
49	"VARDIM"	2	4	12	2	4	12	10	8	10
50	"YFITU"	31	7	23	15	3	11	14	22	24

Table 3 compares the BFGS QN approach with the new QN method (n=500).

NO.	Test Fun.	New QN Method			NH1 QN Method			BFSG QN Method		
	N	CPU	NOI	NOF	CPU	NOI	NOF	CPU	NOI	NOF
1	"ARGLINA"	106	2	8	113	55	165	438	24	26
2	"BEALE"	6	1	2	1	1	2	2	1	2
3	"BIGGS6"	26	2	8	13	2	8	300	13	15
4	"BOX3"	32	1	3	29	1	3	131	1	3
5	"BRKMEC"	431	1	3	421	1	3	1737	1	3
6	"BROWNAL"	10374	24	72	16606	37	111	39928	23	25
7	"BROWNBS"	3607	6	18	1346	2	6	72468	40	42
8	"BROWNDEN"	948	5	15	219	5	15	404325	2500	2502
9	"CLIFF"	24435	13	39	2514	5	15	85578	51	53
10	"DENSCHNA"	5	1	3	1	1	3	2	1	3
11	"DENSCHNC"	25100	13	39	2584	5	15	99817	59	61
12	"DENSCHND"	566427	25	75	243158	47	141	22571935	2500	2502
13	"DENSCHNE"	40948	22	66	2477	5	15	24544	41	43
14	"DENSCHNF"	13929	6	18	2666	5	15	116033	180	182
15	"DIXON3DQ"	21	2	8	90	61	183	302	32	34
16	"DJTL"	6	1	3	1	1	3	1	1	3
17	"EIGENALS"	32	2	8	183	61	183	318	32	34
18	"EIGENBLS"	232942	74	222	139119	181	543	2924265	2500	2502
19	"ERRIMROS"	30604	9	27	5284	6	18	1547131	1433	1435
20	"EXPFIT"	2328	28	84	717	33	99	5453	149	151
21	"FLETCHER"	1122562	246	738	277619	246	738	141163	87	89
22	"GENHUMPS"	2120	26	78	34673	1799	5397	60173	2026	2028
23	"GROWTHLS"	2714	15	47	715	16	50	917	13	15

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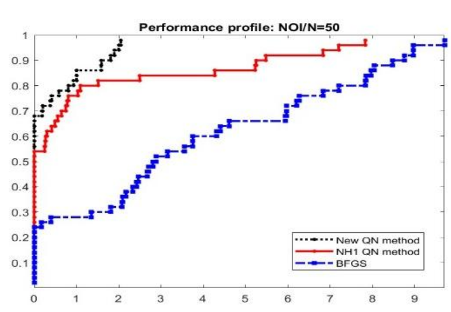
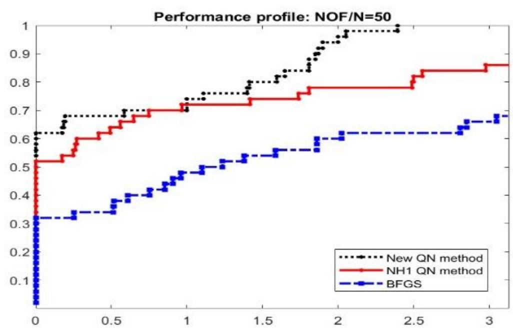
24	"HAIRY"	21491	11	33	3307	6	18	46494	76	78
25	"HEART6LS"	1367	24	72	543	34	102	6525	250	252
26	"HELIX"	244	2	8	65	2	8	15665	461	463
27	"HILBERTA"	117461	50	150	2764086	2500	2518	157689	106	108
28	"HILBERTB"	65504	27	81	34367	30	90	358364	246	248
29	"HIMMELBB"	5	1	3	1	1	3	1	1	3
30	"HIMMELBF"	12073	6	18	1042799	2500	7502	1439412	2500	2502
31	"HIMMELBG"	12160	6	18	1031996	2500	7502	1460745	2500	2502
32	"HIMMELBH"	11988	6	18	1046114	2500	7502	1452095	2500	2502
33	"HUMPS"	54668	32	96	18221	44	132	107364	183	185
34	"JENSMP"	104468	62	186	2462	5	15	82823	138	140
35	"KOWOSB"	1599	1	3	393	1	3	550	1	3
36	"LOGHAIRY"	1390760	2500	7502	1705	3	9	139817	232	234
37	"MANCINO"	2171312	4	12	2237774	4	12	11006570	8	10
38	"MARATOSB"	34390	76	228	37519	77	231	6115611	2500	2502
39	"MEXHAT"	6859	16	48	8497	16	48	46214	18	20
40	"OSBORNEB"	452	7	21	388	6	18	1491	4	6
41	"PALMER1C"	578	27	81	717	32	96	28897	224	226
42	"PALMER2C"	586	26	78	761	34	102	25340	191	193
43	"PALMER7C"	8730	372	1116	892	38	114	39009	296	298
44	"PALMER8C"	2622	110	330	875	38	114	36625	275	277
45	"POWELLSQ"	7700	32	96	1836	7	21	301972	249	251
46	"SINEVAL"	3	1	3	1	1	3	4	1	3
47	"SISSER"	19578	44	132	17890	38	114	168585	69	71
48	"TOINTQOR"	58	14	42	20	6	18	4694	168	170
49	"VARDIM"	27	4	12	24	4	12	138	4	6
50	"YFITU"	242	10	32	89	3	11	282113	2500	2502



The first figure: shows the NOI with (n=50)



The first figure: shows the NOF with (n=50)

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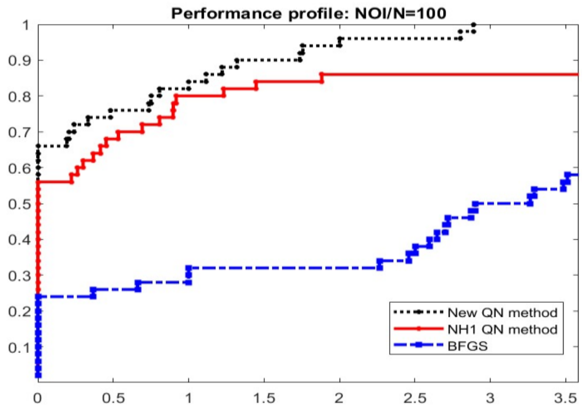

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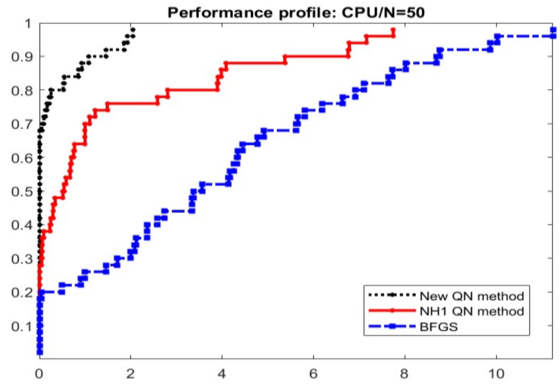

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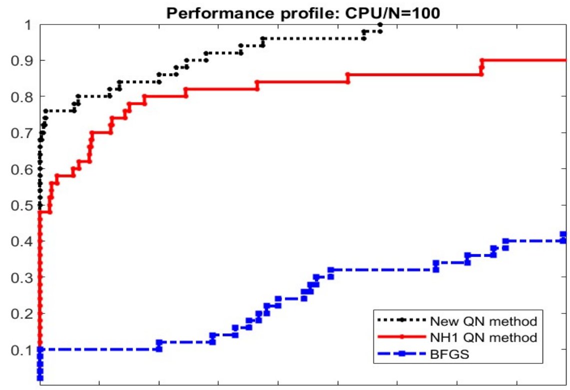

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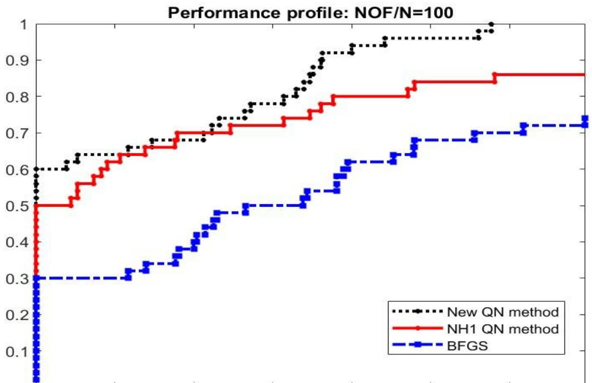
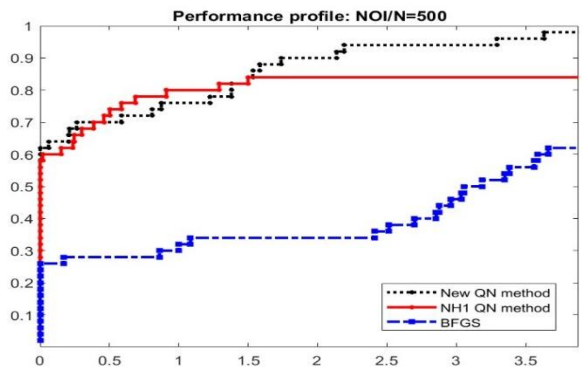


The first figure: shows the NOF with (n=100)



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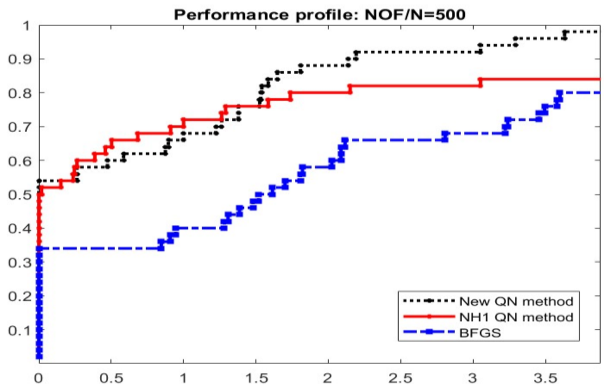
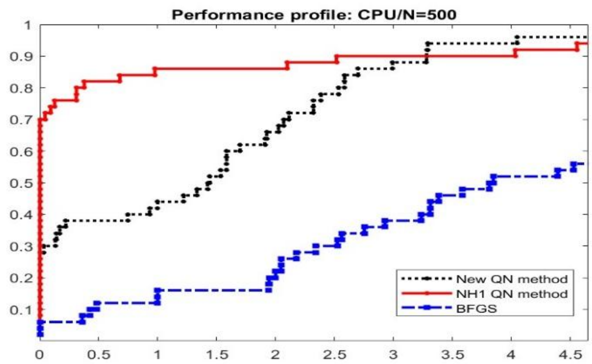


The first figure: shows the NOF with (n=500)

The first figure: shows the CPU with (n=500)



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