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## Conics Through Inequivalent 5-Arcs in $PG(2, 32)$

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## RESEARCH ARTICLE

# Conics Through Inequivalent 5-Arcs in $PG(2, 32)$

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## ABSTRACT

A  $k$ -arcs is usually defined to be a set of  $k$  points in the projective plane such that some lines meets  $K$  in two points. A conic is an irreducible plane quadric curve with six terms. The first aim of the paper is to find the projectively inequivalent 5-arcs and 6-arcs in the finite projective plane over the Galois field of order two, and find the subgroups of  $PGL(3, 32)$  that are fixing these arcs. The second aim is to determine the conics form that passes through these 5-arcs and 6-arcs, and then parameterized. As an application to our results over Galois field of order two, the connection between projective linear code and arc in the finite projective plane was taken advantage determine the number of non-equivalent projective MDS linear codes of length five, dimension three and one error correcting. Also, the dual codes of these codes are introduced. Finally, the weight distribution, covering radius and number of correcting of these projective MDS linear codes and its duals are computed. The GAP programming tools were used to implement the algorithms used in this research.

**Keywords:** Arc, Conic, Finite projective space, Finite field, Group action

## Introduction

Let  $F_q$  be a Galois field of  $q = p^n$  elements ( $p$  is prime). A projective space of dimension  $n$  over a field  $F_q$  is a set of non-zero subspaces of  $F_q^{n+1}$ , points, lines, planes, ..., with rank 1, 2, 3, ..., and denoted by  $PG(n, q)$ . The dimension of a subspace of  $PG(n, q)$  is one less than the rank of a subspace of  $F_q^{n+1}$ .

In the projective plane  $PG(2, q)$  there are  $q^2 + q + 1$  points dually lines and there are  $q + 1$  points on every line dually  $q + 1$  lines through every point.

Hirschfeld gave the general structure of conic and cubic forms in the projective space, and gave its relations with arcs, but the exact forms of these conic and cubic inequivalent ones are remained, see.<sup>1</sup> Sadeh in his thesis gave the conics through 5-arcs in projective plane of order eleven,<sup>2</sup> and then found the parameterization form of these conic. Al-Zangana did the same as Sadeh over the field of order 19,23,25,27, see<sup>3</sup> and the references mentioned therein. So, the first aim of this article is to classified the 5-arcs and 6-arcs, and then to parameterize the conics that passes through

these inequivalent 5-arcs and 6-arcs in the projective plane over  $F_{32}$ . As an information, the history of classification is very rich, dating back to the middle of the last century and could mention the following sources as a sample for review.<sup>4-6</sup>

The relation between projective linear codes and arcs already known and appear in many researches that deals with both concepts.<sup>7</sup> Because of the importance of coding theory, many researchers have appeared in the research area who are interested in studying this relationship in general as well as in particular on specific fields.<sup>8-10</sup> So, the second aim is to transform the getting arcs into linear codes by given the generator matrix, weight distribution, and covering radius. Also, the by taking the advantage of the corresponding between projective line and conic, the inequivalent 5-sets on projective line are computed, see.<sup>11</sup> This research is considered a continuation of another research that we have completed on the finite projective line over  $F_2$ , which is under publication.

The GAP programming has been used to the all calculation in this paper.<sup>12</sup>

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### Definitions and basic concept

**Definition 1:** A group  $G$  acts on a set  $K$  if there is a map  $\delta : K \times G \rightarrow K$ , such that if  $e$  is the identity and  $g_1, g_2 \in G$ , then for any  $x \in K$  the action of  $G$  on  $k$  is

(i)  $\delta(x, e) = x$ ; (ii)  $\delta(\delta(x, g_1), g_2) = \delta(x, g_1g_2)$ <sup>1</sup>.

**Definition 2:** The orbit of a set  $S$  is  $SG = \{\delta(S, g), g \in G\}$  a subset of  $k$ , the stabilizer of  $S$  is  $G_S = \{g \in G, \delta(S, g) = S\}$ , a subgroup of  $G$ .<sup>1</sup>

**Definition 3:** Let  $\rho_1$  and  $\rho_2$  be two projective spaces of  $n$ -dimension. A projectivity  $\varphi : \rho_1 \rightarrow \rho_2$  is a bijection given by a non-singular matrix  $A$  such that  $P(X') = P(X)\varphi$  if and only if  $tX' = XA$ , where  $t \in F_q \setminus \{0\}$ . Write  $\varphi = M(A)$ , then  $\varphi = M(\lambda A)$  for any  $\lambda \in F_q \setminus \{0\}$ .<sup>1</sup>

**Theorem 1** (Fundamental Theorem of Projective Space): If  $\{P_1, \dots, P_{n+2}\}$  and  $\{P'_1, \dots, P'_{n+2}\}$  are sets of  $n + 2$  points of  $PG(n, q)$  such that no  $n + 1$  points chosen from the same set lie in a hyperplane, then there exists a unique projectivity  $\rho$  such that  $P'_i = P_i\rho, \forall i = 1, \dots, n + 2$ .<sup>1</sup>

According to **Theorem 4**, for  $n = 1$ , there is a unique projectivity transforming any three distinct points on a line to any other three, and for  $n = 2$ , there is a unique projectivity transforming any four distinct points no three on a line to any other four points no three on a line.

**Definition 4:** An  $k$ -arc,  $K$  or  $(k, 2)$ -arc in  $PG(2, q)$  is a set of  $k$  points, no three of which are colinear. A  $k$ -arc is complete if it is not contained in  $(k + 1)$ -arc.<sup>1</sup>

**Definition 5:** A line  $l$  of  $PG(2, q)$  is an  $d$ -secant of  $k$ -arc,  $K$  if  $|l \cap K| = d$ . In particular 1-secant is called unisecant, 2-secant is a bisecant, 3-secant is a trisecant and 0-secant is an external line.<sup>1</sup>

Let  $c_i$  be the number of points of  $PG(2, q) \setminus K$  that lies on  $i$  bisecants of  $K$ . The parameter  $c_i$  has the following formula for every 5-arcs:<sup>1</sup>

$$c_0 = (q - 4)(q - 5) + 1 \quad c_1 = 10(q - 4) \quad c_2 = 15 \tag{1}$$

**Note:** Every  $k$ -arc,  $K$  is complete whenever  $c_0 = 0$ ; that is,  $K$  is complete if every point lies on some bisecant of  $K$ .<sup>1</sup>

**Definition 6:** Let  $F$  be a homogeneous polynomial with indeterminate variables  $X_0, X_1, X_2 \in F_q$ . Let

$X = [x_0, x_1, x_2]$  and  $F(X) = F(x_0, x_1, x_2)$ . A projective plane curve or a plane quadric is

$$V(F) = \{P(X) \in PG(2, q) | F(X) = 0\},$$

where  $F$  is a form of degree two,

$$F = \sum_{0 \leq i \leq j \leq 2} a_{ij}X_iX_j \tag{2}$$

The points  $P(X) \in V(F)$  is called a rational point. A nonsingular plane quadric is called a conic.<sup>1</sup>

**Lemma 2:**

- i. A conic is an irreducible plane quadric.<sup>1</sup>
- ii. Any plane quadric through a 5-arc is non-singular.<sup>1</sup>
- iii. All conics are projectively unique.<sup>1</sup>
- iv. In  $PG(2, q)$  with  $q \geq 4$ , there is a unique conic through a 5-arc.<sup>1</sup>
- v. There is one-to-one correspondence between a set of point on  $PG(1, q)$  and an arc of the same size on a conic in  $PG(2, q)$ ,  $t \leftrightarrow (t^2, t, 1)$ .<sup>1</sup>

### 5-Arc and 6-Arc in $PG(2, 32)$

In  $PG(2, 32)$  there are 1057 points and lines, 33 points on each line and 33 lines passing through each point.

The points and lines are generated by a nonsingular matrix

$$T = C(f) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \omega^2 & \omega^{23} & \omega^2 \end{bmatrix},$$

such that, the points

$$P(i + 1) = P(1, 0, 0)T^i \tag{3}$$

and the lines

$$l_{i+1} = l_1T^i, \tag{4}$$

$i = 0, \dots, 1056$ , where  $l_1$  is the line passing through points  $[x, y, z]$  with third coordinate equal to zero. The lines listed in the **Table 1** by listed the its points in numeral form depending **Eqs. (3)** and **(4)**.

From the Fundamental Theorem of Projective Geometry, the frame  $\mu = \{1, 2, 3, 948\}$ , where

$$U_0 = P(1) = (1, 0, 0), \quad U_1 = P(2) = (0, 1, 0),$$

$$U_2 = P(3) = (0, 0, 1), \quad U = P(948) = (1, 1, 1).$$

**Table 1.** The lines of  $PG(2, 32)$ .

$l_1$	$l_2$	...	$l_{1056}$	$l_{1057}$
1	2	...	1056	1057
2	3	...	1057	1
62	63	...	60	61
108	109	...	106	107
130	131	...	128	129
278	279	...	276	277
284	285	...	282	283
288	289	...	286	287
297	298	...	295	296
309	310	...	307	308
317	318	...	315	316
335	336	...	333	334
359	360	...	357	358
375	376	...	373	374
411	412	...	409	410
418	419	...	416	417
459	460	...	457	458
474	475	...	472	473
491	492	...	489	490
528	529	...	526	527
563	564	...	516	517
577	578	...	575	576
659	660	...	657	658
689	690	...	687	688
712	713	...	710	711
723	724	...	721	722
797	798	...	795	796
800	801	...	798	799
802	803	...	800	801
867	868	...	865	866
895	896	...	893	893
922	923	...	920	921
1014	1015	...	1012	1013

is protectively the unique 4-arc in  $PG(2, 32)$ . The stabilizer group of  $\mu$  is  $S_4$  (symmetric group of order 4).

**Theorem 3:** *There are eleven inequivalent incomplete 5-arcs with parameter  $c_0 = 757$ ,  $c_1 = 280$ , and  $c_2 = 15$ .*

**Proof:** To construct distinct 5-arcs from  $\mu$ , the points which are lies on the bisecants of  $\mu$  are excluded from the space; that is, points of index zero, in addition to the four points excluded of the frame. The number of excluded points was 187, so the group  $S_4$  acts on the set 870 points of  $PG(2, 32)$  and partition it into 40 disjoint orbits. There were 35 orbits of length 24 and 5 orbits of length 6. The 40 distinct 5-arcs are obtained by selecting the first element from each orbit and added it to the frame. These 5-arcs are  $\mu \cup \{x\}$ ,  $x \in \{5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 18, 19, 21, 22, 23, 24, 27, 29, 30, 31, 32, 33, 35, 43, 44, 46, 47, 51, 58, 60, 61, 75, 78, 84, 85, 92, 93, 100, 177\}$ .

After checking the projectivities between these arcs, only eleven of them were inequivalent as given in Table 2 with its stabilizer group types.

**Table 2.** Inequivalent 5-arcs in  $PG(2, 32)$ .

$A_i$	5-arc	Fifth point $P(i)$ of 5-arc	Stabilizer
$A_1$	{1,2,3,948, 5}	$P(5) = [\omega^{20}, \omega^{30}, 1]$	$I$
$A_2$	{1,2,3,948, 6}	$P(6) = [\omega^5, \omega^{21}, 1]$	$I$
$A_3$	{1,2,3,948, 8}	$P(8) = [\omega^{24}, \omega^9, 1]$	$V_4$
$A_4$	{1,2,3,948,11}	$P(11) = [\omega^{10}, \omega^{24}, 1]$	$I$
$A_5$	{1,2,3,948,14}	$P(14) = [\omega^{19}, \omega^{30}, 1]$	$I$
$A_6$	{1,2,3,948,18}	$P(18) = [\omega, \omega^8, 1]$	$I$
$A_7$	{1,2,3,948,19}	$P(19) = [\omega^4, \omega^{10}, 1]$	$V_4$
$A_8$	{1,2,3,948,21}	$P(21) = [\omega^{24}, \omega^{25}, 1]$	$I$
$A_9$	{1,2,3,948,31}	$P(31) = [\omega^{10}, \omega^5, 1]$	$V_4$
$A_{10}$	{1,2,3,948,60}	$P(60) = [\omega^{11}, \omega^{19}, 1]$	$V_4$
$A_{11}$	{1,2,3,948,61}	$P(61) = [\omega, \omega^2, 1]$	$V_4$

The value of  $c_i$ ,  $i = 0, 1, 2$  can be deduce easily from Eq. (1). Since  $c_0 \neq 0$ , so these arcs are incomplete. ■

**Theorem 4:** *There are 1037 inequivalent incomplete 6-arcs with stabilizer group types 862 of type identity, 30 of type  $Z_2$ , 60 of type  $Z_3$ , 65 of type  $Z_2 \times Z_2$ , 5 of type  $S_3$ , and 15 of type  $S_4$ , and four types of  $c_i$ ,  $i = 0, 1, 2, 3$ , as follows: fifteen with parameter  $c_0 = 624$   $c_1 = 396$ ,  $c_2 = 24$ ,  $c_3 = 7$ , 130 with parameter  $c_0 = 628$   $c_1 = 384$ ,  $c_2 = 36$ ,  $c_3 = 3$ , 390 with parameter  $c_0 = 630$   $c_1 = 378$ ,  $c_2 = 42$ ,  $c_3 = 1$ , and 502 with parameter  $c_0 = 631$   $c_1 = 375$ ,  $c_2 = 45$ ,  $c_3 = 0$ .*

**Proof:** The same technique in Theorem 3 is used to find these 6-arcs and GAP programming is used to run the used algorithms. ■

**Example 1:**

- i. The set  $\mathcal{H}_\infty = A_1 \cup \{40\}$  is 6-arc with stabilizer group of type identity, and  $c_0 = 631$   $c_1 = 375$ ,  $c_2 = 45$ ,  $c_3 = 0$ .
- ii. The set  $\mathcal{H}_2 = A_2 \cup \{17\}$  is 6-arc with stabilizer group of type  $Z_2$ , and  $c_0 = 630$   $c_1 = 378$ ,  $c_2 = 42$ ,  $c_3 = 1$ .
- iii. The set  $\mathcal{H}_3 = A_4 \cup \{101\}$  is 6-arc with stabilizer group of type  $Z_2$ , and  $c_0 = 630$   $c_1 = 378$ ,  $c_2 = 42$ ,  $c_3 = 1$ .
- iv. The set  $\mathcal{H}_4 = A_5 \cup \{300\}$  is 6-arc with stabilizer group of type  $Z_2$ , and  $c_0 = 630$   $c_1 = 378$ ,  $c_2 = 42$ ,  $c_3 = 1$ .
- v. The set  $\mathcal{H}_5 = A_8 \cup \{211\}$  is 6-arc with stabilizer group of type  $S_3$ , and  $c_0 = 628$   $c_1 = 384$ ,  $c_2 = 36$ ,  $c_3 = 3$ .

**Conics through inequivalent 5-Arcs**

By Lemma 2, there is a unique conic through each 5-arc, and since all 5-arcs,  $A_i$ ,  $i = 1, \dots, 11$ , are passing through the frame  $\mu$ , thus the conic form of  $C_{A_i}$  take

**Table 3.** Conics form through 5-arc  $\mathcal{A}_i$ .

$C_{\mathcal{A}_i}$	$V(F_{\mathcal{A}_i})$
Rational point of $V(F_{\mathcal{A}_i})$	
$C_{\mathcal{A}_1}$	$V(X_0X_1 + \omega^{14}X_0X_2 + \omega^{13}X_1X_2)$ 1, <b>2</b> , 3, 5, 40, 56, 90, 165, 170, 252, 281, 415, 431, 485, 511, 565, 697, 705, 707, 756, 785, 813, 854, 890, 900, 930, 940, <b>948</b> , 950, 975, 994, 1013, 1019
$C_{\mathcal{A}_2}$	$V(X_0X_1 + \omega^9X_0X_2 + \omega^{16}X_1X_2)$ 1, <b>2</b> , 3, <b>6</b> , 17, 24, 38, 119, 121, 122, 158, 235, 291, 293, 361, 382, 400, 402, 421, 445, 468, 488, 502, 521, 527, 587, 612, 753, 757, 919, <b>948</b> , 1036, 1051
$C_{\mathcal{A}_3}$	$V(X_0X_1 + \omega^{22}X_0X_2 + \omega^7X_1X_2)$ 1, <b>2</b> , 3, <b>8</b> , 27, 140, 164, 237, 251, 263, 306, 349, 384, 438, 481, 519, 562, 566, 583, 628, 630, 661, 686, 699, 767, 772, 881, 898, 913, <b>948</b> , 978, 983, 1045
$C_{\mathcal{A}_4}$	$V(X_0X_1 + \omega^5X_0X_2 + \omega^2X_1X_2)$ 1, <b>2</b> , 3, <b>11</b> , 41, 42, 47, 101, 104, 225, 255, 282, 307, 311, 338, 391, 407, 540, 584, 610, 626, 635, 648, 663, 809, 816, 850, 860, 894, <b>948</b> , 966, 969, 1017
$C_{\mathcal{A}_5}$	$V(X_0X_1 + \omega^3X_0X_2 + \omega^{29}X_1X_2)$ 1, <b>2</b> , 3, <b>10</b> , 14, 18, 53, 79, 188, 190, 249, 300, 374, 378, 454, 500, 539, 674, 684, 734, 748, 783, 841, 848, 871, 875, 897, 926, <b>948</b> , 953, 974, 1033, 1044
$C_{\mathcal{A}_6}$	$V(X_0X_1 + \omega^3X_0X_2 + \omega^{29}X_1X_2)$ 1, <b>2</b> , 3, 10, 14, <b>18</b> , 53, 79, 188, 190, 249, 300, 374, 378, 454, 500, 539, 674, 684, 734, 748, 783, 841, 848, 871, 875, 897, 926, <b>948</b> , 953, 974, 1033, 1044
$C_{\mathcal{A}_7}$	$V(X_0X_1 + \omega^{20}X_0X_2 + \omega^8X_1X_2)$ 1, <b>2</b> , 3, <b>19</b> , 32, 57, 71, 128, 156, 179, 199, 227, 339, 379, 392, 436, 456, 461, 490, 574, 671, 710, 745, 761, 765, 879, 906, 938, 943, <b>948</b> , 988, 1039, 1053
$C_{\mathcal{A}_8}$	$V(X_0X_1 + \omega^{29}X_0X_2 + \omega^3X_1X_2)$ 1, <b>2</b> , 3, <b>21</b> , 92, 95, 103, 134, 153, 174, 202, 211, 250, 377, 478, 507, 516, 544, 558, 560, 647, 769, 771, 794, 825, 866, 887, <b>948</b> , 954, 963, 1008, 1032, 1042
$C_{\mathcal{A}_9}$	$V(X_0X_1 + \omega^2X_0X_2 + \omega^5X_1X_2)$ 1, <b>2</b> , 3, 7, <b>31</b> , 39, 52, 59, 80, 91, 110, 206, 365, 451, 457, 462, 496, 513, 595, 620, 631, 651, 685, 692, 744, 768, 840, 855, <b>948</b> , 999, 1010, 1011, 1012
$C_{\mathcal{A}_{10}}$	$V(X_0X_1 + \omega^7X_0X_2 + \omega^{22}X_1X_2)$ 1, <b>2</b> , 3, 16, 51, <b>60</b> , 64, 82, 89, 204, 229, 256, 292, 333, 545, 548, 585, 599, 666, 693, 747, 758, 775, 852, 911, 921, 929, <b>948</b> , 991, 1009, 1029, 1031, 1041
$C_{\mathcal{A}_{11}}$	$V(X_0X_1 + \omega X_0X_2 + \omega^{18}X_1X_2)$ 1, <b>2</b> , 3, 58, <b>61</b> , 76, 88, 115, 126, 221, 315, 447, 463, 489, 530, 535, 550, 623, 728, 743, 764, 784, 806, 817, 883, 884, 931, 936, <b>948</b> , 949, 952, 996, 1043

**Note:** The bolding number in Table 3 are just the points of the 5-arcs  $\mathcal{A}_i$ .

the form

$$C_{\mathcal{A}_i} = V(F_{\mathcal{A}_i}) = X_0X_1 + \alpha X_0X_2 - (\alpha + 1)X_1X_2, \alpha \in F_q.$$

Therefore, by substituting the fifth point of each 5-arcs  $\mathcal{A}_i$  into  $C_{\mathcal{A}_i}$ , the parameter  $\alpha$  was found. Table 3 contains equation forms of  $V(F_{\mathcal{A}_i})$  with its rational points.

### Parameterization of the conic $C_{\mathcal{A}_i}$

Let  $C_{\mathcal{A}_i}$  be a conic, from Lemma 2, all the conics  $C_{\mathcal{A}_i}$  are projectively equivalent to the conic  $C^*$ , where

$$C^* = V(X_1^2 - X_0X_2) = \{P(t^2, t, 1) \mid t \in F_q \cup \{0\}\}$$

that is,  $C_{\mathcal{A}_i} \cong C^*$ ,  $[x_0, x_1, x_2]M_{\mathcal{A}_i} = [t^2, t, 1]$ , where  $x_0, x_1, x_2$  indeterminate variables and  $M_{\mathcal{A}_i}$  the matrix transformation of  $C_{\mathcal{A}_i}$  to  $C^*$ .

The rational points of  $C^*$  are  $\{1, 3, 31, 120, 238, 262, 274, 306, 312, 445, 448, 494, 500, 503, 512, 579, 681, 747, 760, 763, 794, 843, 883, 910, 940, 943, 948, 992, 1007, 1034, 1052, 1054\}$ .

Now, the parameterization of  $C_{\mathcal{A}_i}$  is finding by the following way

$$[x_0, x_1, x_2] \begin{bmatrix} \omega^3 & 0 & 0 \\ 0 & 0 & \omega^{23} \\ \omega^{16} & \omega^{11} & \omega^6 \end{bmatrix} = [t^2, t, 1],$$

$$\omega^3x_0 + \omega^{16}x_2 = t^2 \tag{5}$$

$$\omega^{11}x_2 = t \tag{6}$$

$$\omega^{23}x_1 + \omega^6x_2 = 1 \tag{7}$$

From Eq. (6), the following equation is obtained

$$x_2 = \omega^{-11}t \tag{8}$$

**Table 4.** Conics parameterization.

$C_{A_i}$	$\mathcal{M}_{A_i}$ : matrix trans. Of $C_{A_i}$ to $C^*$
Parameterization of $C_{A_i}$	
$C_{A_1}$	$\begin{bmatrix} \omega^3 & 0 & 0 \\ 0 & 0 & \omega^{23} \\ \omega^{16} & \omega^{11} & \omega^6 \end{bmatrix}$ $\{P(\omega^{-3}(t^2 - \omega^5t), \omega^{-23}(1 - \omega^{26}t), \omega^{-11}t)\}$
$C_{A_2}$	$\begin{bmatrix} \omega^{23} & 0 & 0 \\ 0 & 0 & \omega^{20} \\ \omega^8 & \omega^3 & \omega^{29} \end{bmatrix}$ $\{P(\omega^{-23}(t^2 - \omega^5t), \omega^{-20}(1 - \omega^{26}t), \omega^{-3}t)\}$
$C_{A_3}$	$\begin{bmatrix} \omega^{23} & 0 & 0 \\ 0 & 0 & \omega^{29} \\ \omega^{30} & \omega^{25} & \omega^{20} \end{bmatrix}$ $\{P(\omega^{-23}(t^2 - \omega^5t), \omega^{-29}(1 - \omega^{26}t), \omega^{-25}t)\}$
$C_{A_4}$	$\begin{bmatrix} \omega^{16} & 0 & 0 \\ 0 & 0 & \omega^3 \\ \omega^{18} & \omega^{13} & \omega^8 \end{bmatrix}$ $\{P(\omega^{-16}(t^2 - \omega^5t), \omega^{-3}(1 - \omega^{26}t), \omega^{-13}t)\}$
$C_{A_5}$	$\begin{bmatrix} \omega^{22} & 0 & 0 \\ 0 & 0 & \omega^7 \\ \omega^{20} & \omega^{15} & \omega^{10} \end{bmatrix}$ $\{P(\omega^{-22}(t^2 - \omega^5t), \omega^{-7}(1 - \omega^{26}t), \omega^{-15}t)\}$
$C_{A_6}$	$\begin{bmatrix} \omega^{22} & 0 & 0 \\ 0 & 0 & \omega^7 \\ \omega^{20} & \omega^{15} & \omega^{10} \end{bmatrix}$ $\{P(\omega^{-22}(t^2 - \omega^5t), \omega^{-7}(1 - \omega^{26}t), \omega^{-15}t)\}$
$C_{A_7}$	$\begin{bmatrix} \omega^{19} & 0 & 0 \\ 0 & 0 & \omega^{28} \\ \omega^{27} & \omega^{22} & \omega^{17} \end{bmatrix}$ $\{P(\omega^{-19}(t^2 - \omega^5t), \omega^{-28}(1 - \omega^{26}t), \omega^{-22}t)\}$
$C_{A_8}$	$\begin{bmatrix} \omega^7 & 0 & 0 \\ 0 & 0 & \omega^2 \\ \omega^{10} & \omega^5 & 1 \end{bmatrix}$ $\{P(\omega^{-7}(t^2 - \omega^5t), \omega^{-2}(1 - \omega^{26}t), \omega^{-5}t)\}$
$C_{A_9}$	$\begin{bmatrix} \omega^7 & 0 & 0 \\ 0 & 0 & 1 \\ \omega^{12} & \omega^7 & \omega^2 \end{bmatrix}$ $\{P(\omega^{-7}(t^2 - \omega^5t), (1 - \omega^{26}t), \omega^{-7}t)\}$
$C_{A_{10}}$	$\begin{bmatrix} \omega^9 & 0 & 0 \\ 0 & 0 & \omega^{14} \\ 1 & \omega^{26} & \omega^{21} \end{bmatrix}$ $\{P(\omega^{-9}(t^2 - \omega^5t), \omega^{-14}(1 - \omega^{26}t), \omega^{-26}t)\}$
$C_{A_{11}}$	$\begin{bmatrix} \omega^{11} & 0 & 0 \\ 0 & 0 & \omega^{18} \\ \omega^{29} & \omega^{24} & \omega^{19} \end{bmatrix}$ $\{P(\omega^{-11}(t^2 - \omega^5t), \omega^{-18}(1 - \omega^{26}t), \omega^{-24}t)\}$

So, substituted Eq. (8) in Eqs. (5) and (7) the following are deduced:

$$x_0 = \omega^{-3}(t^2 - \omega^5t),$$

$$x_1 = \omega^{-23}(1 - \omega^{26}t).$$

Therefore, by the same way, the ten others have been founded. The details are given in Table 4.

According to Lemma 2, get that  $C_{A_i} \cong C^*$ , thus  $A_i.M_{A_i} = \mathcal{B}_i \subseteq C^*$ , where  $\mathcal{M}_{A_i}$  is the matrix transfor-

**Table 5.** Transforming  $\mathcal{B}_i$  to  $f_j^*$ .

$\mathcal{B}_i \cong f_i^*$	$\mathcal{M}_{\mathcal{B}_i}$ : Matrix transformation
$\mathcal{B}_1 \cong f_6^* = \{1, 3, 948, 238, 579\}$	$\begin{bmatrix} 0 & 0 & \omega^{15} \\ 0 & \omega^{21} & 0 \\ \omega^{27} & 0 & 0 \end{bmatrix}$
$\mathcal{B}_2 \cong f_9^* = \{1, 3, 948, 849, 943\}$	$\begin{bmatrix} \omega^6 & \omega^6 & \omega^6 \\ 0 & 1 & 0 \\ \omega & \omega^{24} & \omega^{16} \end{bmatrix}$
$\mathcal{B}_3 \cong f_7^* = \{1, 3, 948, 238, 445\}$	$\begin{bmatrix} \omega^{19} & 0 & 0 \\ 0 & \omega^{23} & 0 \\ 0 & 0 & \omega^{27} \end{bmatrix}$
$\mathcal{B}_4 \cong f_4^* = \{1, 3, 948, 238, 31\}$	$\begin{bmatrix} 0 & 0 & \omega^{17} \\ 0 & \omega^{25} & 0 \\ \omega^2 & \omega^2 & \omega^2 \end{bmatrix}$
$\mathcal{B}_5 \cong f_3^* = \{1, 3, 948, 238, 503\}$	$\begin{bmatrix} \omega^{18} & \omega^{14} & \omega^{10} \\ 0 & \omega^{19} & 0 \\ \omega^{28} & 0 & 0 \end{bmatrix}$
$\mathcal{B}_6 \cong f_2^* = \{1, 3, 948, 238, 794\}$	$\begin{bmatrix} 0 & 0 & \omega^{15} \\ 0 & \omega^{23} & 0 \\ 1 & \omega^{28} & \omega^{25} \end{bmatrix}$
$\mathcal{B}_7 \cong f_{10}^* = \{1, 3, 948, 503, 943\}$	$\begin{bmatrix} \omega^9 & 0 & 0 \\ 0 & \omega^6 & 0 \\ \omega^{19} & \omega^{11} & \omega^3 \end{bmatrix}$
$\mathcal{B}_8 \cong f_5^* = \{1, 3, 948, 238, 760\}$	$\begin{bmatrix} \omega^3 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{bmatrix}$
$\mathcal{B}_9 \cong f_8^* = \{1, 3, 948, 894, 503\}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \omega^4 & \omega^2 & 1 \end{bmatrix}$
$\mathcal{B}_{10} \cong f_{11}^* = \{1, 3, 948, 306, 1034\}$	$\begin{bmatrix} \omega^{20} & 0 & 0 \\ 0 & \omega^3 & 0 \\ 1 & \omega^{24} & \omega^{17} \end{bmatrix}$
$\mathcal{B}_{11} \cong f_1^* = \{1, 3, 948, 238, 894\}$	$\begin{bmatrix} \omega^{22} & \omega^{21} & \omega^{20} \\ 0 & \omega^{26} & 0 \\ \omega & 0 & 0 \end{bmatrix}$

mation of  $C_{A_i}$  to  $C^*$  in Table 4.

$$\mathcal{B}_1 = \{1, 3, 31, 760, 1054\}, \mathcal{B}_2 = \{1, 3, 31, 500, 940\},$$

$$\mathcal{B}_3 = \{1, 3, 31, 1007, 503\}, \mathcal{B}_4 = \{1, 3, 31, 943, 763\},$$

$$\mathcal{B}_5 = \{1, 3, 31, 579, 1052\}, \mathcal{B}_6 = \{1, 3, 31, 579, 794\},$$

$$\mathcal{B}_7 = \{1, 3, 31, 494, 120\}, \mathcal{B}_8 = \{1, 3, 31, 948, 1052\},$$

$$\mathcal{B}_9 = \{1, 3, 31, 894, 306\}, \mathcal{B}_{10} = \{1, 3, 31, 262, 940\},$$

$$\mathcal{B}_{11} = \{1, 3, 31, 1034, 883\}.$$

From Theorem Theorem 3, these arcs  $\mathcal{B}_i$  are projectively equivalent to  $f_j^*$  as given in Table 5.

The matrix  $K_{A_i}$  constructed by multiply the two matrices  $\mathcal{M}_{A_i}$  and  $\mathcal{M}_{\mathcal{B}_i}$ , where  $K_{A_i}$  transform  $A_i$  to one of  $f_j^*$ , as given in Table 6.

**Theorem 5:** On  $PG(1, 32)$ , there are 11 projectively inequivalent 5-sets, summarized in Table 7 with their stabilizer group.

**Proof:** From Lemma 2 there is one-to-one correspondence between a 5-set on  $PG(1, 32)$  and the 5-arc  $f_i^*$ ; that is,  $t \leftrightarrow (t^2, t, 1)$ . Hence, deduced the following inequivalent 5-sets,  $f_i$  on the projective line

**Table 6.** Transforming  $\mathcal{A}_i$  to  $f_i^*$ .

$\mathcal{A}_i \cong f_i^*$	$K_{\mathcal{A}_i}$
$\mathcal{A}_1 \cong f_6^*$	$\begin{bmatrix} 0 & 0 & \omega^{18} \\ \omega^{19} & 0 & 0 \\ \omega^2 & \omega & 1 \end{bmatrix}$
$\mathcal{A}_2 \cong f_9^*$	$\begin{bmatrix} \omega^{29} & \omega^{29} & \omega^{29} \\ \omega^{21} & \omega^{13} & \omega^5 \\ \omega^{23} & 0 & 0 \end{bmatrix}$
$\mathcal{A}_3 \cong f_7^*$	$\begin{bmatrix} \omega^{11} & 0 & 0 \\ 0 & 0 & \omega^{25} \\ \omega^{18} & \omega^{17} & \omega^{16} \end{bmatrix}$
$\mathcal{A}_4 \cong f_4^*$	$\begin{bmatrix} 0 & 0 & \omega^2 \\ \omega^5 & \omega^5 & \omega^5 \\ \omega^{10} & \omega^5 & 1 \end{bmatrix}$
$\mathcal{A}_5 \cong f_3^*$	$\begin{bmatrix} \omega^9 & \omega^5 & \omega \\ \omega^4 & 0 & 0 \\ 0 & 0 & \omega^{30} \end{bmatrix}$
$\mathcal{A}_6 \cong f_2^*$	$\begin{bmatrix} 0 & 0 & \omega^6 \\ \omega^7 & \omega^4 & \omega \\ \omega^{10} & 0 & 0 \end{bmatrix}$
$\mathcal{A}_7 \cong f_{10}^*$	$\begin{bmatrix} \omega^{28} & 0 & 0 \\ \omega^{16} & \omega^8 & 1 \\ 0 & 0 & \omega^{20} \end{bmatrix}$
$\mathcal{A}_8 \cong f_5^*$	$\begin{bmatrix} \omega^{10} & 0 & 0 \\ 0 & 0 & \omega^3 \\ \omega^{13} & \omega^7 & \omega \end{bmatrix}$
$\mathcal{A}_9 \cong f_8^*$	$\begin{bmatrix} \omega^7 & 0 & 0 \\ \omega^4 & \omega^2 & 1 \\ \omega^2 & \omega^2 & \omega^2 \end{bmatrix}$
$\mathcal{A}_{10} \cong f_{11}^*$	$\begin{bmatrix} \omega^{29} & 0 & 0 \\ \omega^{14} & \omega^7 & 1 \\ \omega^7 & \omega^7 & \omega^7 \end{bmatrix}$
$\mathcal{A}_{11} \cong f_1^*$	$\begin{bmatrix} \omega^2 & \omega & 1 \\ \omega^{19} & 0 & 0 \\ 0 & 0 & \omega^{18} \end{bmatrix}$

**Table 7.** The inequivalent 5-sets.

$f_i$	The 5-set	SG-Type
$f_1$	$\{\infty, 0, 1, \omega, \omega^2\}$	$V_4 = \langle \omega^2/t, t + \omega^2/t + 1 \rangle$
$f_2$	$\{\infty, 0, 1, \omega, \omega^3\}$	$I$
$f_3$	$\{\infty, 0, 1, \omega, \omega^4\}$	$I$
$f_4$	$\{\infty, 0, 1, \omega, \omega^5\}$	$I$
$f_5$	$\{\infty, 0, 1, \omega, \omega^6\}$	$I$
$f_6$	$\{\infty, 0, 1, \omega, \omega^{10}\}$	$I$
$f_7$	$\{\infty, 0, 1, \omega, \omega^{16}\}$	$V_4 = \langle \omega/t, t + \omega/t + 1 \rangle$
$f_8$	$\{\infty, 0, 1, \omega^2, \omega^4\}$	$V_4 = \langle \omega^4/t, t + \omega^4/t + 1 \rangle$
$f_9$	$\{\infty, 0, 1, \omega^2, \omega^8\}$	$I$
$f_{10}$	$\{\infty, 0, 1, \omega^4, \omega^8\}$	$V_4 = \langle \omega^8/t, t + \omega^8/t + 1 \rangle$
$f_{11}$	$\{\infty, 0, 1, \omega^7, \omega^{19}\}$	$V_4 = \langle \omega^7/t, t + \omega^7/t + 1 \rangle$

$PG(1, 32)$ , and its stabilizer group type (SG-Type) are founded.

**Remark 1:** Amongst the 1027 of 6-arcs stated in Theorem 4, only 52 of them passes through the conics  $C_{\mathcal{A}_i}$  given in Section 4. Therefore, there are only 52 inequivalent 6-sets in the projective line  $PG(1, 32)$ . For example, the 6-arc in Example 11,  $\mathcal{H}_1$  lie on the conic  $C_{\mathcal{A}_1}$ ,  $\mathcal{H}_2$  lie on the conic  $C_{\mathcal{A}_2}$ ,  $\mathcal{H}_3$  lie on the conic  $C_{\mathcal{A}_4}$ ,  $\mathcal{H}_4$  lie on the conic  $C_{\mathcal{A}_5}$ ,  $\mathcal{H}_5$  lie on the conic  $C_{\mathcal{A}_8}$ .

### Applications of the arcs to coding theory

In this section, a connection between projective liner codes over the Galois field and the getting arcs in  $PG(2, 32)$  is even as applications to the results in this paper. The following basic definitions in coding theory are necessary.

#### Definition 7:

- i. A linear  $[n, k, d]_q$ -code  $C$  is a subspace of  $n$ -dimensional vector space  $F_q^n$  with dimension  $k$  and minimum distance  $d$ , where  $d(x, y) = |i : x_i \neq y_i|$ , and

$$d(C) = \min \{d(x, y) \mid x, y \in C, x \neq y\}$$

The vectors  $v \in C$  are called the codewords.<sup>13</sup>

- ii. The dual code of  $C$ , denoted  $C^\perp$ , is the set of vectors of  $F_q^n$  which are orthogonal to every codeword in  $C$ ; that is,  $C^\perp = \{v \in F_q^n \mid \sum_{i=1}^n v_i u_i = 0, \forall u \in C\}$ .<sup>13</sup>
- iii. If  $C$  is an  $[n, k, d]_q$ -codes, let  $A_i$  be the number of codewords  $v$  such that  $d(v, 0) = i$ . The numbers  $A_0, A_1, \dots, A_n$  are called the weight distribution of  $C$ .
- iv. The covering radius  $\rho = \rho(C)$  of a linear code  $C$  is  $\rho = \max_{x \in F_q^n} \min_{c \in C} d(x, c)$ .<sup>13</sup>

#### Definition 8:

- i. A generator matrix  $G$  of a  $[n, k, d]_q$ -code is a  $k \times n$  matrix whose rows form a basis of code.<sup>13</sup>
- ii. A linear code for which any two columns of a generator matrix are linearly independent is called a projective code.<sup>13</sup>
- iii. A linear code which can correct  $e$  error of any received codewords is called an  $e$ -error correcting code.<sup>13</sup>

Any linear code  $C$  which has minimum distance  $d$  can correct  $e = \lfloor (d - 1)/2 \rfloor$  errors, where  $\lfloor m \rfloor$  denotes the integer part of  $m$ .<sup>13</sup> The distance for any linear  $[n, k, d]_q$ -code  $C$  always satisfies the singleton bound  $d \leq n - k + 1$ .<sup>13</sup>

In coding theory, increases the number of errors  $e$  that can be detected or corrected it is one of the important goals since it is making the transmission is more reliable. Since  $e$  is as large as possible whenever  $d$  is the largest possible so, should maximize the minimum distance  $d$  is become the goal. Therefore, codes

which achieve the singleton bound is regarded good codes.

**Definition 9:** Let  $C$  be a linear  $[n, k, d]_q$ -code. Then  $C$  is an MDS (Maximum Distance Separable) code if  $d = n - k + 1$ .<sup>13</sup>

Due to the importance of MDS codes, many authors have studied construction methods of MDS codes for example see.<sup>14,15</sup> Also, there were authors focused on the bounds of linear codes for certain finite fields.<sup>16</sup>

**Theorem 6:** A projective linear  $[n, k, d]_q$ -code  $C$  exists if and only if there exists an  $(n, n - d)$ -arc in  $PG(k - 1, q)$ .<sup>7</sup>

**Corollary 7:** There are only eleven projective MDS  $[5, 3, 3]_{32}$ -codes  $PC_{A_i}$  and projective dual MDS  $[5, 2, 4]_{32}$ -codes.

**Proof:** By Theorem 6, from each inequivalent 5-arcs  $\mathcal{A}_i$  in Theorem 3, the following basis  $B(\mathcal{A}_i)$  to  $[5, 3, 3]_{32}$ -code  $PC_{A_i}$ ,  $i = 1, \dots, 11$  are constructed.

$$\begin{aligned}
 B(\mathcal{A}_1) &= \{(1, 0, 0, 1, \omega^{20}), (0, 1, 0, 1, \omega^{30}), (0, 0, 1, 1, 1)\}, \\
 B(\mathcal{A}_2) &= \{(1, 0, 0, 1, \omega^5), (0, 1, 0, 1, \omega^{21}), (0, 0, 1, 1, 1)\}, \\
 B(\mathcal{A}_3) &= \{(1, 0, 0, 1, \omega^{24}), (0, 1, 0, 1, \omega^9), (0, 0, 1, 1, 1)\}, \\
 B(\mathcal{A}_4) &= \{(1, 0, 0, 1, \omega^{10}), (0, 1, 0, 1, \omega^{24}), (0, 0, 1, 1, 1)\}, \\
 B(\mathcal{A}_5) &= \{(1, 0, 0, 1, \omega^{19}), (0, 1, 0, 1, \omega^{30}), (0, 0, 1, 1, 1)\}, \\
 B(\mathcal{A}_6) &= \{(1, 0, 0, 1, \omega), (0, 1, 0, 1, \omega^8), (0, 0, 1, 1, 1)\}, \\
 B(\mathcal{A}_7) &= \{(1, 0, 0, 1, \omega^4), (0, 1, 0, 1, \omega^{10}), (0, 0, 1, 1, 1)\}, \\
 B(\mathcal{A}_8) &= \{(1, 0, 0, 1, \omega^{24}), (0, 1, 0, 1, \omega^{25}), (0, 0, 1, 1, 1)\}, \\
 B(\mathcal{A}_9) &= \{(1, 0, 0, 1, \omega^{10}), (0, 1, 0, 1, \omega^{15}), (0, 0, 1, 1, 1)\}, \\
 B(\mathcal{A}_{10}) &= \{(1, 0, 0, 1, \omega^{11}), (0, 1, 0, 1, \omega^{19}), (0, 0, 1, 1, 1)\}, \\
 B(\mathcal{A}_{11}) &= \{(1, 0, 0, 1, \omega), (0, 1, 0, 1, \omega^2), (0, 0, 1, 1, 1)\}.
 \end{aligned}$$

All these codes have the same weight distribution which is  $A_0 = 1, A_1 = 0, A_2 = 0, A_3 = 310, A_4 = 4495, A_5 = 27962$ , and covering radius  $\lambda = 2$ . Since  $d = 3$  for all these codes, so  $e = 1$ .

The dual codes of each  $PC_{A_i}$ ,  $i = 1, \dots, 11$  are projective MDS  $[5, 2, 4]_q$ -codes with the following bases,  $B^\perp(\mathcal{A}_i)$ .

$$\begin{aligned}
 B^\perp(\mathcal{A}_1) &= \{(1, 1, 1, 1, 0), (\omega^{20}, \omega^{30}, 1, 0, 1)\}, \\
 B^\perp(\mathcal{A}_2) &= \{(1, 1, 1, 1, 0), (\omega^5, \omega^{21}, 1, 0, 1)\}, \\
 B^\perp(\mathcal{A}_3) &= \{(1, 1, 1, 1, 0), (\omega^{24}, \omega^9, 1, 0, 1)\},
 \end{aligned}$$

$$\begin{aligned}
 B^\perp(\mathcal{A}_4) &= \{(1, 1, 1, 1, 0), (\omega^{10}, \omega^{24}, 1, 0, 1)\}, \\
 B^\perp(\mathcal{A}_5) &= \{(1, 1, 1, 1, 0), (\omega^{19}, \omega^{30}, 1, 0, 1)\}, \\
 B^\perp(\mathcal{A}_6) &= \{(1, 1, 1, 1, 0), (\omega, \omega^8, 1, 0, 1)\}, \\
 B^\perp(\mathcal{A}_7) &= \{(1, 1, 1, 1, 0), (\omega^4, \omega^{10}, 1, 0, 1)\}, \\
 B^\perp(\mathcal{A}_8) &= \{(1, 1, 1, 1, 0), (\omega^{24}, \omega^{25}, 1, 0, 1)\}, \\
 B^\perp(\mathcal{A}_9) &= \{(1, 1, 1, 1, 0), (\omega^{10}, \omega^{15}, 1, 0, 1)\}, \\
 B^\perp(\mathcal{A}_{10}) &= \{(1, 1, 1, 1, 0), (\omega^{11}, \omega^{19}, 1, 0, 1)\}, \\
 B^\perp(\mathcal{A}_{11}) &= \{(1, 1, 1, 1, 0), (\omega, \omega^2, 1, 0, 1)\}.
 \end{aligned}$$

The weight distribution is  $A_0 = 1, A_1 = 0, A_2 = 0, A_3 = 0, A_4 = 155, A_5 = 868$ , and covering radius  $\lambda = 3$ . Since  $d = 4$  for all these codes, so  $e = 1$ .

**Remark 2:** The codes in Corollary 7 can be extended by its length since the arcs in Theorem 9 are not complete.

## Results and discussion

The inequivalent 5-arcs through the standard frame in the projective plane over the even field of order 32 are founded, which were eight out of fourteen ones. Then these arcs are used to find the conics form passes through these inequivalent 5-arcs. There are 1037 inequivalent incomplete 6-arcs with its stabilizer group, where it was found that the largest stabilizer group has a size twenty-four (symmetric group of order four,  $S_4$ ). The parameterization of the conic forms in the projective plane over the finite field of order thirty-two have founded using Algebra Theory. Finally, introduce these arcs as projective linear codes, where it was deduced that these codes are of types Maximum Distance Separable Codes, MDS.

## Conclusion

There are eleven incomplete 5-arcs in the projective plane  $PG(2, 32)$ , five of them have stabilizer group of kind  $V_4$  and six have identity group, so there are eleven conics through these arcs. Also, the parameterization of these conics had been founded. From the one-to-one corresponding between sets on the projective line  $PG(1, 32)$  and arcs in  $PG(2, 32)$ , there are eleven inequivalent 5-sets. There are eleven projective MDS  $[5, 3, 3]_{32}$ -codes and projective MDS  $[5, 2, 4]_{32}$ -codes with extension property of these codes with respect to length of the codes.

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## Authors' declaration

- Conflicts of Interest: None.
- We hereby confirm that all tables in the manuscript are ours.
- No animal studies are present in the manuscript.
- No human studies are present in the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee at Mustansiriyah University.

## Authors' contribution statement

E. B. conceptualized and designed the study and supervised the overall research process. Z. A. conducted the computational work and implemented the programming aspects of the research. E. B. and Z. A. performed the mathematical proofs and carried out the theoretical analysis. E. B. critically reviewed and revised the manuscript for important intellectual content.

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## قطوع من خلال أقواس-5 الغير متكافئة في

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قسم الرياضيات، كلية العلوم، الجامعة المستنصرية، بغداد، العراق.

### الخلاصة

المخروط هو منحنى رباعي مستو غير قابل للاختزال وله ستة حدود. عادةً ما يتم تعريف أقواس- على أنها مجموعة من من النقاط في المستوى الإسقاطي بحيث يلتقي بعض الخطوط بـ  $k$  في نقطتين. لقد تم تبين أنه هنالك مخروطاً وحيداً يمر من خلال كل أقواس- . الأقواس- غير المتكافئة المارة من خلال الإطار القياسي في المستوى الإسقاطي على حقل جالوا من الرتبة الثانية والثلاثين، أسست وهي أحد عشر مع زمرة مثبتاتها، حيث يوجد خمسة أقواس- 5 مع زمرة مثبتة من نوع زمرة كلاين من الرتبة أربعة وستة أقواس- 5 لها الزمرة المحايدة. وبذلك يتم تأسيس الشكل المخروطي الذي يمر عبر هذه الأقواس- 5. ثم تم إعادة تمثيل كل مخروطي مع الخط إسقاطي على . كتطبيق لنتائجنا، تم الاستفادة من العلاقة بين الترميز الخطي الإسقاطي والأقواس في المستوى الإسقاطي عبر لتحديد عدد الترميزات MDS الخطية الإسقاطية الغير متكافئة ذات الطول خمسة والبعث الثالث وتصحيح خطأ واحد. كما تم تقديم الترميزات المزدوجة لهذه الترميزات والتي كانت بطول خمسة ومن البعد الثاني واحد لتصحيح الخطأ. وأخيراً، تم حساب توزيع الوزن، غطاء نصف القطر وعدد التصحيحات لهذه الرموز MDS الخطية الإسقاطية و الترميزات المزدوجة لها.

**الكلمات المفتاحية:** قوس، مخروط، فضاء إسقاطي منتهي، حقل منتهي، فعل الزمرة.