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Flexible Bayesian Quantile Regression on New Class of Error Distribution

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Abstract*:* Quantile regression provides a flexible framework for modeling heterogeneous covariate effects across the conditional distribution of a response variable, yet existing Bayesian quantile regression methods often suffer from inefficiency in estimation accuracy and variable selection, particularly under non-Gaussian errors and high-dimensional settings. In this paper, we propose two new Bayesian quantile regression approaches: a Bayesian quantile regression model with a g-prior (NBQRg) and a Bayesian Lasso quantile regression model (NBLQR) based on the epsilon asymmetric Laplace distribution (EALD) as error term for quantile regression model. The proposed methods are evaluated through extensive simulation studies under a variety of scenarios, including sparse, very sparse, and dense coefficient structures, as well as settings with strong predictor correlations. Data are generated under normal, heavy-tailed, and skewed error distributions, and performance is assessed using the median of mean squared error (MMSE) alongside false positive and false negative rates for variable selection. The simulation results demonstrate that NBQRg consistently achieves lower MMSE than classical quantile regression and existing Bayesian counterparts across all quantiles and error distributions considered. Furthermore, NBLQR exhibits superior variable selection performance, yielding lower false negative rates and competitive false positive rates, particularly in sparse and highly correlated designs. Convergence diagnostics confirm stable posterior inference and efficient mixing of the proposed MCMC algorithms. Overall, the proposed Bayesian quantile regression methods offer substantial improvements in estimation accuracy, robustness, and variable selection, making them well suited for complex and high-dimensional regression problems.
Based on the results obtained through simulation applied to real data, the proposed method using the EALD error distribution yields better results compared to the classical method and the BQR method. Furthermore, comparison with the LASSO procedure proved its efficiency in estimating the parameters of the proposed model. Additionally, the Zellner's g-prior model was adopted, as it reduces the variance of the parameter estimates for the proposed regression model, thus enhancing the model's accuracy and increasing its explanatory power.
Keywords: Epsilon asymmetric Laplace distribution; Quantile regression; Bayesian estimation; Zellner's g-prior; Simulation.

Introduction: Quantile regression model suggest a regular strategy for investigating how the predictor variables effect the location, Shape, and Scale of the entire response variable distribution. QR is a comprehensive approach for estimating conditional quantile functions of the response variable rather than just its conditional mean (Koenker and Bassett, 1978; Galvao JR, 2009). Recently, ALD is garnering more and more interest within the Bayesian literature (Yu and Moyeed, 2001). Bayesian quantile regression model constructed through the structure model of asymmetric Laplace distribution (ALD) which is considered as extension to the classic Laplace distribution but with skewness parameter that controls how the probability density function of the underlying random variable captures the tail behavior. The ALD of random variable y is given by

$$f(y;\tau,\sigma)=\frac{\tau(1-\tau)}{\sigma}\exp\left(-\frac{1}{\sigma}\rho_\tau(y-\mu)\right), y\in\mathbb{R} \qquad (1)$$

Where, $\mu\in\mathbb{R}$ is the location parameter that satisfy $\Pr(y\le\mu)=\tau$, $\sigma>0$ is the scale parameter, $\tau\in(0,1)$ is the skewness parameter, and $\rho_\tau(u)=u(\tau-I(u<0))$ is the check function. The ALD can be presented as location-scale mixture normal distribution and this is an attractive property that result in explicit formulas for ALD and make it easier for implementing MCMC simulations (Kotz et al. 2001). This mixture of normal distributions leads to a proper posterior distribution (Kozumi

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and Kobayashi, 2011). Also, note that maximizing the likelihood function of (1) corresponds to minimizing for the above check function. Nonetheless, the ALD presents significant drawbacks as an error model for quantile regression. Therefore, we have introduced the epsilon asymmetric Laplace distribution (EALD) as an alternative to the asymmetric Laplace distribution for the error term. The EALD distribution is more flexible in its tail behavior and therefore heavier than the ALD distribution and can be formed if $\tau = \frac{1+\epsilon}{2}$ in density function (2). The probability density function of random variable, y that follows ALD is given by,

$$f(y;\epsilon,\mu,\sigma) = \frac{1-\epsilon^2}{4\sigma} exp \begin{cases} -\frac{1+\epsilon}{2}(\frac{y-\mu}{\sigma}) \ ; \ y \geq \mu \\ \frac{1-\epsilon}{2}\left(\frac{y-\mu}{\sigma}\right); \ y < \mu \end{cases} \qquad (2)$$

Where, $\mu \in \mathbb{R}$ is the location parameter, $\sigma > 0$ is the scale parameter, and $-1 < \epsilon < 1$ is the skewness parameter. So, we can say that the probability mass below and above the mode in (2) are $\frac{1-\epsilon}{2}$ and $\frac{1+\epsilon}{2}$ respectively. Visually, for the purpose of comparison between ALD and EALD, the following figure represents different graphs at different values of asymmetry parameters τ and ϵ. Also, the mean and variance of random variable y are $E(y) = \mu - \frac{4\sigma\epsilon}{1-\epsilon^2}$ and $var(y) = \frac{8(1+\epsilon^2)}{(1-\epsilon^2)^2}$, respectively.

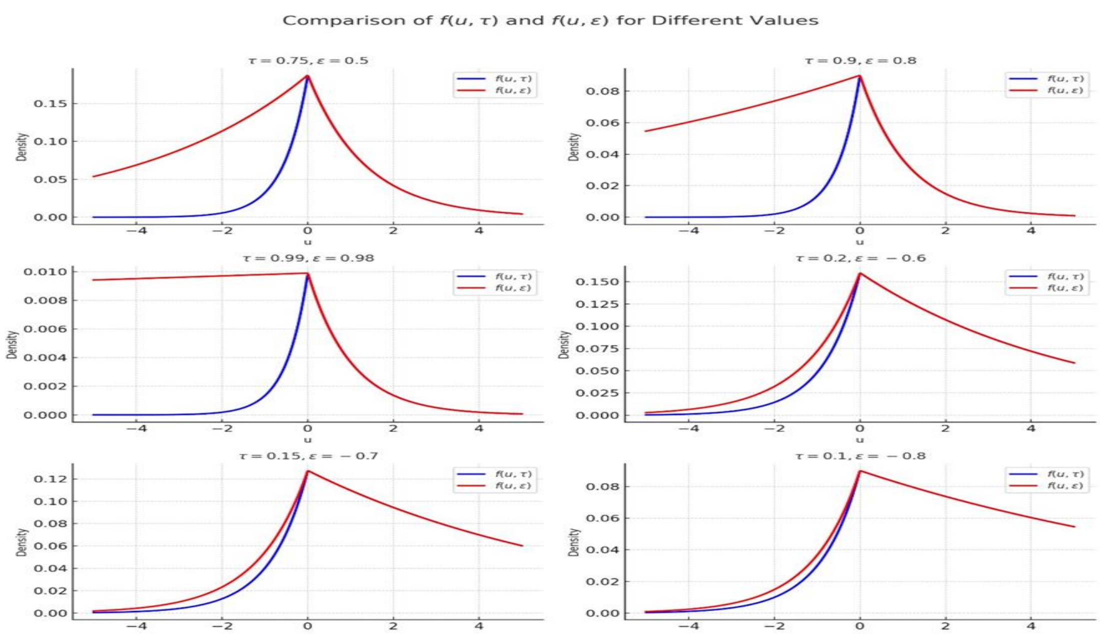

Figure 1: different EALD and ALD graphs with different ϵ and τ values

From Figure 1, obviously that EALD with different values of skewness parameter ϵ is more flexible than ALD with different vales of skewness parameter τ in term of fatness (heavy) tails that cover more outliers. Because of this flexible property of EALD, we developed new structure for the check function of quantile regression in term of ϵ skewness parameter (Arellano-Valle, 2005; Elsalloukh et al., 2005; Elsalloukh, 2008; Elsalloukh, 2009). Also, Wichitaksorn et al., 2014 introduced a skewed family of distributions as alternative for ALD of error terms, but the results present the same limitation of ALD as error term. Bayesian QR with new scale mixture has developed based on skewed family of distribution to overcome the problem of fixed skewness parameter that controls the skewness and percentiles (Yan and Kottas, 2025). Another literature on Bayesian quantile regression have introduced the skew exponential power family of distributions that does not include ALD as special (Zhu and Zinde-Walsh, 2009; Komunjer ,2007).

To this end, we proposed new class of skewed family EAL distributions that includes the ALD as special case to fill the limited scope of using ALD as error distribution in Bayesian QR. Based on EALD, scale mixture representation has developed

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and a skewness parameter, $\epsilon \in (-1,1)$ has presented to allows us to achieve a distribution with greater flexibility in skewness and tail behavior compared to the AL distribution, see (Yan and Kottas, 2025) for more information.
In section 2, we introduced the structure and the relative properties of QR based on EALD. In section 3, ……….

1. Quantile Regression Based on EALD

In this section, we consider the quantile regression in the presence of epsilon (ϵ) parameter that controls the tails of the under tying distribution, where, $-1 < \epsilon < +1$ controls the asymmetry of the distribution. the quantile regression model is defined as follows

$$y_{i,\epsilon} = x_i'\beta(\epsilon) + e_i(\epsilon), \qquad (3)$$

The following is the proposed check function based on (2) ,

$$\rho_\epsilon(\mathrm{e}) = e\left(\frac{1+\epsilon}{2} - I(e<0)\right)$$

$$\rho_\epsilon(\mathrm{e}) = \begin{cases} \left(\frac{1+\epsilon}{2}\right)e \ ; & e \geq 0 \\ -\left(\frac{1-\epsilon}{2}\right)e \ ; & e < 0 \end{cases} \qquad (4)$$

$$= \sum_{i=1}^{n} w_{i,\epsilon}|e|$$

Where, the $w_{i,\epsilon}$ is the weight function defined by,

$$w_{i,\epsilon} = \begin{cases} \frac{1+\epsilon}{2} \ for\ e \geq 0 \\ \frac{1-\epsilon}{2} \ for\ e < 0 \end{cases} \quad \dots (5)$$

Note that the total of weights equals to one, i.e., $\frac{1+\epsilon}{2} + \frac{1-\epsilon}{2} = 1$

As alternative to asymmetry parameter $0 < \tau < 1$ in quantile regression, we can write the optimization problem in terms of epsilon parameter (ϵ) as follows,

$$\mathrm{Q}_{\frac{1+\epsilon}{2}}(\mathrm{y}|\mathrm{x}) = \operatorname{argmin} \sum_{\mathrm{i}:\mathrm{y_i} \geq x_i'\beta_\epsilon} \frac{1+\epsilon}{2}|\mathrm{y_i} - \mathrm{x_i'}\beta_\epsilon| + \sum_{\mathrm{i}:\mathrm{y_i} < x_i'\beta_\epsilon} \frac{1-\epsilon}{2}|\mathrm{y_i} - \mathrm{x_i'}\beta_\epsilon|$$

$$= \operatorname{argmin} \quad E\left[\ \rho_{\frac{1+\epsilon}{2}}(\mathrm{y_i} - \mathrm{x_i'}\beta_\epsilon)\ \right]$$

$$= \operatorname{argmin} \sum_{\mathrm{i}:\mathrm{y_i} \geq x_i'\beta_\tau} \rho_{\frac{1+\epsilon}{2}}(e_\epsilon)$$

Here $x_i'\beta_\epsilon$ is the $\frac{1+\varepsilon}{2}$ quantile

Also, we need $P_r\ (e_{i,\epsilon} < 0) = \frac{1+\varepsilon}{2}$ holds, i.e. the $(\frac{1+\varepsilon}{2})$ of quantile $e_{i,\epsilon}$ is zero. So for the purpose of finding the quantiles, we minimize $E\ [\ \rho_{\frac{1+\epsilon}{2}}(\mathrm{y_i} - \mathrm{x_i'}\beta_\epsilon)\ ]$ with respect to $\hat{q} = \mathrm{x_i'}\beta_\epsilon$, as follows

$$E\ [\ \rho_{\frac{1+\epsilon}{2}}(y - \hat{q})\ ] = \int_{-\infty}^{+\infty} \rho\,(\mathrm{y} - \hat{q})\, d\,F(y)$$

$$= (\frac{1+\epsilon}{2} - 1) \int_{-\infty}^{\hat{q}} \rho\,(\mathrm{y} - \hat{q})\, d\,F(y) + (\frac{1+\epsilon}{2}) \int_{\hat{q}}^{+\infty} \rho\,(\mathrm{y} - \hat{q})\, d\,F(y)$$

By taking the derivation for this expectation with respect to $q^\wedge$ and set it to zero, we get

$$= (\frac{1+\epsilon}{2} - 1)\ [-F(\hat{q})] + (\frac{1+\epsilon}{2})\,[1 - F(\hat{q})] = 0$$

$$= -\ (\frac{1+\epsilon}{2})\ F(\hat{q}) + F(\hat{q}) - \left(\frac{1+\epsilon}{2}\right) + (\frac{1+\epsilon}{2})\,F(\hat{q}) = 0$$

$$F(\hat{q}) - \left(\frac{1+\epsilon}{2}\right) = 0$$

$$\hat{q} = F^{-1}\left(\frac{1+\epsilon}{2}\right).$$

Where,

$$Q\left(\frac{1+\epsilon}{2}\right) = inf\,\{\,y\colon F(y) \geq \frac{1+\epsilon}{2}\}$$

Moreover, the check (loss) function $\rho_{\frac{1+\epsilon}{2}}(e_{i,\epsilon})$ includes the check function in (9) as special case i.e, we have,

1- If $\epsilon = -1 \quad \leftrightarrow \quad \tau = 0$

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2- If $\epsilon = +1 \quad \leftrightarrow \quad \tau = 1$
3- If $\epsilon = 0 \quad \leftrightarrow \quad \tau = 1/2$

It can be noted easily that the minimum of the loss function (3) equivalents to the maximum of the likelihood function under the epsilon asymmetric Laplace distribution (2).

2. Conjugate priors for Bayesian QR Based on EALD

The attractive property of asymmetric Laplace distribution is regarding of the representation of this distribution as location-scale mixture of normal distributions, (Kotz et al., 2001; Reed and Yu, 2009; Kozumi and Kobayashi, 2011). This mixture representation has been employed for sampling from MCMC algorithms which have been used to construct posterior simulation algorithms. The epsilon asymmetric Laplace distribution is developed constructively through an extension of an asymmetric Laplace distribution location-scale mixture representation. Now, recall the pdf of EALD and assume that we have two independent random variables, $z \sim \exp(1)$ and $u \sim N(0,1)$. The error term of quantile regression can be written as mixture representation of normal distributions as follows,

$$e = \theta z + \varphi \sqrt{z}\, u$$

Where,

$$\theta = \frac{-4\epsilon}{1-\epsilon^2} \quad \text{and} \quad \varphi = \sqrt{\frac{8}{1-\epsilon^2}}$$

It is very easy to prove that random variable e has the following mean and variance,

$$E(e) = \frac{-4\epsilon}{1-\epsilon^2} \quad \text{and} \quad v(e) = \frac{8(1+\epsilon^2)}{(1-\epsilon^2)^2}$$

Based on this result, the response variable y_i is defined as follows,

$$y_i = x_i'\beta_\epsilon + \theta z_i + \varphi\sqrt{z_i}\, u_i \qquad \ldots(6)$$

Hence, $y_i|z_i$ is normally distributed with mean $x_i'\beta_\epsilon + \theta z_i$ and variance $\varphi^2 z_i$ .The joint density of y_i is defined as,

$$f(y_i|z_i) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\,\varphi^2 z_i}} \exp\left(-\frac{(y_i - x_i'\beta_\epsilon + \theta z_i)^2}{2\,\varphi^2 z_i}\right), \quad \ldots(7)$$

Alhamzawi in 2013, discussed and modified the Bayesian framework for quantile regression with Zellner g_ϵ – prior that assigns for each quantile level a different prior. By following the work of Alhamzawi in 2013, we modify the framework Bayesian quantile regression but under EALD. Also, for computational speed we also discuss the employing of lasso-type method in Bayesian QR with EALD. Next subsection discusses that.

3.1 *Zellner g_ϵ – prior*

the standard zellner' s g-prior naturally is not applicable with QR because the ALD is not a member of exponential family and then there is no normal likelihood function to conjugate with the same logic applies to the EALD case.so, how we can plug in the zellner' s g_e -prior with QR , the solution is to make a modification to prior variance of β this modification provides different prior for each quantile level through using $\boldsymbol{g_\tau}$ instead of g so, since the QR provides model structure changes with τ ,we must adapt new zellner' s g-prior for β,

$$\beta \sim N\left(0, g_\tau^{-1}\sigma^2 (X^T X)^{-1}\right) \ldots\ldots(8)$$

The prior covariance of $\beta'/\, g_\tau^{-1}\sigma^2 (X^T X)^{-1}$ can adapt different uncertainty or beliefs of each quantile level, . Given ϵ , v , E[σ] , β_{0P} Zellner prior in (8) , and $\pi(\sigma) \propto \sigma^{-1}$. Also by following smith and kohn in 1996 we set g= 100 , the joint conditional posterior distribution is defined as follows,

$$\pi(\beta,\sigma|\underline{v}, y, x) \propto L(y|\beta,\sigma,v)\,\pi(\beta|\sigma,v)p(v|\sigma)p(\sigma), \ldots(9)$$

Be following Zellner in (1983) and Al- hamzawi in (2013) , the MCMC (algorithm has constructed to be updated by the following parameter:

Updating β : since we depends on the likelihood,

$$y - (-\epsilon v) \sim N(x\beta, v^{-1}) \text{ and the Zellner prior}$$

$\beta \sim N(0, g\,(x^T v x)^{-1})$ which are normal, the conjugate is a normal, and the posterior mean is

$$\mu = \left(x^T v x + \frac{1}{g} (x^T v x)\right)^{-1} x^T v\,(y - (-\epsilon v))$$

$$= \left(1 + \frac{1}{g}\right)(x^T v x)^{-1} x^T v\,(y - (-\epsilon v))$$

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$$= \frac{g}{1+g} \quad (\mathrm{x}^{\mathrm{T}} \mathrm{v}\,\mathrm{x}))^{-1}\ \mathrm{x}^{\mathrm{T}}\mathrm{v}\,(y-(\epsilon v))$$

$$\sum = ((\mathrm{x}^{\mathrm{T}}\mathrm{v}\,\mathrm{x}) + \frac{1}{g}(\mathrm{x}^{\mathrm{T}}\mathrm{v}\,\mathrm{x}))^{-1} = \frac{g}{1+g} \quad (\mathrm{x}^{\mathrm{T}}\mathrm{v}\,\mathrm{x}))^{-1}$$

And since the likelihood $\sim N(0\,, 2\sigma\, v^{-1}\ )$, then

$$\sum = \frac{2\sigma\, g}{1+g} \quad (\ \mathrm{x}^{\mathrm{T}}\mathrm{v}\,\mathrm{x})^{-1}$$

So , β $is\ updataing\ as \quad N_p\,(\mu\,,\Sigma\,)$
Updating σ :

$$\sigma|\,\beta\,,v \quad \sim\ Inverse-Gamma$$

$$\left[\frac{3n+p}{2}\ ,\ \frac{1}{4}\ (y-x\beta-(\epsilon v))^{-1}\ \ v\ \ \big(y-x\beta-(\epsilon v)\big)+\frac{1}{4g}\ \beta^{T}\ (\ \ \mathrm{x}^{\mathrm{T}}\mathrm{v}\,\mathrm{x})\,\beta+(1-\ \epsilon^2)\sum_{i=1}^{n} v_i\ \right]$$

See alhamzawi 2013 for more details .
Updating v :
The conditional distribution for v_i ; i=1 ,2, …., n is defined as follows,

$$v_i \quad \propto\ v_i{}^{-1}\ \exp\left\{-\frac{1}{2}\ \ (v_i{}^{-1}\ \ c_1^2\ + v_i\ c_2^2\ \right\}\ \ ,$$

Where $c_1^2\ =\ \left[(\ y_i\ -\ x_i^T\ \beta\,)^2\ \ +\ \ \frac{\beta^T\ x_i\ \ x_i^T\ \ \beta}{g}\right]/2\,\sigma \quad ,\ and\quad c_2^2 = \frac{1}{2\,\sigma}$

3. Simulation Study

We compare the performance of the proposed methods: the new Bayesian quantile regression with g prior distribution referred to as 'NBQRg' and the new Bayesian Lasso quantile regression referred to as 'NBLQR' with three existing methods, including the frequentest quantile regression approach (QR), Bayesian quantile regression with g prior distribution (BQRg), Bayesian Lasso quantile regression (BLQR). Following Zou and Hastie (2005) and Li and Lin (2010), methods are evaluated based on the median of mean squared error (MMSE). We simulate 30 training observations and 300 testing observations from the linear regression model,

$$y_i = x'\,\beta +\ \epsilon_i$$

The design matrix $X = (\boldsymbol{x_1}, \cdots, \boldsymbol{x_n})$, where $\boldsymbol{x}_i = (x_{i1}, \cdots, x_{ip})'$ is simulated from the multivariate normal distribution $N(\mathbf{0}, \boldsymbol{\Sigma})$, where the $(i,j)^{th}$pairwise correlation element of $\boldsymbol{\Sigma}$ is $0.5^{|i-j|}$.
We simulate the data with three different scenarios by varying the error distribution (ϵ_i) :

1. $e_i \sim \quad N(0,3)$
2. $e_i \sim t_{(3)}$ distribution with three degrees of freedom.
3. $e_i \sim \chi^2{}_{(3)}$ distribution with three degrees of freedom.

The dependent variable is centered and the predictors are standardized to have zero means and unit variances before applying the comparison between the methods. We consider three choices of error (0.0, 0.5 and 0.9) which is corresponding to 0.5, 0.75, and 0.95 quantiles. The prior specifications are the same as those in section 2. We set $c = d = r_1 = r_2 = 0.1$ and $g = np$. We ran the developed MCMC algorithms for 15,000 iterations, after a burn-in period of 1000 iterations. Based on the credible intervals, we listed the average false positive rates (AFPR, the proportion of incorrectly selected insignificant covariates) and average false negative rates (AFNR, the proportion of incorrectly removed significant covariates) for the Bayesian Lasso models. The convergence of the proposed algorithm is assessed using the trace plots, histograms, autocorrelation plots and the Potential Scale Reduction Factor (PSRF) (Brooks and Gelman, 1998). Due to the space limit, we only present these plots for Simulation one. Similar plots are also observed for

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the other simulation studies not shown here.

3-1-1: Simulation 1 (Sparse recovery problem)

To test the performance of the proposed methods, we consider a simple sparse recovery problem. Here, we set $\beta = (1,1,1,0,0,0,0,0,0,0)^T$. The results of Simulation are summarized in Tables 1 and 2. In Table 1, we compare MMSE results of NBQRg, BQRg and QR for Simulation averaged over 100 replecations. In the parentheses are standard deviations (SD) of the MSEs. The results of MMSE shows that the proposed Bayesian quantile regression approach (NBQRg), consistently achieved the lowest MMSE for all quantiles under consideration when the error is normal. Similar results are found for the other error distributions $t_{(3)}$ distribution with three degrees of freedom and $x^2{}_{(3)}$ distribution. From Table 2, we can see that the NBLQR produces better results than BLQR. Table 2 comparing the MMSE results of NBLQR and BLQR for Simulation averaged over 100 replecations.. The main takeaway is that the proposed Bayesian Lasso quantile regression approach (NBQRg), consistently achieved the lowest MMSE for all quantiles under and error distributions under consideration.

Table 1: Comparing MMSE of NBQRg, BQRg and QR for Simulation obtained by 100 simulation study. In the parentheses are standard deviations (SD) of the MSEs.

	Method	N(0,3) MMSE	SD	$t_{(3)}$ MMSE	SD	$\chi^2{}_{(3)}$ MMSE	SD
ϵ = 0.0	NBQRg	0.73110	0.07097	0.77826	0.43676	2.81808	2.10159
	BQRg	1.70704	1.17568	1.05253	0.47239	3.10701	2.24252
	QR	2.33224	1.26948	1.55294	0.90562	3.96201	2.45913
ϵ = 0.50	NBQRg	1.63180	0.76713	1.05448	0.73539	2.35079	1.59832
	BQRg	1.80913	0.82596	1.16179	0.83153	2.57561	1.69207
	QR	2.54318	1.50723	1.23102	0.85809	3.30639	1.86996
ϵ = 0.90	NBQRg	1.34972	0.59944	1.16847	0.88493	2.39645	1.82532
	BQRg	1.50590	0.69273	1.26580	0.96567	2.62051	1.95455
	QR	2.30822	1.73642	1.56796	1.20649	3.67565	2.57315

Table 2: Comparing MMSE of NBLQR and BLQR for Simulation obtained by 100 simulation study. In the parentheses are standard deviations (SD) of the MSEs.

	Method	N(0,3) MMSE	SD	$t_{(3)}$ MMSE	SD	$\chi^2{}_{(3)}$ MMSE	SD
ϵ = 0.0	NBLQR	0.45059	0.07069	0.77566	0.46046	1.05429	0.30790
	BLQR	0.54467	0.26133	0.83179	0.63725	1.07985	0.65083
ϵ = 0.50	NBLQR	0.79522	0.37991	0.78175	0.28093	1.01464	0.41696
	BLQR	0.81529	0.36938	0.84149	0.37061	1.08364	0.61845
ϵ = 0.90	NBLQR	0.85972	0.43507	0.65821	0.44917	1.29938	1.10095
	BLQR	2.64080	0.48572	2.58145	0.50461	2.83842	0.74868

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In Table 3: FPR and FNR for the Bayesian Lassos (NBLQR and BLQR) in Simulation based on the credible intervals. FPR is the false positive rate and FNR is the false negative rate. All results are averaged over 100 replications

		N(0, 3)		$t_{(3)}$		$\chi^2_{(3)}$	
	Method	FPR	FNR	FPR	FNR	FPR	FNR
$\epsilon = 0.0$	NBLQR	0.09	0.05	0.10	0.06	0.07	0.06
	BLQR	0.08	0.04	0.07	0.07	0.05	0.08
$\epsilon = 0.50$	NBLQR	0.11	0.05	0.08	0.08	0.11	0.15
	BLQR	0.09	0.08	0.10	0.10	0.13	0.21
$\epsilon = 0.90$	NBLQR	0.08	0.07	0.11	0.21	0.17	0.10
	BLQR	0.14	0.13	0.19	0.26	0.26	0.07

As we can see from Table 3, compared with BLQR, NBLQR produces comparable or better FPR and produces better FNR in Simulation. It has the smallest FPR in 5 out of 9 simulation setups and has the smallest FNR in 7 out of 9 simulation setups. The histograms of Simulation based on posterior samples of 14,000 iterations are illustrated in Figures 1. These histograms show that the poster samples are in fact the desired stationary univariate normal. The trace plots of Simulation based on posterior samples of 14,000 iterations are illustrated in Figures 2 and 3. These plots are offer a good visual diagnostic of the mixing property.

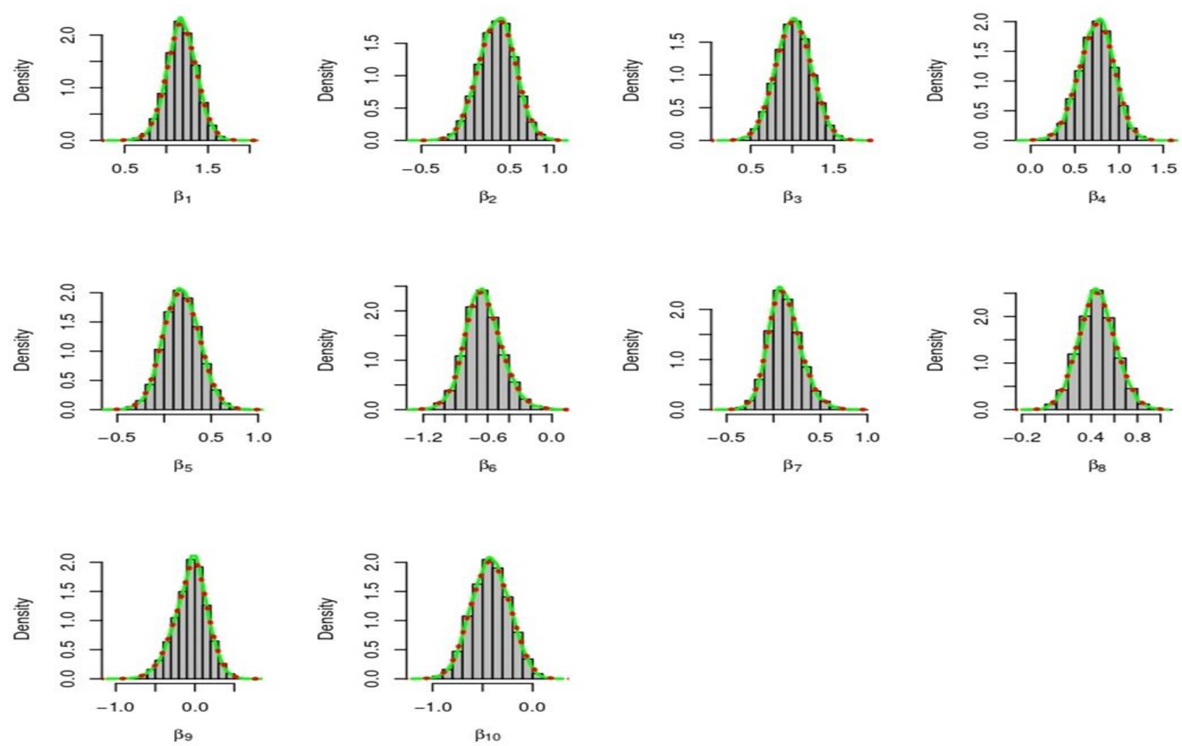

Figure 1: Histograms based on posterior samples of NBQRg for Simulation under the median when the error is normal.

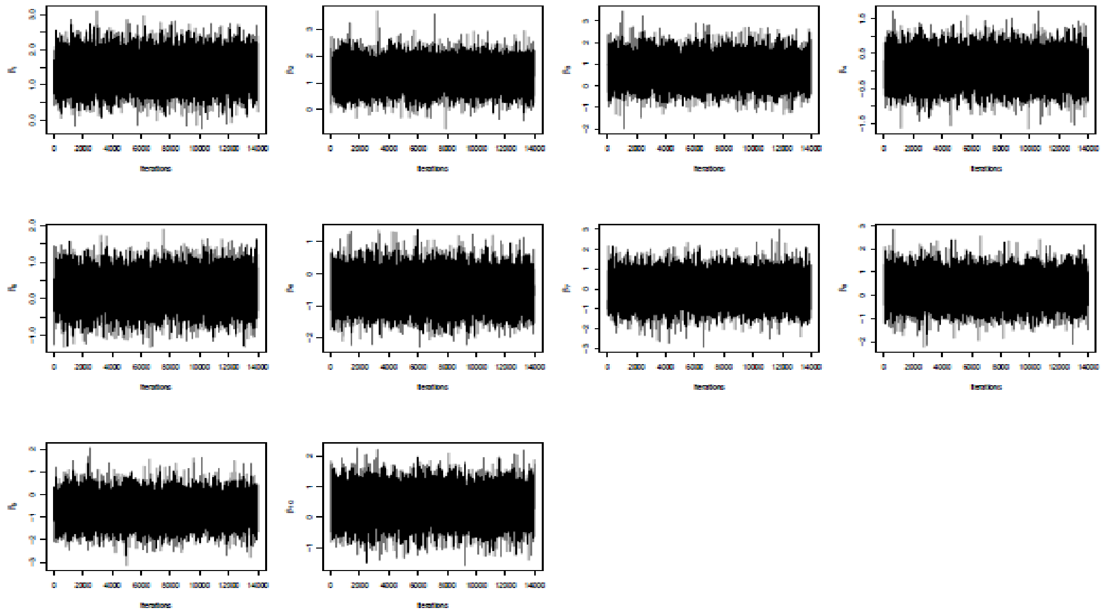

Figure 2: Trace plot based on posterior samples of NBQRg for Simulation under the median when the error is normal.

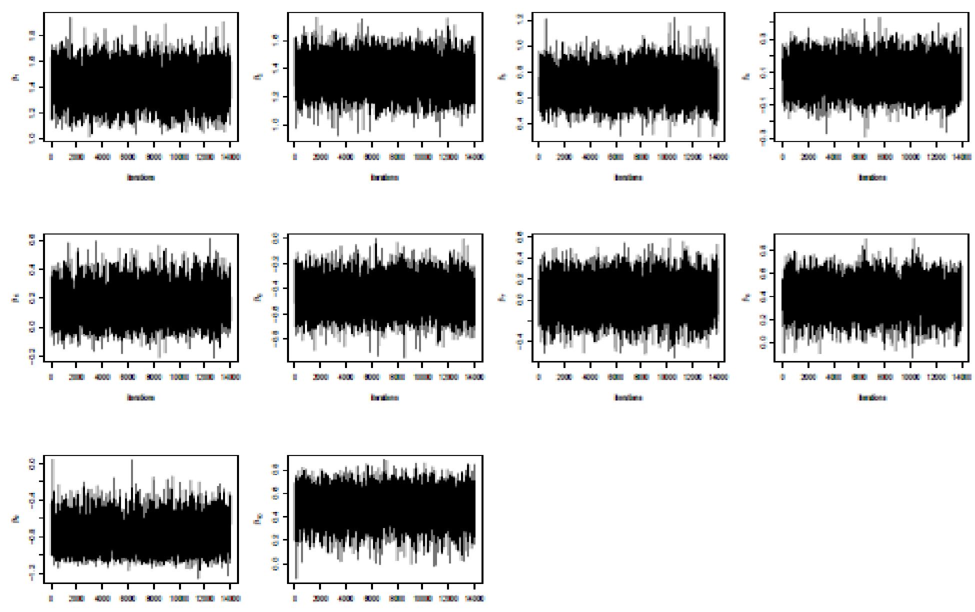

Figure 3: Trace plot based on posterior samples of NBLQR for Simulation under the median when the error is normal

4. Conclusions

Based on the theoretical part and simulation results, the following overall conclusions can be drawn. Across all simulation scenario, sparse and under different error distributions (normal, heavy-tailed t(3), and skewed $\chi 2(3)$ the proposed new Bayesian quantile regression with g-prior (NBQRg) consistently outperforms the competing methods, including classical quantile regression (QR) and the existing Bayesian quantile regression with g-prior (BQRg). This superiority is clearly reflected in the median of mean squared error (MMSE), where NBQRg achieves the lowest values for nearly all quantiles considered (τ=0.5,0.75,0.95) and under all error distributions. These results indicate that NBQRg provides more accurate and robust estimation, particularly in non-Gaussian settings and at extreme quantiles, where classical QR and standard Bayesian approaches tend to deteriorate in performance. In terms of variable selection, the new Bayesian Lasso quantile regression (NBLQR) demonstrates clear advantages over the traditional BLQR. Across the majority of simulation setups, NBLQR attains lower false negative rates (FNR) while maintaining comparable or smaller false positive rates (FPR), indicating a better balance between identifying truly important covariates and avoiding spurious ones. This advantage is especially pronounced in sparse and very sparse recovery problems, where correct identification of nonzero coefficients is crucial. Moreover, the MCMC diagnostic results including trace plots, posterior histograms, autocorrelation functions, and MPSRF values close to one confirm good mixing behavior and reliable convergence of the proposed algorithms. Overall, the findings strongly support the effectiveness, stability, and robustness of the proposed Bayesian quantile regression frameworks, making them attractive alternatives for high-dimensional and nonstandard data settings.

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