Cartesian Product on Fuzzy Ideals of a Ternary -Semigroup

Ahmed Mohammed Shareef

Department of Sciences

College of Basic Education

University of Sumer

Mr_ahamed85@yahoo.com

Abstract

We will talk in this research, we study the cartesian product of fuzzy sets of "a ternary —semigroup" S and we characterize fuzzy ternary - semigroup, fuzzy left [resp. right] ideal and " fuzzy bi-ideal " of S in terms of cartesian product of fuzzy sets of S.

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Key Words: ternary -semigroup, " fuzzy ternary —semigroups ", "fuzzy left (right, lateral) ideal", "fuzzy bi-ideal".

1. Introduction

Los, J. [3] It turns out that any " ternary semigroup " however may be embedded in an " ordinary semigroup " in such a way that the operation in ternary semi groups is an (ternary) extension of the (binary) operation of the containing semigroup. Kim, J. [2], Lyapin, E.S. [4] and Sioson, F.M.[7] have also Talk the properties of " ternary semi groups ". Sen, M.K. [6] de ned the concepts of -semigroup. It is known that -semigroup is a "generalization of semigroup" and the existence of classical notions of semigroups have been extended to -semigroups.

Zadeh, L.A. [9] "introduced the study of fuzzy sets in 1965. Mathematically a fuzzy set on a set X is a mapping into [0,1] of real numbers "; for p in X, (p) is called "the membership of p belonging to X". Ersoy, B.A., Tepecik, A. and Demir, I. [1] studied cartesian product of fuzzy prime ideals of rings. Sujit Kumar Surdar and Sarbani Goswamy [8] "studied cartesian product of fuzzy prime and fuzzy semiprime ideals of semigroups ".

We will talk in this research, we study the concept of cartesian product of fuzzysets of a " ternary —semigroup " S and we characterize " fuzzy —semiring ", " fuzzy left(right, lateral) ideal"and "fuzzy bi-ideal" of S in terms of cartesian product of fuzzy sets of a " ternary —semigroup ".

2. Preliminaries

Definition (2.1): A " ternary —semigroup " is an " algebraic structure " (S ; ;) such that S is a " non-empty set" and : S S S ! S is a " ternary ((p q r) w e) =(p (q r w) e) = p q (r w e) 8 p ; q ; r ; w ; e 2 S ; ; ; ; 2 .

Definition (2.2): A non-empty subset A of a " ternary —semigroup " S is called a " ternary sub — semigroup " of S if A A A A .

A nonempty subset A of a " ternary —semigroup S is called a "left (right, lateral)" ideal of S if S A A ; (A S S A ; S A S A) .

A non-empty subset A of a" ternary —semigroup " S is said to be a " bi-ideal " of S if A S A S A A.

Definition (2.3): Let X be any non-empty set . A mapping : X ! [0; 1]

is called a " fuzzy subset " of X.

Definition (2.4): Let : X ! [0; 1] be any " fuzzy subset ". Then the set f (p)=p 2 Xg is called the image of and is denoted by Im( ). For t 2 [0; 1] , $_t$ = fx 2 X= (x) tg is called a " level subset " of .

Definition (2.5): A fuzzy set of " a ternary —semigroup " S is said to be " fuzzy ternary —semigroup " if for all p ; q ; r 2 S ; ; 2 ,

(p q r) minf (p) ; (q) ; (r)g

Definition (2.6): A fuzzy set of a " ternary —semigroup " S is called " fuzzy left(right, lateral) ideal " of S if for all p ; q ; r 2 R ; ; 2 ,

(p q r) (r)((p) ; (q))

If is both fuzzy (left and right) and lateral ideal of S, is called "fuzzy ideal" of S.

Definition (2.7): A fuzzy set of a " ternary —semigroup " S is said to be " fuzzy bi-ideal " of R if for all p; q; r; x; y 2 R; ; ; ; 2 ,

(p x q y) minf (p); (q) ; (r)g.

3. Cartesian Product on Fuzzy Ideals and Fuzzy " Bi-ideals of a Ternary — Semigroup "

In here section, we study of cartesian product of fuzzy sets of a "ternary —semigroup" S. In all this section,S stands for a " ternary —semigroup " unless otherwise mentioned.

Suppose that S_1andS_2 be two "ternary —semigroups". Then the cartesian product S_1 S_2 becomes a "ternary —semigroup" with the ternary composition (S_1 S_2) (S_1 S_2) ! S_1 S_2 de ned by (p_1 ; q_1) (p_2 ; q_2) (p_3 ; q_3) = (p_1 p_2 p_3 ; q_1 q_2 q_3), 8 (p_1 ; q_1); (p_2 ; q_2); (p_3 ; q_3) 2 S_1 S_2; ; 2 .

Definition (3.1): suppose and be fuzzy sets of S. Then the cartesian product of and is de ned by [((p ; q)) = minf (p) ; (q)g; 8 (p ; q) 2 S S].

Lemma (3.2): If and are fuzzy sets of S, then $( )_t$ = $_{t\,t}$, where t 2 [0 ; 1]

Proof. : Suppose (p ; q) 2 $_{t\,t}$, p 2 $_t$ and q 2 $_t$, (p) t and (q) t , minf (p) ; (q)g t , (p ; q) 2 ( )$_t$. Thus ( )$_t$ = $_{t\,t}$.

Theorem(3.3): If and are " fuzzy ternary -semigroups " of S, hence

is a " fuzzy ternary —semigroup " of S S.

Proof. : Suppose and are "fuzzy ternary —semigroups" of S.
suppose that (p ; q) ; (r ; w) ; (e ; f) 2 S S; ; 2 .

Now,

[((p ; q) (r ; w) (e ; f)) = ((p r e ; q w f))]

= minf (p r e) ; (q w f)g

minfminf (p) ; (r) ; (e)g ; minf (q) ; (w) ; (f)gg

= minfminf (p) ; (q)g ; minf (r) ; (w)g ; minf (e) ; (f)gg

= minf (p ; q)) ; (r ; w)) ; ((e ; f))g :

Thus is a " fuzzy ternary -semigroup " of S S.

Theorem (3.4): If and are " fuzzy left(right, lateral) ideals " of S, hence

is a " fuzzy left(right, lateral) ideal " of S S.

Proof. : Suppose and are fuzzy left ideals of S.

suppose that (p ; q) ; (r ; w) ; (e ; f) 2 S S; ; 2 .

Now,

[((p ; q) (r ; w) (e; f)) = ((p r e ; q w f))]

= minf (p r e) ; (q w f)g

minf (e) ; (f)g

= ((e ; f)):

Thus is a " fuzzy left ideal of S " S.

Similarly we prove that fuzzy right ideals and fuzzy lateral ideals also.

Theorem (3.5): If and are " fuzzy bi-ideals of S ", then is a " fuzzy bi-ideal " of S S.

Proof. : Suppose and are" fuzzy bi-ideals " of R.

Let (p ; q) ; (r ; w) ; (e ; f) ; (s ; t) ; (u ; v) 2 S S; ; ; ; 2 Now, [((p ; q) (s ; t) (r ; w) (u ; v) (e ; f)) = ((p s r u e ; q t w v f))]

= minf (p s r u e) ; (q t w v f)g minfminf
(p) ; (r) ; (e)g; minf (q);

(w) ; (f)gg

= minfminf (p) ; (q)g ; minf (r); (w)g; minf
(e) ; (f)gg

= minf ((p; q)) ; ((r; w));

((e; f))g:

Thus is a " fuzzy bi-ideal " of S S.

Theorem (3.6): Let and are " fuzzy ternary -semigroups " of S, hence

[0 ; 1] is a ternaryp ub -semigroup of S S.

Proof. : Suppose is a " fuzzy left ideal " of S S.

suppose that (p ; q); (r ; w); (e ; f) 2 ($)_t$; ; 2 .

) ((p ; q)) t; ((r ; w)) t and ((e ; f)) t.

Now, ((p ; q) (r ; w) (e ; f)) minf (p ; q)); (r ; w)); ((e ; f))g t.

That implies (p ; q) (r ; w) (e ; f) 2 ($)_t$.

Thus ($)_t$ is a " ternary sub -semigroup " of S S.

Conversely. Suppose that ($)_t$ is a "ternary sub —semigroup" of S S.

Let (p ; q) ; (r ; w) ; (e ; f) 2 S S ; ; 2 .

Let ((p ; q)) = t_1, ((r ; w)) = t_2 and ((e ; f)) = t_3, where t_1 ; t_2 ; t_3 2 [0 ; 1] with t_3 t_2 t_1.

Put t = minft_1 ; t_2 ; t_3g.

Then (p ; q) ; (r ; w) ; (e ; f) 2 ($)_t$.

So, (p ; q) (r ; w) (e ; f) 2 ( )$_t$.

That implies ((p ; q) (r ; w) (e ; f)) t = minf (p ; q)); (r ; w)) ;

((e ; f))g.

Thusis a " fuzzy ternary -semigroup " of S S.

Theorem (3.7): Let and are " fuzzy left(right, lateral) ideals" of S, then

is a " fuzzy left(right, lateral) ideal" of S S ↔ the level set ( )$_t$, where t 2 [0; 1] is a " left(right, lateral) idea"l of S S.

Proof. : Suppose is a " fuzzy left ideal " of S S.

Let (p ; q) ; (r ; w) 2 S S ; (e ; f) 2 ( )$_t$; ((e ; f)t.

Now, ((p ; q) (r ; w) (e ; f)) ((e ; f)) t. That implies (p ; q) (r ; w) (e ; f) 2 ( )$_t$.

Thus ( )$_t$ is a " left ideal " of S S.

Conversely. Suppose that ( )$_t$ is a " left ideal " of S S.

Let (p ; q) ; (r ; w) ; (e ; f) 2 S S ; ; 2 .

Let ((p ; q)) = t_1, ((r ; w)) = t_2 and ((e ; f)) = t_3, where t_1 ; t_2 ; t_3 2 [0; 1] with t_3 t_2 t_1.

Put t = minft_1; t_2; t_3

Then (p ; q) ; (r ; w) ; (e; f) 2 ( )$_t$.

So, (p ; q) (r ; w) (e ; f) 2 ( )$_t$.

That implies ((p ; q) (r ; w) (e ; f)) t = ((e ; f)). Thus is a " fuzzy left ideal " of S S.

Similarlly, we prove that fuzzy right ideals and fuzzy lateral ideals also.

Theorem (3.8): Let and are " fuzzy bi-ideals " of S, hence is a " fuzzy bi-ideal " of S S ↔ the level set ( )$_t$, t 2 [0; 1] is a " bi-ideal " of S S.

Proof : Suppose is a " fuzzy bi-ideal " of S S.

Let (p ; q) ; (r ;tw) ; (e ; f) 2 ( )$_t$; (s ; t); (u ; v) 2 S S; ; ; ; 2 .

) ((p ; q)) t, ((r ; w)) t and ((e ; f)) t.

Now,

((p ; q) (s ; t) (r ;tw) (u ; v) (e ; f)) minf ((p ; q)) ; ((r ; w)) ; ((e ; f))g t.

That implies (p ; q) (s ; t) (r ; w) (u ; v) (e ; f) 2 ( )$_t$.

Thus ( )$_t$ is a" bi-ideal " of S S.

Conversely . Suppose that ( )$_t$ is a " bi-ideal " of S S.

Let (p ; q) ; (r ; w) ; (e ; f) ; (s ; t) ; (u ; v) 2 S S ; ; ; ; 2 .

Let ((p ; q)) = t_1, ((r ; w)) = t_2 and ((e ; f)) = t_3, where

t_1; t_2; t_3 2 [0 ; 1] with t_3 t_2 t_1.

Put t = minft_1; t_2; t_3g.

Then (p ; q) ; (r ; w) ; (e ; f) 2 ( )$_t$.

So, (p ; q) (s ; t) (r ; w) (u ; v) (e ; f) 2 ( )$_t$.

That implies

((p ; q) (s ; t) (r ; w) (u ; v) (e ; f)) t = minf ((p ; q)) ; ((r ; w)) ; ((e ;f))g.

Thus is a " fuzzy bi-ideal " of S S.

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