

Simplified Hotel System by Linking Matrices and *e*-Abacus Diagram

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Abstract

In the direction of a world that has started implementing its concepts in a new way where sustainability permeates or touches on a part of our everyday lives as a crucial component to set this notion apart from others. Altaie, Mahmood, and Alhyali came up with a new model in the direction of a multi-layer diagram (but only in the positive direction) in this study, which is regarded as a development of the good idea reached in presenting a link between chemistry and the *e*-abacus diagram. This made us consider the possibility of extending this multi-layer diagram in the negative direction as well. These concepts were best connected and applied in matrices that had to fulfill specific requirements, which made it possible to create a straightforward, useful hotel model. We will be able to easily determine how many rooms are occupied or vacant thanks to this program.

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1. Introduction

In algebra, a matrix is a rectangular array of numbers, symbols, or expressions, arranged in rows and columns. For examples:

$$\begin{bmatrix} 2 & -1 \\ 0 & \cos 60 \end{bmatrix}, \begin{bmatrix} -3 & -4 \\ 0 & \sqrt{50} \\ e^{8x} & -2.7 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \dots$$

Many researchers have addressed important aspects of its characteristics and properties over decades, including: [1]-[4], this is the first part of the linking process. The second part is: let *h* be a non-negative integer, a sequence $\tau = (\tau_1, \tau_2, \dots, \tau_m)$ of *h* is called a partition if $\tau_k \geq \tau_{k+1}, \forall 1 \leq k \leq m - 1$, and $\tau_1 + \tau_2 + \dots + \tau_m = h$, [5]. Every partition can be represented, as James [6] did, by a diagram later called the *e*-abacus, where $e \geq 2$ and so that $\alpha_i = \tau_i + b - 1, \forall 1 \leq i \leq b$. The set $\alpha = \{\alpha_1, \alpha_2, \dots, \alpha_b\}$ is said to be the set of α - numbers for τ . Every α will be represented by (\bullet) and the rest of the sites by $(-)$

Table 1. *e*-Abacus Diagram

Runner 1	Runner 2	...	Runner <i>e</i>
0	1	...	<i>e</i> -1
<i>e</i>	<i>e</i> +1	...	2 <i>e</i> -1
2 <i>e</i>	2 <i>e</i> +1	...	3 <i>e</i> -1
⋮	⋮	...	⋮

In order to deal optimally with this diagram, Mahmood [7] defined the main e-abacus, which requires that the weight of the first location should always be - in order to know the hash value with complete accuracy. For examples:

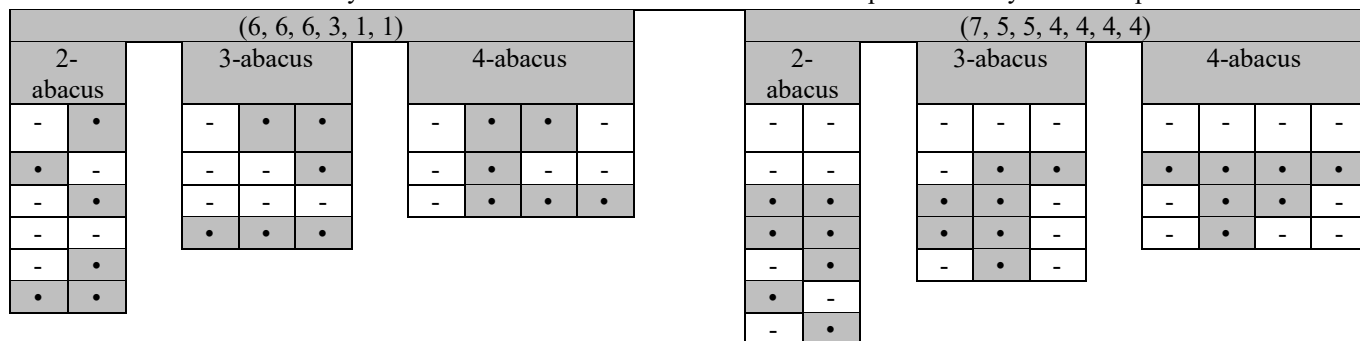


Figure 1. Two selected partitions and each one with e-abacus diagrams.

2. Multi-Layer e-Abacus Diagram

For many years, the e-abacus was limited to dealing with beads or slashes or the possibility of dispensing with 1 or zero through graph theory, where Ali and Mahmood [8] were able to clarify this feature, and through it the question was whether this diagram can be used with cases other than 0 and 1? The answer was yes when Altaie, Mahmood and Alhyali [9] were able through the chemical model that was used and the integers were equal to and greater than zero, which gave birth to their definition called the multi-layer e-abacus but only in the positive direction. To illustrate this point, the chemical process used was roughly a square matrix (LCAO no. of Carbon) [10], of amplitude v , with two conditions: the first location must always be zero, and the remaining locations must be positive integers greater than or equal to zero. The e-abacus (first or main floor) is calculated using the values mentioned in the matrix, but only using the numbers 0 and 1, As for the rest of the numbers in the matrix, they will take their place in the e-abacus (the second floor) according to the rule that whoever was occupied by number 1 (or •) on the first floor can occupy the same location on the second floor with another 1 (or •), so the total according to the same location will be 2, and so on for the rest of the locations, and the same thing for the rest of the floors. For example, if we have a matrix of a (random) chemical symbol in the following form:

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Figure 2. LCAO no. of a specific type of Cyclohexane and its modification

Figure 3 corresponds to the case of the 9-abacus diagram, and since this diagram takes only - and • at each time, a hotel model was proposed where we note the largest number that appeared within the LCAO and then design the e-abacus one after the other until we reach the largest number that appeared and always takes - and • at each time. Thus, we will have the following:

Cyclohexane																	
9-abacus (main diagram=first) (65, 57, 49, 48, 40, 32, 24, 18, 17, 9, 9)									9-abacus (second diagram) (42, 34, 26, 9, 9)								
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
•	•	-	-	-	-	-	-	-	•	•	-	-	-	-	-	-	-
-	•	-	•	-	-	-	-	-	-	-	-	-	-	-	-	-	-
-	•	-	-	-	-	-	-	-	-	•	-	-	-	-	-	-	-
-	•	-	-	-	-	-	-	-	-	•	-	-	-	-	-	-	-
-	•	-	-	-	-	-	-	-	-	•	-	-	-	-	-	-	-
-	•	-	•	-	-	-	-	-	-	-	-	-	-	-	-	-	-

-	-	-	•	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
-	-	-	•	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

Figure 3. 9-abacus diagrams of Cyclohexane

So the question remains: can we make this multi-layer in the negative direction as well?

3. Multi-Layer e-abacus diagram (negative and positive direction)

The question of finding an application to make a diagram in the negative direction remained on our minds until we were able to find it through the usual matrices, but we must make them consistent with the concepts of representation theory, and this means choosing the matrices according to the following:

1. The first position in the array must be equal to zero; (we will mark it in red) so that we can later know the value of partition.
2. The matrix values in this case must be positive or negative integers with zero.
3. It is possible to choose any matrix with any capacity.

The process in the positive direction is clear, as we mentioned earlier. However, in the negative direction, the value does not have to be 1 (or •) in the e-abacus (main diagram = first). Then, the negative value in the diagram (below the main diagram) is reduced until it reaches its value in the remaining diagrams, always in the negative direction. For examples:

	2-abacus (-4) diagram	2-abacus (-3) diagram	2-abacus (-2) diagram	2-abacus (-1) diagram	2-abacus (main) diagram	2-abacus (+1) diagram
Value of Partition	(2)	(4, 2)	(4, 2)	(4, 2)	(2, 2, 1)	(1)
$\begin{bmatrix} 0 & 2 \\ -4 & 1 \\ 1 & -3 \end{bmatrix}$	$\begin{bmatrix} - & - \\ \bullet & - \\ - & - \end{bmatrix}$	$\begin{bmatrix} - & - \\ \bullet & - \\ - & \bullet \end{bmatrix}$	$\begin{bmatrix} - & - \\ \bullet & - \\ - & \bullet \end{bmatrix}$	$\begin{bmatrix} - & - \\ \bullet & - \\ - & \bullet \end{bmatrix}$	$\begin{bmatrix} - & \bullet \\ - & \bullet \\ \bullet & - \end{bmatrix}$	$\begin{bmatrix} - & \bullet \\ - & - \\ - & - \end{bmatrix}$

Figure 4. A matrix and its corresponding 3-abacus in its various possible layers

	3-abacus (-3) diagram	3-abacus (-2) diagram	3-abacus (-1) diagram	3-abacus (main) diagram	3-abacus (+1) diagram	3-abacus (+2) diagram
Value of Partition	(6)	(5, 1)	(5, 5, 1)	(5, 3, 2, 2)	(7, 2)	(2)
$\begin{bmatrix} 0 & -2 & 3 \\ 1 & 0 & 1 \\ -3 & -1 & 2 \end{bmatrix}$	$\begin{bmatrix} - & - & - \\ - & - & - \\ \bullet & - & - \end{bmatrix}$	$\begin{bmatrix} - & \bullet & - \\ - & - & - \\ \bullet & - & - \end{bmatrix}$	$\begin{bmatrix} - & \bullet & - \\ - & - & - \\ \bullet & \bullet & - \end{bmatrix}$	$\begin{bmatrix} - & - & \bullet \\ \bullet & - & \bullet \\ - & - & \bullet \end{bmatrix}$	$\begin{bmatrix} - & - & \bullet \\ - & - & - \\ - & - & \bullet \end{bmatrix}$	$\begin{bmatrix} - & - & \bullet \\ - & - & - \\ - & - & - \end{bmatrix}$

Figure 5. Another matrix and its corresponding 2-abacus in its various possible layers

From what was mentioned, the convergence led us to think about presenting a simplified hotel model where each of the above locations is like a room on a certain floor. If it is occupied, we put a •, and if it remains empty, a mark is placed -.

4. Conclusion

The mechanism adopted to converge the matrices and the e-abacus diagram will be completely useful in considering that we have a hotel model consisting of several floors, taking into account the following matters:

1. The possibility of considering the mandatory vacancy in the first location and for each diagram is like the maintenance or control room located on each floor, and this is one of the requirements of hotel work.
2. The number of spaces will make it easy to control the number of rooms that are not occupied, and the same is true if we know the number of •, this means control and knowing the number of rooms that have been occupied. We have prepared the following program which maps the occupied rooms to any floor, whether above ground or underground:

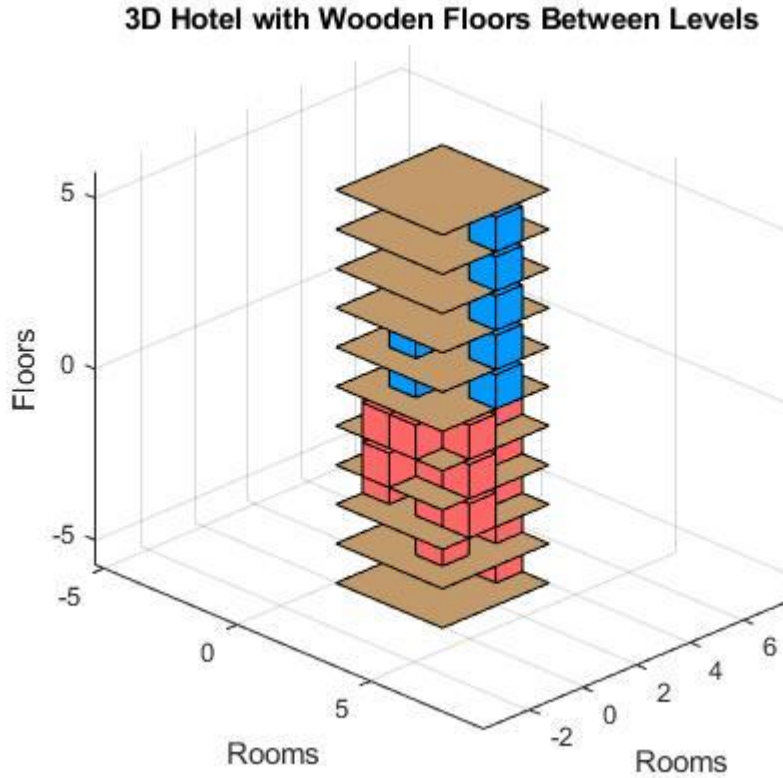


Figure 6. 3D Hotel Representation

3. It is quite natural that the ideal situation for occupying all the rooms on one floor is for them all to contain one • in every location except the first location, the benefit of which we mentioned in 1.
4. This method enabled us to know the value of the partition on each floor with ease and without complications.
5. Depending on the resources 4, we will be able to find relationships that are basically for the diagram. For example, we can draw a Young Diagram [5] or find a graph [8] for it.
6. If it is found on a floor that all the rooms were occupied - and thus as a hotel service, something can be done to reduce expenditure and focus on the floors that have occupied rooms.
7. The only problem we face is that we know that the room on which floor is occupied, but we could not determine the number of people occupying it because the single site specifies only a bead unless a later paragraph is added on the same site to specify that the largest range for any room is from 1 to s, and whenever one of them is occupied, we ask about the value of s to know the total number. We **expect** that solving this problem is possible if we study the cases of each floor and its units (rooms according to their occupants) separately. For example, Let us have a floor in a hotel with rooms occupied according

to the following numbers arranged in an array $\begin{bmatrix} 0 & 2 & 0 & 0 \\ 0 & 4 & 3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}$ so its 4-abacus diagram can be represented as:

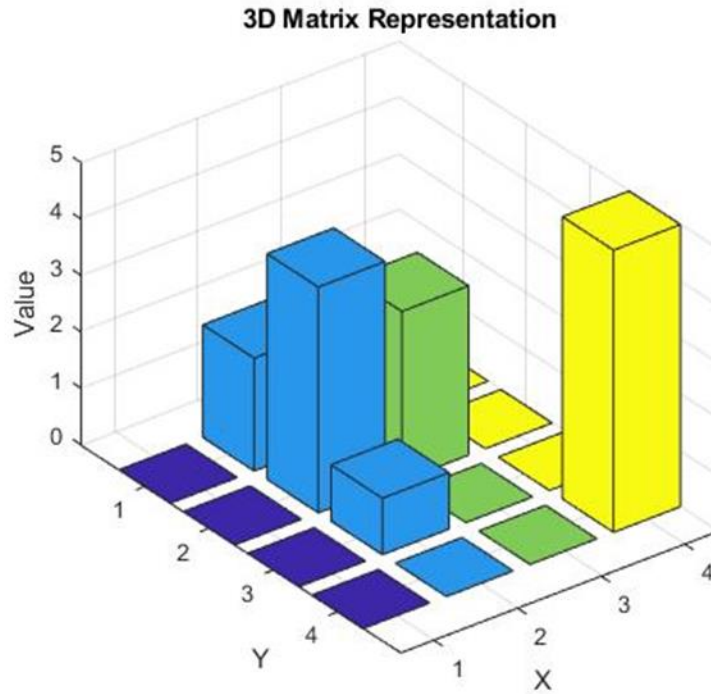


Figure 7. 3D Matrix Representation of a case for each room

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6. Declarations

6.1 Ethics approval and consent to participate

Not applicable.

6.2 Consent for publication (Bold, 12 pt)

Not applicable.

6.3 Availability of Data and Materials (Bold, 12 pt)

Data will be provided upon receiving a valid request.

6.4 Conflicts of interest (Bold, 12 pt)

The authors declare that there is no conflict of interest.

6.5 Funding (Bold, 12 pt)

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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نظام فندقى مبسط من خلال الربط بين المصفوفات والمخطط المعداد من النمط - e

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الخلاصة:

نحو عالم بدأ بتطبيق أفكاره نحو نهج جديد تدخل فيه الاستدامة أو تمس جانباً من جوانب حياتنا اليومية كجانب مهم وحيوي لتمييز هذه الفكرة عن غيرها. في هذا البحث، الذي يُعد تطويراً للفكرة الجيدة التي توصل إليها الطائي ومحمود والحيالي في عرض رابط بين الكيمياء ومخطط المعداد من النمط e، توصلوا إلى نموذج جديد يتجه نحو مخطط متعدد الطبقات (ولكن في الاتجاه الإيجابي حصراً)، مما دفعنا إلى التفكير في إمكانية توسيع هذا المخطط متعدد الطبقات في الاتجاه السلبي أيضاً. وقد وُجد أفضل ربط وتطبيق لهذه الأفكار في مصفوفات يجب أن تلبى شروطاً معينة، وبالتالي إمكانية تقديم نموذج فندقى عملي بسيط. سيمكننا هذا التطبيق من معرفة عدد الغرف الشاغرة أو المشغولة بسهولة.