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The Conditional Value at Risk as a Risk Constraint in Optimal Investment Portfolio: An Analytical Study

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Abstract: This study aims to measure the value-at-risk (VaR) of a group of investment portfolios and use it as a constraint in portfolio composition. This was achieved by utilising the conditional value at Risk (CVaR), also known as the mean value at Risk (Mean-VaR), which is a proxy for risk in investment portfolios. The study resorted to returns for the period (2012-2024) and employed SPSS statistical software. For accomplishing its objective, this study adopted the main hypothesis: "The conditional value at risk is a reliable measure of risk and therefore gives an account for a constraint on the risk of the optimal investment portfolio."
The study reached several conclusions, the most significant of which is that the conditional value-at-risk indicates a financial constraint on the risk of the optimal investment portfolios during the study's periods. The most prominent recommendations from the study are that investment specialists, particularly investment portfolio managers, should embrace techniques and models for calculating and estimating the various aspects of risk involved in investment decision-making, including portfolio risk.

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القيمة المعرضة للخطر المشروطة كقيد للمخاطر في المحفظة الاستثمارية المثلى: دراسة تحليلية

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المستخلص

تهدف هذه الدراسة إلى قياس القيمة المعرضة للخطر (VaR) لمجموعة من المحافظ الاستثمارية، واستخدامها كقيد في تكوين المحفظة، وقد تم ذلك باستخدام القيمة المعرضة للخطر المشروطة (CVaR)، والمعروفة أيضًا باسم متوسط القيمة المعرضة للخطر(Mean-VaR) ، والتي تُعد مؤشرًا للمخاطر في المحافظ الاستثمارية، اعتمدت الدراسة على عوائد للفترة (2012-2024) واستخدمت برنامج SPSS الإحصائي، ولتحقيق ذلك تبنت الدراسة الفرضية الرئيسية التالية(تُعد القيمة المعرضة للخطر المشروطة مقياسًا موثوقًا للمخاطر، وبالتالي فهي تُفسر قيدًا على مخاطر المحفظة الاستثمارية المثلى).

توصلت الدراسة إلى عدة استنتاجات، أهمها أن القيمة المعرضة للخطر المشروطة تُشير إلى قيد مالي في مخاطر المحافظ الاستثمارية المثلى خلال فترة الدراسة، أما أبرز التوصيات التي انبثقت عن الدراسة هي ضرورة أن يتبنى المتخصصون في الاستثمار ولا سيما مديرو محافظ الاستثمار أساليب ونماذج لحساب وتقدير مختلف جوانب المخاطر المرتبطة بقرارات الاستثمار بما في ذلك محفظة الاستثمار.

الكلمات المفتاحية: المحفظة الاستثمارية، القيمة المعرضة للخطر، القيمة المعرضة للخطر المشروطة

1. Introduction

Regularly, Investors are vulnerable to a range of risks, and without a thorough and sound analysis of these risks, investments may bring about losses. Therefore, effective risk control is a must. At the institutional level, regulatory bodies have begun imposing restrictions to limit exposure to several types of risks. For example, the 1993 G30 meeting and the 1995 Basel Committee on Banking Supervision set guidelines and recommendations for market risk management. The 1998 Basel Accord emphasised credit risk management, and several models were proposed to measure risk. However, market risk is the most important risk, defined as the possibility that an investor will incur losses due to factors that affect performance. It is also a systematic risk that cannot be thrown out through diversification. This differs from so-called unsystematic risks, which are unique to certain companies. The main types of market risk include interest rate risk, commodity risk, and currency risk, among others (Colevici et al., 2019: 31).

With this type of risk spreading across financial markets, assessing exposure to market risks has become one of the most important tasks for financial institutions and specialists. Several models typically drive this, as market risks faced by financial institutions during sharp market fluctuations can trigger radical changes in investment values, which, in turn, influence company performance. On that ground, market risk measures have begun to estimate the worst possible financial loss for an investment over a specific period of time, at a specific level of reliability (or probability). (Chan et al., 2007: 556).

It is substantial to mention that studies in the literature of financial management and its applications in the recent period have been specific about value and the extent of its risk exposure, with the emergence of engineering and re-engineering concepts and developments in techniques and methods for estimating the potential risks of investment returns, which are known as VaR models. (Rachev & Khindanova, 2002: 8) In 1994, J.P. Morgan published a risk control methodology known as RiskmetricsTM, which was primarily based on a modern measure of financial risk called VaR, which is one of the approved measures that represents an option for measuring market risks, which regulatory bodies and institutions can use to figure out capital requirements (Chan et al., 2007: 556). This methodology was widely regarded as a masterpiece in financial risk management and quickly gained widespread popularity. Over the past few years, VaR has become a fundamental component of market risk management for many financial institutions. It is used as an internal risk management tool and has been adopted by the Basel Committee as an international regulatory standard. VaR is a simple risk measure used by financial institutions to assess the exposure of their portfolios to market risk. The main feature of VaR is aggregating potential losses that may occur with a certain probability over a specific time period into a single value. It is based on the so-called profit-or-loss distribution in the model proposed by RiskMetrics (Lamantia et al., 2006: 1).

Accordingly, this study employed 13 time periods, each 1 year, i.e., 13 investment portfolios comprising 11 company stocks. This study was divided into a set of main paragraphs—the first paragraph, as shown, contained an introduction to the study. The second paragraph included literature reviews. The third paragraph covered the development of hypotheses, while the fourth paragraph comprised the methodology followed in the study. The fifth paragraph focused on the results. The sixth paragraph brought about the end of this study under the title "discussions".

The problem of the study revolves around the increasing risks facing investors and the diversity of risk measures. Therefore, the study aimed to measure the value-at-risk for a group of investment portfolios and use it as a constraint in the formation of the investment portfolio, using the conditional value-at-risk (CVaR). For this reason, the study adopted a main hypothesis: the conditional value-at-risk is a reliable measure of risk and thus represents a constraint on the optimal investment portfolio's risk.

2. Literature Review

2-1. Value at risk: Value at Risk (VaR) was not a conventionally used term before the mid-1990s, but its origins date back much further. The mathematics underlying VaR was considerably improved in the context of portfolio theory by Harry Markowitz and others. Several terms were coined before VaR, especially during the 1990s, including Dollars at Risk (DaR), Income at Risk (IaR), Capital at Risk (CaR), Earnings at Risk (EaR), and Value at Risk (VaR). All of these terms are based on VaR. DaR has been criticised for not being comprehensive enough for all businesses. CaR has been found at fault with not using capital in their models. LaR and (R) Terms that are not related to the overall risk, especially market risk.

Hence, VaR was employed because it encompassed the aforementioned concepts (Glyn, 2002: 24). Even though efforts initially focused on designing optimal portfolios, market risk and its associated changes are central to VaR calculation. VaR is a risk measurement method that statistically estimates the maximum potential loss of an investment over a specified period at a given confidence level under normal market conditions. VaR is the amount of loss investors would incur over a specified investment period at a confidence level of α-1, expressed as a percentage of units or currency (Astuti & Gunarsih, 2021: 106). It is therefore one possible measurement method for estimating potential losses when the price of an asset or portfolio declines. A portfolio's VaR symbolises the maximum amount an investor could lose over a certain period of time with a given probability (Abada et al., 2014: 15). VaR is the worst-case loss over a target time horizon at a specified confidence level. More accurately, VaR illustrates the percentage distribution of expected gains and losses over the target time horizon (Ryabtsev &Ryabtsev, 2011: 157). The VaR can also be used as a warning to avoid the worst financial risks (Syuhada, 2020: 1). Consequently, the VaR was published through the Bank for International Settlements' regulations and adopted as a standard method for assessing market risk. The abbreviated VaR concept depicts the value of risk at a confidence level of α for a period of t days, meaning that losses exceeding the value of risk occur with a probability of 100% (α − 1) within t days (Kozaki & Sato, 2008: 1226). It is also exploited as a tool to measure insurance companies' exposure to Risk (Chan et al., 2007: 556).

It is also among the most routinely used risk management and assessment tools for measuring the risks of investment assets or investment funds in both highly liquid and less liquid financial markets. The VaR calls for the so-called tail risk, or the risk of severe downside that an investor may be exposed to when holding an asset or portfolio for a specific period of time and at certain confidence levels. In simpler terms, VaR measures the worst possible loss for an investment option. Due to the ease of use and simplicity of the VaR model, this is according to several experimental studies, such as those by (Betcheva, 2005: 39), (Krokhmal & Palmquist, 2002: 2-4), (Dorokhov, 2023: 6). Various market participants (such as policymakers, financial intermediaries, and portfolio or fund managers) have used this tool to address adverse market movements and shocks. Unlike traditional risk measures, VaR provides an aggregate view of portfolio risk, accounting for leverage and asset correlations. Consequently, it is a truly forward-looking risk measure (Ryabtsev, 2011: 158). Under the VaR strategy, the insured portfolio is constructed and often rebalanced so that the portfolio level exceeds a minimum threshold with a given confidence level at each time step (Alipour & Bastani, 2023: 3). The VaR has thus become a standard for risk reporting because it captures an important aspect of risk, namely market risk (Di Clemente & Romano, 2005: 30). The VaR has become widely used, both theoretically and quantitatively, as a potential method for assessing future Risk (Zhang et al., 2018: 5280).

Nevertheless, VaR has faced many criticisms, including its unrealistic assumptions about linear and normal distributions, its sensitivity to estimation and holding period choices, and its inadequacy during crises, especially when correlations between assets vary. The latter point is of particular importance, as the impact of correlations is much greater than

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during periods of market stability (Cho et al., 2022: 197). Besides, some VaR models ignore certain types of correlation, others are illogical, or correlations do not accurately reflect stock markets. Therefore, proposing a model that accurately reflects changes in the stock markets is of paramount importance (Zhang et al., 2018: 5281). Furthermore, Gao & Liu (2009) identify three criticisms: first, the nature of distributions, which does not lead to a comprehensive measure of risk; second, the difficulty with discrete data, especially within investment portfolios; and finally, VaR does not furnish a complete picture of the magnitude of risk, but rather a percentage (Gao & Liu, 2009: 246). Most risk models focus on short-term risks, neglecting the long-term risks that played a role in the financial crisis. Short-term risk forecasting is usually based on manipulating historical returns as inputs, which cannot be converted into long-term outputs. Short-term forecasting is, in effect, a late warning for most illiquid assets that cannot be liquidated in the short term (Molino & Sala, 2021: 1191).

The bright side, particularly in estimating the expected probability density function (VaR) and even the expected probability density function (CvaR), is that once the expected probability density function relevant to the data used is known, the expected probability density function for the stock or portfolio can be estimated. Estimating the probability density function using historical data is not possible without assuming that the historical data used in the estimation represents a good indicator of the future (Molino & Sala, 2021: 1192). If the cumulative density function is known, the VaR is simply the p-th percentile. Since the cumulative density function is unknown in practice, value-of-risk studies primarily focus on estimating it, particularly its tail behaviour. In general, as for calculating VaR (Colivicchi et al., 2019: p. 32), the following elements are included:

1. Probability
2. Time horizon
3. Cumulative distribution function, symbolised by F (x), or its quantity.

On that account, unexpected losses are measured using VaR, which assesses the maximum loss that can be recorded over a specified period of time for a given confidence interval. Generally, this value represents a multiple (k) of the standard deviation:

$VaR = k \times s$

In a probability distribution, the VaR value is the difference between the expected loss and the maximum loss that can be incurred with a given confidence level. The α value represents a very low value (e.g., 1%, 5%, 10%). These coefficients measure the probability of losses exceeding the Risk (Beltrame et al., 2014: p. 54). Value-at-Risk approaches are further classified into two main categories: parametric, nonparametric, and semi-parametric (Abada et al., 2014: 15).

2-1-1. Parametric Framework: The parametric framework hypothesises that the data follow a known and described probability distribution. Market risk management typically refers to a normal distribution. In this case, the relevant change is the portfolio loss, not the market return. The return is characterised by symmetric behaviour and often centres on zero. However, the distribution of losses is not completely symmetrical, with a non-zero mean. Hence, alternatives to the normal distribution, such as the beta distribution, are necessary. Given the distribution parameters, it is possible to obtain the percentage at the desired confidence level. Therefore, the Value of Risk (VaR) is calculated as the difference between the expected loss and the maximum loss for a given confidence level, and the Value of Risk is defined as the percentage α of the loss distribution.

Even though the parametric approach is simple, it is often limited by distributional behaviour, such as thick tails, skewness, and kurtosis. Moreover, before using a parametric distribution, it is necessary to conduct a series of tests to verify whether the parametric model provides a valid basis for interpreting the behavioural effect of the data. This is a very long and expensive process (Beltrame et al., 2014: 55-56). This approach includes two models: the variance/covariance model and the quadratic approximations model. VaR is calculated according to the normal distribution of returns σ as follows:

$VaR\alpha = - \sigma F\text{-}1y$

Where (σ) stands for the variance value over the period, and (F-1y) represents the table value according to a specified probability K. One of the most prominent parametric models is the GARCH model. Our study relied on the assumption of a normal distribution of returns, based on the observation of near-stability in stock prices, leading to stable returns. developed by Bollerslev (1986) from the autoregressive moving average

(ARMA) process. However, this process cannot symbolise asymmetric variance patterns (Chang et al., 2011: 266).

GARCH can be used for stock prices, financial indices, and foreign exchange rates, and there is a growing literature on its applications in asset pricing and risk management. (Chan et al., 2007: 556) It is widely used for modelling and forecasting the daily volatility of financial returns. Estimation and forecasting results in the presence of extreme values have been documented in previous studies. According to the GARCH model, the evolution of the squared variance is illustrated by the following equation (Trucíos et al., 2020: 2583):

$$\sigma 2_t = \omega + \alpha r 2_{t-1} + \beta 2_{t-1}$$

Due to the extreme values experienced by some returns, the following equation was adopted: yc is a constant to ensure Fisher consistency:

$$\sigma 2_t = \omega + \alpha y_c r_c \left(\frac{r2_{t-1}}{\alpha 2_{t-1}}\right) + \alpha 2_{t-1} + \beta 2_{t-1}$$

The GARCH and other approaches are among the procedures deployed in applying different VaR models. Many experts have adopted various models to measure asymmetric power VaR, making specific distributional assumptions. They have concluded that combining methods produces a VaR measure that provides a more accurate forecast of future losses across different investment opportunities. Others have used VaR measures that combine GARCH models with different return distributions, and they have concluded that the model assuming the "t" distribution performs better than other measures. The effectiveness of different VaR measures has also been tested using GARCH models, and it has been shown that the VaR model outperforms using exponential GARCH. Many subsequent studies have examined various GARCH models to identify the most effective VaR model for risk management. Some have attempted to present a generalised VaR model, called Lambda VaR, and demonstrate its competitiveness using a different testing procedure (Ryu et al., 2022: 3-4).

2-1-2. Nonparametric Framework: The parametric approach is closely identical to the nonparametric approach. They both aim to construct an empirical probability distribution and calculate VaR using percentage logic, or scale the distribution to a loss level where the frequency associated with the largest losses is less than or equal to α. The essence of this approach is to allow the data to express itself as much as possible, as VaR is measured without making strong assumptions about the distribution of returns. All nonparametric approaches assume that the near future will be sufficiently similar to the recent past that data from the recent past can be used to predict near-future risks. This is a simulation approach, derived through the application of simulation steps, and includes two models: The Historical Simulation Model and the Monte Carlo Simulation Model.

2-1-2-1. Historical Simulation Model: This traditional method uses historical data to estimate the value at risk (VaR). One form of HS is filtered historical simulation (FHS), which calculates sample ratios from filtered residuals using a barometric model such as AR-GARCH (Peng et al., 2023: 5). The historical simulation method constructs a probability distribution from historical data. The basic theory is that historical data can be easily interpreted even in the future. Because outcomes are predictable, this requires large data sets, which are not always available. To overcome the lack of data, new data can be generated, and specific formulas can reproduce what appear to be random phenomena. The numbers generated by mathematical formulas are called random number generators (RNGs) because they are not completely random. RNGs simulate; at this stage, all risk factors are involved and produce different effects with each simulation. If the number of simulations is large, the likely outcomes will be precisely determined, providing results that are often difficult to achieve using traditional methods. Once a distribution of probability values that can be assumed to be a loss is obtained, the outcome can then be regularised according to size, excluding extreme losses that occur with a frequency less than or equal to α, with the associated maximum loss value at the chosen confidence level. More recently, limitations of VaR, such as the impossibility of predicting maximum future losses based on historical distributions, have been overcome through conditional VaR (expected shortfall), which accounts for average losses that fall within a confidence interval (Beltrame et al., 2014: p56). VaR is measured according to the following formula (Rydell, 2013: 4):

$\} VaR\alpha\ (X) = \inf\{x \in R: P(X > x) \leq (1 - \alpha$

2-2-1-2. Monte Carlo Simulation Model: Monte Carlo simulation is another nonparametric model and one of the most popular with VaR. Nevertheless, it is also the most complicated to implement and generates a large number of scenarios based on assumptions. However, the Monte Carlo (MC) simulation method ignores the widely observed heteroscedasticity in real markets, leading to heteroskedastic return distributions that can underestimate risk. MC assumes that returns follow a specific time-series model, such as the GARCH model, and evaluates VaR numerically via Monte Carlo simulation. As a numerical method, MC imposes few constraints on models. However, the heavy computational load is analysing the impact of changing portfolio weights, which may require several simulations. VC also suffers from the disadvantage of underestimating risk, while MC requires significant computational resources and faces analytical difficulties (Kozaki & Sato, 2008: p. 1226). The variance–covariance (VC method evaluates VaR assuming normally distributed returns. Its name comes from the fact that the variance–covariance (VC matrix of assets plays an important role in the method. Based on portfolio theory, it permits simple analysis and calculations, which facilitated the widespread use of variance–covariance early in practice.

2-1-3. Semi-Parametric Framework: The GJR-GARCH-FHS model is one of the most remarkable semi-parametric approaches. It is parametric in estimating the variance of the GJR-GARCH part and nonparametric in estimating the FHS part. Empirical evidence shows that index returns exhibit random volatility, negative skewness, and excessive kurtosis. It is also well established that volatility groups for high and low volatility revert to their means and respond more strongly to negative than to positive events. One characteristic of the GJR-GARCH-FHS is its ability to accurately predict variances over short and long time horizons (Molino & Sala, 2021: 1191).

The CVaR is a risk measure that can be calculated parametrically, non-parametrically, or semi-parametrically. CVaR, also known as expected default, or ES, has gained significant attention in the financial risk management literature as an effective risk indicator. CVaR at level α represents the conditional expectation of losses in the top α% (1 - α), and is intended to be a better risk indicator than Value at Risk (VaR).

While the use of VaR partly accounts for the 2007-2008 financial crisis, CVaR is more attractive than VaR because it accounts for the contribution of very rare but very large losses. CVaR is also a robust risk measure. CVaR is robust, but several observations are needed to estimate it accurately because it is a tail statistic. However, historical data older than 5 years may be irrelevant due to the non-stationarity of the return distribution. Therefore, from a practical perspective, data-driven portfolio optimisation involves estimated statistics that are subject to potentially significant estimation errors. Such observations have been made in the context of mean-variance optimisation (Lim et al., 2011: 164). Additionally, the conditional value-at-risk (CovAr) measure has emerged as one of the most important metrics that accounts for the level of extreme tail dependence among financial institutions at specific significance levels (P 6-10) (Tobias & Brunnermeier, 2016). $Pr(Xi \leq VaRi\ q) = q\%$

Where (X) is the series of crises of the returns of financial institution (i), and since VaR is positive, a rise in this value signifies an increase in the financial institution's exposure to risk.

2-2. Portfolio Model and Selection: Investment portfolio managers face a set of challenges, including constructing an investment portfolio and adjusting its weightings based on the returns and risks of each asset. The investment portfolio manager conducts this under an advisory, guidance, or portfolio management contract (Lee & Sohn, 2023: 44). Besides, modern portfolio theory seeks to allocate assets to maximise the expected risk premium per unit of risk, within the framework of mean and variance. Focusing on standard deviation means that investors weigh the probability of negative returns equally with that of positive returns, which is inconsistent with the reality that returns are typically abnormal, skewed, and have a heavy tail. Furthermore, there is ample evidence indicating that losses and gains are often treated asymmetrically. In response to these problems, optimal portfolio selection models have been developed to maximise expected returns while accounting for a risk constraint rather than standard deviation. These models are better suited to the individual perception of risk and the constraints encountered by management (Scheller & Auer, 2018: 2009).

It is worth noting that the problem of maximising the total expected utility of consumption or wealth over a given time period [0, T] is often formulated at time t = 0, when the investor has initial wealth X0. The problem is how to allocate funds between investment and/or consumption over a given time period (Yiu, 2004: 1219). Hence, the portfolio allocation problem is to allocate assets and determine the amount to borrow or lend to achieve the maximum expected level of final wealth. Specialists introduce a constraint into the basic portfolio selection problem that focuses on the desired level of the Risk (VaR) dollar value. Therefore, the constraint takes the following form (Scheller & Auer, 2018: 2009):

(1- α) }≤w(0) – w(T,p) ≥ VaR{Po

Where P0 (0) stands for the regular probability conditional on the information available at time 0 for portfolio p, and thus the constraint states that the probability of a loss exceeding the VaR level set by the investor should not be greater than one minus the confidence level α, for the optimal portfolio, as it maintains the condition of the previous equation, which will directly reflect investors' risk aversion because it is related to both the VaR level and the associated confidence level (Scheller & Auer, 2018: 2009).

2-2-1. Building an Ideal Investment Portfolio: The basis of portfolio construction concerns rational financial decisions that balance return and risk. The essence of portfolio theory revolves around the impact of considered diversification. An investor can choose his optimal portfolio by identifying his indifference curves in the form of the efficient set, then selecting the portfolio located in the northwest corner of the investor's indifference curves, as it lies on the efficient frontier and within the feasible region of available portfolios (Alexander et al., 2001: 149). On the efficient frontier for investment portfolios, whether composed of two or more assets, the optimal portfolio is the possible portfolio along a straight line for two assets or a curve connecting two assets for more than two assets (Brigham & Ehrhardt, 2014: 983). In line with the framework of many empirical studies and in accordance with the previous constraint that permits the establishment of a rule for selecting an optimal investment portfolio, and since the expected return on an investment portfolio is r(p) for portfolio p in period T, the expected final wealth from investing in p is:

$$E0(W(T, p)) = (W(0) + B)(1 + r(p)) - B(1 + r f)$$

3. **Developing Hypotheses:** Investors seek investment returns commensurate with their acceptable risk profile. Building and diversifying an investment portfolio is one way to reduce risk. Conventionally, investment risk is measured through standard deviation or variance. With the adoption of more appropriate models and approaches, CVaR has settled into a more effective method for capturing Risk after VaR, which is used as a constraint within an investment portfolio. Therefore, the study developed a basic hypothesis centred on the following question: Is CVaR a reliable measure of risk, and does it constitute a constraint in constructing the optimal investment portfolio?

4. Methodology

Sample and Measurement: In this study, reliance was placed on returns calculated from the closing prices of a sample of companies listed on the Iraq Stock Exchange for the period 2012-2024. At various confidence levels of 99%, 95%, and 90%, to calculate both VaR and CVaR, in addition to the main indicators (realised profits and losses) for each stock within the investment portfolio and according to the periods, the period was divided into thirteen periods and investment portfolios were built with a value of 10 million dinars for each portfolio for a period of time, from 11 shares of the companies' shares with equal weights and (12) observations to arrive at results that prove or deny the main hypothesis of the study. Appendix (1) displays the descriptive statistics for the profits and losses of the investment portfolios, multiplied by the value of each portfolio, for each holding period, in preparation for their inclusion in the VaR calculation. Table (1) brings to light some descriptive statistics for both VaR and CVaR for the shares of the 11 companies in the study sample and for the 13 periods extracted from Appendix (1), which encompasses the values of profits and losses for the companies' shares.

Table (1): Descriptive Statistics for both VaR and CVaR

		VaR	VaR AMOUNTS	CVaR	VaR AMOUNTS
Confidence Level 90%	MAX	-2,834,733	-28,347,334,768	-5,144,206	-51,442,059,248
	MIN	-11,762,234	-117,622,338,691	-16,790,196	-167,901,959,515
	AVERAGE	-7,138,710	-71,387,099,347	-9,557,120	-95,571,197,651
	SD	3,032,628	30,326,281,364	3,805,321	38,053,205,446
Confidence Level 95%	MAX	-4,926,721	-49,267,208,075	-6,447,384	-64,473,844,233
	MIN	-16,313,150	-163,131,501,187	-21,560,654	-215,606,542,791
	AVERAGE	-9,338,893	-93,388,927,445	-11,739,390	-117,393,899,707
	SD	3,717,504	37,175,037,615	4,837,521	48,375,205,246
Confidence Level 99%	MAX	-6,270,121	-62,701,206,288	-6,447,384	-64,473,844,233
	MIN	-20,511,153	-205,111,534,471	-21,560,654	-215,606,542,791
	AVERAGE	-11,259,291	-112,592,905,255	-11,739,390	-117,393,899,707
	SD	4,591,061	45,910,607,928	4,837,521	48,375,205,246

The researcher prepares the Table based on EXCEL results.

5. Results: In this part, an analysis of both VaR and CVaR will be carried out according to the three confidence levels: 90%, 95%, and 99%, as follows:

Value at Risk VaR

As far as historical simulations of investment portfolio returns are concerned, it turned out to be clear, through the data in Table (2), which displays the 13 periods under which the investment portfolios for the 11 companies' stocks were constructed, that the largest value at risk achieved during these periods was in the third period, at -11,762 thousand VaR indicates that losses will exceed this threshold with a probability of (1−α), meaning the maximum loss incurred by the investment portfolio at a 90% confidence level. Therefore, there is a 10% probability that the portfolio will incur losses greater than -11,762 thousand. This is because when an amount of 10 million dinars is invested within the third period (portfolio), this is the maximum loss that will be incurred within this probability. At a 95% confidence level and a 5% significance level, and based on the data in Table 3, we also find that the third portfolio had the largest VaR, which amounted to -16,313 thousand. Thus, there is a 5% probability that this portfolio will incur losses greater than this value. Table 4 also showed the VaR at a 99% confidence level and a 1% probability that the third portfolio within the periods achieved the largest VaR, which amounted to -20,511 thousand, indicating that there is only a 1% probability that this portfolio will incur losses greater than -20,511 thousand.

The lowest loss the investment portfolios will encounter during the 13th period will be in the second period, as the value-at-risk reached -2,835 thousand, indicating the lowest losses the investment portfolios will face at a 90% confidence level. Therefore, there is a 10% probability that this portfolio will incur a loss greater than -2,835 thousand. Therefore, investing 10 million dinars in the second portfolio will incur the lowest loss among the portfolios (periods), according to the data in the Table. At a 95% confidence level and a 5% significance level, we also find that the second portfolio had a VaR of -4,927 thousand. Therefore, there is only a 5% probability that the second portfolio will incur losses exceeding the minimum of -4,927 thousand, according to Table 3. The second portfolio also achieved the lowest VaR of -6,270 thousand at a 99% confidence level. Consequently, there is only a 1% probability that this portfolio will incur losses greater than the minimum of -6,270 thousand, as referred to in the results of Table 4.

Conditional Value at Risk (CVaR): In this study, the data in Table 2 and the section on the results of calculating the Conditional Value at Risk (CVaR) show that there was no limit on the loss ceiling, unlike Value at Risk (VaR).

Rather, we calculate the actual loss of investment portfolios when the loss exceeds the VaR limit. The results are presented based on CVaR, historical simulations, and various confidence levels. At a 90% confidence level, the third portfolio had the largest actual loss under CVaR, amounting to -16,790 thousand. Therefore, the expected losses within this portfolio are very high in the worst cases, meaning that risks are high in crises and losses are large in bad market conditions. There is only a 10% probability that the third portfolio's losses will exceed this loss. As is the case with VaR, the third investment portfolio also achieved the largest CVaR at a 95% confidence level of 21,561 thousand, meaning that there is only a 5% probability that the third portfolio will achieve a CVaR greater than 21,561 thousand, as shown in Table (3), and also in Table (4), and at a 99% confidence level, the third portfolio achieved the largest CVaR of 21,561 thousand. What is remarkable here, upon examining the calculation process, is that some values, specifically at the 95% and 99% confidence levels, are equal. There are several causes, including a small number of maximum values, the maximum loss occurring before reaching a 95% confidence level, the absence of additional losses after a certain limit, or a short distribution in the tail (bell) of CVaR.

The lowest CVaR at the 90% confidence level, realised across the 13 periods, was also in the second period, at -5,144 thousand, as shown in Table 2. Hence, there is only a 10% probability that losses will exceed the minimum limit within this portfolio. That is, the second portfolio will incur a loss estimated at -5,144 thousand when the investment portfolio value is 10 million dinars. There is only a 10% probability that this value will be exceeded. This represents that the risks are relatively low even in times of crises. Therefore, the portfolio is safer and more stable compared to the remaining portfolios. As for the 95% and 99% confidence levels, the situation and reasons are the same as for the highest value. The second portfolio achieved the lowest value, reaching -6,447 thousand. Therefore, there is a 5% and 1% probability, respectively, that the second portfolio will incur losses exceeding the minimum limit, as shown in Tables 4 and 3. The tables also included the various confidence levels, VaR AMOUNTS, and CVaR AMOUNTS, whose rankings do not differ across investment portfolios, as both VaR and CVaR are expressed as a percentage of the portfolio's capital, i.e., the portfolio's value.

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Table (2): Values at Risk for the Periods Group, 90% Confidence Level

period / V	VaR	VaR AMOUNTS	CVaR	CVaR AMOUNTS
period 1	-8,599,275	-85,992,748,154	-14,594,355	-145,943,546,632
period 2	-2,834,733	-28,347,334,768	-5,144,206	-51,442,059,248
period 3	-11,762,234	-117,622,338,691	-16,790,196	-167,901,959,515
period 4	-10,495,935	-104,959,351,307	-11,688,310	-116,883,099,152
period 5	-8,288,636	-82,886,356,131	-10,311,714	-103,117,136,596
period 6	-9,810,521	-98,105,205,057	-13,282,801	-132,828,006,159
period 7	-9,881,238	-98,812,376,384	-11,578,127	-115,781,268,802
period 8	-6,193,664	-61,936,635,146	-7,820,281	-78,202,813,676
period 9	-3,414,897	-34,148,971,675	-5,694,524	-56,945,235,000
period 10	-4,675,645	-46,756,450,769	-5,641,640	-56,416,399,028
period 11	-4,981,577	-49,815,766,248	-6,847,534	-68,475,344,024
period 12	-3,414,080	-34,140,799,871	-6,042,714	-60,427,138,554
period 13	-8,450,796	-84,507,957,308	-8,806,156	-88,061,562,075

The researcher prepared the Table based on EXCEL results.

The following figure shows the degree of agreement or discrepancy between VaR and CVaR, at a 90% confidence level.

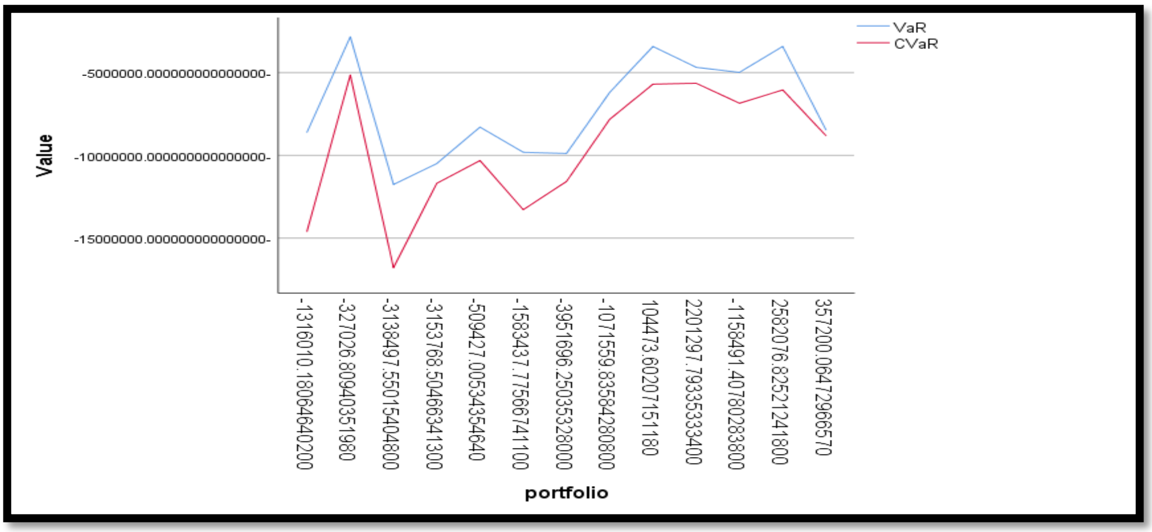

Figure prepared by the researcher based on the values data and using the SPSS statistical program.

Table (3): Values at Risk for the Period Group at the 95% Confidence Level.

period	VaR	VaR AMOUNTS	CVaR	VaR AMOUNTS
period 1	-14,064,451	-140,644,512,425	-19,893,389	-198,933,888,704
period 2	-4,926,721	-49,267,208,075	-7,319,057	-73,190,570,984
period 3	-16,313,150	-163,131,501,187	-21,560,654	-215,606,542,792

Tikrit Journal of Administrative and Economic Sciences, Vol. 22, No. 73, Part (2): 735-755
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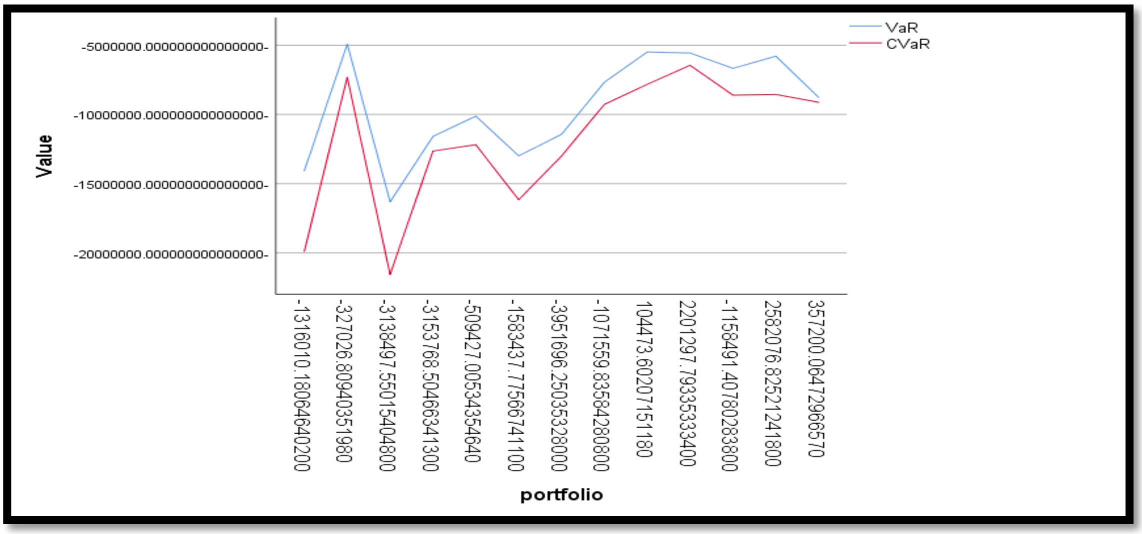
period	VaR	VaR AMOUNTS	CVaR	VaR AMOUNTS
period 4	-11,592,661	-115,926,605,161	-12,644,804	-126,448,039,070
period 5	-10,124,362	-101,243,618,240	-12,185,232	-121,852,320,153
period 6	-12,994,743	-129,947,434,242	-16,163,373	-161,633,725,329
period 7	-11,435,525	-114,355,245,774	-13,004,150	-130,041,499,083
period 8	-7,674,830	-76,748,300,599	-9,274,794	-92,747,944,444
period 9	-5,481,493	-54,814,929,713	-7,824,830	-78,248,298,870
period 10	-5,561,065	-55,610,654,507	-6,447,384	-64,473,844,233
period 11	-6,671,317	-66,713,174,803	-8,609,704	-86,097,036,227
period 12	-5,791,144	-57,911,435,635	-8,558,417	-85,584,167,751
period 13	-8,774,144	-87,741,436,426	-9,126,282	-91,262,818,557

The researcher prepared the Table based on Excel results.

The following figure shows the degree of congruence or variance between VaR and CVaR at the 95% confidence level.



The researcher prepared the figure based on the value data using SPSS.

Table (4): Values at Risk for the Periods Group at the 99% confidence level

period	VaR	VaR AMOUNTS	CVaR	VaR AMOUNTS
period 1	-18,727,601	-187,276,013,449	-19,893,389	-198,933,888,704
period 2	-6,840,590	-68,405,898,402	-7,319,057	-73,190,570,984
period 3	-20,511,153	-205,111,534,471	-21,560,654	-215,606,542,792
period 4	-12,434,375	-124,343,752,289	-12,644,804	-126,448,039,070
period 5	-11,773,058	-117,730,579,771	-12,185,232	-121,852,320,153
period 6	-15,529,647	-155,296,467,112	-16,163,373	-161,633,725,329
period 7	-12,690,425	-126,904,248,421	-13,004,150	-130,041,499,083
period 8	-8,954,802	-89,548,015,675	-9,274,794	-92,747,944,444

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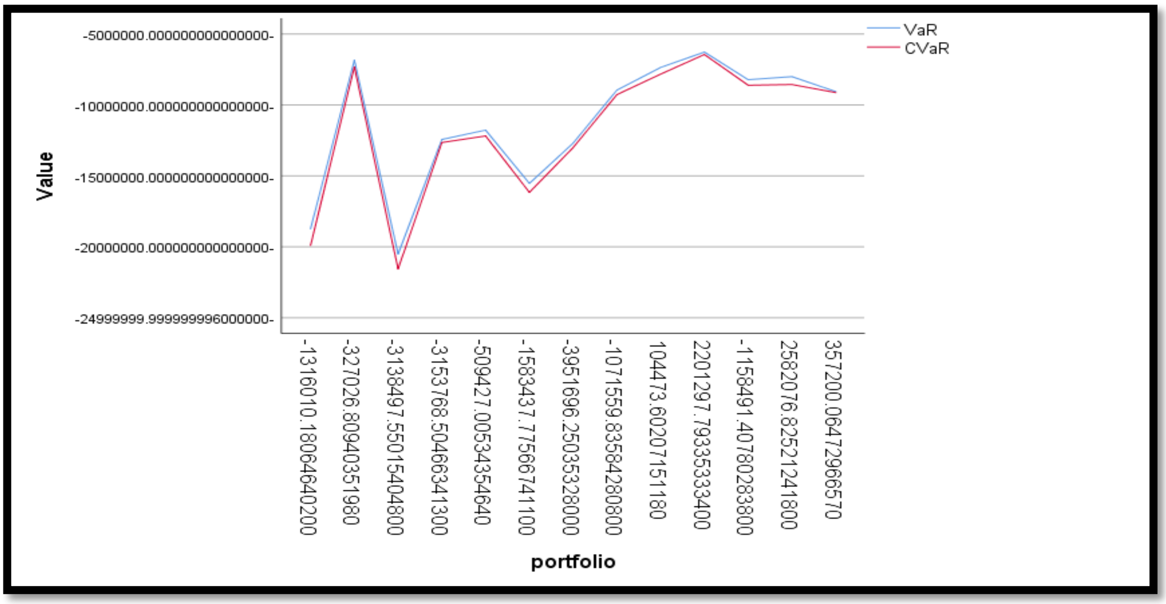
period	VaR	VaR AMOUNTS	CVaR	VaR AMOUNTS
period 9	-7,356,163	-73,561,625,038	-7,824,830	-78,248,298,870
period 10	-6,270,121	-62,701,206,288	-6,447,384	-64,473,844,233
period 11	-8,222,026	-82,220,263,942	-8,609,704	-86,097,036,227
period 12	-8,004,962	-80,049,621,328	-8,558,417	-85,584,167,751
period 13	-9,055,854	-90,558,542,131	-9,126,282	-91,262,818,557

The researcher prepared the Table based on EXCEL results.

The following figure discloses the degree of agreement or discrepancy between VaR and CVaR at a 99% confidence level.



The researcher prepared the figure based on the value data using SPSS.

Discussion: Through calculating both the Value at Risk (VaR) and the Conditional Value at Risk (CVaR) for different confidence levels (90%, 95%, and 99%), it turned out to be clear that the Conditional Value at Risk (CVaR) speaks for a constraint in constructing optimal investment portfolios. Tables (1, 2, 3, and 4) indicate that the Conditional Value at Risk (CVaR) now symbolises a greater amount of loss than the Value at Risk (VaR). Therefore, the average Value at Risk (VaR), i.e., the potential loss at the 90% level, is put into 9,557,120 dinars. The average at the 95% level was 11,739,390 dinars, while at the 99% level, it was 11,739,390 dinars. Therefore, the main hypothesis of the study, which states that the Conditional Value at Risk (CVaR) is a reliable measure of risk and, consequently, serves as a constraint on the risk of the optimal investment portfolio, is accepted, while the alternative hypothesis is rejected. On that ground, investment specialists, especially investment portfolio managers, must adopt techniques and models

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to calculate and estimate various aspects of risk in investment decisions comprising the investment portfolio.

Descriptive Statistics for Profits and Losses of the name of the investment portfolios during each holding period as a percentage of the value of the investment portfolio.

Period	V/Stock	Stock1	Stock2	Stock3	Stock4	Stock5	Stock6	Stock7	Stock8	Stock9	Stock10	Stock11
1	MAX	1,612,681	914,923	1,264,841	884,110	820,132	2,330,939	1,392,621	636,257	1,119,179	914,342	1,435,482
	MIN	-5,108,256	-2,148,140	-2,461,331	-806,889	-336,301	-1,775,483	-1,064,835	-3,677,248	-1,521,918	-1,112,256	-4,752,347
	AVERAGE	-355,432	-316,480	-7,610	-39,690	0	181,878	-52,100	-228,241	-9,862	51,036	-549,371
2	MAX	1,656,047	833,816	1,442,496	1,449,340	859,424	833,816	941,872	1,335,314	1,758,907	1,397,619	1,230,601
	MIN	-2,310,632	-2,170,645	-1,885,912	-702,043	-3,051,567	-843,811	-1,283,812	-1,185,597	-1,887,941	-1,033,784	-2,231,436
	AVERAGE	119,441	-366,076	40,633	263,771	-239,683	-1,242	-179,803	52,879	-19,295	-27,158	61,525
3	MAX	2,894,132	1,773,340	1,407,496	5,358,269	2,316,320	1,289,704	2,180,022	1,076,307	3,028,110	1,139,443	1,231,327
	MIN	-3,990,753	-2,809,024	-2,605,311	-4,744,580	-2,261,242	-1,053,605	-1,209,526	-5,978,370	-1,636,294	-943,107	-2,309,056
	AVERAGE	-629,917	-418,933	-185,430	-539,893	-282,241	-96	-3,506	-665,533	70,124	21,341	-214,076
4	MAX	2,100,715	3,364,722	1,631,953	2,052,631	1,473,247	540,672	1,793,409	2,651,078	1,970,172	1,127,955	2,014,217
	MIN	-3,101,549	-3,249,891	-2,363,888	-976,090	-2,363,888	-565,704	-2,570,451	-3,545,450	-2,541,757	-2,181,560	-1,973,594
	AVERAGE	-491,857	-533,581	-451,175	-14,537	-387,928	-39,464	-428,184	-261,093	-373,434	-295,022	-232,814
5	MAX	3,610,133	1,967,103	1,372,011	1,216,969	1,112,256	0	1,941,560	1,251,631	1,872,115	1,743,534	1,119,179
	MIN	-1,112,256	-1,603,427	-1,625,189	-930,904	-1,582,240	-540,672	-2,058,521	-2,411,621	-3,036,824	-2,682,640	-1,905,183
	AVERAGE	146,742	-177,145	-130,985	98,275	91,412	-44,626	-213,583	-151,334	-254,696	-118,973	-211,228
6	MAX	1,251,631	2,548,922	1,823,216	1,053,605	2,559,334	0	3,237,871	2,992,429	3,639,654	971,637	2,876,821
	MIN	-759,859	-1,941,560	-2,177,235	-1,521,918	-2,047,944	-44,626	-1,670,541	-3,001,046	-1,133,287	-1,686,227	-2,876,821
	AVERAGE	-64,082	-243,237	-120,153	-166,034	22,893	-3,433	-112,709	-169,989	85,466	-283,517	-376,101
7	MAX	416,727	689,929	1,625,189	1,579,030	889,475	112,996	1,431,008	800,427	5,663,955	909,718	3,361,087
	MIN	-2,929,871	-2,513,144	-1,967,103	-2,699,196	-870,114	-224,729	-1,431,008	-4,054,651	-3,591,410	-1,251,631	-3,718,267
	AVERAGE	-413,105	-740,456	-522,957	-186,859	-30,295	-264	-213,057	-774,921	-242,493	-333,705	-548,743
8	MAX	1,236,140	2,711,528	741,080	1,823,216	953,102	0	909,718	0	3,302,417	3,513,979	4,855,078
	MIN	-526,437	-3,889,358	-1,431,008	-2,231,436	-759,859	-1,358,015	-1,000,835	-1,670,541	-2,076,394	-2,513,144	-1,941,560
	AVERAGE	23,854	-250,277	-314,593	32,978	-18,874	-436,931	16,349	-261,428	430,972	-53,645	-16,133
9	MAX	1,372,011	2,876,821	741,080	531,098	476,280	0	2,513,144	1,053,605	3,170,960	1,743,534	2,513,144
	MIN	-1,112,256	-1,541,507	-741,080	-1,198,012	-930,904	-1,076,307	-1,643,031	-1,053,605	-2,199,384	-833,816	-1,957,446
	AVERAGE	-34,802	83,464	-24,199	-121,803	-35,645	-220,492	119,835	120,138	349,240	-127,467	239,047
10	MAX	2,336,149	6,190,392	1,941,560	512,933	3,839,589	408,220	2,787,134	4,595,323	2,876,821	1,431,008	5,260,931
	MIN	-1,100,009	-1,670,541	-1,397,619	-702,043	-2,876,821	-512,933	-723,207	-1,335,314	-4,480,247	-1,053,605	-500,104
	AVERAGE	-61,128	251,385	51,210	-94,928	261,062	-56,417	356,899	402,184	250,246	194,582	726,970
11	MAX	870,114	251,385	1,823,216	1,177,830	261,062	2,744,368	723,207	606,246	2,901,543	714,590	1,808,189
	MIN	-779,615	-2,231,436	-645,385	-674,413	-582,689	-3,877,655	-1,397,619	-1,335,314	-2,513,144	-984,401	-1,836,647
	AVERAGE	-46,292	-328,344	53,584	142,935	-145,389	-537,530	-45,862	-243,428	-66,676	-38,104	275,345
12	MAX	3,593,740	1,541,507	4,519,851	3,029,495	2,744,368	2,231,436	2,947,995	1,670,541	3,469,986	5,245,245	3,453,112
	MIN	-1,022,788	-1,541,507	-1,541,507	-855,222	-1,315,764	-1,431,008	-1,124,780	-953,102	-1,823,216	-3,237,871	-1,743,534
	AVERAGE	601,000	77,459	94,724	637,540	269,311	-333,264	34,003	-137,303	604,549	327,066	742,682
13	MAX	3,400,823	1,177,830	8,803,587	2,496,547	2,006,707	2,231,436	1,053,605	1,335,314	2,246,462	3,528,214	3,199,429
	MIN	-1,201,443	-1,335,314	-2,478,362	-3,091,045	-1,448,309	-2,231,436	-2,231,436	-2,513,144	-1,376,214	-1,957,446	-3,101,549
	AVERAGE	93,438	-96,758	694,839	315,676	192,365	-227,454	-256,209	-322,458	380,246	-90,443	204,668

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