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Bayesian composite quantile regression for longitudinal count data

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Abstract*:* This paper develops a composite quantile regression framework for the analysis of longitudinal count data. Unlike classical regression approaches that assume a continuous response, the proposed method explicitly accounts for the discrete nature of count outcomes while preserving their inherent smooth structure. The model is constructed using a flexible representation based on a growing mixture of asymmetric distributions, which allows the conditional distribution of the response variable to be captured across multiple quantiles. To facilitate Bayesian inference, a structured Gibbs sampling algorithm is derived for parameter estimation. The performance of the proposed approach is carefully evaluated through extensive simulation studies, demonstrating its robustness and efficiency under various data-generating scenarios. Furthermore, the methodology is applied to a real dataset from the field of neurology, illustrating its practical relevance and interpretability. Comparative analysis with existing models highlights the advantages of the proposed composite quantile regression approach for longitudinal count data.

Keywords: Asymmetric Laplace, Gibbs sampling, longitudinal count data, Markov chain, Monte Carlo, Poisson process, quantile regression.

Introduction: Composite Quantile Regression (CQR), proposed by Zou and Yuan (2008), describes the relationship between the predictor variables and multiple conditional quantiles of the response variable. It represents an important alternative to classical regression methods and extends the traditional single-quantile regression (QR) framework. By combining information from several quantiles, CQR achieves more efficient and reliable estimation compared to standard quantile regression (Koenker and Bassett, 1978; Barrodale and Roberts, 1973; Koenker and d'Orey, 1987). Consequently, CQR demonstrates greater robustness, particularly in the presence of contaminated or non-normal error distributions.

Suppose that we have a sample $(\boldsymbol{y}, X)$ where $\boldsymbol{y} = (y_1, \cdots, y_n)'$ denoted the outcome of interest and $X = (\boldsymbol{x}_1, \cdots, \boldsymbol{x}_n)'$ denoted the corresponding covariate matrix. Then, the conditional θ_k the quantile $(\theta_k = k/(K+1), k = 1, \cdots, K)$ of $Q_{y_i}(\theta_k \mid \boldsymbol{x}_i) = b_{\theta_k} + \boldsymbol{x}_i'\boldsymbol{\beta}$, where b_{θ_k} is the quantile intercept and $\boldsymbol{\beta} = (\beta_1, \cdots, \beta_p)'$.The CQR estimators of $\boldsymbol{b}_\theta = (b_{\theta_1}, \cdots, b_{\theta_K})$ and $\boldsymbol{\beta}$ can be estimated by minimizing (Zou and Yuna,2008)

$$\min_{b_{\theta_1},\cdots,b_{\theta_K},\boldsymbol{\beta}} \sum_{k=1}^{K} \left\{\sum_{i=1}^{n} \rho_{\theta_k}\left(y_i - b_{\theta_k} - \boldsymbol{x}_i'\boldsymbol{\beta}\right)\right\} \qquad (1)$$

The quantile loss function $\rho_{\theta_k}(t) = 0.5|t| + (\theta_k - 0.5)t$ is not differentiable at zero (see figure 1), it isn't allowed to obtain an explicit solution to CQR problem in 1. Acquiring CQR requires applying a unique algorithm like the simplex algorithm, the smoothing algorithm, and the interior point algorithm (for example see, Bradic et al., 2011; Cannon, 2018; Guo et al., 2013; Jiang et al., 2012, 2013; Kai et al., 2010, 2011; Kong and Xia, 2014; Tang et al., 2012; Xu et al., 2017; Zhao and Xiao, 2014; Zheng et al., 2017). The large sample of CQR has been thoroughly observed and studied (Kai et al., 2010; Zou and Yuan, 2008).

The Bayesian method provides identical conclusions despite the number of examinations and monitoring being small. The use of the Bayesian method evaluates CQR modals through the asymmetric Laplace distribution already shown in papers by Huang and Chen 2015, Tian et al. 2017, Alhamzawi 2016, and Alhamzai and Alsaadi 2022.

CQR holds many advantages, one of which is the ability to work with a model that counts the outcome variable. Count models are a discrete subset outcome regression model. The count model is well-known among other models and can be used in various scientific fields, such as medicine, economics, social studies, and criminology.

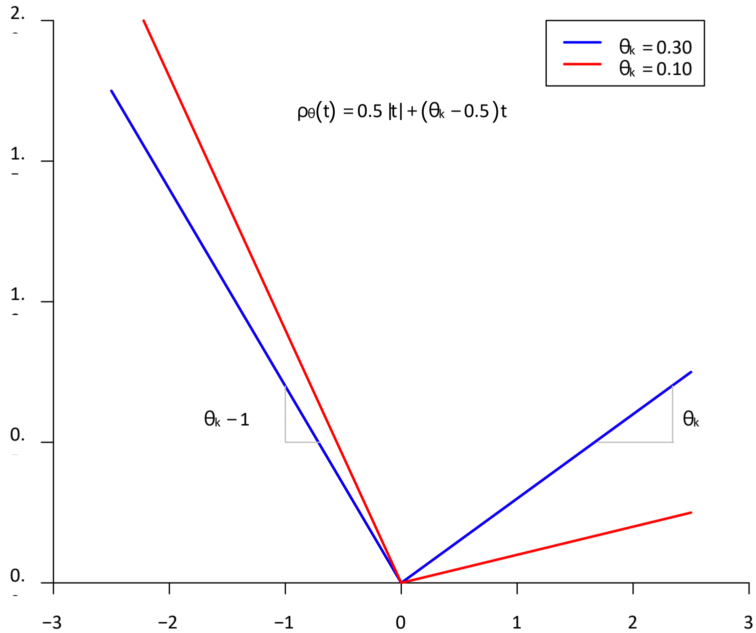

Figure (1) : Quantile regression check function.

But CQR is not so well grounded in the literature counting data. The problem comes from the identical count outcomes and the check function, and their not having continuous quantiles. Regarding the single quantile, Silva and Machado 2005 presented a smoothing technique, as the jittered technique used to solve such obstacles and also to obtain QR estimators of counts (see Silva and Machado, 2005). As for the composite quantiles, Wang et al. 2021, acquire the smoothing technique of the jittered technique and also evaluate the CQR estimator for panel count data. Through this search paper, we present the Bayesian composite quantile regression and reciprocal lasso BCQRRLasso. The manageable yet structured, the algorithm of Gibbs sampling was presented to the posterior estimation through employing the normal-exponential combination of an asymmetric-lapse distribution. The simulation examples and the real data set indicated this algorithm.

2 Methods

2.1. Model Framework

Let $(\boldsymbol{x}'_{ij}, \boldsymbol{y}_{ij})$: $(i = 1, \dots, N\ ; j = 1, \dots, n_i)$, denote longitudinal observations, where $\boldsymbol{y}_{ij}$ represents the response measured for the subject i at the time point j, and $\boldsymbol{x}'_{ij}$ is a $k \times 1$ covariate vector corresponding to the design matrix X_i. The responses are assumed to arise from a Poisson counting process, so that $\boldsymbol{y}_{ij}$ follows a Poisson distribution, as motivated by the framework of (Jorgenson DW, 1961). For such longitudinal count data, a Poisson linear model is commonly employed to describe the relationship between the expected response and the associated covariates. The general formulation of the Poisson longitudinal model is given by:

$$Y_{ij} \sim poisson\,(\mu_{ij}),$$
$$\mu_{ij} = exp\{\boldsymbol{x}'_{ij}\boldsymbol{\beta} + \omega'_{ij}\varphi_i\}$$

The variable formulation indicates the longitudinal count data model

$$z_{ij} = b_{\theta_k} + \boldsymbol{x}'_{ij}\boldsymbol{\beta} + \omega'_{ij}\varphi_i + \epsilon_{ij}, \qquad i = 1, \dots, N, j = 1, \dots, n_i \qquad (2)$$

z_{ij} indicates jth the latent variable of the ith individual, β is $k \times 1$ I' to an untold number of fixed effects parameters,ω'_{ij} is a row vector 1× l aim line of undetermined parameters effects, when φ_i is $l \times 1$ vector of unknown random-effects parameters and ϵ_{ij} represents the flaws which are thought to be i.i.d then following ALD (0,σ, θ) the latent variable θ_k can compose and formulate the z_{ij} quantile level linear mixed quantile functions, as follows:

$$Q_{z_{ij}}\left(\theta_k \middle| \boldsymbol{x}'_{ij}, \omega'_{ij}\right) = b_{\theta_k} + \boldsymbol{x}'_{ij}\boldsymbol{\beta} + \omega'_{ij}\varphi_i \qquad (3)$$

In the expression mentioned above, the latent variables are unobserved, while the count variables are observed. Conditions were sufficiently supplied for the asymptotic inference of the parameters of the $Q_{y_i}(\theta_k \mid \boldsymbol{x}_i)$. The positive support and continuity of the pdf of the $\boldsymbol{y}$ conditional on the $\boldsymbol{x}$ are conditions of concern. The counting process with its non-negative numeric produced Y, and that did not meet both of the conditions. Fluctuating variable Y and uniform noise to generate a continuous variable can solve this problem. The fluctuating responses,$\boldsymbol{y}^*_{ij} = \boldsymbol{y}_{ij} + u_{ij}$, where $u_{ij} \sim unif(0,1)$, can be modeled and have valued quantiles like

$$Q_{y^*_{ij}}\left(\theta \middle| x_{ij}, \omega_{ij}\right) = \theta_k + exp\left\{b_{\theta_k} + \boldsymbol{x}'_{ij}\boldsymbol{\beta} + \omega'_{ij}\varphi_i\right\} \qquad (4)$$

Both models 3 and 4 can be acquired due to them being equal, using a monotonic transformation.

$$z_{ij} = \begin{Bmatrix} \ln\left(y^*_{ij} \_ \theta_k\right), & for\ \boldsymbol{y}^*_{ij} > \theta_k \\ \ln(\zeta) & for\ \boldsymbol{y}^*_{ij} \le \theta_k \end{Bmatrix} \qquad (5)$$

Equation 5 can be written as the above due to ζ being a small positive number, and we can take ζ$= 10^{-5}$as

$$Q_{z_{ij}}\left(\theta \middle| \boldsymbol{x}_{ij}, \omega_{ij}\right) = \ln\left\{Q_{y^*_{ij}}\left(\theta \middle| \boldsymbol{x}_{ij}, \omega_{ij}\right) - \theta_k\right\}$$

Considering the different responses presented through irregular count responses of inconsistent jittering variables, the model was applied to the M jittering data $Y + U^{(h)}$, to $h = 1, \dots, M$.The parameters of the underlying regression were calculated as follows:

$$\widehat{\boldsymbol{\beta}}_P = \sum_{h=1}^{M} \widetilde{\boldsymbol{\beta}}_p^{(h)}$$

the $\tilde{\beta}_p^{(h)}$ is a calculated acquisition of the simulation of the jittered response data $Y + U^{(h)}$. It is possible to estimate the credible internal of regular calculations by averaging the credible intervals of M estimates as a substitute for deriving the expression of the asymptotic covariance matrix, as specified in the estimated conditional quantile function, as given below $\hat{Q}_{y_{ij}}\left(\theta \middle| \boldsymbol{x}_{ij}, \omega_{ij}\right) = \left\lceil \hat{Q}_{y^*_{ij}}\left(\theta \middle| \boldsymbol{x}_{ij}, \omega_{ij}\right) - 1 \right\rceil = \left\lceil \theta_k + exp\left\{b_{\theta_k} + \boldsymbol{x}'_{ij}\boldsymbol{\beta} + \omega'_{ij}\varphi_i - 1\right\} \right\rceil$

If the discretization schemeis $\lceil . \rceil$, It is possible to provide the complete set of the latent variables z_{ij}. The condition of α_i for $i = 1, \dots, N$ and $j = 1, \dots, n_i$, supposing they are independently and identically distributed in accord to ALD $\left(\boldsymbol{x}'_{ij}\boldsymbol{\beta} + \omega'_{ij}\varphi_i, \sigma, \theta\right)$.

$$f_z\left(z_{ij} \middle| \beta, \varphi_i, \sigma\right) = \frac{\theta_k(1-\theta_k)}{\sigma_k} exp\left\{-p_{\theta_k}\left(\frac{z_{ij} - \boldsymbol{x}'_{ij}\boldsymbol{\beta} - \omega'_{ij}\varphi_i}{\sigma_k}\right\}\right.$$

The conditional quantile is calculated, θ , As a stable and familiar in advance, the causes presented induce auto correction, among the observations on the same subject.
Supposing $\varphi_i \overset{idd}{\sim} f_\varphi(\varphi_i|\sum)$ and ϵ_{ij} to be on independent terms, thus φ_i are also independent.
If $z_i = ((z_{i1}, \dots, z_{in_i})$ and $f_z(z_i|\boldsymbol{\beta}, \varphi_i, \sigma) = \prod_{j=1}^{n_i} f_z(z_{ij}|\boldsymbol{\beta}, \varphi_i, \sigma)$ are both the latent responses are the conditional density on i^{th} subject conditional on the effect φ_i. The density of the complete latent data of $(z_i, \varphi_i.)$, as for $= 1, \dots, N,$, is provided through:

$$f(z_i, \varphi_i|\boldsymbol{\beta}, \sigma, \sum)) = f_z(z_i|\boldsymbol{\beta}, \varphi_i, \sigma) f_\varphi(\varphi_i|\sum)$$

If $\boldsymbol{z} = (z_1, \dots, z_N)$ and $\boldsymbol{\varphi} = (\varphi_1, \dots, \varphi_N)$ the joint latent data density of $(z, \varphi)$for N individuals is as it follows:

$$f(z, \varphi|\beta, \sigma, \sum) = \prod_{i=1}^{N} f_z(z_i|\beta, \varphi_i, \sigma) f_\varphi(\varphi_i|\sum) \qquad (6)$$

2.2 The Hierarchical Model
Presenting AL after utilizing a normal exponential mixture, presuming the priors $\beta \sim \pi(\beta), \sigma \sim \pi(\sigma), \sum \sim \pi(\sum)$, we will achieve the next model of hierarchical Bayesian quantile regression:

$$\begin{aligned} & z_{ij} = b_{\theta_k} + \boldsymbol{x}'_{ij}\boldsymbol{\beta} + \omega'_{ij}\varphi_i + \delta\alpha_{ij} + \tau\sqrt{\sigma\alpha_{ij}u_{ij}} \\ & \alpha_{ij} \sim \varepsilon\left(\frac{1}{\sigma}\right), \qquad u_{ij} \sim N(0,1), \\ & \qquad \beta \sim \pi(\beta), \qquad \sigma \sim \pi\,(\sigma) \\ & \varphi_i|\sum \sim f_\varphi(\varphi_i|\sum), \qquad \sum \sim \pi(\sum). \end{aligned} \qquad (7)$$

To acquire the posterior distribution of the parameters, we can apply the Bayes theorem

$$f(\beta, \sigma, \sum, \varphi|z) \propto f(z, \varphi|\beta, \sigma, \sum)\pi(\beta)\pi\,(\sigma)\pi(\sum),$$

Our interest has been the conclusion, considering the fixed effects parameters $\boldsymbol{\beta}$, and that can be achieved through acquiring marginal distribution. The parameters $\sigma, \sum, \varphi$ can be combined from $f(\beta, \sigma, \sum, \varphi|z)$ like the following :

$$f(\boldsymbol{\beta}|z) = \int \dots \int f(z, \varphi|\boldsymbol{\beta}, \sigma, \sum)\, d_\varphi\, d_\sigma\, d\sum.$$

Selecting the suitable preceding distribution for every parameter is the following step in the calculation. Cautious must be taken when choosing priors, as problems might occur that will sabotage the calculations and inference process. The reciprocal independent random effects parameter, the normal prior $N(0, \emptyset^2 I)$ is taken. The prior of φ_i holds an interpretation provided, the normal priors imply l_2 and l_1 penalty selecting an AL distribution as prior could press penlization. The prior could be implied to be $N(0, \sum)$. However, assuming the layout could have additional parameters to be calculated immoderate parameters. The prior of $\emptyset^2$, is considered an Inverse Gamma (IG) distribution, along with shape parameter q_1 and scale parameter q_2

$$\varphi_i \sim N(0, \emptyset^2 I), \qquad \emptyset^2 \sim IG(q_1, q_2)$$

In the fixed effects case β, it is reasonable choice to calculate the normal prior of a zero mean leads. The contrasted size of the fixed effects might lead to the poor performance of the prior. As an alternative, the lapse prior of the fixed effect will be used

$$\pi(\beta|\lambda) = \prod_{t=1}^{k} \frac{\lambda}{2} \exp\,(-\lambda|\beta_t), \quad \lambda \geq 0.$$

The Laplace prior is an induction to the ridge prior, that is an alterative to Lasso model, which can be generated as the below

$$\prod_{t=1}^{k} \frac{\lambda}{2} \exp(-\lambda|\beta_t) = \prod_{t=1}^{k} \int_0^{\infty} N(\beta_t; 0, c_t^2)\, \varepsilon\left(c_t^2; \frac{\lambda^2}{2}\right) dc_t;$$
$$c_t^2 \sim \varepsilon\left(\frac{\lambda^2}{2}\right), \quad \lambda^2 \sim Gamma\,(a_1, a_2).$$

An Inverse Gamma prior is set by the scale parameter, $\sigma \sim IG(s_1, s_1)$ this strongly update every repetition of the Gibbs sampling model algorithm. In this case it is not necessary to tune σ to acquire decent affirmation rates, like in MCMC. Sampling applying the algorithm of metropolis hastings.

The latest literature discussed the importance of the values of the parameter int the Inverse Gamma. It is university criticized to have both up to 0.01 rate and shape parameters, due to it allocating insignificant weight to smaller values of parameter. By stating that flat prior at $\sigma$and $\emptyset^2$shape parameter to -0.5, while the set parameter is set to 0, it is an irregular distribution of IG, that reserve compounding a suitable distribution of IG and unclear identity. All the parameters with latent variable z, are mentioned in the following joint posterior distribution:

$$\begin{aligned} f(\boldsymbol{\beta}, \varphi, \alpha, \sigma, \emptyset^2, c^2, \lambda^2|z) \;\propto\; & f(z|\beta, \varphi, \alpha, \sigma)\, f(\alpha|\sigma)\, f(\sigma) \\ & \times\; f(\beta|c^2)\, f(c^2|\lambda^2)\, f(\lambda^2) \\ & \times\; f(\varphi|\emptyset^2)\, f(\emptyset^2), \end{aligned}$$

If $\boldsymbol{z} = (\boldsymbol{z}_{11}, \dots, \boldsymbol{z}_N, n_N)$, $\boldsymbol{\alpha} = (\alpha_{11}, \dots, \alpha_N, n_N)$, $\boldsymbol{\beta} = (\boldsymbol{\beta}_1, \dots, \boldsymbol{\beta}_k), \boldsymbol{\varphi} = (\varphi_1, \dots, \varphi_N)\ and\ c^2 = (c_1^2, \dots, c_k^2)$ after substituting the distribution of probability function in the preceding equation, it generates as the bellow:

$$f(\boldsymbol{\beta}, \varphi, \alpha, \sigma, \emptyset^2, \lambda^2|z)$$
$$\propto \left\{ \prod_{i=1}^{N} \left\{ \prod_{k=1}^{K} \prod_{j=1}^{n_i} \left(2\pi\tau^2\sigma\alpha_{ij}\right)^{-\frac{1}{2}} exp\left[-\frac{1}{2\tau_k^2\sigma\alpha_{ij}}\left(z_{ij} - b_{\theta_k} - \boldsymbol{x}'_{ij}\boldsymbol{\beta} - \omega'_{ij}\varphi_i - \delta\alpha_{ij}\right)^2 \exp\left(-\frac{\alpha_{ij}}{\sigma}\right)\right]\right\}\right\}$$
$$\propto (2\pi\emptyset^2)^{-1/2} exp\left[-\frac{1}{2}\frac{\varphi_i'\varphi_i}{\emptyset^2}\right]$$
$$\times \frac{1}{\sigma^{(s_1+1)}} exp\left[-\frac{s_2}{\sigma}\right]$$
$$\times \frac{1}{\sigma^{(q_1+1)}} exp\left[-\frac{q_2}{\emptyset^2}\right]$$
$$\times (2\pi)^{-\frac{k}{2}} \left|D_c^{-\frac{1}{2}}\right| exp\left[-\frac{1}{2}\beta' D_c^{-\frac{1}{2}}\beta\right]$$
$$\times \left\{\prod_{h=1}^{k} exp\left[-\frac{\lambda^2}{2}c_h^2\right]\right\}$$
$$\times (\lambda^2)^{a_1 - 1} exp[-a_2\lambda^2]$$

since the posterior distribution lacks a controllable form. Bayesian inference is employed to perform Monte Carlo simulation using the Markov chain, this results in calculation of parameters. Instead of sampling every component separately, we think of within block sampling for (β, φ). They used a linear mixed effects model and sampled both fixed and random effects from their respective conditional posterior distributions. The potential correlation between the components of β and φ is taken into consideration by this block sampling of parameters. We sample β from an updated normal distribution conditional on φ , and similarly, φ is sampled from an updated normal distribution conditional on β . The generalized inverse Gaussian (GIG) distribution is updated, and the latent variable α is sampled from this distribution. The expression in equation 5 is used to produce the latent data variable . To get the latent variable z in this procedure, jittering variables from a uniform distribution are sampled at each iteration and used. Because the conditional quantiles of this variable are continuous, it may be modeled by a linear combination of covariates. The uniform jittering variables have no practical significance, and with each repetition, new values are generated while discarding the previous ones. A sample of the parameters σ and $\emptyset^2$ is taken from the most recent IG distributions. On the other hand, $c_t^2$and λ^2 are samples from updated gamma and GIG distributions, respectively. A thorough explanation of the Gibbs sampling formula.

Algorithm

(1)Sample $z|y$ according to the Equation 5

$$z_{ij} = \begin{cases} \ln\left(y^*_{ij\_}\theta_k\right), & for\ \boldsymbol{y}^*_{ij} > \theta_k \\ \ln(\zeta) & for\ \boldsymbol{y}^*_{ij} \leq \theta_k \end{cases}$$

Where, $\boldsymbol{y}^*_{ij} = \boldsymbol{y}_{ij} + u_{ij}\ and\ \ u_{ij}$ are sampled from unif(0,1);

ζ= 10^{-5} and θ is the quantile level.

(2)Sample $\alpha_{ij}|z, \beta, \varphi_i, \sigma \quad \sim \quad GIG(\frac{1}{2}, \rho_1, \rho_2)$ for $i = 1, \ldots, N, j = 1, \ldots, n_i$ where,

$$\rho_1 = \frac{(z_{ij} - \boldsymbol{x}'_{ij}\boldsymbol{\beta} + \omega'_{ij}\varphi_i)^2}{\tau^2\sigma} \qquad \text{and} \qquad \rho_2 = \frac{\delta^2}{\tau^2\sigma} + \frac{2}{\sigma}.$$

(3)Sample $\sigma|z, \alpha, \beta, \varphi, \emptyset^2 \ \sim\ IG((\tilde{s}_1, \tilde{s}_2),\ \ where,$

$$\tilde{s}_1 = \frac{n_i N}{2} + s_1\ and \quad \tilde{s}_2 = \frac{1}{2\tau^2}\sum_{i=1}^{N}\sum_{j=1}^{n_i}\frac{(z_{ij} - \boldsymbol{x}'_{ij}\boldsymbol{\beta} - \omega'_{ij}\varphi_i - \delta\alpha_{ij})^2}{\alpha_{ij}} + s_{2.}$$

(4)Sample $\boldsymbol{\beta}$ conditional upon φ from the distribution $\beta|z, \alpha, \varphi, \sigma, c^2 \sim N(\tilde{\beta}, \tilde{B})$,

$\tilde{B}^{-1} = (\sum_{i=1}^{N}\sum_{k=1}^{K}\frac{x'_i\left(D^2_{\alpha_i}\right)^{-1}x_i}{\tau^2_k n_i})\ and\ \ \tilde{\beta} = \widetilde{B}(\sum_{i=1}^{n}\sum_{k=1}^{K}\frac{x'_i\left(D^2_{\alpha_i}\right)^{-1}(z_i - \omega'_i\varphi_i - \delta\alpha_i)}{\tau^2_k n_i})$

Where,D_{α_i} is the diag ($\sqrt{\sigma\alpha_{i1}}$,…,$\sqrt{\sigma\alpha_{in_i}}$) $matrix.$

(5)Sample $c^2_h|\beta_h \ \sim\ GIG\left(\frac{1}{2}, \rho_3, \rho_4\right)\ for\ h = 1, \ldots, k,\ where, \rho_3 = \beta^2_h\ and\ \rho_4 = \lambda^2.$

(6)Sample $\lambda^2|c^2_h \ \sim\ Gama\ (\tilde{a}_1, \tilde{a}_2), where,\ \tilde{a}_1 = k + a_1\ and\ \tilde{a}_2 = \sum_{h=1}^{k}\frac{c^2_h}{2} + a_{2\,.}$

(7)Sample $\varphi\ contitional\ on\ \beta\ from\ the\ distribution\ \varphi_i|z, \alpha, \beta, \sigma, \emptyset^2\ \sim N\left(\tilde{a}, \tilde{A}\right) for\ i = 1, \ldots, N, where,\ \ k = 1, \ldots, K$

$$\tilde{A}^{-1} = \left(\frac{\omega'_i(D^2_{\alpha_i})^{-1}\omega_i}{\tau_k^{2n_i}} + \frac{1}{\emptyset^2}I_l\right)\ and\ \ \tilde{a} = \tilde{A}(\frac{\omega'_i(D^2_{\alpha_i})^{-1}(z_i - \boldsymbol{x}'_i\boldsymbol{\beta} - \delta\alpha_i)}{\tau_k^{2n_i}})$$

(8)Sample $\emptyset^2|\varphi\ \sim IG(\widetilde{q_1}, \tilde{q}_2), where, \widetilde{q_1} = \frac{n_i N}{2} + q_1\ \ and\ \tilde{q}_2 = \sum_{i=1}^{n_i}\frac{\varphi'_i\varphi_i}{2} + q_{2\,.}$

3. Simulation studies for longitudinal count data

Simulation study 1

We generate data from the following linear mixed regression model for count data

$$z_{ij} = x_{1ij}\beta_1 + x_{2ij}\beta_2 + x_{3ij}\beta_3 + x_{4ij}\beta_4 + x_{5ij}\beta_5 + \varphi_i + \varepsilon_{ij}, \qquad (8)$$

For $= 1, \ldots, N = 30$, and $j = 1, \ldots, n_i = 10$. We simulate $x_{1ij}\ x_{2ij}, x_{3ij}, x_{4ij}\ \ , x_{5ij}$ from the standard uniform distribution andφ_i from the standard normal distribution. The regression coefficients vector β is set as $\beta = (\beta_1, \beta_2, \beta_3, \beta_4, \beta_5)' = (1,1,2,2,2)'$. We consider three types of the error distribution:$N(0,1)$, the Student's t distribution with three degrees of freedom $t(3)$, and the Chi-squared distribution with three degrees of freedom$\chi^2_{(3)}$. The counts $y_{11}, \ldots, y_{Nni}$ were generated according to Poisson process. Hence, the counts were Poisson random variables withconditional mean given by

$$\mu_{ij} = \exp\left\{x_{1ij}\beta_1 + x_{2ij}\beta_2 + x_{3ij}\beta_3 + x_{4ij}\beta_4 + x_{5ij}\beta_5 + \varphi_i\right\}$$

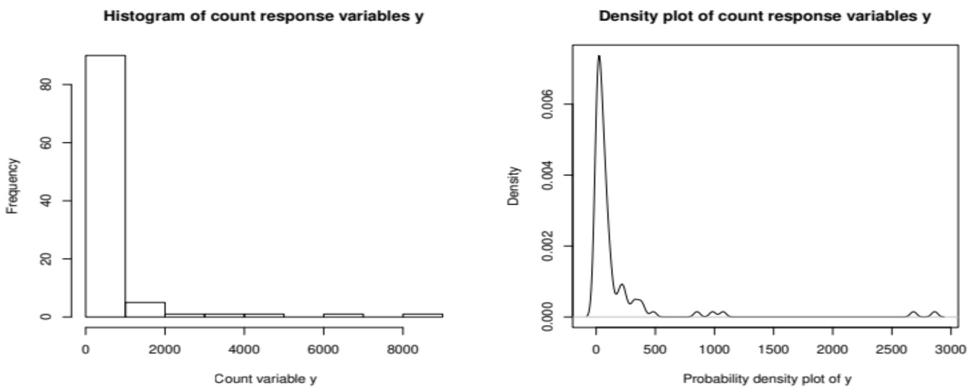

Figure 2: Plot the response count variable for simulation study 1 when the error is normal.
The histogram and the plot of the density for the count variable y is given in Figure (2). We run 100 simulations. In each simulation, we simulate a training

Table 1: Average posterior means and standard deviations for Simulation Study 1.

		Methods			
Error distribution	Parameter	True β	BCCQRLong	BQRLong	QRLong
N(0, 1)	β_1	1	0.997(0.38)	1.134(0.44)	1.115(0.14)
N(0, 1)	β_2	1	1.005(0.33)	1.119(0.46)	0.988(0.17)
N(0, 1)	β_3	2	1.945(0.31)	1.831(0.35)	2.166(0.13)
N(0, 1)	β_4	2	2.078(0.54)	2.235(0.41)	2.173(0.19)
N(0, 1)	β_5	2	2.045(0.40)	2.114(0.46)	1.863(0.11)
$t(3)$	β_1	1	0.999(0.41)	1.113(0.36)	1.026(0.14)
$t(3)$	β_2	1	1.007(0.49)	1.119(0.42)	0.967(0.15)
$t(3)$	β_3	2	1.962(0.42)	1.853(0.39)	2.185(0.21)
$t(3)$	β_4	2	2.091(0.43)	2.216(0.37)	2.156(0.19)
$t(3)$	β_5	2	2.064(0.40)	2.114(0.43)	1.853(0.11)
$\chi^2_{(3)}$	β_1	1	0.999(0.43)	1.099(0.36)	1.113(0.18)
$\chi^2_{(3)}$	β_2	1	1.009(0.46)	1.115(0.42)	0.964(0.17)
$\chi^2_{(3)}$	β_3	2	1.961(0.40)	1.879(0.39)	2.213(0.21)
$\chi^2_{(3)}$	β_4	2	2.093(0.42)	2.169(0.37)	2.221(0.17)
$\chi^2_{(3)}$	β_5	2	2.032(0.35)	2.189(0.43)	1.836(0.11)

set with $N = 30, n_i = 10$ observations and a testing set with $N = 30, n_i = 30$ observations. We compare the proposed Bayesian composite quantile regression model for longitudinal data, quantile regression with longitudinal data and Bayesian quantile regression for longitudinal data. The results are summarized in Table 1. We can observe that the regression coefficient values estimated by our method are very close to the true values. Our method provides average posterior means of the regression coefficients close to their true values.

Simulation study 2

This simulation study is similar to simulation study 1 except that the regression coefficients vector β is set as $\beta = (\beta_1, \beta_2, \beta_3, \beta_4, \beta_5)' = (0.5,1,1,2,2.5)'$. The histogram and the plot of the density for the count variable y is given in

Figure (3). The results are summarized in Table 2. We can observe that the regression coefficient values estimated by our method are very close to the true values. Our method provides average posterior means of the regression coefficients close to their true values.

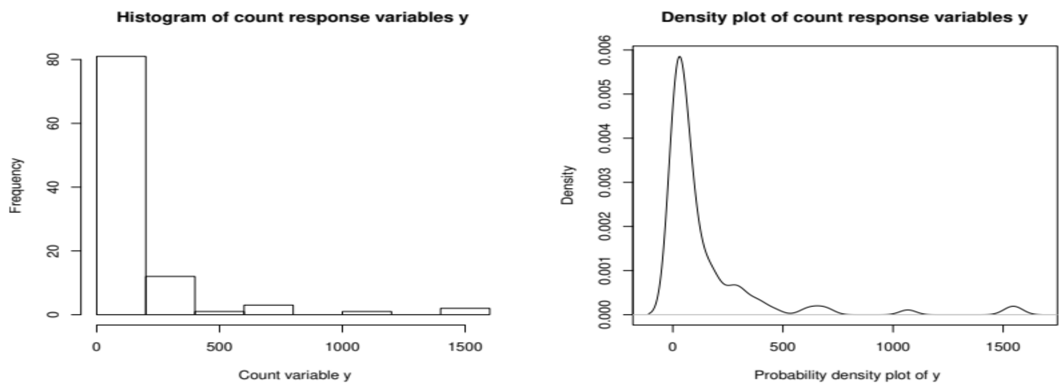

Figure 3: Plot the response count variable for simulation study 2 when the error is normal

Table 2: Average posterior means and standard deviations for Simulation Study 1 .

		Methods			
Error distribution	Parameter	True β	BCCQRLong	BQRLong	QRLong
$N(0,1)$	β_1	0.5	0.511(0.49)	0.498(0.47)	0.531(0.18)
$N(0,1)$	β_2	1	1.015(0.51)	1.198(0.53)	1.119(0.17)
$N(0,1)$	β_3	1.5	1.493(0.49)	1.611(0.53)	1.662(0.20)
$N(0,1)$	β_4	2	2.001(0.48)	2.117(0.57)	2.310(0.23)
$N(0,1)$	β_5	2.5	2.489(0.51)	2.611(0.47)	2.692(0.14)
$t(3)$	β_1	0.5	0.498(0.61)	0.623(0.42)	0.555(0.22)
$t(3)$	β_2	1	1.002(0.53)	1.125(0.51)	1.018(0.18)
$t(3)$	β_3	1.5	1.499(0.49)	1.618(0.49)	1.632(0.13)
$t(3)$	β_4	2	2.002(0.55)	2.119(0.59)	2.199(0.16)
$t(3)$	β_5	2.5	2.499(0.61)	2.378(0.50)	2.731(0.14)
$\chi^2_{(3)}$	β_1	0.5	0.495(0.44)	0.511(0.47)	0.577(0.19)
$\chi^2_{(3)}$	β_2	1	1.009(0.52)	1.211(0.53)	1.017(0.18)
$\chi^2_{(3)}$	β_3	1.5	1.499(0.47)	1.614(0.56)	1.599(0.18)
$\chi^2_{(3)}$	β_4	2	1.998(0.46)	2.119(0.59)	2.247(0.14)
$\chi^2_{(3)}$	β_5	2.5	2.511(0.43)	2.613(0.51)	2.689(0.16)

4. Kidney Failure Disease

In this paper , we dealt with a set of explanatory variables (diagnoses of specialist doctors) that were obtained from Al-Hussein educational hospital's (artificial kidney center)in the center of Dhi Qar Governorate, which is concerned with (kidney failure) disease. the condition. In this aspect, reliance was made on real data taken from Al-Hussein educational hospital's (artificial kidney center), in order to analyze the relationship between the number of times of dialysis (the dependent variable) and a set of diagnoses (independent variables) specified by the specialist doctors for the patient's condition. The total data obtained from the hospital by the researcher was 200 observations for 50 patients, with each patient having 4 observations during one month. Where the first observation represents the diagnoses (independent

variables) during the first week of the month (for the first case), and the (dependent variable) represents the number of times of dialysis during the same week (for the first patient). The second observation represents the diagnoses (independent variables) during the second week of the month, and the (dependent variable) represents the number of times of dialysis during the same week (for the same patient). The third observation represents the diagnoses (independent variables) during the third week of the month (the dependent variable) and the number of times of dialysis during the same week (for the same patient). The fourth observation represents the diagnoses (independent variables) during the fourth week of the month, and the (dependent variable) represents the number of times of dialysis during the same week (for the same patient). And so with the rest of the cases. These data were obtained from Al-Hussein educational hospital's (artificial kidney center) in Dhi Qar Governorate. A group of specialized doctors was consulted to classify the most important factors affecting the patient's condition.
Where the variables were classified as follows:
Y= The number of times of dialysis
X1: Gender (male = 1, female = 2)
X2: Cr Mg/dl
X3: Urea Mg/dl
X4: ALP U/I
X5: ALT U/I
X6: PHO Mg/dl
X7: Age (years)
X8: disease duration (days)

6. Conclusion

In this paper, we provide a Bayesian hierarchical model for composite quantile regression with a reciprocal lasso penalty. We provide a simple and effective Gibbs sampling strategy for Bayesian composite quantile regression for count data based on a mixture representation of the asymmetric Laplace distribution. Using simulated studies and renal failure disease data, we demonstrated that the proposed technique outperforms in terms of MSE.

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