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An economic and econometric analysis of the wheat production function in Samarra District during the 2023 production season

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Abstract: This research aims to estimate the wheat production function in Samarra district using field data for a sample of wheat farmers in Samarra district amounting to (65) farms with sprinkler irrigation (40 sprinklers) representing (8%) of the sample population in order to analyse the impact of different production factors on production quantity, and determine the optimal combination of resources to maximise profits. The research assumed the existence of a positive, statistically significant relationship between wheat production quantity and several production variables, including: quantity of seeds, quantity of fertiliser, number of hours of human work, number of hours of automated work, quantity of irrigation water, and cost of pesticides. The traditional production function was used in its logarithmic form and analysed standardly using the ordinary least squares (OLS) method. The estimated model demonstrated statistical and standard significance, with an adjusted coefficient of determination of 0.812, indicating that the production variables can explain 81% of the changes in production entered into the model. The optimal combination of production elements was also reached, which achieves a production of (1046.7 kg/dunum) and a net profit of (401,590.4 dinars/dunum), which is much higher than the actual profit achieved. The research concluded that wheat farms in the research area operate at the first stage of the production function (decreasing returns), suggesting the possibility of improving efficiency through resource redistribution. There is also a clear gap between the actual and optimal use of production elements. The research recommended directing farmers to use optimal combinations of production inputs to achieve the highest production and profitability.

تحليل اقتصادي وقياسي لدالة إنتاج القمح في قضاء سامراء خلال موسم الإنتاج لعام 2023

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المستخلص

يهدف هذا البحث إلى تقدير دالة إنتاج محصول القمح في قضاء سامراء باستخدام بيانات ميدانية لعينة من مزارعي محصول القمح في قضاء سامراء بلغت (65) مزرعة للري بالرش (مرشة) شكلت نسبة (8%) من مجتمع العينة من أجل تحليل أثر العوامل الإنتاجية المختلفة على كمية الإنتاج، وتحديد التوليفة المثلى للموارد بهدف تعظيم الأرباح. وافترض البحث وجود علاقة موجبة ذات دلالة إحصائية بين كمية إنتاج القمح وبعض المتغيرات الإنتاجية مثل: كمية البذار، كمية السماد، عدد ساعات العمل البشري، عدد ساعات العمل الآلي، كمية مياه الري، وتكلفة المبيدات. تم استخدام دالة الإنتاج التقليدية بصيغتها اللوغاريتمية وتحليلها قياسياً باستخدام طريقة المربعات الصغرى العادية (OLS). أثبت النموذج المقدر معنويته الإحصائية والقياسية، حيث بلغ معامل التحديد المعدل (0.812)، مما يدل على أن 81% من التغيرات في الإنتاج يمكن تفسيرها من خلال المتغيرات الإنتاجية المدخلة في النموذج. كما تم التوصل إلى التوليفة المثلى للعناصر الإنتاجية والتي تحقق إنتاجاً قدره (1046.7) كغم/دونم (وربما صافياً يبلغ 401,590.4 دينار/دونم، وهو أعلى بكثير من الربح الفعلي المحقق واستنتج البحث ان مزارع القمح في منطقة البحث تعمل ضمن المرحلة الأولى من دالة الإنتاج (عائد متناقص) مما يشير إلى وجود إمكانية لتحسين الكفاءة من خلال إعادة توزيع الموارد. وكذلك وجود فجوة واضحة بين الاستخدام الفعلي والامثل لعناصر الإنتاج. وأوصى البحث بضرورة توجيه المزارعين لاستخدام التوليفات المثلى للمدخلات الإنتاجية لتحقيق أعلى إنتاج وربحية.

الكلمات المفتاحية: مرونة الإنتاج، التوليفة المثلى، الموارد الإنتاجية.

Introduction

Wheat is a major strategic crop in Iraq, serving as a pillar of food security amid agricultural, climatic, and economic challenges. Salah al-Din Governorate is distinguished by its significant contribution to the local production of this crop; however, productivity remains below the desired level. Wheat production constitutes approximately (50%) of global grain production, which includes, in addition to wheat, barley, rice, yellow corn, white corn, millet, oats, and rye. The rising global demand for wheat and its growth rate, driven by population growth, are matched by a low rate of supply growth. This has exacerbated problems worldwide, especially in food-deficient countries that face a widening food gap. This calls for the development of sound agricultural policies and the implementation of

effective measures to increase grain production, especially wheat, as their situation is heading towards a serious food crisis. Wheat ranks first among cereal crops worldwide, and in Iraq (wheat, barley, rice, and yellow corn). Iraq also has a diverse climate across its territory, with three distinct regions: the north, the centre, and the south. This diversity results in differences in crop profitability, especially wheat. The cultivated area of wheat represents approximately (43.4%) of the average area under agricultural exploitation and approximately (50.11%) of the area cultivated with grains. This calls for analysing the factors determining production using economic and econometric tools to diagnose shortcomings and determine the optimal resource combination.

Research Problem: Wheat is a major strategic crop that contributes to achieving food security in Iraq, particularly in the Samarra District, a key agricultural area in Salah al-Din Governorate. Despite efforts to increase production, the situation remains characterised by low economic efficiency in the use of productive resources, negatively impacting agricultural production and income levels.

The problem is that many farmers do not rely on scientific principles when allocating resources, leading to waste of production inputs such as seeds, fertilisers, water, and labour, and preventing them from achieving optimal production levels that maximise profits or minimise costs. The lack of a quantitative (standard) analysis of the production function also limits the ability to assess the impact of each production element.

Hence, the need for a precise economic and standard analysis of the wheat production function in Samarra District emerges. This analysis aims to determine the relationship between production quantity and the variables involved in the production process, to identify the optimal resource combination that achieves the desired economic efficiency and supports production decision-making based on realistic quantitative data.

Significance of the Research: The importance of this research stems from the economic and social significance of wheat, which serves as the cornerstone of food security in Iraq and is a strategic crop on which the country's agricultural policies depend.

In light of the challenges facing the agricultural sector, such as scarce resources, high production costs, and low production efficiency, this study highlights the efficiency of resource use in wheat cultivation in Samarra

District, one of the most prominent agricultural districts in Salah al-Din Governorate.

The importance of the study also lies in its ability to:

1. Provide a quantitative (standard) tool to aid agricultural decision-making by determining the impact of each production element on the quantity of output.
2. Contribute to identifying the optimal combination of inputs that achieves the highest possible productivity, thereby increasing profitability and reducing resource waste.
3. Provide agricultural decision-makers with quantitative indicators that help direct technical, financial, and advisory support more efficiently.
4. Fill a local research gap by presenting recent field data (for the year 2023) from an active agricultural area that has not received sufficient modern standardised studies.

Therefore, this research contributes to improving the efficiency of agricultural production and to directing agricultural policies towards more sustainable, productive paths in rural areas.

Research Objectives: The research seeks to achieve the following objectives:

1. Estimating the wheat production function in Samarra District using standard methods.
2. Identifying the most important inputs affecting wheat production, such as land, labour, seeds, irrigation, and fertilisers.
3. Measuring the flexibility of production factors and determining the type of return to scale.
4. Evaluating the technical efficiency of wheat farmers in the study area.
5. Providing policy recommendations to improve productivity and optimal resource utilisation in wheat cultivation.

Research Hypothesis: The study assumes that there is a statistically significant positive relationship between wheat production and certain production variables, including seed quantity, fertiliser quantity, human labour hours, mechanical labour hours, irrigation water quantity, and pesticide costs.

Temporal Boundaries of the Search: The study covers the 2023 agricultural season. Field data was collected and analysed for a sample of 65 wheat farmers in Samarra District, using sprinkler irrigation (40 sprinklers), representing (8%) of the sample population. This study aimed to analyse the

impact of various production factors on output and to determine the optimal resource combination to maximise profits.

Spatial Boundaries of the Search: The study is limited to Samarra District, Salah al-Din Governorate, Iraq, due to its significant agricultural importance and its status as one of the major centres of wheat production in the governorate. The district was selected due to its diverse production conditions and the availability of a wide range of farmers, which enables generalisation of the study results within the governorate.

Study methodology: Cross-sectional data for the 2023 agricultural production season were used. The data were collected from a random sample of wheat farmers in Salah al-Din Governorate through field questionnaires administered to 65 farms using sprinkler irrigation (40 sprinkler farms). All data were entered into SPSS 26 for analysis.

Theoretical Framework

The Concept of Production: Production is a fundamental concept in economics and represents one of the basic pillars of economic activity. It refers to the process of transforming inputs (productive resources) into outputs (goods and services) using specific means and technologies. Production has two main dimensions: the quantitative dimension, which concerns the quantity of outputs, and the economic dimension, which focuses on the efficient use of resources and maximising returns or minimising costs. (Naja and Ayed, 2015: 71) Production is defined economically as: the process by which economic resources (land, labour, capital, organisation) are transformed into products of economic value that satisfy human needs directly or indirectly. Some economists have defined it as creating a new benefit or increasing an existing benefit through a change in form, time, or place, thereby contributing to meeting the demand for goods and services. The basic and actual goal of production theory is to research the conditions of technical and economic efficiency, in addition to other main goals, including finding or increasing an existing benefit, i.e., searching and investigating new benefits that did not exist to meet the needs of individuals. In this regard, benefit can be divided into (spatial benefit) through transporting the commodity from its place of production to its place of consumption or transporting it from a place where its benefit decreases to a place where its benefit increases, (formal benefit) we obtain this benefit from transforming the form of the material from a form that is useless or of little

benefit to a form that is more useful, such as transforming soil elements into a crop.

In contrast, the other type of benefit (Time utility) is obtained by storing crops until a time when the benefit of these goods is greater. In contrast, the last type is (ownership utility), which is the transfer of ownership of the commodity from one person to another who will benefit from it more. (Naji, 2016: 16), And basically the most important main goal of the production process is to reach the maximum possible profit (maximising profits) and this is not achieved unless the production is efficient, as the efficiency of production cannot be measured except through the production function, which we will talk about later in some detail (Hidayah & Susanto, 2013: 623).

The most important types of production functions

The traditional polynomial production function: The term "function" is used in mathematics to indicate the existence of a relationship between the explanatory variables (independent variables) and the dependent variable (Al-Ruwais, 2009: 43). The production function can be defined mathematically as a rule for determining each value in a set of other variables called the "range of the function." It is also known as the mathematical expression between the amount of production factors used to produce a particular commodity and the amount of production obtained (Dwivedi, 2016:184). It is a quantitative or mathematical expression of the relationship between inputs and outputs by clarifying the number of units produced as a function of the units of production factors used. The general formula for the production function is:

$$Y = f(X)$$

Where Y is the dependent variable (volume of production), X is the independent variable (production factor).

The range of the function consists of each level of production (Y) produced by each level of production factors. (X) is used, and the function is used mathematically to indicate the relationship between the explanatory variables (independent) and the dependent variable. (Debertin, 2012: 14). The production function can also be defined as determining the maximum output that can be achieved through the use of a specific or limited amount of production resources, and it is determined according to the available technical knowledge and engineering level. (Samuelson, 2006: 110). The

production function can also be defined as the relationship between a combination of productive resources and the maximum output obtainable from that combination at a specific technological level (Al-Nusour, 2009: 196). The term "function" has been used in mathematics to clarify the relationship between explanatory variables and the dependent variable, i.e., the natural production of a given capacity, over a specific period of time, excluding prices for both inputs and outputs. The production function illustrates the ratios in which the factors of production are combined to produce a product. Therefore, there are several functions equal to the number of ways in which the elements of production can be combined and transformed into output (Hamad, M. S., & Shabib, 2025: 2).

The polynomial production function can be written in the following form:

$$Y = f(X_1, X_2, X_3, X_4 \dots, X_n) \dots \dots \dots 1$$

where Y is the level of production and (X₁, X₂, -----, X_n) The units used from the different production elements, the production volume (Y) is determined according to the employed quantities of production resources (X_n) and this is done by using a combination of production elements while keeping the other elements constant (Al-Ruwais, 2009: 43-44), and the production function can be expressed either in the form of a spreadsheet or a mathematical equation that shows the possible quantities of production that are produced from a group of inputs (production resources) while keeping the other factors constant. The production function can also be expressed graphically (Cary, 2017:96).

The above function can be written in linear form by converting both sides to the natural logarithm:

$$\ln Y = \ln a_0 + a_1 \ln X_1 + a_2 \ln X_2 + a_3 \ln X_3 + a_4 \ln X_4 \dots + a_n \ln X_n \dots \dots \dots -2$$

To obtain economic derivatives and for mathematical ease, Equation 2 is converted to exponential form as follows:

$$Y = a_0 X_1^{a_1} X_2^{a_2} X_3^{a_3} X_4^{a_4} \dots \dots \dots X_n^{a_n} \dots \dots \dots 3$$

From the traditional production function, the optimal quantities of production elements can be extracted:

To extract the optimal quantities, we form an objective function by constraining the cost function and adding it to the production function using the Lagrange multiplier λ, as follows, prepared by the researcher based on (Deberton, 2012: 138-140):

$$Y = a_0 X_1^{a1} X_2^{a2} X_3^{a3} X_4^{a4} \dots X_n^{an} \dots \dots \dots 1$$

$$C = V_1X_1 + V_2X_2 + V_3X_3 + V_4X_4 \dots \dots \dots + V_n X_n \dots \dots \dots 2$$

$$C - V_1X_1 - V_2X_2 - V_3X_3 - V_4X_4 \dots \dots \dots - V_n X_n = 0$$

$$L = a_0 X_1^{a1} X_2^{a2} X_3^{a3} X_4^{a4} \dots \dots \dots X_n^{an} + \lambda(C - V_1X_1 - V_2X_2 - V_3X_3 - V_4X_4 \dots \dots \dots - V_n X_n) \text{ aim function} \dots \dots \dots 3$$

Where L: objective function, a0: constant term, a1, an: function parameters (elasticities of production elements)

λ: Landa (Lagrange multiplier), C: total costs, V1, Vn: prices of production factors.

From the objective function, we take the first partial derivative for each production factor as follows: $\frac{\partial L}{\partial X_1} = a_0 a_1 X_1^{a1-1} X_2^{a2} X_3^{a3} X_4^{a4} \dots \dots \dots X_n^{an} - \lambda V_1 = 0$

$$a_0 a_1 X_1^{a1-1} X_2^{a2} X_3^{a3} X_4^{a4} \dots \dots \dots X_n^{an} = \lambda V_1 \dots \dots \dots 4$$

$$\frac{\partial L}{\partial X_2} = a_0 a_2 X_1^{a1} X_2^{a2-1} X_3^{a3} X_4^{a4} \dots \dots \dots X_n^{an} - \lambda V_2 = 0$$

$$a_0 a_2 X_1^{a1} X_2^{a2-1} X_3^{a3} X_4^{a4} \dots \dots \dots X_n^{an} = \lambda V_2 \dots \dots \dots 5$$

$$\frac{\partial L}{\partial X_3} = a_0 a_3 X_1^{a1} X_2^{a2} X_3^{a3-1} X_4^{a4} \dots \dots \dots X_n^{an} - \lambda V_3 = 0$$

$$a_0 a_3 X_1^{a1} X_2^{a2} X_3^{a3-1} X_4^{a4} \dots \dots \dots X_n^{an} = \lambda V_3 \dots \dots \dots 6$$

$$\frac{\partial L}{\partial X_4} = a_0 a_4 X_1^{a1} X_2^{a2} X_3^{a3} X_4^{a4-1} \dots \dots \dots X_n^{an} - \lambda V_4 = 0$$

$$a_0 a_4 X_1^{a1} X_2^{a2} X_3^{a3} X_4^{a4-1} \dots \dots \dots X_n^{an} = \lambda V_4 \dots \dots \dots 7$$

$$\frac{\partial L}{\partial X_n} = a_0 a_n X_1^{a1} X_2^{a2} X_3^{a3} X_4^{a4} \dots \dots \dots X_n^{an-1} - \lambda V_n = 0$$

$$a_0 a_n X_1^{a1} X_2^{a2} X_3^{a3} X_4^{a4} \dots \dots \dots X_n^{an-1} = \lambda V_n \dots \dots \dots 8$$

$$\frac{\partial L}{\partial \lambda} = C - V_1X_1 - V_2X_2 - V_3X_3 - V_4X_4 \dots \dots \dots - V_n X_n = 0 \dots \dots \dots 9$$

Then we swear. Substitute Equation 4 into Equation 5 to find the expansion path equation and then find the value of X1 in terms of X2 as follows:

$$\frac{a_0 a_1 X_1^{a1-1} X_2^{a2} X_3^{a3} X_4^{a4} \dots \dots \dots X_n^{an}}{a_0 a_2 X_1^{a1} X_2^{a2-1} X_3^{a3} X_4^{a4} \dots \dots \dots X_n^{an}} = \frac{\lambda V_1}{\lambda V_2} \Rightarrow \frac{a_1 X_2}{a_2 X_1} = \frac{V_1}{V_2}$$

$$a_1 X_2 V_2 = a_2 X_1 V_1 \text{ (expansion path equation between X1 and X2)}$$

$$X_1 = \frac{a_1 X_2 V_2}{a_2 V_1} \Rightarrow \therefore V = \frac{a_1 V_2}{a_2 V_1} \Rightarrow \therefore X_1 = V X_2 \dots \dots \dots 10$$

Equation 10 represents the value of X1 in terms of X2

Divide Equation 6 by Equation 5 to find the expansion path equation, then find the value of X3 in terms of X2, and so on for the remaining production factors as follows:

$$\frac{a_0 a_3 X_1^{a_1} X_2^{a_2} X_3^{a_3-1} X_4^{a_4} \dots X_n^{a_n}}{a_0 a_2 X_1^{a_1} X_2^{a_2-1} X_3^{a_3} X_4^{a_4} \dots X_n^{a_n}} = \frac{\lambda V_3}{\lambda V_2} \Rightarrow \frac{a_3 X_2}{a_2 X_3} = \frac{V_3}{V_2}$$

$$a_3 X_2 V_2 = a_2 X_3 V_3 \text{ (expansion path equation between X2 and X3)}$$

$$\Rightarrow X_3 = \frac{a_3 X_2 V_2}{a_2 V_3} \Rightarrow \therefore V = \frac{a_3 V_2}{a_2 V_3} \Rightarrow \therefore X_3 = V X_2 \text{ ----- 11}$$

Equation 11 represents the value of X3 in terms of X2

Divide Equation 7 by Equation 5 and find the equation of the expansion path, then find the value of X4 in terms of X2

$$\frac{a_0 a_4 X_1^{a_1} X_2^{a_2} X_3^{a_3} X_4^{a_4-1} \dots X_n^{a_n}}{a_0 a_2 X_1^{a_1} X_2^{a_2-1} X_3^{a_3} X_4^{a_4} \dots X_n^{a_n}} = \frac{\lambda V_4}{\lambda V_2} \Rightarrow \frac{a_4 X_2}{a_2 X_4} = \frac{V_4}{V_2}$$

$$a_4 X_2 V_2 = a_2 X_4 V_4 \text{ (expansion path equation between X2 and X4)}$$

$$\Rightarrow X_4 = \frac{a_4 X_2 V_2}{a_2 V_4} \Rightarrow \therefore V = \frac{a_4 V_2}{a_2 V_4} \Rightarrow \therefore X_4 = V X_2 \text{ ----- 12}$$

Equation 12 represents the value of X4 in terms of X2

Divide Equation 8 by Equation 5 and find the expansion path equation, then find the value of Xn in terms of X2

$$\frac{a_0 a_n X_1^{a_1} X_2^{a_2} X_3^{a_3} X_4^{a_4} \dots X_n^{a_n-1}}{a_0 a_2 X_1^{a_1} X_2^{a_2-1} X_3^{a_3} X_4^{a_4} \dots X_n^{a_n}} = \frac{\lambda V_n}{\lambda V_2} \Rightarrow \frac{a_n X_2}{a_2 X_n} = \frac{V_n}{V_2}$$

$$a_n X_2 V_2 = a_2 X_n V_n \text{ (expansion path equation between X2 and Xn)}$$

$$\Rightarrow X_n = \frac{a_n X_2 V_2}{a_2 V_n} \Rightarrow \therefore V = \frac{a_n V_2}{a_2 V_n} \Rightarrow \therefore X_n = V X_2 \text{ ----- 13}$$

Equation 13 represents the value of Xn in terms of X2

Now substitute Equations 10, 11, 12, and 13 into Equation 9 (derivative of λ) to find the value of X2:

$$C = V_1(V X_2) + V_2 X_2 + V_3(V X_2) + V_4(V X_2) + \dots + V_n(V X_2)$$

$$C = V X_2 \Rightarrow X_2 = C/V \text{ ----- 14 (X2 is the optimal quantity of)}$$

Substitute Equation 14, which represents the value of X2, into Equation 10 to find the optimal quantity of X1, into Equation 11 to find the optimal quantity of X3, into Equation 12 to find the optimal quantity of X4, and into Equation 13 to find the optimal quantity of Xn.

After finding the optimal quantities of production elements, substitute them into production function 1 to find the optimal quantity of production Y.

Results and Discussion:

Estimating the Traditional Production Function for Wheat Crops in Samarra District for the 2023 Production Season: This study analysed

field data and results from a sample of 130 wheat farmers in Samarra District for the 2023 production season. This was done by estimating their production function to determine the nature of the relationship between crop production per unit area and the production elements included in the estimated function. It also sought to identify which explanatory (independent) factors had the greatest influence on the dependent variable and then to examine that influence across the entire study sample.

To determine the impact and change caused by the independent factors included in the estimated model, (Y), which represents wheat crop production estimated in (kg/dunum), was adopted as a dependent variable in the proposed production function model, providing the most representative picture of the reality of farm work under the conditions facing farmers to produce this crop. As for the independent variables that most influence wheat production, seven explanatory variables were used: cultivated area, amount of nitrogen fertiliser, amount of phosphate fertiliser, amount of seeds, amount of chemical pesticides, hired and family labour, and number of ploughing hours.

Description and formulation of the mathematical model used:

After identifying the dependent and independent variables in the multiple regression equation, we used the ordinary least squares (OLS) method, which is the most widely applied method due to its simplicity and provides the best unbiased linear estimate (BLUE) (Gujarati, 2004: 82).

The model includes several variables, constants, and parameters. The variables represent the elements of production, the intercept indicates the overall technical impact of those elements, and the parameters represent their elasticities. Based on this, we can describe the production function model, which was formulated to study the relationship between production and the factors that affect it. Therefore, the relationship form for the estimated production function is as follows:

$$Y = f(X_1, X_2, X_3, X_4, X_5, X_6, X_7)$$

After subjecting the model to the assumptions of economic Theory and to statistical and econometric tests, it was found to be consistent with economic logic and to pass those tests. In light of this, a standard model was formulated to study the relationship between production and the factors influencing it, as follows (Fraser, 2002:61):

$$\ln Y = \ln B_0 + B_1 \ln X_1 + B_2 \ln X_2 + B_3 \ln X_3 + B_4 \ln X_4 + B_5 \ln X_5 + B_6 \ln X_6 + B_7 \ln X_7 + u_i$$

By studying the various formulas to determine the appropriate relationship among the variables in the production function, the double logarithmic formula above was adopted for its consistency with economic, statistical, and econometric logic. To obtain economic derivatives and for mathematical ease, the above equation was converted from its double-logarithmic form to exponential form as follows:

$$Y = B_0 \cdot X_1^{B_1} \cdot X_2^{B_2} \cdot X_3^{B_3} \cdot X_4^{B_4} \cdot X_5^{B_5} \cdot X_6^{B_6} \cdot X_7^{B_7}$$

Where:

Y: Wheat crop production quantity (kg/dunum).

X1: Seed quantity (kg/dunum).

X2: Nitrogen fertiliser quantity (Urea) (kg/dunum).

X3: Compound phosphate fertiliser quantity (DAP) (kg/dunum).

X4: Number of irrigation hours (hours/dunum).

X5: Quantity of pesticides applied (litres/dunum).

X6: Number of working hours (rented and family) (man/day/dunum).

X7: Number of ploughing hours (hours/dunum).

U_i: Random error term, which includes all variables affecting wheat crop production and not included in the model, such as climatic, environmental, and technical conditions, etc.

Wheat Crop Production Function Results for Sprayer (40): The results of estimating the production function in the study area, as shown in Table (1), show that the estimated parameters of production elements (seed quantity, nitrogen fertiliser quantity, phosphate fertiliser quantity, irrigation hours, pesticide quantity, labor hours (family and hired), and plowing hours) were found to be statistically significant according to the t-test at a significance level of 0.01. The F-test also demonstrated the significance of the estimated model and its suitability for the nature of the data, as the calculated F-value of 204.146 confirmed the model's overall significance. The adjusted coefficient of determination (R^2) value indicates that the independent variables included in the estimated production function explain approximately 81% of the changes in production, and that approximately 19% of these changes are due to other factors not included in the estimated model, whose effects were absorbed by the random variable (U_i).

Table (1): Results of estimating the wheat crop production function for sprayer 40

Variables	Parameters	Statistical tests
C	3.30 (2.407)*	$R^2 = (0.818)$
X ₁	0.066 (3.472)*	$R^2 = (0.812)$
X ₂	0.065 (2.136)*	F = (1623.568)
X ₃	0.494 (1.759)*	D.W = (1.897)
X ₄	0.016 (1.983)*	*t
X ₅	0.046 (2.051)*	
X ₆	0.125 (3.240)*	
X ₇	0.127 (2.229)*	

Source: Prepared by the researcher using SPSS 26 based on questionnaire data.

After the estimated model passed the statistical tests (t, F, R^2), the standard tests were confirmed to overcome the standard problems (second-order problems), and it became clear that the model used met the acceptance criteria and was reliable in explaining the phenomenon under study. The estimated model shows the impact of the aforementioned economic variables on the date of production of the study sample, using a multiple regression model with a double-logarithmic specification. It shows a statistically significant direct relationship between the production quantity and the elements affecting it, represented by each of (the quantity of seeds, the quantity of nitrogen fertiliser, the quantity of phosphate fertiliser, the number of irrigation hours, the quantity of pesticides, the number of working hours (family and hired), and the number of ploughing hours). The production elasticities indicate that an increase of (0.01) in the number of units of each of the production elements entering into the aforementioned production

process, assuming other factors are constant, leads to an increase in production by (0.066%, 0.065%, 0.494%, 0.016%, 0.046%, 0.125%, 0.127%) for each of them, respectively. To know the relative importance of each of these production elements in production, we divide the elasticity of each production element by the total elasticity and multiply it by 100. It is evident that the contribution of phosphate fertiliser, which amounts to (49.4%), is higher than its counterpart, meaning that it has a greater impact than other variables in increasing wheat productivity. This is followed by the contribution of the number of ploughing hours, which amounts to (12.7%), while the number of family and hired work hours ranked third with a contribution of (12.5%). Then, the seed quantity variable comes in fourth place in terms of its positive impact on wheat productivity, with a contribution of (6.6%). Then, nitrogen fertiliser recorded fifth place with a contribution of (6.5%), and then the pesticide quantity variable came in sixth place with a contribution of (4.6%). Therefore, the Directorate of Agriculture in the governorate is required to ensure the provision of high-quality seeds and chemical pesticides in the required quantities to farmers, as these have a significant impact on increasing wheat production in this governorate. Finally, the number of irrigation hours came in at a contribution of (1.6%), which is less influential than other variables in increasing wheat productivity in the research area. Through the total elasticities of the estimated model, which amounts to (0.939), which means that the wheat farms in the research area are operating in the first production stage with decreasing returns to scale, meaning that increasing the quantities of production resources by (0.01) is accompanied by an increase in the quantity of the final wheat crop by a percentage less than (0.939), which means that it provides the possibility of increasing the total production in a decreasing manner when added in fixed proportions with the possibility of expanding production

The optimal combination of production factors that achieves profit-maximising production volume (Sprinkler 40): The optimal combination of production resources (seed quantity, nitrogen fertiliser quantity, phosphate fertiliser quantity, irrigation hours, pesticide quantity, labor hours (family and hired), and plowing hours) that achieves profit-maximising production volume is achieved through the estimated production function and the cost constraint to obtain the objective function (aiming function) (the normal profit function π), as follows:

$$\ln y = 3.30 + 0.066 \ln X_1 + 0.065 \ln X_2 + 0.494 \ln X_3 + 0.016 \ln X_4 \\ + 0.046 \ln X_5 + 0.125 \ln X_6 \\ + 0.127 \ln X_7$$

For mathematical derivations, the function is transformed from double logarithmic form to exponential form as follows:

$$Y = 27.112 X_1^{0.066} X_2^{0.065} X_3^{0.494} X_4^{0.016} X_5^{0.046} X_6^{0.125} X_7^{0.127} \text{ -----(1)}$$

The Lagrange model is derived from the estimated production function, converted to exponential form and linked to the budget constraint equation, which specifies the prices of the productive resources contributing to the production process, using the Lagrange multiplier, whose economic value is equal to 1. The prices of these resources are determined by the prices of the production inputs.

$$\pi = P_y \cdot (A X_1^{b_1} X_2^{b_2} X_3^{b_3} X_4^{b_4} X_5^{b_5} X_6^{b_6} X_7^{b_7}) - \lambda (X_1 P_{X_1} + X_2 P_{X_2} + \\ X_3 P_{X_3} + X_4 P_{X_4} + X_5 P_{X_5} + X_6 P_{X_6} + X_7 P_{X_7} - \bar{C}) \text{ aim equation for profit}$$

$$\pi = 850 (27.112 X_1^{0.066} X_2^{0.065} X_3^{0.494} X_4^{0.016} X_5^{0.046} X_6^{0.125} X_7^{0.127}) \\ - \lambda (850 X_1 + 600 X_2 + 900 X_3 + 6000 X_4 + 20000 X_5 \\ + 15000 X_6 + 25000 X_7 - 488125) \text{ --- (3)}$$

Where:

π : The normal profit function

P_y : The price of the product (the average price of wheat allocated by the state is 850 dinars/kg)

A: The technical factor

$X_1, X_2, X_3, X_4, X_5, X_6, X_7$: The production factors (the quantity of seeds, the quantity of nitrogen fertiliser, the quantity of phosphate fertiliser, the number of irrigation hours, the quantity of pesticides, the number of hours of work (family and hired), the number of ploughing hours)

P_{X_1} : The price of seeds is 850 dinars/kg

P_{X_2} : The price of nitrogen fertiliser (the average price of nitrogen fertiliser is 600 dinars/kg)

P_{X_3} : The price of phosphate fertiliser (the average price of phosphate fertiliser is 900 dinars/kg)

P_{X_4} : The price of irrigation (the average Irrigation cost: 6,000 dinars per dunum)

P_{X_5} : Price of chemical pesticides (average price of chemical pesticides: 20,000 dinars per litre)

PX6: Labour wage (average wage of workers on wheat farms: 15,000 dinars per man per day)

PX7: Price of ploughing hours (average ploughing cost: 25,000 dinars per hour per dunum)

b7, b6, b5, b4, b3, b2, b1: Function parameters (partial elasticities of production factors)

C: Cost (cost per dunum: 488,125 dinars per dunum)

λ: Landa (Lacragrange multiplier)

By applying the profit maximisation condition from the profit function (VMPx = Px), we must derive the profit function for the production factors and Landa X1, X2, X3, X4, X5, X6, X7, λ) as follows:

$$\frac{\partial \pi}{\partial X_1} = (850)(27.112)(0.066) X_1^{-0.934} X_2^{0.065} X_3^{0.494} X_4^{0.016} X_5^{0.046} X_6^{0.125} X_7^{0.127} - 850 \lambda = 0$$

$$= (1520.983 X_1^{-0.934} X_1^{0.066} X_2^{0.065} X_3^{0.494} X_4^{0.016} X_5^{0.046} X_6^{0.125} X_7^{0.127}) - 850 \lambda = 0 \quad (4)$$

$$\frac{\partial \pi}{\partial X_2} = (850)(27.112)(0.065) X_1^{0.066} X_2^{-0.935} X_3^{0.494} X_4^{0.016} X_5^{0.046} X_6^{0.125} X_7^{0.127} - 600 \lambda = 0$$

$$= (1497.938 X_1^{0.066} X_2^{-0.935} X_3^{0.494} X_4^{0.016} X_5^{0.046} X_6^{0.125} X_7^{0.127}) - 600 \lambda = 0 \quad (5)$$

$$\frac{\partial \pi}{\partial X_3} = (850)(27.112)(0.494) X_1^{0.066} X_2^{0.065} X_3^{-0.506} X_4^{0.016} X_5^{0.046} X_6^{0.125} X_7^{0.127} - 900 \lambda = 0$$

$$= (11384.328 X_1^{0.066} X_2^{0.065} X_3^{-0.506} X_4^{0.016} X_5^{0.046} X_6^{0.125} X_7^{0.127}) - 900 \lambda = 0 \quad (6)$$

$$\frac{\partial \pi}{\partial X_4} = (850)(27.112)(0.016) X_1^{0.066} X_2^{0.065} X_3^{0.494} X_4^{-0.984} X_5^{0.046} X_6^{0.125} X_7^{0.127} - 6000 \lambda = 0$$

$$= (368.723 X_1^{0.066} X_2^{0.065} X_3^{0.494} X_4^{-0.954} X_5^{0.046} X_6^{0.125} X_7^{0.127})$$

$$= 6000 \lambda - - - - (7)$$

$$\frac{\partial \pi}{\partial X_5}$$

$$= (850)(27.112)(0.046) X_1^{0.066} X_2^{0.065} X_3^{0.494} X_4^{0.016} X_5^{-0.954} X_6^{0.125} X_7^{0.127})$$

$$- 20000 \lambda = 0$$

$$= (1060.079 X_1^{0.066} X_2^{0.065} X_3^{0.494} X_4^{0.016} X_5^{-0.954} X_6^{0.125} X_7^{0.127})$$

$$= 20000 \lambda - - - - (8)$$

$$\frac{\partial \pi}{\partial X_6}$$

$$= (850)(27.112)(0.125) X_1^{0.066} X_2^{0.065} X_3^{0.494} X_4^{0.016} X_5^{0.046} X_6^{-0.875} X_7^{0.127})$$

$$- 15000 \lambda = 0$$

$$= (2880.65 X_1^{0.066} X_2^{0.065} X_3^{0.494} X_4^{0.016} X_5^{0.046} X_6^{-0.875} X_7^{0.127})$$

$$= 15000 \lambda - - - - (9)$$

$$\frac{\partial \pi}{\partial X_7}$$

$$= (850)(27.112)(0.127) X_1^{0.066} X_2^{0.065} X_3^{0.494} X_4^{0.016} X_5^{0.046} X_6^{0.125} X_7^{-0.437})$$

$$- 25000 \lambda = 0$$

$$= (2926.740 X_1^{0.066} X_2^{0.065} X_3^{0.494} X_4^{0.016} X_5^{0.046} X_6^{0.125} X_7^{-0.437})$$

$$= 25000 \lambda - - - - (10)$$

$$\frac{\partial \pi}{\partial \lambda}$$

$$= (850 X_1 + 600 X_2 + 900 X_3 + 6000 X_4 + 20000 X_5 + 15000 X_6$$

$$+ 25000 X_7 - 488125) = 0 - - - (11)$$

By dividing all equations (4, 6, 7, 8, 9, 10) by equation (5) in order, we obtain the expansion path equation between them as follows:

$$= \frac{(1520.983 X_1^{-0.934} X_1^{0.066} X_2^{0.065} X_3^{0.494} X_4^{0.016} X_5^{0.046} X_6^{0.125} X_7^{0.127})}{(1497.938 X_1^{0.066} X_2^{-0.935} X_3^{0.494} X_4^{0.016} X_5^{0.046} X_6^{0.125} X_7^{0.127})}$$

$$= \frac{850 \lambda}{600 \lambda}$$

$$= \frac{1520.983 X_2}{1497.938 X_1} = \frac{850 \lambda}{600 \lambda} \Rightarrow 912589.8 X_2 = 1273247.3 X_1 \quad \text{expansion path}$$

equation

$$X_1 = 0.716 X_2 \text{ ----- (12)}$$

By dividing equation (6) by equation (5), we obtain the expansion path equation between X3 and X2 as follows:

$$\begin{aligned} &= \frac{(11384.328 X_1^{0.066} X_2^{0.065} X_3^{-0.506} X_4^{0.016} X_5^{0.046} X_6^{0.125} X_7^{0.127})}{(1497.938 X_1^{0.066} X_2^{-0.935} X_3^{0.494} X_4^{0.016} X_5^{0.046} X_6^{0.125} X_7^{0.127})} \\ &= \frac{900 \lambda}{600 \lambda} \\ &= \frac{(11384.328 X_2)}{(1497.938 X_3)} = \frac{900 \lambda}{600 \lambda} \Rightarrow 6830596.8 X_2 = 1348144.2 X_3 \quad \text{expansion} \end{aligned}$$

path equation

$$X_3 = 5.066 X_2 \text{-----} (13)$$

By dividing equation (7) by equation (5), we obtain the expansion path equation between X4 and X2 as follows:

$$\begin{aligned} &= \frac{(368.723 X_1^{0.066} X_2^{0.065} X_3^{0.494} X_4^{-0.954} X_5^{0.046} X_6^{0.125} X_7^{0.127})}{(1497.938 X_1^{0.066} X_2^{-0.935} X_3^{0.494} X_4^{0.016} X_5^{0.046} X_6^{0.125} X_7^{0.127})} \\ &= \frac{6000 \lambda}{600 \lambda} \\ &= \frac{(368.723 X_2)}{(1497.938 X_4)} = \frac{6000 \lambda}{600 \lambda} \Rightarrow 221233.8 X_2 = 8987628 X_4 \quad \text{expansion path} \end{aligned}$$

equation

$$X_4 = 0.024 X_2 \text{-----} (14)$$

By dividing equation (8) by equation (5), we obtain the expansion path equation between X5 and X2 as follows:

$$\begin{aligned} &= \frac{(1060.079 X_1^{0.066} X_2^{0.065} X_3^{0.494} X_4^{0.016} X_5^{-0.954} X_6^{0.125} X_7^{0.127})}{(1497.938 X_1^{0.066} X_2^{-0.935} X_3^{0.494} X_4^{0.016} X_5^{0.046} X_6^{0.125} X_7^{0.127})} \\ &= \frac{20000 \lambda}{600 \lambda} \\ &= \frac{(1060.079 X_2)}{(1497.938 X_5)} = \frac{20000 \lambda}{600 \lambda} \Rightarrow 636047.4 X_2 = 29958760 X_5 \quad \text{expansion} \end{aligned}$$

path equation

$$X_5 = 0.0212 X_2 \text{-----} (15)$$

By dividing equation (9) by equation (5), we obtain the expansion path equation between X6 and X2 as follows:

$$\begin{aligned} &= \frac{(2880.65 X_1^{0.066} X_2^{0.065} X_3^{0.494} X_4^{0.016} X_5^{0.046} X_6^{-0.875} X_7^{0.127})}{(1497.938 X_1^{0.066} X_2^{-0.935} X_3^{0.494} X_4^{0.016} X_5^{0.046} X_6^{0.125} X_7^{0.127})} \\ &= \frac{15000 \lambda}{600 \lambda} \end{aligned}$$

$$= \frac{(2880.65 X_2)}{(1497.938 X_6)} = \frac{15000 \lambda}{600 \lambda} \Rightarrow 1728390 X_2 = 22469070 X_6 \quad \text{expansion}$$

path equation

$$X_6 = 0.076 X_2 \text{-----} (16)$$

By dividing equation (10) by equation (5), we obtain the expansion path equation between X7 and X2 as follows:

$$\begin{aligned} &= \frac{(2926.740 X_1^{0.066} X_2^{0.065} X_3^{0.494} X_4^{0.016} X_5^{0.046} X_6^{0.125} X_7^{-0.437})}{(1497.938 X_1^{0.066} X_2^{-0.935} X_3^{0.494} X_4^{0.016} X_5^{0.046} X_6^{0.125} X_7^{0.127})} \\ &= \frac{25000 \lambda}{600 \lambda} \\ &= \frac{(2926.740 X_2)}{(1497.938 X_7)} = \frac{25000 \lambda}{600 \lambda} \Rightarrow 1756044 X_2 = 37448450 X_7 \quad \text{expansion path} \end{aligned}$$

equation

$$X_7 = 0.046 X_2 \text{-----} (17)$$

By substituting equations (17, 16, 15, 14, 13, 12) in equation (11), we obtain the optimal amount of nitrogen fertiliser:

$$\begin{aligned} &(850(0.716 X_2) + 600 X_2 + 900(5.066 X_2) + 6000(0.024 X_2) \\ &\quad + 20000(0.021 X_2) + 15000(0.076 X_2) \\ &\quad + 25000(0.046 X_2) - 488125) = 0 \end{aligned}$$

$$608.6 X_2 + 600 X_2 + 4559.4 X_2 + 144 X_2 + 420 X_2 + 1140 X_2 + 1150 X_2 - 488125 = 0$$

$$X_2 = \frac{488125}{8622} = 56.613 \text{ kg/donum}$$

The optimal amount of nitrogen fertilizer to maximize profits

We substitute the optimal amount of nitrogen fertiliser (50.94) in equations (12, 13, 14, 15, 16, 17) to find the optimal amounts of (the amount of seeds, the amount of phosphate fertiliser, the number of irrigation hours, the amount of pesticides, the number of work hours (family and rented), the number of plowing hours) as follows:

$X_1 = 0.716 (56.613) = 40.534$ kg/dunum (The optimal amount of seeds that maximises profits)

$X_3 = 5.066 (56.613) = 286.801$ kg/dunum (The optimal amount of phosphate fertiliser to maximise profits)

)The optimal number of profits maximising returns(Irrigation/dunum $X_4 = 0.246 (56.613) = 13.926$

$X_5 = 0.021 (56.613) = 1.188$ litre/dunum (The optimal amount of pesticides that maximises profits)

$X_6 = 0.076 (56.613) = 4.302$ hour/dunum (The optimal number of working hours that maximise profits)

$X_7 = 0.046 (56.613) = 2.604$ hours/dunum (The optimal number of ploughing hours that maximise profits)

We substitute the optimal quantities of resources that were reached in the estimated production function (1) to obtain the optimal quantity of production that maximises profits as follows:

$$Y = 27.112 (40.534)^{0.066} (56.613)^{0.065} (286.801)^{0.494} (13.926)^{0.016} (1.188)^{0.046} (4.302)^{0.125} (2.604)^{0.127}$$

$$Y = 27.112 (1.276) (1.299) (16.369) (1.043) (1.007) (1.200) (1.129)$$

$$= 1046.724 \text{ kg/dunum (profit-maximising production quantity)}$$

The profit achieved when producing the maximum profits is:

$$\pi = 850 (1046.724) - (850 * 40.534 + 600 * 56.613 + 900 * 286.801 + 6000 * 13.926 + 20000 * 1.188 + 15000 * 4.302 + 25000 * 2.604)$$

$$\pi = 889715.4 - (34453.9 + 33967.8 + 258120.9 + 83556 + 23760 + 64530 + 65100)$$

$$\pi = 889715.4 - 563488.6$$

$$326226.8 = \text{dinars/dunum (maximum profit)}$$

It is clear from the above that the optimal combinations of the above-mentioned production elements reached (40.534 kg, 56.613 kg nitrogen fertiliser), (286.801 kg phosphate fertiliser), (13.926 irrigation), (1.188 liters of pesticides), (4.302 number of hours of family and hired work), (2.604 number of hours of plowing) per dunum, and in order, this leads to an increase in production and its reaching the optimal quantity that maximises profits, amounting to (1046.724) kg/dunum, and thus achieving economic profits amounting to (326226.8) dinars/dunum.

Comparison of the optimal product behaviour with the actual behaviour in the farms of the study area for the production season (2023): The results of the economic analysis of a sample of wheat production farms showed that there was variation in the quantities of all production resources used at different rates, while production costs remained the same. The optimal combinations, compared to the actual quantities used, were as shown in Table: (2)

Table (2): Actual and optimal combinations of profit-maximising resources for the study sample

Details	Actual sample	The optimal combination at the profit-maximising production volume
Total production volume (kg/dunum)	875	1046.724
Seed quantity (kg/dunum)	33	40.534
Nitrogen fertiliser quantity (urea) (kg/dunum)	60	56.613
Compound phosphate fertiliser quantity (DPF) (kg/dunum)	197	286.801
Number of irrigations (1/dunum)	18	13.926
Quantity of chemical pesticides (litres/dunum)	0.747	1.188
Number of workers (man/day)	9	4.302
Number of ploughing hours (hour/dunum)	5	2.604
Revenue (dinar/dunum)	743750	889715.4
Total costs (dinar/dunum)	488125	488125
Net revenue (profit) (dinar/dunum)	255625	401590.4

Source: Prepared by the researcher based on questionnaire data and the production function estimated using SPSS 26.

From the previous Table, it is clear that the optimal combinations at the profit-maximising production volume of the aforementioned production resources amounted to (40,534 kg, 56,613 kg nitrogen fertiliser, 286,801 kg phosphate fertiliser, 13,926 irrigations, 1,188 liters of pesticides, 4,302 hours of family and hired labor, 2,604 hours of plowing) per dunum, respectively, which gives a production volume of (1046) kg/dunum, and a net profit of (401,590.4) dinars/dunum. From the above, it is clear that the profit achieved at the profit-maximising production volume is higher than the actual profits of wheat farmers. Therefore, it is necessary to reduce the use of nitrogen fertiliser (urea) and compound phosphate fertiliser (DAP), and the number of irrigations and workers, on the one hand. On the other hand, it is necessary

to increase the use of seeds and chemical pesticides, as they are important for increasing production and thus maximising farmers' profits.

Suggestions:

1. The results showed that phosphate fertiliser had the greatest impact on production with an elasticity ratio of 0.494, followed by the number of ploughing hours at 0.127, and family and hired labour at 0.125. We conclude that the fertiliser supplier is the most significant supporter of wheat production in Samarra District.
2. Wheat farms in the study area operate at the first stage of the production function (decreasing returns), suggesting potential for improving efficiency through resource reallocation.
3. There is a clear gap between the actual and optimal use of production factors, particularly pesticide use, manual labour, and irrigation, leading to a loss in production and profitability.
4. A maximum profit of (326,226.8) dinars per dunum can be achieved using the optimal combination, compared to an actual profit of only about (255,625) dinars per dunum.

Recommendations:

1. Farmers should be guided to use optimal combinations of production inputs (especially seeds and pesticides) to achieve maximum production and profitability.
2. Over-reliance on labour and irrigation should be reduced, as their actual use exceeds optimal levels, leading to higher costs without significant production gains.
3. Expand agricultural extension programs and training on economic efficiency in resource use, particularly in fertilisation, pest control, and tillage.
4. Secure certified seeds and effective pesticides at reasonable prices to ensure optimal production levels within available resources.
5. Decision-makers can use the results of this analysis to develop targeted local agricultural plans to improve production efficiency and reduce the food gap.

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