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## New Optimal Formulas to Conjugate Gradient Method for Image Noise Reduction

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## RESEARCH ARTICLE

# New Optimal Formulas to Conjugate Gradient Method for Image Noise Reduction

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## ABSTRACT

A key component of conjugate gradient (CG) algorithms is the directions used in the CG approach. Because of their low memory requirements, these directed methods have proven effective in various applications, especially image processing. For this study, we have developed a new conjugate gradient coefficient based on the well-known quadratic model, that will accelerate the convergence and increase the accuracy of the CG approach. Because the new method uses the second order curvature and provides improved direction. So, the global convergence and critical descent characteristics of the final algorithm provide reliable performance in a range of scenarios. The results of extensive numerical testing on picture restoration show that the new formulae perform noticeably better than the current techniques. In particular, the performance of the suggested conjugate gradient (CG) scheme has been better than that of the conventional Fletcher-Reeves (FR) conjugate gradient approach. This development is a key tool in the field of computational imaging and optimization as it not only increases computing efficiency but also boosts picture restoration, image quality and its accuracy.

**Keywords:** Conjugate gradient methods, Image processing, Image restoration problems, Line search methods, New formula conjugate gradient, Theoretical analysis, Unconstrained optimization

## Introduction

The target of this study is to provide a range of repetitive methods for solving optimization problems that involve an aim function known as an edge preserving regularization (EPR) function. The focus of this study is to develop a family of iterative approaches to optimization problems that incorporate an objective function which preserves edges in the data i.e. (EPR) functional. An adaptive median filter (AMF) can be utilized to detect and eliminate impulse noise,<sup>1,2</sup> which uses two equations to focus pixels which are may be contaminated. The true picture is symbolized by  $X$  and  $A = \{1, 2, 3, \dots, M\} \times \{1, 2, 3, \dots, N\}$  is an index set of  $X$ . Let's define  $N \subset A$  as the collection of indices corresponding to the identified noisy pixels during the initial stage.  $P_{i,j}$  is the group of nearest neighbors of a pixel at position

$(i, j) \in A$ ,  $y_{i,j}$  is the intended value of the picture over a position  $(i, j)$ , and  $u_{i,j} = [u_{i,j}]_{(i,j) \in N}$  is a vertical vector for length of  $c$  ordered lexicographically. Where  $c$  is denoted to the number of items in  $N$ .<sup>2</sup> The 2<sup>nd</sup> equation is used to recover noise pixels by minimized the following function:

$$f_u(u) = \sum_{(i,j) \in N} \left[ |u_{i,j} - y_{i,j}| + \frac{\beta}{2} \left( 2 \times S_{i,j}^1 + S_{i,j}^2 \right) \right] \quad (1)$$

Where  $\beta$  denotes the tuning parameter for regularization, and:  $S_{i,j}^1 = 2 \sum_{(m,n) \in P_{i,j} \cap N^c} \phi_\alpha(u_{i,j} - y_{m,n})$ ,  $S_{i,j}^2 = \sum_{(m,n) \in P_{i,j} \cap N} \phi_\alpha(u_{i,j} - y_{m,n})$  where  $\phi_\alpha = \sqrt{\alpha + x^2}$ ,  $\alpha > 0$  is an example of a potential function that keeping the edges that can be used to minimize some of smooth function in order to restore noisy pixels in an optimization process. The inclusion

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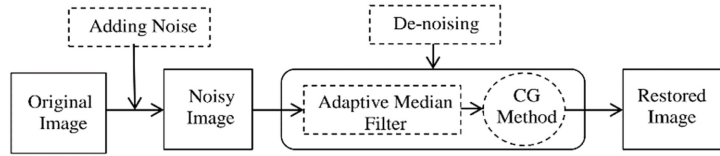


Fig. 1. Flow chart of our experiments.

of the unsmooth data fitting term is unnecessary at this stage.

$$f_{\alpha}(u) = \sum_{(i,j) \in N} [(2 \times S_{i,j}^1 + S_{i,j}^2)] \quad (2)$$

The technique used in the research work can also be explained according to the diagram shown in Fig. 1:

There are many works proposed to clean the noise see for instance the schemes proposed in.<sup>2,3</sup> One of the most significant iterative procedures for eliminating impulse noise is the conjugate gradient (CG) technique:

$$\text{Min } f_{\alpha}(u), \text{ for all } u \in R^n \quad (3)$$

$f_{\alpha}(u) : R^n \rightarrow R$  denote to the smooth function. To refresh your memory, conjugate gradient algorithms iteratively update their sequence of points using the following recursive formula:

$$u_{k+1} = u_k + \alpha_k d_k \quad (4)$$

Here  $d_k$  represents a direction for line search and  $\alpha_k$  represents the step length, both of which are typically achieved by an appropriate exact line search as:

$$\alpha_k = -\frac{g_k^T d_k}{d_k^T Q d_k} \quad (5)$$

For more See.<sup>3</sup> Typically, the step length which selected to test in a Wolfes line search requirement:

$$f(x_k + \alpha_k d_k) \leq f(x_k) + \delta \alpha_k g_k^T d_k \quad (6)$$

$$d_k^T g(x_k + \alpha_k d_k) \geq \sigma d_k^T g_k \quad (7)$$

Here  $0 < \delta < \sigma < 1$ . For more details.<sup>4</sup> Numerous authors have dedicated extensive time to studying the convergence patterns of the formulae mentioned, particularly when utilized in specific line search configurations. The conjugate gradient algorithms provide the following search axes:

$$d_{k+1} = -g_{k+1} + \beta_k d_k \quad (8)$$

where  $\beta_k$  is a Numerical value.<sup>5</sup> Take a look at the FR formula, which was first suggested by Fletcher-Reeves (FR) approach<sup>6</sup> and is written as follows:

$$\beta_k^{FR} = \frac{\|g_{k+1}\|^2}{\|g_k\|^2} \quad (9)$$

The gradient-method is a way to identify the closest local minimum of a variable function, assuming that the function's gradient can be calculated. This outcome encouraged more research on conjugate gradient techniques for recursive issues. Hence, it could effectively handle complex iterative issues on a vast scale.<sup>7</sup> In some unique circumstances for convex functions, the convergence of the Fletcher-Reeves (FR) approach has been demonstrated.

Starting with Polyak,<sup>8</sup> the FR technique has good global convergence qualities, but in practical calculations it often performs poorly. The Dai-Yuan (DY)<sup>9</sup> and Hestenes-Stiefel (HS)<sup>10</sup> methods have comparable experiences. The conjugacy constraint was added to achieve the theoretical and computational benefits of efficient CG algorithms presented in<sup>11</sup>:

$$d_{k+1}^T Q s_k = 0 \quad (10)$$

It uses an accurate line search, which is crucial for mathematical experiments and convergence analysis, the derived CG method for unconstrained optimization.<sup>12,13</sup> Conjugacy-based optimization techniques solve huge issues.

The quadratic model is being used to build new conjugate gradient parameters that will increase the accuracy and rate of convergence of the CG method. By leveraging the second-order curvature and offering modified equations, the new method may provide improved direction and step length adjustments. As a consequence, picture restoration would be more successful. Consequently, a robust optimization process is needed.

The rest of this paper is organized as follows. First, a new random parameter is given to present a conjugate gradient method, and then the global convergence of our method is proved under appropriate conditions. Then, some numerical experiments are implemented. Finally, the application of the new

method in the regression model is presented. The conclusions we have reached are presented.

### Deriving new coefficient conjugate

The new conjugate gradient coefficients are constructed by applying an alternative denominator in:

$$\beta_k = \frac{g_{k+1}^T Q s_k}{d_k^T Q s_k} \tag{11}$$

where  $Q$  is Hessian matrix which introduced by Hideaki and Yasushi in 2011.<sup>5</sup> So, by use a Taylor's formula for  $f(u)$  this obtains:

$$f(u) = f(u_{k+1}) + g_{k+1}^T (u - u_{k+1}) + 0.5 (u - u_{k+1})^T Q (u_k) (u - u_{k+1}) \tag{12}$$

Here  $Q$  denoted to Hessian matrix. Putting  $u = u_k$  and find the derivative as:

$$g_{k+1} = g_k + Q(u_k) s_k \tag{13}$$

During Eqs. (12) and (13), that's go to have:

$$s_k^T Q(u_k) s_k = 2/3 s_k^T y_k + 2/3 (f_k - f_{k+1}) \tag{14}$$

Using simple algebra, based on the Eq. (14) and  $(\alpha_k)$ , as:

$$d_k^T Q(x_k) s_k = \frac{2}{3} \frac{\alpha_k (g_k^T d_k)^2}{s_k^T y_k + 2/3 (f_{k+1} - f_k)} \tag{15}$$

Using Eq. (15) in Eq. (11), then obtain:

$$\beta_k = \frac{g_{k+1}^T y_k}{\frac{2}{3} \frac{\alpha_k (g_k^T d_k)^2}{(s_k^T y_k + 2/3 (f_{k+1} - f_k))}} \tag{16}$$

May express Eq. (16) as follows if the step-length is exact:

$$\beta_k = \frac{\|g_{k+1}\|^2}{\frac{2}{3} \frac{\alpha_k (g_k^T d_k)^2}{(s_k^T y_k + 2/3 (f_{k+1} - f_k))}} \tag{17}$$

Again, using the exact line search with Eq. (14), So the equation Eq. (17) reduces to:

$$\beta_k = \frac{\|g_{k+1}\|^2}{\frac{2}{3} \frac{\alpha_k (g_k^T d_k)^2}{(-s_k^T g_k + 2/3 (f_{k+1} - f_k))}} \tag{18}$$

And

$$\beta_k = \frac{\|g_{k+1}\|^2}{\frac{2}{3} \frac{\alpha_k (g_k^T d_k)^2}{(\alpha_k g_k^T g_k + 2/3 (f_{k+1} - f_k))}} \tag{19}$$

As a proposal result, the so-called BY, becomes obvious.

The algorithmic steps for the derived method are summarized as:

Initialization. Given  $x_0 \in R^n$ ,  $\delta \in (0, 1)$ ,  $\sigma \in (\delta, 1)$ , set  $d_0 = -g_0$  and  $k = 0$ .

Stage 1. If  $\|g_k\| \leq \epsilon$  then stop.

Stage 2. Novelty  $\alpha_k$  by 6 and 7.

Stage 3. Let  $x_{k+1} = x_k + \alpha_k d_k$ , and calculate  $\beta_k$  by 17-19.

Stage 4. Calculate  $d_{k+1} = -g_{k+1} + \beta_k d_k$ .

Stage 5. Usual  $k = k + 1$  and go to stage 1.

### Theoretical analysis

Using the following, this part will demonstrate the globally convergence of new Algorithm.

Assumptions:

(i) For any  $x \in R^n$ , then  $\Omega = \{x \in R^n / f(x) \leq f(x_1)\}$  is bounded set.

(ii)  $f$  is smooth function and there found a scalar  $L > 0$  with any  $x \in R^n$ , the gradient of  $f$  satisfy:

$$\|g(\gamma) - g(\chi)\| \leq L \|\gamma - \chi\|, \forall \gamma, \chi \in R \tag{20}$$

For more details.<sup>14-16</sup>

**Theorem 1:** If use a proposed method to construct  $\{x_k\}$  and  $\{d_k\}$ , then:

$$d_{k+1}^T g_{k+1} < 0 \quad \text{and} \quad d_{k+1}^T g_{k+1} = \beta_k d_k^T g_k \tag{21}$$

**Proof:** Clearly, since  $d_k = -g_k$  the essential  $d_1^T g_1 < 0$ . For any  $k$  be for that  $d_k^T g_k < 0$ . From Eqs. (9) and (10), that is easy to obtain:

$$\begin{aligned} d_{k+1}^T g_{k+1} &= -g_{k+1}^T g_{k+1} + \beta_k d_k^T g_{k+1} \\ &= -\beta_k \frac{2}{3} \frac{\alpha_k (g_k^T d_k)^2}{(s_k^T y_k + 2/3 (f_{k+1} - f_k))} + \beta_k d_k^T g_{k+1} \end{aligned} \tag{22}$$

which ensures:

$$d_{k+1}^T g_{k+1} = \beta_k \left[ d_k^T g_{k+1} - \frac{2}{3} \frac{\alpha_k (g_k^T d_k)^2}{(s_k^T y_k + 2/3 (f_{k+1} - f_k))} \right] \tag{23}$$

The result of adding Eqs. (17) and (22) is:

$$d_{k+1}^T g_{k+1} = \beta_k d_k^T g_k \tag{24}$$

Most definitely  $d_k^T g_k < 0$ , which result:

$$d_{k+1}^T g_{k+1} < 0 \tag{25}$$

proof is finished.

Based on the assumption, Polyak<sup>8</sup> has proven this lemma which is necessary to prove the global convergence.

**Lemma 1:** Suppose Assumption holds, let BY Algorithms, where  $d_{k+1}$  is a directional for descent, and the step length  $\alpha_k$  is obtained using the Wolfe\_conditions. Therefore.

$$\sum_{k=1}^{\infty} \frac{(g_k^T d_k)^2}{\|d_k\|^2} < \infty \tag{26}$$

**Theorem 2:** If  $\alpha_k$  the step length is calculated via relation Wolfe conditions, then:

$$\lim_{k \rightarrow \infty} \inf \|g_k\| = 0 \tag{27}$$

**Proof:** Assume by opposition formula Eq. (27) is False. Then for each k, that's will find an  $r > 0$  for which:

$$\|g_{k+1}\| > r \tag{28}$$

But from search duration as  $d_{k+1} + g_{k+1} = \beta_k d_k$  and squaring each side, this obtains:

$$\|d_{k+1}\|^2 + \|g_{k+1}\|^2 + 2d_{k+1}^T g_{k+1} = (\beta_k)^2 \|d_k\|^2 \tag{29}$$

Utilizing Eq. (24) to Eq. (29) implies:

$$\|d_{k+1}\|^2 = \frac{(d_{k+1}^T g_{k+1})^2}{(d_k^T g_k)^2} \|d_k\|^2 - 2d_{k+1}^T g_{k+1} - \|g_{k+1}\|^2 \tag{30}$$

When Eq. (30) is divided by  $(d_{k+1}^T g_{k+1})^2$ , this obtains:

$$\begin{aligned} \frac{\|d_{k+1}\|^2}{(d_{k+1}^T g_{k+1})^2} &= \frac{\|d_k\|^2}{(d_k^T g_k)^2} - \frac{\|g_{k+1}\|^2}{(d_{k+1}^T g_{k+1})^2} - \frac{2}{d_{k+1}^T g_{k+1}} \\ &\leq \frac{\|d_k\|^2}{(d_k^T g_k)^2} - \left( \frac{\|g_{k+1}\|}{(d_{k+1}^T g_{k+1})} + \frac{1}{\|g_{k+1}\|^2} \right) + \frac{1}{\|g_{k+1}\|^2} \\ &\leq \frac{\|d_k\|^2}{(d_k^T g_k)^2} + \frac{1}{\|g_{k+1}\|^2} \end{aligned} \tag{31}$$

As a result, this came to:

$$\frac{\|d_{k+1}\|^2}{(d_{k+1}^T g_{k+1})^2} \leq \sum_{i=1}^{k+1} \frac{1}{\|g_i\|^2} \tag{32}$$

Assume there is  $c_1 > 0$  such that  $\|g_i\| \geq c_1$  for each  $i \in n$  exists. Then:

$$\frac{\|d_{i+1}\|^2}{(d_{i+1}^T g_{i+1})^2} < \frac{i+1}{c_1^2} \tag{33}$$

Then finally get:

$$\sum_{k=1}^{\infty} \frac{(g_k^T d_k)^2}{\|d_k\|^2} = \infty \tag{34}$$

Likewise, by Lemma 1, it's obtained  $\lim_{k \rightarrow \infty} \inf \|g_k\| = 0$  Satisfied. As same result you get for other formulas.

### Results and discussion

The results of the numerical analysis presented in this part demonstrate that New is successful in lowering the amount of salt-and-pepper impulse noise.

**Table 1.** The results of FR, BY1, BY2 and BY3 Modes.

Picture	Noise level r (%)	FR-Mode			BY1-Mode			BY2-Mode			BY3-Mode		
		N.I.	N.F.	PSNR (dB)	N.I.	N.F.	PSNR (dB)	N.I.	N.F.	PSNR (dB)	N.I.	N.F.	PSNR (dB)
Le	50	82	153	30.5529	11	28	31.744	11	28	32.0271	11	29	32.3841
	70	81	155	27.4824	11	29	29.945	11	29	29.9836	8	21	31.4021
	90	108	211	22.8583	11	30	29.0193	9	23	28.9238	6	15	29.4954
Ho	50	52	53	30.6845	10	21	35.4017	10	24	36.2411	12	25	35.1207
	70	63	116	31.2564	8	20	34.0135	8	19	33.9268	13	29	32.3862
	90	111	214	25.287	9	24	32.8338	9	23	32.1561	6	15	32.8677
El	50	35	36	33.9129	8	18	33.4417	10	26	33.4324	11	25	32.9012
	70	38	39	31.864	7	16	32.5717	10	25	32.3061	12	27	32.3492
	90	65	114	28.2019	9	23	32.1158	8	19	31.6646	12	29	31.3845
c512	50	59	87	35.5359	14	23	35.762	10	22	36.4822	8	17	36.5513
	70	78	142	30.6259	10	24	33.0018	10	23	33.3559	10	20	32.5345
	90	121	236	24.3962	12	30	29.1896	12	29	29.8672	10	19	28.5947

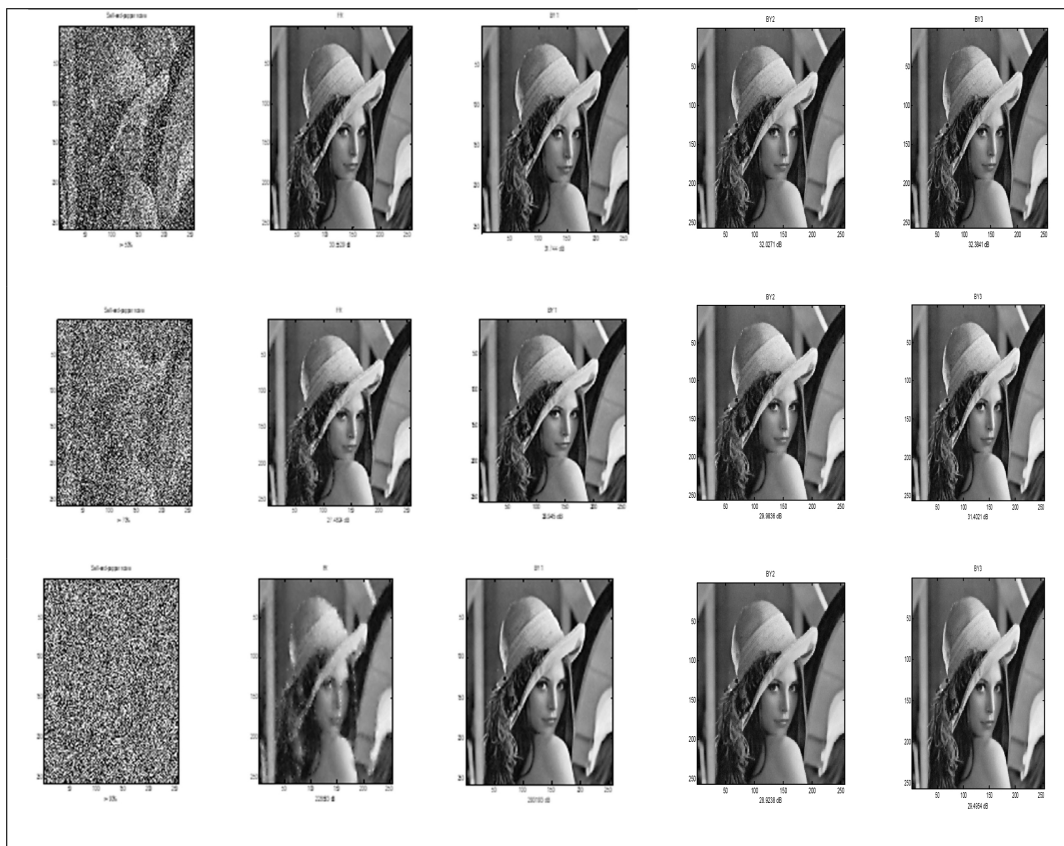


Fig. 2. Shows the outcomes of methods FR, BY1, BY2 and BY3 of  $256 \times 256$  Lena\_image.

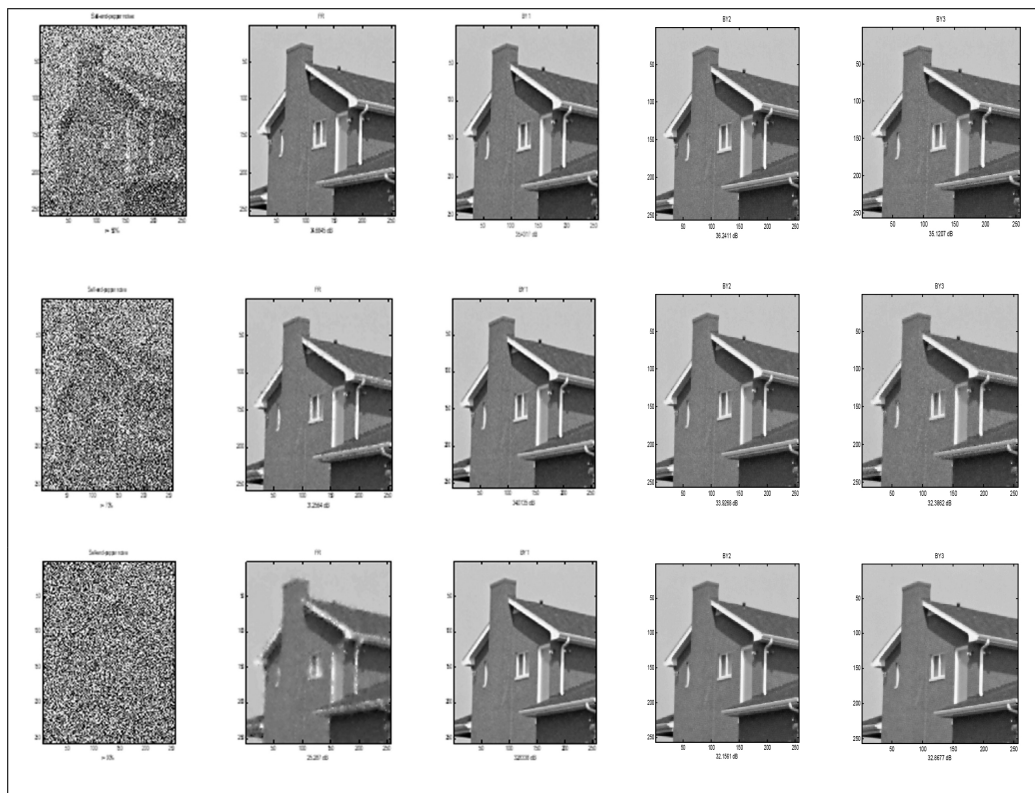


Fig. 3. Shows the outcomes of methods FR, BY1, BY2 and BY3 of  $256 \times 256$  House image.



Fig. 4. Shows the outcomes of methods FR, BY1, BY2 and BY3 of  $256 \times 256$  Elaine image.

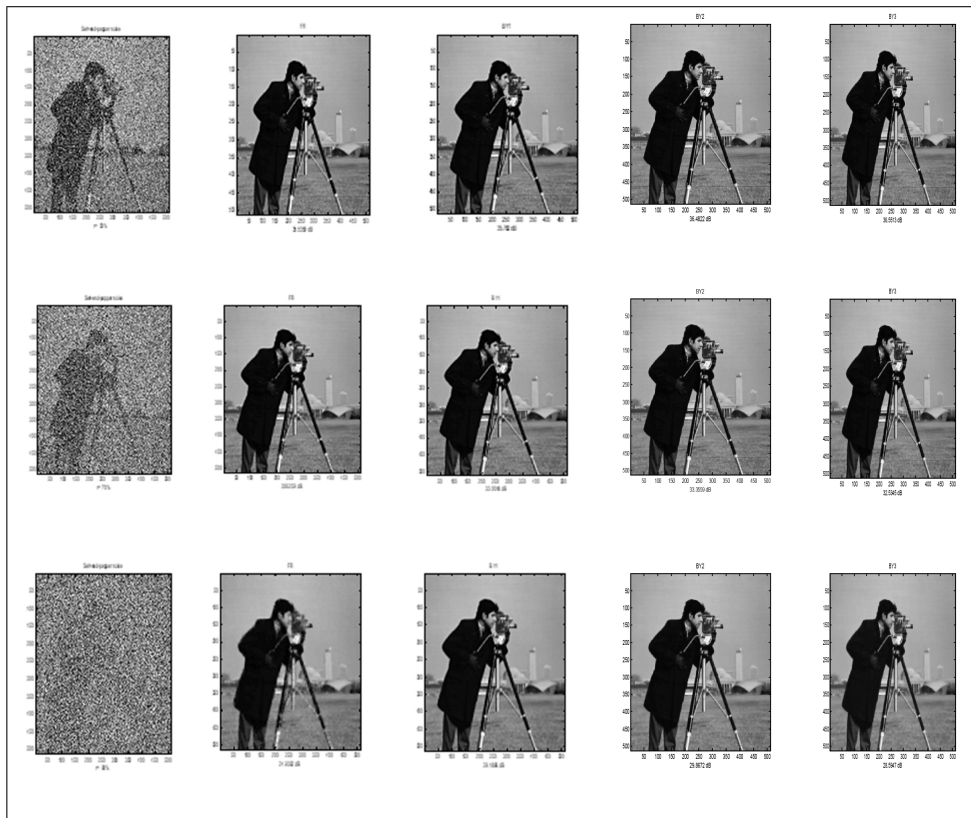


Fig. 5. Shows the outcomes of methods FR, BY1, BY2 and BY3 of  $256 \times 256$  Cameraman\_image.

In our trials, both new and previously unexplored approaches are examined. The whole process of writing and running the programs is carried out using MATLAB r2017a.<sup>17</sup> The following are the conditions for when either approach should be stopped:

$$\frac{|f(u_k) - f(u_{k-1})|}{|f(u_k)|} \leq 10^{-4}$$

$$\text{and } \|f(u_k)\| \leq 10^{-4} (1 + |f(u_k)|) \quad (35)$$

The test shots consist of Lena, Home, Cameraman, and Elaine. The PSNR, an abbreviation for peak signal-tonoise ratio, serves as a quantitative measure that can be utilized to evaluate the effectiveness of the restoration procedure:

$$PSNR = 10 \log_{10} \frac{255^2}{\frac{1}{MN} \sum_{i,j} (u_{i,j}^r - u_{i,j}^*)^2} \quad (36)$$

In this particular instance,  $u_{i,j}^r$  and  $u_{i,j}^*$  stand for the pixel values of the reconditioned picture with an original image, in a sequential manner. This study reports an PSNR of the recuperate image as well as a number of the function evaluations (NF) and iterations (NI) required for denoising procedure. Can see that in Figs. 2 to 5 also Table 1 explains that, for the great majority of the test photos, the new methodology is substantially faster than the FR method. The PSNR values generated by New and FR techniques are also comparable, as can be shown. Conclusion. When it comes to removing impulsive noise from photographs, Table 1 demonstrates that the recommended approaches perform better than the FR method about the numbers of iterations, function assessments, and maximum signal to noise ratio. For more see.<sup>18–21</sup>

Based on the number of iterations and function evaluation, the figures show that the suggested algorithms performed better than the traditional FR method. This indicates that the new approaches compete favourably with the current approach because they are ranked lower than the FR technique.

## Conclusion

Finally, three novel conjugate gradient approaches (BY1, BY2, and BY3) that suggest several choices for the conjugacy parameter have been presented, along with a new modified conjugate gradient formula. The additional options are intended to improve image processing operations, especially in applications involving picture restoration. We established the global convergence features of these novel techniques using Wolfe line search conditions. The results of

our extensive simulation tests showed that the BY1, BY2, and BY3 approaches improve computing efficiency on unconstrained optimization problems by drastically reducing the number of iterations and function evaluations needed. Furthermore, it was demonstrated that these techniques outperformed the conventional conjugate gradient method in terms of picture quality restoration.

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Compliance with Ethical Standards.

## Authors' declaration

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are ours. Furthermore, any Figures and images, that are not ours, have been included with the necessary permission for republication, which is attached to the manuscript.
- No animal studies are present in the manuscript.
- No human studies are present in the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee at University of Mosul.

## Authors' contribution statement

B.A.H. completed the mathematical derivation and scientific proofs, while M.W.T. carried out the programming part of the research and extracted the numerical results and both worked the drafting of the manuscript.

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# الصيغ الجديدة المثالية لطريقة التدرج المترافق لتقليل التشويش الصور

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## الخلاصة

يعد المكون الرئيسي في خوارزميات التدرج المترافق (CG) هو الاتجاهات المستخدمة في هذه النهج. ونظراً لمتطلبات الذاكرة المنخفضة، فقد أثبتت هذه الأساليب الموجهة فعاليتها في مجموعة واسعة من التطبيقات، خاصة في مجال معالجة الصور. في هذه الدراسة، قمنا بتطوير معامل جديد للتدرج المترافق استناداً إلى النموذج التربيعي المعروف. والذي من شأنه تسريع التقارب وزيادة دقة نهج CG. لأن الطريقة الجديدة تستخدم انحناء الدرجة الثانية وتوفر اتجاهات متحركة. لذلك، توفر خصائص التقارب الشامل وخاصة الانحدار في الخوارزمية الجديدة أداءً موثوقاً في مجموعة متنوعة من المسائل. وقد أظهرت نتائج الاختبارات العددية المكثفة في استعادة الصور أن الصيغ الجديدة تحقق أداءً أفضل بشكل ملحوظ مقارنةً بالتقنيات القياسية الحالية. على وجه الخصوص، كان أداء مخطط التدرج المترافق المقترحة أفضل من الطريقة التقليدية لنهج التدرج المترافق التقليدي فليتشر-ريفز (FR). ويشكل هذا التطور أداة أساسية في مجال التصوير الحسابي والتحسين، حيث انه لا يزيد من كفاءة الحوسبة فحسب، بل يعزز أيضاً جودة ودقة استعادة الصور.

**الكلمات المفتاحية:** التحسين غير المقيد، التحليل النظري، أساليب التدرج المترافق، طرق البحث الخطي، صيغة التدرج المترافق الجديدة، مسائل معالجة الصور، معالجة الصور.