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On Soft Strongly \mathfrak{b}^* –Closed Sets and Soft Strongly \mathfrak{b}^* –Continuous Functions in Soft Topological Spaces

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Article Info.	Abstract
<p><i>Article history:</i></p> <p>Received 24 September 2024</p> <p>Accepted 25 November 20224</p> <p>Publishing 30 March 2026</p>	<p>In this paper, we define the notion of new class of soft sets, called soft strongly \mathfrak{b}^* –closed and soft strongly \mathfrak{b}^* –open sets. We introduce the soft strongly \mathfrak{b}^* –continuous functions and study several properties of images and inverse images and define open map and closed map. In addition, we introduce a several characterizations and properties and for better understanding examples have been addressed.</p>
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1. Introduction and Preliminaries

Molodtsov [1], initiated a novel concept of soft set theory, which is a completely new approach for modeling vagueness and uncertainty. He successfully applied the soft set theory to several directions such as smoothness of functions, game theory, Riemann Integration, and theory of measurement. K. Kannan [2] introduced the soft g -closed soft sets in a soft topological space. I. Arockiarani and A. Arokialancy [3] defined soft β -open sets and continued to study weak forms of soft open sets in soft topological space. Akdag and Ozkan [4, 5] introduced the soft α -open, soft \mathfrak{b} -open and studied continuous function of them. Hameed, S. Z., Hussein, A. K [6] defined the soft \mathfrak{bc} –open set. The nc Open- Sets in Topological Space study by [7]. Some Results on Fuzzy w -Local Covering Dimension Function in Fuzzy Topological Space introduced by H. Sadiq and M. A.Khalik in [8]. The soft \mathfrak{b}^* –closed, $s\mathfrak{b}^*$ –continuous, $sS\mathfrak{b}^*$ –separation axioms, $sS\mathfrak{b}^*$ –compactness and $sS\mathfrak{b}^*$ –connectedness in soft topological spaces are studied by Saif at el. in [7, 8, 9, 10, 11,12]. A. Poongothai, R. Parimelazhagan introduced $s\mathfrak{b}^*$ -closed [13], strongly \mathfrak{b}^* –continuous functions [14]. N. Gomathi [15] introduced a $S\mathfrak{b}^*$ homeomorphism.

In this work, we introduce soft versions of strongly \mathfrak{b}^* –open sets and strongly \mathfrak{b}^* –closed sets, as well as soft strongly \mathfrak{b}^* –continuous functions. We investigate the properties of these sets and functions, presenting new results in this field. Additionally, we explore the relationships between soft continuity, soft α -continuity, soft \mathfrak{b} -continuity, and other types of soft continuity in the context of soft strongly continuous functions. Using counterexamples, we demonstrate the distinctness of these various mapping types.

Definition 1.1: [1] Let D be an initial universe set, $P(D)$ the power set of D , and Λ a set of parameters. A pair (L, Λ) (briefly L), where L is a function mapping Λ to $P(D)$, is referred to as a soft set over D . We will be represented the collection of all soft sets over D as $SS(D, \Lambda)$.

Definition 1.2: [14] The soft set $L \in SS(D, \Lambda)$, where $L(\nabla) = \emptyset$, for every $\nabla \in \Lambda$ is referred to as A-null soft set of $SS(D, \Lambda)$ and is represented as $\tilde{\emptyset}$. The soft set $L \in SS(D, \Lambda)$, where $L(\nabla) = D$, for every $\nabla \in \Lambda$ is referred to as the A-absolute soft set of $SS(D, \Lambda)$ and is represented as \tilde{D} .

Definition1.3: [15] Let T be a collection of soft sets over D , then T is referred to as soft topology on D if it satisfies the following axioms:

- (1) $\tilde{\emptyset}$ and \tilde{D} s included in T
- (2) The union of any subcollection of soft sets of T is included in T
- (3) The intersection of any two soft sets in T is included in T .

The triplet (D, T, Λ) is called a soft topological space (briefly STS) over D and abbreviated as D .

Definition 1.4: [15] Let (D, T, Λ) be a soft space over D , then the members of T are referred to as soft open sets in T .

Definition 1.5: [15] Let (D, T, Λ) be a soft space over D . A soft set M over D is referred to as be a soft closed set in D , if its relative complement M' belongs to T .

Definition 1.6: [15] Let (D, T, Λ) be a STS and $M \in SS(D, \Lambda)$. Then

- (1) The soft closure of M is the soft set

$$cl(M) = \cap \{O : O \in T^c, M \subseteq O\}.$$
- (2) The soft interior of M is the soft set

$$int(M) = \cup \{O : O \in T, O \subseteq M\}.$$

Definition 1.7: A soft set M of an $STS (D, T, \Lambda)$ is referred to as

- (1) Soft α - open [4] if $M \subset int(cl(int(M)))$,
- (2) Soft preopen [3] if $M \subset int(cl(M))$,
- (3) Soft semi - open [16] if $M \subset cl(int(M))$,
- (4) Soft β -open [3] if $M \subset cl(int(cl(M)))$,
- (5) Soft \mathbf{b} –open [5] if $M \subset int(cl(M)) \cup cl(int(M))$,
- (6) Soft \mathbf{b}^{**} –open [17] if $M \subset int(cl(int(M)) \cup cl(int(cl(M))))$.

Definition 1.8: [2] Let (D, T, Λ) be an STS . A subset P of D is referred to as soft generalized closed in D if $cl(P) \subseteq O$ whenever $P \subseteq O$ where O is soft open set in D . we represented as by sg – closed.

Definition 1.9: [18] Let (D, T, Λ) be an STS . A subset P of D is referred to as soft g^* – closed in D if $cl(P) \subseteq O$ whenever $P \subseteq O$ where O is soft g –open set in D . we represented as by sg^* – closed.

Definition 1.10: [19] A soft set P is referred to as soft ω – closed set in an $STS (D, T, \Lambda)$, if $cl(P) \subseteq M$ whenever $P \subseteq M$ and M is soft *semi* –open set in D . The relative complement of P is referred to as soft ω –open in D .

Definition 1.11: [7] A soft set L of an $STS (D, T, \Lambda)$ is referred to as a soft \mathbf{b}^* – closed (or simply sb^* –closed) if $int(cl(L)) \subseteq O$, whenever $L \subset O$ and O is soft \mathbf{b} –open. The relative complement of L is referred to as soft \mathbf{b}^* –open (or simply sb^* –open) in D .

Definition 1.12: A soft mapping $f: D \rightarrow W$ is referred to as soft β – continuous [20] (resp., soft α – continuous [4], soft \mathfrak{b} – continuous [5] and soft \mathfrak{b}^* – continuous [8]) if the inverse image of each soft open set in W is a soft β – open (resp., soft α – open, soft \mathfrak{b} – open and soft \mathfrak{b}^* – open) set in D .

2. Soft strongly \mathfrak{b}^* – closed sets

Throughout this section, we introduce and explore the soft strongly \mathfrak{b}^* – closed set and some of characteristics. As well as we examine the relationship between it and another soft closed sets in \mathcal{STS} .

Definition 2.1: A soft set P for an \mathcal{STS} (D, T, Λ) is named a soft strongly \mathfrak{b}^* – closed (or simply $s\mathfrak{S}\mathfrak{b}^*$ – closed) if $cl(int(P)) \subseteq O$, in case of $P \subset O$ and O is an $s\mathfrak{b}$ – open in D .

Theorem 2.2: If O $s\mathfrak{S}\mathfrak{b}^*$ – closed and soft open sets then it is an $s\mathfrak{b}$ – closed.

Proof. Let O is an $s\mathfrak{S}\mathfrak{b}^*$ – closed and soft open. Since $s\mathfrak{S}\mathfrak{b}^*$ – closed this implies $O \subseteq M$ and M is $s\mathfrak{b}$ – open, then $cl(int(M)) \subseteq M$. But $int(cl(M)) \cap cl(int(M)) \subseteq cl(int(M)) \subseteq M$. Hence, M is $s\mathfrak{b}$ – closed.

The next example indicates that the converse of Theorem 2.2 need not be true in general.

Example 2.3: Let $D = \{o_1, o_2, o_3\}$, $\Lambda = \{\nabla_1, \nabla_2, \nabla_3\}$ and $T = \{\tilde{\emptyset}, \tilde{D}, M_1, M_2, M_3, \dots, M_{15}\}$

where $M_1, M_2, M_3, \dots, M_{15}$ are soft set over D . Identify as:

$$M_1 = \{(\nabla_1, \{o_1, o_2\}), (\nabla_2, \{o_3\}), (\nabla_3, \{o_1, o_3\})\},$$

$$M_2 = \{(\nabla_1, \{o_2\}), (\nabla_2, \{o_1, o_2\}), (\nabla_3, \{o_1, o_2\})\},$$

$$M_3 = \{(\nabla_2, \{o_2\}), (\nabla_3, \{o_1\})\},$$

$$M_4 = \{(\nabla_1, \{o_1, o_2\}), (\nabla_2, D), (\nabla_3, D)\},$$

$$M_5 = \{(\nabla_1, \{o_3\}), (\nabla_2, \{o_1, o_3\}), (\nabla_3, \{o_2\})\},$$

$$M_6 = \{(\nabla_2, \{o_3\})\},$$

$$M_7 = \{(\nabla_1, D), (\nabla_2, \{o_1, o_3\}), (\nabla_3, D)\},$$

$$M_8 = \{(\nabla_2, \{o_1\}), (\nabla_3, \{o_2\})\},$$

$$M_9 = \{(\nabla_1, \{o_2, o_3\}), (\nabla_2, D), (\nabla_3, \{o_1, o_2\})\},$$

$$M_{10} = \{(\nabla_1, \{o_2, o_3\}), (\nabla_2, \{o_1, o_3\}), (\nabla_3, \{o_1, o_2\})\},$$

$$M_{11} = \{(\nabla_2, \{o_1, o_3\}), (\nabla_3, \{o_2\})\},$$

$$M_{12} = \{(\nabla_1, \{o_2\}), (\nabla_2, D), (\nabla_3, \{o_1, o_2\})\},$$

$$M_{13} = \{(\nabla_1, \{o_2\}), (\nabla_2, \{o_1\}), (\nabla_3, \{o_1, o_3\})\},$$

$$M_{14} = \{(\nabla_1, \{o_1, o_2\}), (\nabla_2, \{o_1, o_3\}), (\nabla_3, D)\} \text{ and}$$

$$M_{15} = \{(\nabla_1, \{o_2\}), (\nabla_2, \{o_3\}), (\nabla_3, \{o_1\})\}.$$

Therefore, T is an \mathcal{STS} on D ,

and soft closed sets are $\tilde{D}, \tilde{\emptyset}, M_1^c, M_2^c, M_3^c, \dots, M_{15}^c$.

Let us take $O = \{(\nabla_1, \{o_2, o_3\}), (\nabla_2, \{o_1, o_2\}), (\nabla_3, \{o_1, o_2\})\}$, therefore O is an $s\mathbf{b}$ –closed which is not $sS\mathbf{b}^*$ –closed.

Theorem 2.4: Each soft closed set is soft strongly \mathbf{b}^* –closed set.

Proof. Let O is a soft closed in D , M be an sb –open where $O \subseteq M$. Now, $cl(O) \subseteq M$. Therefore $cl(int(O)) \subseteq cl(O) \subseteq M$. Therefore, O is an $sS\mathbf{b}^*$ –closed.

The next example indicates that the converse of Theorem 2.4 need not be true in general.

Example 2.5: Consider $D = R$, and $\Lambda = Z$, let W be the soft usual topology on R and $O(\nabla) = (1, 3)$ for all $\nabla \in \Lambda$. Therefore O is $sS\mathbf{b}^*$ –closed but is not soft closed set. Since $(1, 3)$ not soft closed set in soft usual topology.

Theorem 2.6: A soft set M is an $sS\mathbf{b}^*$ –closed if and only if $cl(int(M)) - M$ does not contain any non-empty sb –closed sets.

Proof. Assume that O be a nonempty sb –closed subset of $cl(int(M))$ such that $O \subseteq cl(int(M)) - M$ then $O \subseteq cl(int(M)) \cap M^c$. Therefore, $O \subseteq cl(int(M))$ and $O \subseteq M^c$. Since O^c is sb –open set and M is an $s\mathbf{b}^*$ –closed set, $cl(int(M)) \subseteq O^c$. Therefore, $O \subseteq [cl(int(M))]^c$. Then, $O \subseteq [cl(int(M))] \cap [cl(int(M))]^c = \emptyset$. Therefore, $O = \emptyset$ and this meaning that $cl(int(M)) - M$ contains no nonempty an sb –closed sets. Conversely, let $M \subseteq O$ is an sb –open. Suppose that $cl(int(M))$ is not set forth in O , then $[cl(int(M))]^c$ is nonempty sb –closed and so on in $cl(int(M)) - M$, which is a contradiction. Therefore, $cl(int(M)) \subseteq O$ and consequently, M is an $s\mathbf{b}^*$ –closed set.

Theorem 2.7: Let $W \subseteq \tilde{W} \subseteq D$, if \tilde{W} is $sS\mathbf{b}^*$ –closed set relative to W and \tilde{W} is soft open and $sS\mathbf{b}^*$ –closed in (D, T) , then \tilde{W} is $sS\mathbf{b}^*$ –closed set in (D, T) .

Proof. Let $\tilde{W} \subseteq O$ and \tilde{W} is an sb –open set in D , $\tilde{W} \subseteq W \subseteq D$. Therefore $\tilde{W} \subseteq \tilde{W}$ and $\tilde{W} \subseteq O$. This meaning that $\tilde{W} \subseteq \tilde{W} \cap O$. Since \tilde{W} is an $sS\mathbf{b}^*$ –closed set relative to W , then $cl(int(\tilde{W})) \subseteq O$. $\tilde{W} \cap cl(int(\tilde{W})) \subseteq \tilde{W} \cap O$ which meaning that $\tilde{W} \cap cl(int(\tilde{W})) \subseteq O$. Therefore, $[\tilde{W} \cap cl(int(\tilde{W}))] \cup [cl(int(\tilde{W}))]^c \subseteq O \cup [cl(int(\tilde{W}))]^c$. which meaning that

$(\tilde{W} \cup [cl(int(\tilde{W}))]^c) \cap (cl(int(\tilde{W})) \cup [cl(int(\tilde{W}))]^c) \subseteq (O, \Lambda) \cup [cl(int(\tilde{W}))]^c$. Therefore, $(\tilde{W} \cup [cl(int(\tilde{W}))]^c) \subseteq O \cup [cl(int(\tilde{W}))]^c$. Since \tilde{W} is an $sS\mathbf{b}^*$ –closed in D , we have $cl(int(\tilde{W})) \subseteq O \cup [cl(int(\tilde{W}))]^c$. As well as $\tilde{W} \subseteq \tilde{W}$ meaning that $cl(int(\tilde{W})) \subseteq int(cl(\tilde{W}))$. Then $cl(int(\tilde{W})) \subseteq int(cl(\tilde{W})) \subseteq O \cup [cl(int(\tilde{W}))]^c$. Therefore, $cl(int(\tilde{W})) \subseteq O$. Since $cl(int(\tilde{W}))$ is not set forth in $[cl(int(\tilde{W}))]^c$. Therefore, \tilde{W} is an $sS\mathbf{b}^*$ –closed set relative to D .

Theorem 2.8: Every $s\alpha$ –closed set is $sS\mathbf{b}^*$ –closed.

Proof. Assume that M is an $s\alpha$ –closed set and O is a soft open set where $M \subseteq O$. For M is an $s\alpha$ –closed set. Therefore, $sacl(M) \subseteq O$. Now, $cl(int(M)) \subseteq cl(int(sacl(M))) \subseteq M \cup cl(int(sacl(M))) = sacl(M) \subseteq O$. For each soft open is an sb –open. Therefore, M is a $sS\mathbf{b}^*$ –closed set in D .

The next examples, indicates that the converse of Theorem 2.8. need not be true in general.

Example 2.9: Suppose $D = R$, $\Lambda = Z$, T be the usual topology on R and $M(a) = (1, \infty)$ for all $a \in \Lambda$. Therefore M is sSb^* –closed which is not an $s\alpha$ –closed set. Since $cl(int(cl(1, \infty))) = [1, \infty) \not\subseteq (1, \infty)$. Therefore, that is not a $s\alpha$ –closed.

Example 2.10: Consider $D = \{o_1, o_2, o_3\}$, $\Lambda = \{\nabla_1, \nabla_2, \nabla_3\}$ and (D, T, Λ) be STS . Consider the STS given in Example 2.3. $T = \{\tilde{\emptyset}, \tilde{D}, M_1, M_2, M_3, \dots, M_{15}\}$. Therefore, let us take $O = \{(\nabla_1, \{o_2\}), (\nabla_2, \{o_3\})\}$ is an sSb^* –closed, which is not a $s\alpha$ –closed.

Theorem 2.11: Let $M \subseteq \Sigma \subseteq D$ and supposed that M be an sSb^* –closed in D , then M is sSb^* –closed relative to Σ .

Proof. Given that $M \subseteq \Sigma \subseteq D$ and M is a sSb^* –closed in D . Let $\Sigma \cap O \subseteq M$ where O is sb –open set in D . For M is an sSb^* –closed $O \subseteq M$ implies that $O \subseteq cl(int(M))$ (i.e) $(\Sigma \cap O) \subseteq \Sigma \cap cl(int(M))$ where $\Sigma \cap cl(int(M))$ is interior of closure of M in Σ . Thus, M is sSb^* –closed relative to Σ .

Theorem 2.15: If P and O are sSb^* –closed sets assigned on D , then their intersection $P \cap O$ is an sSb^* –closed.

Proof. Let P and O be sSb^* –closed sets and M be an sb –open set in D such that $P \cap O \subseteq M$. We have $cl(int((P) \cap O)) \subseteq cl(int((P))) \cap cl(int((O))) \subseteq M$. Consequently, $P \cap O$ is an sSb^* –closed.

Remark 2.16: The union of two sSb^* –closed sets need not to become sSb^* –closed set.

Example 2.17: Let $D = \{h_1, h_2, h_3\}$, $\Lambda = \{\nabla_1, \nabla_2, \nabla_3\}$ and (D, T, Λ) be STS .

Consider the STS given in Example 2.3: $T = \{\tilde{\emptyset}, \tilde{D}, M_1, M_2, M_3, \dots, M_{15}\}$. If we take

$O = \{(\nabla_1, \{h_1, h_2\}), (\nabla_2, \{h_3\}), (\nabla_3, D)\}$ and $P = \{(\nabla_1, \{h_1\}), (\nabla_2, \{h_1, h_3\}), (\nabla_3, \{h_2, h_3\})\}$ which are sSb^* –closed. Therefore $O \cup P$ is not an sSb^* –closed.

Remark 2.18: In Figure 1 we shown that every sSb^* –closed is $s\beta$ –closed, and every sSb^* –closed is sB^{**} –closed. Similarly, every sg^* –closed set is sSb^* –closed.

The following example indicates that the converses of these implications are not true in general.

Example 2.19: Let $D = \{o_1, o_2, o_3\}, \Lambda = \{\nabla_1, \nabla_2, \nabla_3\}$ and (D, T, Λ) be STS . Consider the STS given in Example 2.3. $T = \{\tilde{\emptyset}, \tilde{D}, M_1, M_2, M_3, \dots, M_{15}\}$. Therefore, the set

$M = \{(\nabla_1, \{o_1\}), (\nabla_2, \{o_1, o_3\}), (\nabla_3, \{o_2\})\}$ is an $s\beta$ –closed which is not an sSb^* –closed. As well as the set $O = \{(\nabla_1, \{o_1, o_2\}), (\nabla_2, \{o_3\}), (\nabla_3, \{o_1, o_3\})\}$ is an sB^{**} –closed but is not sSb^* –closed.

Finally, $P = \{(\nabla_1, \{o_1\}), (\nabla_2, \{o_1, o_3\}), (\nabla_3, \{o_1, o_3\})\}$ is an sSb^* – closed which is not an sg^* –closed.

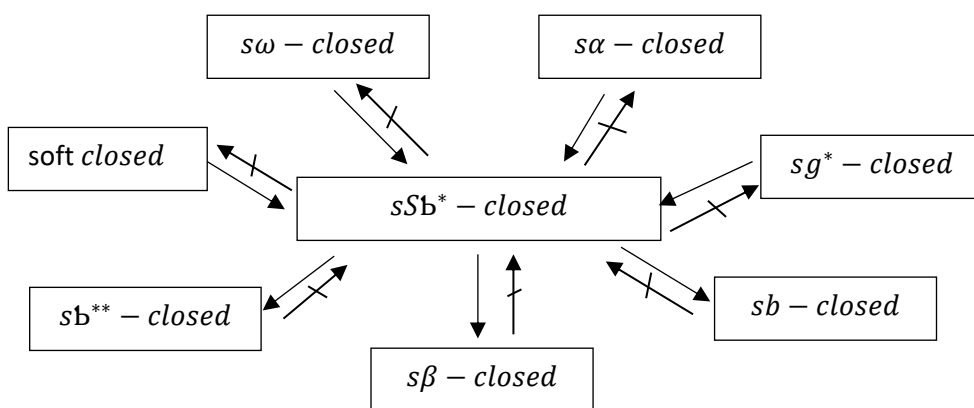


Figure 1 Relationships of soft strongly b^* –closed

3. Soft strongly b^* –open set

Throughout this section, we present and inspect the concept of the soft strongly b^* –open set in soft topological spaces. We examine some of the basic results of the soft strongly b^* –open.

Definition 3.1: A soft set S is named soft strongly b^* –open set (or simply sSb^* –open) if it’s complement S^c is an sSb^* –closed.

Theorem 3.2: A soft set M of STS D is an sSb^* –open if and only if $O \subseteq int(cl(M))$ in case of O is sb –closed and $O \subseteq M$.

Proof. Let M be an sSb^* –open. Therefore M^c is a sSb^* –closed. Let O be an sb –closed set such that $O \subseteq M$. Therefore $M^c \subseteq O^c$. Since M^c is a sSb^* –closed, $cl(int(M)^c) \subseteq O$ taking complement on both sides, then $O \subseteq int(cl(M))$. Conversely, let O^c is set forth in $int(cl(M))$ in case of $O \subseteq M$ and O is soft closed in D . Suppose P be an sb –open set containing M^c , then $P^c \subseteq int(cl(M)^c)$. By taking the complement of both sides, we arrive at $cl(int(M)^c) \subseteq P$. Consequently, M^c is an sSb^* –closed. Therefore, M is an sSb^* –open.

Theorem 3.3: Let (D, T, Λ) be an STS . Then the following statements are true in general:

- (i) Every soft open is sSb^* –open.
- (ii) Every sa –open is sSb^* –open.

(iii) Every $s\omega$ –open is sSb^* –open.

(iv) Every sSb^* –open set is $s\omega$ –open.

Proof. Clear.

Definition 3.4: Let (D, T, Λ) be an \mathcal{STS} . A soft set $M \subseteq D$ is named a soft strongly b^* –neighborhood (briefly $sSb^* - nbd$) for a point $c \in D$ if there exists sSb^* –open set O where $c \in O \subseteq M$.

Definition 3.5: Let (D, T, Λ) be an \mathcal{STS} . A subset $M \subseteq D$ is named a soft strongly b^* –neighborhood (sSb^* –neighborhood) of $O \subseteq D$ if there exists an sSb^* –open set P where $O \subseteq P \subseteq M$. The family for all soft strongly b^* –neighborhood for a point $c \in D$ is a soft strongly b^* –neighborhood system of c and it is referred to as by $sSb^*N(c)$.

Theorem 3.6: Let (D, T, Λ) be an \mathcal{STS} , for each point $c \in D$, then we have the following results:

(i) For every $c \in D, sSb^*N(c) \neq \tilde{\emptyset}$.

(ii) $M \in sSb^*N(c) \implies c \in M$.

(iii) $M \in sSb^*N(c), M \subseteq O \implies O \in sSb^*N(c)$.

(iv) $M \in sSb^*N(c) \implies$ there exists $O \in sSb^*N(c)$ such that $O \subseteq M$ and $O \in sSb^*N(z)$ for every $z \in O$.

Proof.

(i) For D is an sSb^* –open set, it is a sSb^* –neighborhood for each $c \in D$. Consequently, $sSb^*N(c) \neq \tilde{\emptyset}$ for every $c \in D$.

(ii) If $M \in sSb^*N(c)$, then M is an sSb^* –neighborhood of c . By Definition of sSb^* –neighborhood, $c \in M$.

(iii) Let $M \in sSb^*N(c)$ and $M \subseteq O$. Therefore, \exists an sSb^* –open set P where $c \in P \subseteq M$, since $M \subseteq O, c \in P \subseteq O$. Therefore, O is a sSb^* –nbd of c . Consequently, $O \in sSb^*N(c)$.

(iv) If $M \in sSb^*N(c)$, then $c \in O \subseteq M$, for O is sSb^* –open set, therefore it is sSb^* –nbd of each its points. Hence, $O \in sSb^*N(z)$ for every $z \in O$.

Definition 3.7: Let M be soft subset for $\mathcal{STS} D$. Then, the soft strongly b^* –interior of M is

$$sSb^*int(M) = \cup \{O : O \text{ is an } sSb^* \text{ –open set and } O \subset M\}.$$

Definition 3.8: Let M be soft subset for $\mathcal{STS} D$. Then, the soft strongly b^* –closure of M is assigned as the intersection for all soft strongly b^* –closed set containing M , that is

$$sSb^*cl(M) = \cap \{O : O \text{ is an } sSb^* \text{ –closed set and } M \subset O\}.$$

Definition 3.9: Let M be a soft subset of D . A point $c \in D$ is stated to become an sSb^*int point of M if M is an sSb^* –neighborhood of c .

Proposition 3.10: Let S be a soft subset of D . Then

$$sSb^*int(S) = \cup \{c : c \text{ is an interior-point of } S\}.$$

Proof. Assume $S \subseteq D$, then $c \in sSb^*int(S)$

$\Leftrightarrow c \in \cup \{O: O \text{ is an } sSb^* \text{ -open set and } O \subset S\}$.

$\Leftrightarrow \exists$ an sSb^* -open set O where $c \in O \subseteq S$.

$\Leftrightarrow S$ is an sSb^* - *nb*d of the point c .

$\Leftrightarrow c$ is an sSb^*int point of S .

Consequently, $sSb^*int(S) = \cup \{c : c \text{ is an interior-point of } S\}$.

Theorem 3.11: Let (D, T, Λ) be an STS in D , the following statements are hold for sSb^*int .

- (a) $sSb^*int(D) = D$ and $sSb^*int(\emptyset) = \emptyset$.
- (b) $sSb^*int(O) \subseteq (O)$.
- (c) If M is any sSb^* -open set forth in O , then $M \subseteq sSb^*int(O)$.
- (d) If $O \subseteq M$, then $sSb^*int(O) \subseteq sSb^*int(M)$.
- (e) $sSb^*int(sSb^*int(O)) = sSb^*int(O)$.
- (f) $sSb^*int(D - (O)) \subseteq D - (sSb^*cl(O))$.
- (g) $sSb^*int((O) - M) \subseteq sSb^*int(O) - sSb^*int(M)$.

Proof. Omitted.

Theorem 3.12: If M and O are soft subsets for a space D , then:

- (i) $sSb^*cl(D) = D$ and $sSb^*cl(\emptyset) = \emptyset$.
- (ii) $M \subseteq sSb^*cl(M)$.
- (iii) If P is any sSb^* -closed set containing M , then $sSb^*cl(M) \subseteq P$.
- (iv) If $M \subseteq P$, then $sSb^*cl(M) \subseteq sSb^*cl(P)$.
- (v) $sSb^*cl(M) = sSb^*cl(sSb^*cl(M))$.

Proof. Omitted.

4. Soft strongly b^* -continuous functions

Throughout this section, we present the soft strongly b^* -continuous, soft strongly b^* -irresolute, soft strongly b^* -closed maps and soft strongly b^* -open maps. We examine some of their characteristics.

Definition 4.1: A mapping $g: D \rightarrow W$, from $STS (D, T, \Lambda)$ into $STS (W, \Omega, \theta)$, is referred to as soft strongly b^* -continuous (or simply sSb^* -continuous) if the inverse image of every soft open set in W is an sSb^* -open set in D .

Theorem 4.2: Every soft continuous function is sSb^* -continuous functions.

Proof. Let $g: D \rightarrow W$ be a soft continuous function. Let M be any soft open set in W . Therefore, the inverse image $g^{-1}(M)$ is soft open. Since each soft open set is an sSb^* -open set. For $g^{-1}(M)$ is an sSb^* -open. Hence, g is a sSb^* -continuous functions.

The next example indicates that the converse of Theorem 4.2 need not be true in general.

Example 4.3: Assume that $D = W = R$, and $\Lambda = \{\nabla\}$, $\theta = \{\varrho\}$ and γ be the indiscrete topology on R and W be the discrete topology on R . Identify $u: D \rightarrow W$ and $h: \Lambda \rightarrow \theta$ by $g(o) = o$ and $h(\nabla) = \varrho$ for all $o \in D$, and mapping $(D, T, \Lambda) \rightarrow (W, W, \theta)$ is a soft mapping. Therefore, g is a sSb^* -continuous but g is not a soft continuous.

Theorem 4.4: Every sSb^* -continuous function is sb -continuous function.

Proof. Suppose $g: D \rightarrow W$ be a sSb^* -continuous, O be a soft open. Therefore, the inverse image $g^{-1}(O)$ is sSb^* -open in D . For each sSb^* -open is sb -open set, and for that $g^{-1}(O)$ is sb -open. Hence, g is sb -continuous.

The next example indicates that the converse of Theorem 4.4 need not be true in general.

Example 4.5: Consider $D = \{o_1, o_2, o_3\}$, $W = \{y_1, y_2, y_3\}$, $\Lambda = \{\nabla_1, \nabla_2, \nabla_3\}$ and $\theta = \{\varrho_1, \varrho_2, \varrho_3\}$. On considering that (D, T, Λ) and (W, I, θ) be STS identify $u: D \rightarrow W$ and $h: \Lambda \rightarrow \theta$ as

$$u(o_1) = \{y_2\}, \quad u(o_2) = \{y_1\}, \quad u(o_3) = \{y_3\},$$

$$h(\nabla_1) = \{\varrho_2\}, \quad h(\nabla_2) = \{\varrho_1\}, \quad h(\nabla_3) = \{\varrho_3\}.$$

Consider the STS given in Example 2.3.

$$T = \{\tilde{\emptyset}, \tilde{D}, M_1, M_2, M_3, \dots, M_{15}\},$$

$$I = \{\tilde{\emptyset}, \tilde{W}, O\}; \text{ where } O = \{(\varrho_1, \{y_1, y_2\}), (\varrho_2, \{y_3\}), (\varrho_3, W)\},$$

and mapping $(D, T, \Lambda) \rightarrow (W, I, \theta)$ is a soft mapping. Therefore O is soft open set in W ; $g^{-1}(O) = \{(\nabla_1, \{o_3\}), (\nabla_2, \{o_1, o_2\}), (\nabla_3, D)\}$ is sb -open but is not sSb^* -open. Consequently, g is sb -continuous which is not an sSb^* -continuous.

Theorem 4.6: Let $g: D \rightarrow W$ be a mapping from an STS D to STS W . Therefore, the following statements are equivalent.

- (i) g is an sSb^* -continuous,
- (ii) The inverse image of every soft open set in W is an sSb^* -open in D .

Proof.

(i) \Rightarrow (ii) Let $g: D \rightarrow W$ is an sSb^* -continuous and M be a soft open set in W . So, M^c is soft closed in W , then, $g^{-1}(M^c)$ is sSb^* -closed in D . But $g^{-1}(M^c) = D - g^{-1}(M)$. Therefore, $D - g^{-1}(M)$ is sSb^* -closed, and for that $g^{-1}(M)$ is an sSb^* -open in D .

(ii) \Rightarrow (i) Suppose that the inverse image of each soft open set in W is sSb^* -open in D . Let O be any soft closed set in W . Therefore $g^{-1}(O^c)$ is an sSb^* -open in D , i.e., $D - g^{-1}(O)$ is an sSb^* -open and for that $g^{-1}(O)$ is an sSb^* -closed set in D . Hence, g is an sSb^* -continuous.

Theorem 4.7: Every $s\alpha$ -continuous is sSb^* -continuous.

Proof. Let $g: D \rightarrow W$ be an $s\alpha$ –continuous function, M be a soft open in W . Therefore, the inverse image $g^{-1}(M)$ is $s\alpha$ –open in D . For every soft α –open set is sSb^* –open set, and for that $g^{-1}(M)$ is an sSb^* –open set in D . Hence, g is an sSb^* –continuous.

The following example indicate that the converse of the Theorem 4.7 need not be true in general.

Example 4.8: Consider $W = U = R$, and $\Lambda = \{\mathbb{V}\}$, $\theta = \{\varrho\}$. Let J be the indiscrete topology on R and K be the usual topology on R . Identify $u: W \rightarrow U$ and $h: \Lambda \rightarrow \theta$ by $g(o) = o$ and $h(\mathbb{V}) = \varrho$ for all $o \in W$. and mapping $(W, J, \Lambda) \rightarrow (U, K, \theta)$ is a soft mapping. Then, g is a sSb^* –continuous which is not an $s\alpha$ –continuous.

Theorem 4.9: A map $g: D \rightarrow W$ is sSb^* –continuous if and only if the inverse image of every soft closed set in W is sSb^* –closed in D .

Proof. Assume that M be a soft closed in W . Then M^c is a soft open in W . For g is sSb^* –continuous, $g^{-1}(M^c)$ is an sSb^* –open in D , but $g^{-1}(M^c) = D - g^{-1}(M)$ and for that $g^{-1}(M)$ is an sSb^* –closed in D . Conversely, suppose that the inverse image of every soft closed set in W is an sSb^* –closed in D . Let O be a soft open set in W , then O^c is a soft closed in W . By hypothesis, $g^{-1}(O^c) = D - g^{-1}(O)$ is an sSb^* –closed set in D and for that $g^{-1}(O)$ is an sSb^* –open in D . Hence, g is a sSb^* –continuous.

Theorem 4.10: Every $s\omega$ –continuous is sSb^* –continuous.

Proof. Let $g: D \rightarrow W$ is an $s\omega$ –continuous, O be a soft open in W . For g is an $s\omega$ –continuous, $g^{-1}(O)$ is an $s\omega$ –open and consequently, it is an sSb^* –open in D . Therefore, g is an sSb^* –continuous.

Theorem 4.11: Every sSb^* –continuous function is $s\beta$ –continuous.

Proof. Let g be an sSb^* –continuous, M be any soft open in W . Then, the inverse image $g^{-1}(M)$ is an sSb^* –open in D . Since every sSb^* –open set is an $s\beta$ –open set, and for that $g^{-1}(M)$ is an $s\beta$ –open set in D . Thus, g is an $s\beta$ –continuous.

The next example, indicate that the converse of Theorem 4.11 need not be true in general.

Example 4.12: Consider $D = \{o_1, o_2, o_3\}$, $W = \{y_1, y_2, y_3\}$, $\Lambda = \{\mathbb{V}_1, \mathbb{V}_2, \mathbb{V}_3\}$ and $\theta = \{\varrho_1, \varrho_2, \varrho_3\}$. On considering that (D, T, Λ) and (W, \mathbb{U}, θ) be STS , identify $u: D \rightarrow W$ and $h: \Lambda \rightarrow \theta$ as

$$u(o_1) = \{y_1\}, \quad u(o_2) = \{y_3\}, \quad u(o_3) = \{y_2\},$$

$$h(\mathbb{V}_1) = \{\varrho_1\}, \quad h(\mathbb{V}_2) = \{\varrho_2\}, \quad h(\mathbb{V}_3) = \{\varrho_3\}.$$

Consider the STS described in Example 2.3.

$$T = \{\tilde{\emptyset}, \tilde{D}, M_1, M_2, M_3, \dots, M_{15}\},$$

$$\mathbb{U} = \{\tilde{\emptyset}, \tilde{W}, O\}; \text{ where } O = \{(\varrho_1, W), (\varrho_2, \{y_2\}), (\varrho_3, \{y_1, y_2\})\},$$

and mapping $(D, T, \Lambda) \rightarrow (W, \mathbb{U}, \theta)$ is a soft mapping. Therefore O is a soft open set in W ; $g^{-1}(O) = \{(\mathbb{V}_1, D), (\mathbb{V}_2, \{o_3\}), (\mathbb{V}_3, \{o_1, o_3\})\}$ is an $s\beta$ –open set which is not an sSb^* –open. Consequently, g is an $s\beta$ –continuous which is not an sSb^* –continuous.

Definition 4.13: A mapping $g: D \rightarrow W$ is stated to become soft strongly \mathfrak{b}^* -irresolute (briefly $s\mathfrak{Sb}^*$ -irresolute) if the inverse image of each $s\mathfrak{Sb}^*$ -closed set in W is an $s\mathfrak{Sb}^*$ -closed set in D .

Theorem 4.14: A map $g: D \rightarrow W$ is an $s\mathfrak{Sb}^*$ -irresolute if and only if the inverse image of each $s\mathfrak{Sb}^*$ -open set in W is an $s\mathfrak{Sb}^*$ -open in D .

Theorem 4.15: Every $s\mathfrak{Sb}^*$ -irresolute mapping is $s\mathfrak{Sb}^*$ -continuous functions.

Proof. Suppose that $h: D \rightarrow W$ be an $s\mathfrak{Sb}^*$ -irresolute mapping, M is a soft closed set in W . Therefore M is an $s\mathfrak{Sb}^*$ -closed set in W . For h is an $s\mathfrak{Sb}^*$ -irresolute mapping, $h^{-1}(M)$ is an $s\mathfrak{Sb}^*$ -closed set in D . Consequently, h is an $s\mathfrak{Sb}^*$ -continuous function.

Definition 4.16: A mapping $g: D \rightarrow W$ is stated to become $s\mathfrak{Sb}^*$ -open ($s\mathfrak{Sb}^*$ -closed) map if the image of every soft open (soft closed) set in D is $s\mathfrak{Sb}^*$ -open ($s\mathfrak{Sb}^*$ -closed) set in W .

Theorem 4.17: Every soft open map is $s\mathfrak{Sb}^*$ -open map.

Proof. Let $g: D \rightarrow W$ be soft open map, O be soft open set in D . Then $g(O)$ is soft open and consequently, an $s\mathfrak{Sb}^*$ -open in W . Thus, g is an $s\mathfrak{Sb}^*$ -open map.

The following example indicate that the converse of Theorem 4.17 need not be true in general.

Example 4.18: Consider $D = \{o_1, o_2\}$, $W = \{y_1, y_2\}$, $\Lambda = \{\nabla_1, \nabla_2\}$ and $\theta = \{\varrho_1, \varrho_2\}$ and let (D, T, Λ) and $(W, \mathfrak{U}, \theta)$ be \mathcal{STS} , define $u: D \rightarrow W$ and $h: \Lambda \rightarrow \theta$ as

$$u(o_1) = \{y_2\}, \quad u(o_2) = \{y_1\},$$

$$h(\nabla_1) = \{\varrho_1\}, \quad h(\nabla_2) = \{\varrho_2\}.$$

Consider the \mathcal{STS} $T = \{\tilde{\emptyset}, \tilde{D}, M_1, M_2, M_3, M_4\}$ and

$$M_1 = \{(\nabla_1, \{o_1\}), (\nabla_2, \{o_2\})\},$$

$$M_2 = \{(\nabla_1, D), (\nabla_2, \emptyset)\},$$

$$M_3 = \{(\nabla_1, \{o_1\}), (\nabla_2, \emptyset)\} \text{ and}$$

$$M_4 = \{(\nabla_1, D), (\nabla_2, \{o_2\})\}.$$

Let $\mathfrak{U} = \{\tilde{\emptyset}, \tilde{W}, O\}$; $O = \{(\varrho_1, W), (\varrho_2, \{y_2\})\}$, and be mapping $g: (D, T, \Lambda) \rightarrow (W, \mathfrak{U}, \theta)$ be soft mapping. Therefore g is an $s\mathfrak{Sb}^*$ -open map which is not a soft open map as the image of the soft open set

$$M_1 = \{(\nabla_1, \{o_1\}), (\nabla_2, \{o_2\})\} \text{ in } D \text{ is } \{(\varrho_1, \{y_2\}), (\varrho_2, \{y_1\})\} \text{ not a soft open in } W.$$

Theorem 4.19: Let $g: D \rightarrow W$ be soft mapping then each soft closed map is $s\mathfrak{Sb}^*$ -closed map.

Proof. Suppose $g: D \rightarrow W$ is a soft closed map, O be a soft closed set in D . then $g(O)$ is a soft closed and consequently is an $s\mathfrak{Sb}^*$ -closed in W . Therefore, g is an $s\mathfrak{Sb}^*$ -closed.

The next example, indicate that the converse of Theorem 4.19 need not be true in general.

Example 4.20: Consider $D = \{o_1, o_2, o_3\}$, $W = \{y_1, y_2, y_3\}$, $\Lambda = \{\nabla_1, \nabla_2\}$ and $\theta = \{\varrho_1, \varrho_2\}$. On considering that (D, T, Λ) and $(W, \mathfrak{U}, \theta)$ be \mathcal{STS} are assigned $u: D \rightarrow W$ and $h: \Lambda \rightarrow \theta$ as

$$u(o_1) = \{y_2\}, \quad u(o_2) = \{y_1\}, \quad u(o_3) = \{y_3\},$$

$$h(\nabla_1) = \{e_2\}, \quad h(\nabla_2) = \{e_1\}, \quad h(\nabla_3) = \{e_3\}.$$

Let us consider the soft topology $T = \{\tilde{\emptyset}, \tilde{D}, M\}$ with

$$M = \{(\nabla_1, \{o_1, o_2\}), (\nabla_2, \{o_2, o_3\}), (\nabla_3, \{o_1, o_3\})\}.$$

and $\mathcal{U} = \{\tilde{\emptyset}, \tilde{W}, O_1, O_2, O_3\}$; with

$$O_1 = \{(e_1, \{y_1\}), (e_2, \{y_1, y_2\}), (e_3, \{y_3\})\},$$

$$O_2 = \{(e_1, \{y_1, y_3\}), (e_2, \{y_3\}), (e_3, \{y_2\})\},$$

$$O_3 = \{(e_1, \{y_1, y_3\}), (e_2, W), (e_3, \{y_2, y_3\})\}.$$

and $(D, T, \Lambda) \rightarrow (\Sigma, \mathcal{U}, \theta)$ be a soft mapping. Therefore, g is an sSb^* -closed which is not a soft closed as the image of the soft closed set $\{(\nabla_1, \{o_3\}), (\nabla_2, \{o_1\}), (\nabla_3, \{o_2\})\}$ in D is $\{(e_1, \{y_2\}), (e_2, \{y_3\}), (e_3, \{y_1\})\}$ not a soft closed in Σ .

Theorem 4.21: If $g: D \rightarrow W$ is soft closed function and $\mathfrak{z}: W \rightarrow U$ is sSb^* -closed function, then $\mathfrak{z} \circ g$ is an sSb^* -closed function.

Proof. Suppose M is a soft closed set in D . $g(M)$ is a soft closed set in W . For $\mathfrak{z}: W \rightarrow U$ is an sSb^* -closed function, $\mathfrak{z}(g(M))$ is an sSb^* -closed set in U . Therefore, $\mathfrak{z} \circ g$ is an sSb^* -closed map.

Definition 4.22: (i) A mapping $g: D \rightarrow W$ is referred to as soft strongly b^* -homeomorphism (sSb^* -homeomorphism) if g is an sSb^* -open, sSb^* -continuous bijective and g^{-1} is an sSb^* -continuous.

(ii) A property is referred to as a soft strongly b^* -topological property if the property is preserved by sSb^* -homeomorphism maps.

Theorem 4.23: Every soft homeomorphism is sSb^* -homeomorphism.

Proof. Suppose $g: D \rightarrow \Sigma$ be a soft homeomorphism. Then g and g^{-1} are soft continuous, consequently, g is bijective. By Theorem 4.2., every soft continuous is an sSb^* -continuous. Therefore, g and g^{-1} are sSb^* -homeomorphism.

Definition 4.24: A mapping $g: D \rightarrow \Sigma$ is stated to become soft g^* -homeomorphism (sg^* -homeomorphism) if g is bijective, sg^* -open and sg^* -continuous.

Theorem 4.25: Every sg^* -homeomorphism is sSb^* -homeomorphism.

Proof. Suppose $g: D \rightarrow \Sigma$ be soft g^* -homeomorphism. Then g and g^{-1} is a soft g^* -continuous, consequently g is bijective. Since every sg^* -closed is an sSb^* -closed. Therefore, every sg^* -continuous is sSb^* -continuous. Therefore, g and g^{-1} are sSb^* -homeomorphism.

Theorem 4.26: Let $g: D \rightarrow W$ be a bijective sSb^* -continuous map. Then, the following statements are equivalent.

- (i) g is sSb^* -open map,
- (ii) g is sSb^* -homeomorphism,
- (iii) g is sSb^* -closed map.

Proof.

(i) \Rightarrow (ii) Let $g: D \rightarrow W$ be a bijective sSb^* -continuous we show that g^{-1} is an sSb^* -continuous, $\square\square\square O$ be a soft closed set in D . Then $D - O$ is soft open. Since g is an sSb^* -open, $g(D - O)$ is sSb^* -open in W . Consequently, $g(O)$ is an sSb^* -closed in W , g is sSb^* -continuous. Now, $(g^{-1})^{-1}(O) = g(O)$ is an sSb^* -closed in W . Consequently, g^{-1} is an sSb^* -continuous. Therefore g and g^{-1} are sSb^* -continuous. Hence, g is an sSb^* -homeomorphism.

(ii) \Rightarrow (iii) Suppose that g is sSb^* -homeomorphism, then g is bijective sSb^* -continuous and g^{-1} is an sSb^* -continuous. Let O be an sSb^* -closed in D . Then $(g^{-1})^{-1}(O) = g(O)$ is sSb^* -closed in W . Therefore g is sSb^* -closed map.

(iii) \Rightarrow (i) Let g is sSb^* -closed map. Let M is an sSb^* -open in D . Therefore, $D - M$ is an sSb^* -closed in D . Since g is an sSb^* -closed map. $g(D - M)$ is an sSb^* -closed in W . So, $W - g(M)$ is an sSb^* -closed in W . $g(M)$ is an sSb^* -open in W . Consequently, g is an sSb^* -open map.

5. Conclusion

Through this paper, soft *strongly* b^* -closed sets and soft *strongly* b^* -continuous functions are assigned. Some of their characteristics are examined. multiple characterizations, relationships and examples are given. Furthermore, more theoretical examination is required to upgrade a generic framework for practical applications.

References

- [1] D. Molodtsov, "Soft set theory-first results," *Computers and Mathematics with Applications*, vol. 37, no. 4-5, pp. 19-31, 1999.
- [2] K. Kannan, "Soft generalized closed sets in soft topological spaces," *Journal of Theoretical and Appl. Inform. Technology*, vol. 37, no. 1, pp. 17 – 21, 2012.
- [3] I. Arockiarani and A. Arokialancy, "Generalized soft $g\beta$ -closed sets and soft $gs\beta$ -closed sets in soft topological spaces," *International Journal of Mathematical Archive*, vol. 4, no. 2, pp. 1-7, 2013.
- [4] M. Akdag and A. Ozkan, "Soft α -open sets and soft α -continuous functions," *Abstr. Anal. Appl.*, pp. 1-7, 2014.
- [5] M. Akdag and A. Ozkan, "Soft b -open sets and soft b -continuous functions," *Math Sci*, vol. 8, no. 127, pp. 1-9, 2014.
- [6] Hameed, S. Z. and A. K. & Hussein, "On Soft b_c – open Sets in Soft Topological Spaces," *Iraqi Journal of Science*, pp. 238-242, 2020.
- [7] Z. K. Raheem and S. Jasem, "nc-Sets in Topological Space," *Mustansiriyah Journal of Pure and Applied Sciences*, vol. 2, no. 1, pp. 115-123, 2024.
- [8] H. Sadiq and M. A. Khalik, "Some Results on Fuzzy ω -Local Covering Dimension Function in Fuzzy Topological Space," *Mustansiriyah Journal of Pure and Applied Sciences*, vol. 1, no. 2, pp. 11-20, 2023.

- [9] S. Z. Hameed, F. A. Ibrahim and and Essam A. El-Seidy, "On soft b^* -closed sets in soft topological space," *International Journal of Nonlinear Analysis and Applications*, vol. 12, no. 1, pp. 1235-1242, 2021.
- [10] S. Z. Hameed, A. E. Radwan and E. El-Seidy, "On soft b^* - continuous functions in soft topological spaces," *Measurement: Sensors*, vol. 27, no. 2665-9174, 2023.
- [11] S. Z. Hameed, A. E. Radwan and E. El-Seidy, "On Soft Strongly b^* -Separation Axioms," *Iraqi Journal of Science*, vol. 65, no. 6, pp. 3259-3269, 2024.
- [12] S. Z. Hameed, A. E. Radwan and E. El-Seidy, "On Soft Strongly b^* -Compactness and Soft Strongly b^* -Connectedness in Soft Topological Spaces," *Scalable Computing: Practice and Experience*, vol. 25, no. 5, pp. 4335-4342, 2024.
- [13] A. Poongothai and R. Parimelazhagan, " sb^* -closed sets in Topological spaces," *Int. Journal of Math. Analysis*, vol. 6, no. 47, pp. 2325 - 2333, 2012.
- [14] A. Poongothai and R. Parimelazhagan, "Strongly b^* - Continuous functions in topological spaces," *Int. Journal of Computer Applications*, vol. 58, no. 14, pp. 8-11, 2012.
- [15] N. Gomathi, "On Sb^* homeomorphism in topological spaces," *International Journal of Innovative Research in Science, Engineering and Technology*, vol. 5, 2016.
- [16] P. K. Maji, R. Biswas and A. R. Roy, "Soft set theory," *Comput. Math. Appl.*, vol. 45, pp. 555-562, 2003.
- [17] M. Shabir and M. Naz, "On soft topological spaces," *Comput. Math. Appl.*, vol. 61, pp. 1786-1799, 2011.
- [18] B. Chen, "Soft Semi-open sets and related properties in soft topological spaces," *Applied Mathematics and Information Sciences*, vol. 7, no. 1, pp. 287-294, 2013.
- [19] K. Abd-Rabou, I. K. Hafka, K. Barakat and M. El sayed, "Soft $*b$ - open and soft $*b$ - continuous functions," *Mathematical Sciences Letters*, vol. 9, no. 3, pp. 83-88, 2020.
- [20] A. Devika and L. E. Mary, "Soft g^* -closed sets in Soft Topological Spaces," *International Journal of Mathematics Trends and Technology (IJMTT)*, vol. 18, no. 1, pp. 32-39, 2015.
- [21] N. R. Paul, "Remarks on soft ω - closed sets in soft topological spaces," *Boletim da Sociedade Paranaense de Matemática*, vol. 33, no. 1, pp. 183-192, 2015.
- [22] A. K. Kaymakc and Y. Yumak, "Soft β -open sets and their application," *journal of New Theory*, no. 4, pp. 80-89, 2018.
- [23] M. Akdag, A. Ozkan and A., "Soft b -open sets and soft b -continuous functions," *Math Sci*, vol. 8, no. 127, pp. 1-9, 2014.
- [24] Hameed, S. Z., F. A. Ibrahim and and Essam A. El-Seidy, "On soft b^* -closed sets in soft topological space," *International Journal of Nonlinear Analysis and Applications*, vol. 12, no. 1, pp. 1235-1242, 2021.
- [25] Hameed, S. Z., A. E. Radwan and E. A. El-Seidy, "On soft b^* - continuous functions in soft topological spaces," *Measurement: Sensors*, vol. 27, pp. 1-5, 2023.