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RESEARCH ARTICLE

Combined Hybrid ARDL-GARCH-BIGRU Model in Analyzing and Forecasting Currency in Circulation Issued by the Central Bank of Iraq

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Abstract

This research examines the dynamics and forecasting performance of currency in circulation (CIC) in Iraq. It uses a theoretical framework that recognizes long-run equilibrium relationships, short-run adjustment, volatility clustering, and nonlinear patterns. These complexities limit the adequacy of traditional linear models and motivate hybrid approaches. Accordingly, a triple hybrid model is employed. This combines autoregressive distributed lag (ARDL), Generalized Autoregressive Conditional Heteroskedasticity (GARCH) with Generalized Error Distribution (GED) to capture heavy-tailed behavior, and Bidirectional Gated Recurrent Unit (BIGRU). Stationarity is assessed using ADF tests. Residual diagnostic tests confirm heteroskedasticity, thereby justifying the inclusion of a GARCH component. The proposed model is compared with ARDL–GARCH and ARDL–BIGRU benchmarks. Empirical results based on RMSE, MAE, and MAPE demonstrate the statistically superior out-of-sample forecasting performance of the triple hybrid model. Bank deposits negatively affect CIC by absorbing liquidity, while withdrawals positively impact CIC, reflecting the persistence of cash-based transactions. These findings highlight the importance of advanced hybrid models for monetary policy and liquidity management in rent-based economies. Future research should include macroeconomic variables and examine the possible impact of Central Bank Digital Currencies (CBDC).

Keywords: Currency in circulation, ARDL model, GARCH model, BIGRU, Hybrid time series models, Deep learning

1. Introduction

Statistics is a cornerstone of quantitative research and economic analysis, providing a systematic framework that converts raw data into interpretable information, thereby supporting evidence-based decision-making, hypothesis testing, and the creation of accurate models to explain intricate economic relationships. In this regard, time-series models have become essential for analyzing sequentially ordered data, e.g., currency in circulation, exchange rates, and inflation rates, by decomposing them into a combination of core components, including trend, seasonality, and stochastic volatility, thereby providing a basis for forecasting and policy-making. Despite the flexibility of the Autoregressive Distributed Lag (ARDL) model when variables of mixed order of integration (I(0) and I(1)) are involved, standard linear models have a critical drawback in the sense that they are incapable of capturing the nonlinear patterns and complicated interactions inherent in highly volatile economic regions. This is especially the case in the Iraqi context, where money in circulation is subject to sudden volatility driven by oil-price distortions, exchange-rate fluctuations, and frequent changes in monetary policy, rendering traditional models inappropriate for predicting and managing liquidity. It thus presents a strong incentive to develop an innovative hybrid modelling model, which fuses ARDL to derive short-run linear relationships, GARCH to describe nonlinear volatility clusters, and the Bidirectional Gated Recurrent Unit (BIGRU) neural network

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to capture residual nonlinear patterns, and thus enriches the predictive power but at the cost of economic interpretability.

This paper is structured in the following way. Section 2 summarizes the relevant literature, identifies the research gap, and presents the research hypothesis. Section 3 outlines the theoretical models of the ARDL, GARCH, and BIGRU, along with the diagnostic tests conducted before, during, and after estimation. Section 4 presents the empirical use of Central Bank of Iraq data, including the analysis of the effects of the explanatory variable on currency in circulation, a comparative analysis of the individual and hybrid models using BIC and RMSE criteria, and a three-month-ahead forecast expressed in original currency units. Section 5 concludes with effective recommendations for the Central Bank of Iraq to improve liquidity management, based on forward-looking projections that account for potential volatility.

2. Literature review and hypothesis development

(Tian & Ma, 2010) Use the ARDL model to show that the appreciation of the renminbi exchange rate and increased money supply from hot money inflows positively impact Shanghai stock prices following financial liberalization in 2005 (Tuyon et al., 2016). The study uses the ARDL model to examine the long- and short-run effects of investor sentiment proxies on Malaysian stock returns, confirming significant and varying impacts across firm sizes and industries. The study by Al-Hajj et al. (2018) finds that oil price shocks negatively impact Malaysia's stock market returns across most sectors, showing market sensitivity and inefficiency. This is based on a nonlinear ARDL analysis of data from 1990 to 2016.

It was also shown that GARCH implementation in R provides a better framework for modeling financial volatility, offering a highly predictive and useful tool for risk management (Dhahir Al-Mahmood & Markovskaya, 2019). The argument in Liu et al. (2019) is that, to analyze hydrological time series data observed in China, an ARMA-GARCH hybrid model was applied; the authors demonstrated that this hybrid model outperformed traditional models at capturing irregular volatility and improving drought monitoring accuracy. According to Walczak (2019), there are 15 main guidelines for creating Artificial Neural Networks (ANNs), and it is highlighted that carefully selecting inputs and using Pearson correlation and stepwise regression significantly reduce noise and improve generalization. The research also supported the recency effect, in that recent one-to-two-year financial information provided the best predictors, rather than long historical series. Using ARDL and VECM methods, (Abuhabel & Olanrewaju, 2020) analyzed Nigerian quarterly data (1991–2018) and found out that currency in circulation (CIC) and the private sector credit significantly increase money supply (M2) in the long run, and that there exists a bidirectional causality between currency in circulation (CIC) and money supply (M2). The article (Farhan et al., 2022) presented the Central Bank of Iraq's experience using exchange rate and interest rate tools (2004–2018) to monitor inflation and stabilize prices, thereby maintaining a stable monetary policy despite political and economic instability. A hybrid FTS-GRU model that combines fuzzy membership degrees with GRU networks has been proposed and applied to achieve better forecasting results (lower RMSE/MAE) than LSTM, BILSTM, and traditional fuzzy models for nonlinear time series (Arslan, 2023). Nigerian currency in circulation was modeled using ARIMAX with seasonal adjustments by Bamanga and Adams (2023), with a prediction that it would reach 3.4 trillion Naira by the end of 2022. The authors also demonstrated that seasonal correction improves currency prediction in monetary planning. Vergili and Celik (2023) used the ARDL model to analyze the relationship between the Dow Jones Sustainability Emerging Markets Index (DJSEMUP) and the VIX Index, finding a long-term negative equilibrium relationship between the variables. Nounou et al. (2023) applied an ARDL model to annual data (1980–2020) of Morocco to ensure cointegration between public expenditure and GDP despite a combination of I(0)/I(1) integration; the error-correction coefficient used indicated that it would adjust to equilibrium at 48 percent/year. By implementing VAR and impulse-response analysis of Iraqi data (1990–2022) (Al-Wasity & Al-Attabi, 2023), it is clear that exchange-rate shocks primarily impact themselves, and money-supply shocks have a close connection to the measures of the Central Bank policy, and in particular, to the agricultural sector. Abd and Almohana (2024) created a hybrid of ARDL and LSTM to predict the Iraqi stock market (2017–2021), using ARDL to capture the linear dynamics and LSTM to model the residuals. The hybrid ARDL-LSTM achieved lower RMSE/MAPE than the standalone ARDL or ARDL-GRU. Mienye et al. (2024) provided an extensive overview of RNN networks, including LSTM, GRU, BiLSTM, and ESN, their applications in time-series forecasting, natural-language processing, and anomaly detection, and the existing challenges and research trends. Research on the rentier economy in Iraq (ALDulaimi et al., 2024) reported a direct positive correlation between the money supply and GDP, with structural changes in the aggregate money supply allegedly driven by the Central Bank of Iraq's monetary policy framework. In a study (Rashid et al., 2024) tested the connection between money supply, inflation, and exchange rate in Iraq during the period 2003-2023, the relationships between money supply and inflation within the exchange rate are investigated based on an ARDL model, according to the results obtained, one-direction causality between the two exogenous variables and inflation has been identified, so the increase of these exogenous variables is accompanied by an increase in the inflationary pressures. SARIMA was used to assess the performance of the actual and counterfactual Brazilian currency circulation (2000–2023), as shown by de Andrade and da Cruz (2025), where the gap between actual and counterfactual performance is significant because of the pandemic, demonstrating the importance of emergency policies in changing cash-demand patterns significantly. In Awogbemi et al. (2025), the authors analyzed the process of money in circulation in Nigeria by monthly data over 15 years; they used seasonal ARIMA models, and an optimal model was obtained, reflecting non-seasonal and annual-seasonal trends. Diagnostic tests ensured the model's sufficiency, and long-range projections indicate further stability in currency circulation, which is influenced by GDP, inflation, interest rates, and the popularity of digital payments. In Kumar and Mishra (2025), the authors examined the cash-demand paradox in India, in which the levels of digital payments and the demand for physical cash rise simultaneously. They indicate, through an ARDL model, that cash is no longer used for transactions, more cash is being kept as precautionary and store-of-value, and that these changes are driven by income growth, economic uncertainty, and low digital inclusion. Although digitalization is heavily supported by policy, cash remains indispensable to the unbanked population and the informal sector, and it remains functional and psychological. A new method of predicting monthly Currency in Circulation (CIC) was proposed (Glova & Hernandez, 2025) through the use of balance-sheet data issued by the central bank, that is, non-currency assets and liabilities. Their approach leverages high-frequency, readily available central bank accounting data (in contrast to more traditional models, which use only quarterly macroeconomic statistics such as GDP) to produce liquidity predictions that are more timely and responsive. The authors show that this framework yields the correct out-of-sample forecast and provides useful value to central banks in controlling the supply of currency, especially in developing economies where real-time economic indicators may be unavailable. Their work highlights central bank financial statements as a rich source for monetary analysis and operational planning.

The above survey demonstrates that modern trends in the methodology of financial time-series analysis are moving towards the incorporation of statistical models and artificial intelligence algorithms to improve forecast accuracy. In recent literature, hybrid models have consistently outperformed standalone approaches in capturing linear dynamics, nonlinear complexities, and time-varying volatility in monetary data. However, existing studies largely focus on econometric or machine-learning models in isolation, with limited efforts to integrate short-run dynamics, volatility behavior, and nonlinear residual patterns within a unified framework, particularly in emerging monetary systems such as those in Iraq. Based on this premise, the current research develops the following main hypothesis: the hybrid ARDL-GARCH-BIGRU model, i.e. the combination of short-run linear dynamics (ARDL), conditional-variance volatility modelling (GARCH), and bidirectional nonlinear pattern recognition of residuals (BIGRU) as a structured multi-stage hybrid framework applied to Iraqi currency in circulation (IQD), will possess a higher forecasting accuracy than the traditional statistical models implemented to Iraqi currency in circulation (IQD) and determinants of its operation (Central Bank deposits and withdrawals). Such a unified platform is likely to provide a self-modeling framework for the sophisticated monetary behavior that defines cash circulation in Iraq.

3. Methodology

3.1. Data collection

The research is based on monthly time-series data from official publications of the Central Bank of Iraq. The dependent variable (currency in circulation) is measured as an end-of-month stock (cumulative level), while the explanatory variables (deposits and withdrawals) are measured as monthly totals. All variables are expressed in trillion IQD. No seasonal adjustment was applied. The dataset was examined for missing values and outliers, and no significant issues were detected. In order to achieve a clear and reproducible both econometric model and Python-based programming implementation, the important variables have been formally defined and provided with brief abbreviations as given in Table 1 below:

Table 1. Definition of variables.

Abbreviation	Variable Classification	Variable Name
Cic	Dependent (y)	Currency in circulation (IQD)
Dep	Explanatory (x_1)	Commercial banks' deposits with the Central Bank of Iraq (IQD)
Withd	Explanatory (x_2)	Central Bank of Iraq withdrawals (IQD)

Source: Prepared by the researchers.

Table 1. presents the description of variables employed in this study, along with the abbreviations adopted for programming implementation in Python. The dependent variable represents currency in circulation, abbreviated as “cic”. The first explanatory variable denotes commercial banks’ deposits in Iraqi Dinars held at the Central Bank of Iraq and its branches across all governorates, abbreviated as ”dep”. The second explanatory variable represents withdrawals of Iraqi Dinars disbursed by the Central Bank of Iraq and its branches, abbreviated as “withd”.

3.2. *Autoregressive distributed lag model (ARDL)*

The ARDL model represents one of the important standard models in econometrics. It was first introduced by Pesaran and Shin (1995) and subsequently refined in Pesaran et al. (2001) to address a common challenge in time series analysis, namely, the differing orders of integration among economic variables included in the model, and to determine cointegration relationships among them. A key advantage of ARDL lies in its flexibility to accommodate variables that are integrated of order zero I(0) and order one I(1) simultaneously, provided that none of the variables are integrated of order two I(2). Given that this research includes two independent variables, the general specification of the regression model is defined as: Chancharat and Suwannapak (2024)

$$\mathrm{y_t} = B_0 + \lambda\ \mathrm{t} + B_1 x1_\mathrm{t} + B_2 x2_\mathrm{t} + \varepsilon_t \tag{1}$$

where:

$\mathrm{y_t}$: denotes the dependent variable
$x1_\mathrm{t}$: denotes the first independent variable
$x2_\mathrm{t}$: denotes the second independent variable
B_0: represents the constant term (intercept)
B_1: represents the regression coefficient for the first independent variable
B_2: represents the regression coefficient for the second independent variable
λ: denotes the trend coefficient
ε_t: denotes the random error term

The Autoregressive Distributed Lag (ARDL) model equation can be expressed in the following general form:

$$\mathrm{y_t} = \beta_0 + \lambda\mathrm{t} + \sum_{\mathrm{i=1}}^{\mathrm{p}} \phi_\mathrm{i}\mathrm{y_{t-i}} + \sum_{\mathrm{j=0}}^{\mathrm{q_1}} \beta_{1\mathrm{j}}\mathrm{x_{1t-j}} + \sum_{\mathrm{j=0}}^{\mathrm{q_2}} \beta_{2\mathrm{j}}\mathrm{x_{2t-j}} + \varepsilon_\mathrm{t} \tag{2}$$

where:

p: number of lags for the dependent variable.
$\mathrm{q_1}$: number of lags for the first independent variable.
$\mathrm{q_2}$: number of lags for the second independent variable.

3.3. *Generalized autoregressive conditional heteroscedasticity (GARCH)*

Especially in the financial field, where volatility and imbalance are common, the generalized autoregressive conditional heteroskedasticity (GARCH) model has become an important theoretical tool and application for econometric series. Unfortunately, such series often exhibit significant heteroskedasticity, and the distribution of the corresponding residuals is not constant over time. Even models for some common stocks are unreliable. With a deeper understanding of the behavior of dynamic waves, we have overturned traditional models one by one, creating more modern models that better describe their dynamic behavior. Following the pioneering work of Engle (1982) and the further extension of the GARCH model (Bollerslev, 1987), GARCH-type models not only provide greater flexibility in the modeling of traditional financial time-varying models but also enable better prediction of time-varying volatility. By further exploring the numerical results of the GARCH (p, q) model, it can be converted into a probabilistic representation of the future (Ali & Yadgar, 2020a).

The GARCH (p, q) model can be expressed as:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i a_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2,\ a_t = \sigma_t \varepsilon_t \tag{3}$$

where σ_t^2 represents the conditional variance at time t, a_{t-i}^2 denotes the squared past errors, and q is order of ARCH Model, p is order of GARCH Model, and α_i and β_i are the model parameters, which must be greater than 0 and satisfy the condition $\sum \alpha_i + \sum \beta_i < 1$ to ensure model stability and a finite variance (Ekpeyong, 2023).

The most commonly used model, GARCH (1,1), can be written as follows:

$$\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1\ \sigma_{t-1}^2,\ \alpha_1, \beta_1 > 0,\ \alpha_1 + \beta_1 < 1 \tag{4}$$

It is also possible to use the GARCH model alone and include a dummy variable in the variance equation to achieve more precise forecasting results (Bhattacharya & Joshi, 2001).

3.4. Gated recurrent unit (GRU)

The Gated Recurrent Unit (GRU) is a recurrent neural network (RNN) variant introduced by Chung et al. (2014) to dynamically adapt to diverse temporal patterns by capturing long-term dependencies, reducing training time, and accelerating convergence. It features two core gating mechanisms: the update gate, which regulates the retention and updating of past information, and the reset gate, which discards irrelevant historical content. This streamlined architecture integrates memory and control mechanisms within a simpler structure than the Long Short-Term Memory (LSTM) network. The mathematical formulation of the GRU model is expressed as follows: (Pan, 2024; Abd & Almohana, 2024)

$$h_t = (1 - z_t)\ h_{t-1} + z_t \tilde{h}_t \tag{5}$$

$$z_t = \sigma\ (W_z x_t + U_z h_{t-1} + b_z) \tag{6}$$

$$\tilde{h}_t\ = tanh\,(W_h x_t + U_h\,(r_t \odot\ h_{t-1}) + b_h) \tag{7}$$

$$r_t\ = \sigma\ (W_r x_t + U_r h_{t-1} + b_r) \tag{8}$$

Where:

h_t	Hidden state at time t(information extracted up to time t)
z_t	Update gate: determines whether to retain past information or rely on the new candidate state
h_{t-1}	Previous hidden state at time t-1
$\tilde{h}_t$	Candidate hidden state computed via a nonlinear transformation of current inputs and previous state, modulated by the reset gate
σ	Sigmoid activation function, constraining outputs to (0,1) to control information flow: $\sigma\ = \frac{1}{1+e^{-x}}$ Values near 0: gate closes (block information) Values near 1: gate opens (allow information) (Essang et al., 2025)
x_t	Current input at time t
W_z	Weight matrix connecting the input to the update gate
U_z	Weight matrix connecting the previous hidden state to the update gate
$tanh$	Hyperbolic tangent activation function, constraining outputs to (–1,1): $tanh\ = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ Values near 1: add new information, Values near 0: suppress addition, Values near –1: add new information while suppressing previous content (Essang et al., 2025)
W_h	Weight matrix connecting the input to the candidate hidden state
U_h	Weight matrix connecting the previous hidden state to the candidate hidden state
r_t	Reset gate: controls how much past information to forget or discard
$\odot$	Hadamard (element-wise) product between matrices/vectors
W_r	Weight matrix connecting the input to the reset gate
U_r	Weight matrix connecting the previous hidden state to the reset gate

3.5. Bidirectional gated recurrent unit (BIGRU)

The Bidirectional Gated Recurrent Unit (BIGRU) is a variant of Recurrent Neural Networks (RNNs) that extends the conventional GRU by processing sequential data in two directions simultaneously: a forward layer that reads the sequence from start to end, and a backward layer that processes it from end to start, both using GRU units. This dual-directional architecture enables the model to capture contextual information from both past and future states at each time step, yielding deeper and more comprehensive sequence understanding. The BIGRU thus offers high predictive accuracy, structural simplicity, and superior performance in processing textual and sequential data (Zhang et al., 2019).

The mathematical formulation of the Bidirectional Gated Recurrent Unit (BIGRU) model can be expressed as follows:

$$h_t = w_t\ \overrightarrow{h}_t + v_t\ \overleftarrow{h}_t + b_t \tag{9}$$

Where:

w_t	Weight matrix associated with the forward hidden layer state.
v_t	Weight matrix associated with the backward hidden layer state.
$\overrightarrow{h}_t$	Forward hidden state (processed from past to future).
$\overleftarrow{h}_t$	Backward hidden state (processed from future to past).
b_t	Combined hidden state at time t.

The mathematical formulation of the forward hidden state follows the same structure as the standard GRU model presented earlier in Eq. (5). To distinguish the forward direction, a right arrow (→) is placed above the state symbol. The backward hidden state adopts an identical formulation but processes the sequence in reverse order, denoted by a left arrow (←) above the state symbol, as shown below:

$$\overrightarrow{h}_t = GRU\left(x_t, \overrightarrow{h}_{t-1}\right) \tag{10}$$

$$\overleftarrow{h}_t = GRU\left(x_t, \overleftarrow{h}_{t-1}\right) \tag{11}$$

3.6. *Methodology of hybrid models and mathematical formulations*

The Bidirectional Gated Recurrent Unit (BIGRU) is a variant of Recurrent Neural Networks (RNNs) that extends the conventional GRU by processing sequential data in two directions simultaneously: a forward layer that reads the sequence from start to end, and a backward layer that processes it from end to start, both using GRU units. This dual-directional architecture enables the model to capture contextual information from both past and future states at each time step, yielding deeper and more comprehensive sequence understanding. The BIGRU thus offers high predictive accuracy, structural simplicity, and superior performance in processing textual and sequential data (Zhang et al., 2019).

The mathematical formulation of the Bidirectional Gated Recurrent Unit (BIGRU) model can be expressed as follows:

3.7. *First: A double hybrid (ARDL-GARCH) model*

In this hybrid model, which represents a special case, estimation will not be performed in separate stages. Instead, a single-step estimation approach will be adopted by utilizing a GARCH framework comprising two equations: the first is the mean equation, into which the ARDL coefficients, previously identified based on optimal lag orders, are incorporated, the second is the variance equation, where the GARCH orders are specified and the optimal orders are selected based on information criteria (AIC, BIC, HQIC).

The specification of the first hybrid model is formulated as follows:

$$y_t = \mu_t + \varepsilon_t,\ Where\ \varepsilon_t = z_t\, \sigma_t,\ z_t \sim i.i.d.\ (0, 1) \tag{12}$$

$$\hat{y}_t = \hat{\mu}_t, \left(\hat{\mu}_t = \hat{\beta} + \hat{\lambda}\, t + \sum_{i=1}^{p} \widehat{\phi}_i\, y_{t-i} + \sum_{j=0}^{q_1} \hat{\beta}_{1j}\, x_{1\ t-j} + \sum_{j=0}^{q_2} \hat{\beta}_{2j}\, x_{2\ t-j}\right) \tag{13}$$

Where:

$\hat{y}_t$: denotes the final fitted equation,

$\hat{\mu}_t$: represents the mean equation, the first (linear) component, estimated using the ARDL model,

$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$ denotes the variance equation, the second (nonlinear) component, estimated via the GARCH model, which effectively addresses the issue of heteroskedasticity in the residuals

3.8. *Second: A double hybrid (ARDL-BIGRU) model*

Hybrid models are employed to accurately capture all types of patterns in the data. Specifically, the hybrid model integrates ARDL to detect linear patterns and the GRU neural network to identify nonlinear patterns and unexpected complexities. Consequently, their combination enables the model to capture both linear and nonlinear relationships present in the data (Abd & Almohana, 2024).

The operational mechanism of the hybrid model can be summarized as follows:

1. The ARDL model is first applied to determine the relationships among study variables, analyze long-run dynamics, and estimate the speed of error correction. The residuals from this model are retained to represent the unexplained component of the time series behavior and constitute the first (linear) part of the hybrid model equation.
2. The residuals of the ARDL model, together with the original time series values and forecasts, are fed into the Bidirectional Gated Recurrent Unit (BIGRU) neural network. It completely allows the network to receive all the information, thus being able to calculate what inputs are good as well as learn time patterns forward and backward. That is essentially the second (nonlinear) term in the hybrid model equation and it assists us in making predictions of future values with extreme precision. Thus, the final estimate of the hybrid model is obtained by summing the estimates of the linear and nonlinear components, as follows:

$$\hat{z}_t = c_0 + c_1 \hat{v}_{1(Linear\ component\ [\hat{L}_t])} + c_2 \hat{v}_{2(Non-Linear\ component\ [\hat{N}_t])} \tag{14}$$

Where:

- $\hat{z}_t$: denotes the final dependent variable,
- $\hat{v}_{1(Linear\ component\ [\hat{L}_t])}$: represents the first (linear) component, estimated via the ARDL model,
- $\hat{v}_{2(Non-Linear\ component\ [\hat{N}_t])}$: represents the second (nonlinear) component, estimated using the BIGRU model,
- c_0, c_1, c_2: denotes the model coefficients obtained after estimating the equation using the Ordinary Least Squares (OLS) method, which operates on the principle of minimizing the sum of squared residuals to the lowest possible value.

3.9. Third: A triple hybrid (ARDL-GARCH-BIGRU) model

The ARDL-GARCH-BIGRU hybrid model provides a keen examination of the transformation of data with the course of time. It combines three components: ARDL to observe straight-line trends and long-term relationships, GARCH to identify bursts of volatility as well as asymmetric variation, and BIGRU to identify non-linear quirks which go both ways. These two are used together to provide a complete picture of complex time series.

1. The first thing that we do is to determine the relationship between the variables using the ARDL model, examine long-term patterns, and the speed at which the errors are rectified. Things that the ARDL cannot explain, we retain as residuals. These residuals will be the initial linear term of the hybrid equation.
2. Then we manage to put the ARDL residuals into a GARCH model to allow us to capture the changes in volatility that are conditional on past values, having already checked to ensure that there are changes in volatility. The GARCH provides us with more stable residuals which indicate the variation of the variance in a jumpy way. This acute the linear element of the hybrid equation.
3. Finally, we input the GARCH residuals into a Bidirectional Gated Recurrent Unit (BIGRU) neural network to identify any other non-linear trends and curved interactions that were overlooked in the foregoing steps. The BIGRU examines the data both forwards and backward thus delivering the third, non-linear aspect of the hybrid model

Accordingly, the specification of the third hybrid model is formulated as follows:

$$\begin{aligned}\hat{f}_t &= r_0 + r_1\hat{w}_{1_{(Linear\ component\ [\hat{L}_t])}} \\ &\quad + r_2\hat{w}_{2(Non-Linear\ component\ [\hat{N}_t])}\end{aligned} \tag{15}$$

Where:

- $\hat{f}_t$ denotes the final fitted equation.
- $\hat{w}_{1(Linear\ component\ [\hat{L}_t])}$: represents the first component (linear), estimated via ARDL with heteroskedasticity correction performed by GARCH.
- $\hat{w}_{2_{(Non-Linear\ component\ [\hat{N}_t])}}$: represents the second component (nonlinear), estimated via BIGRU.
- $r_0,\ r_1,\ r_2$: are model coefficients obtained through estimation.

In addition to the specific structural formulations outlined above, this research also implements the standard additive hybrid methodology widely adopted in forecasting literature. This approach computes the final forecast by directly summing the linear component ($\hat{L}_t$) and the nonlinear component ($\hat{N}_t$). Consequently, multiple estimation structures are evaluated for each hybrid model. The final specification for each model is selected based on a comparative performance analysis, adopting the method that yields the lowest levels of forecasting errors (such as RMSE, MAE, and MAPE) during the evaluation period.

3.10. Diagnostic tests and model selection criteria

3.10.1. Unit root tests

The unit root tests are applied to ensure that our time series is stationary before we make estimates. Herranz (2017) suggests the Augmented Dickey Fuller test as it is a popular and efficient test (Afriyie et al., 2020). The three primary items required to conduct the test are the number of observations (n), the level of significance (a), and the model of choice.

3.10.2. Normality test (Jarque–Bera)

The Jarque–Bera test is used to verify the presence of normal distribution of the residuals:

- H_0: Residuals are normally distributed
- H_1: Residuals are not normally distributed

With a p -value greater than 0.05, you may accept H_0 and declare that the residual values are normally distributed (Roshanpour et al., 2025).

3.10.3. Residual autocorrelation test (Ljung–Box Q)

This test, as well as ACF and PACF diagnostics, is searching autocorrelation in the residues (Salman & Ansseif, 2025):

- H_0: The distribution of the residual values is random (there is no autocorrelation of the values).
- H_1: Residuals exhibit autocorrelation

3.10.4. Heteroskedasticity test (ARCH Effect)

Heteroskedasticity complicates the efficiency of regression and inference particularly when regression contains sharp changes or shocks (Zubair & Boyi, 2025) The ARCH test uses:

- H_0: Homogeneous variance (no ARCH effect)
- H_1: Heterogeneous variance (ARCH effect present)

3.10.5. Model selection criteria: AIC, BIC, and HQIC

Among competing models, the optimal specification minimizes information criteria (Agiakloglou &

Tsimpanos, 2023):

$$\mathrm{AIC}\left(\mathrm{p}\right) = -2\log\left(\hat{\mathrm{L}}\right) + 2\mathrm{P} \tag{16}$$

$$\mathrm{SBC}\left(\mathrm{p}\right) = -2\log\left(\hat{\mathrm{L}}\right) + \mathrm{p}\log\left(\mathrm{n}\right) \tag{17}$$

$$HQ\left(\mathrm{p}\right) = \ln(\hat{\sigma}_e^2) + \frac{2\ p\ c\ \ln\left(\ln\left(n\right)\right)}{n} \tag{18}$$

where $\hat{\mathrm{L}}$ = maximized likelihood, p = number of parameters, n = sample size.

3.10.6. *Forecast accuracy measures*

According to Hodson (2022), you can use three metrics to understand the prediction effectiveness of your model:

1. RMSE (Root Mean Squared Error): Square root of mean squared residuals, will be of the same units as the data; smaller values are preferable.
2. MAE (Mean Absolute Error): mean size of errors without squaring, suitable where the data has an outlier.
3. MAPE (Mean Absolute Percentage Error): the mean error as a percentage, useful in comparing across models as well as providing an economic feel. The less the values, the more precise is the value. Y

3.10.7. *Using returns variable instead of the original series*

Given the substantial magnitude of the nominal values, logarithmic returns were employed in the analysis to ensure numerical stability, stabilize variance, and mitigate potential econometric issues associated with large-scale time series data. Returns are often used in monetary and financial research, especially when using GARCH models, as they capture the continuously compounded rate of change in the same variable over two consecutive periods. The calculation is usually expressed as follows (Qi & Wang, 2022).

$$r_t = \log\left(\frac{y_t}{y_{t-1}}\right) \tag{19}$$

where y_t is the value of the time series at time t, and y_{t-1} is the value of the time series at time t-1.

4. Results

The research variables are currency in circulation, dinar withdrawals, and dinar deposits. The data are monthly between Jan 2016 and Dec 2025 making it 120 data points. Due to the inability to obtain enough data, the data were divided into 90% for training and 10% for testing to ensure sufficient data for model estimation while retaining a holdout sample for evaluation.

Table 2. Description of the research variables data.

Variable	cic (y)	dep (x1)	withd (x2)
Mean	–	4,897,498,708,333	5,407,531,225,000
Std_Dev	-	1,720,047,674,690	1,568,416,151,959
Min	38,639,751,000,000	1,198,360,000,000	2,635,694,000,000
Max	104,385,660,000,000	8,594,071,000,000	9,398,207,000,000
Skewness	0.19	-0.05	0.20

Source: Prepared by the researchers.

Table 2, provides the descriptive statistics of three monetary aggregates in terms of the Iraqi dinars, bank deposits are around 4.9 trillion dinars, central bank withdrawals stand at 5.4 trillion dinars, and currency in circulation is a cumulative stock variable. The fact that near-zero coefficients of skewness is an initial indication that this data are not in conformity to normal distribution.

Fig. 1, shows that currency in circulation in Iraq is characterized by an upward trend, which indicates a fairly stable direction in its movement. These kinds of observations provide an initial indication of non-stationarity when it comes to the mean.

Fig. 2, reveals that banks' deposits in Iraqi dinars held at the Central Bank exhibit instability, characterized by a pronounced general trend alongside evident volatility over time.

Fig. 3, demonstrates that Iraqi dinar withdrawals to the Central Bank follow a general trend over time, accompanied by persistent fluctuations across different.

Fig. 4, shows that the logarithmic returns of currency in circulation exhibit stable behavior around a constant mean, indicating that stationarity is achieved after applying the logarithmic return transformation.

Fig. 5, indicates that banks' deposits in Iraqi dinars held at the Central Bank achieve stationarity after transforming the series into logarithmic returns.

Fig. 6, demonstrates that Iraqi dinar withdrawals from the Central Bank attain stationarity following the logarithmic return transformation, although persistent fluctuations remain evident over time.

4.1. *Unit root tests*

To evaluate stationarity of time series data, a condition of valid statistical inference and prevention of the phenomenon of spurious regression, the Augmented Dickey Fuller (ADF) test is used. The null hypothesis on which the test is based is that the series is non-stationary, in which there is a unit root. The decision-making is based on the p-value: a p-value below 0.05 results in the rejection of the null

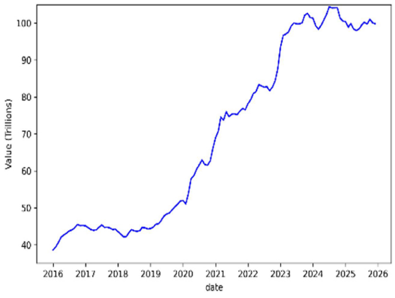

Fig. 1. Currency in circulation level.
Source: Prepared by the researchers.

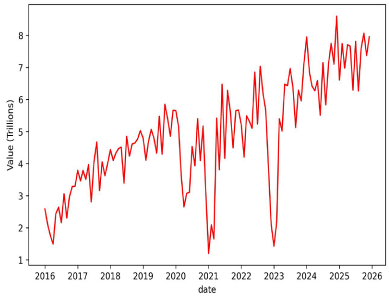

Fig. 2. Deposits level.
Source: Prepared by the researchers.

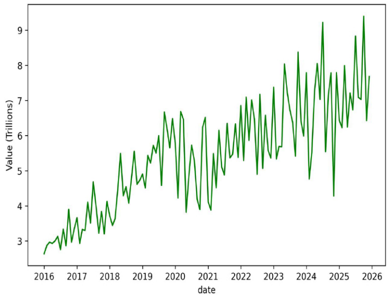

Fig. 3. Withdrawals level.
Source: Prepared by the researchers.

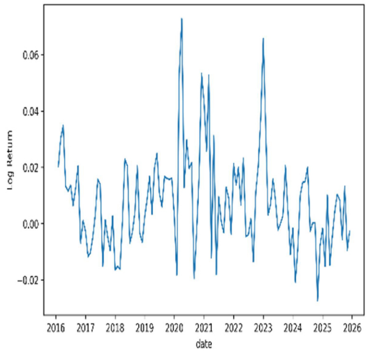

Fig. 4. Logarithmic returns of cic.
Source: Prepared by the researchers.

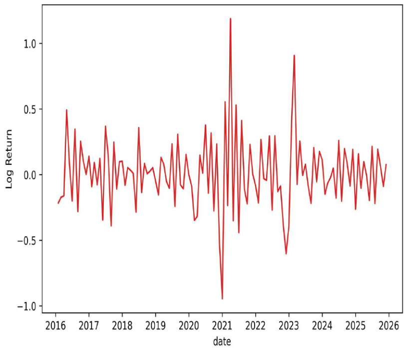

Fig. 5. Logarithmic returns of dep.
Source: Prepared by the researchers.

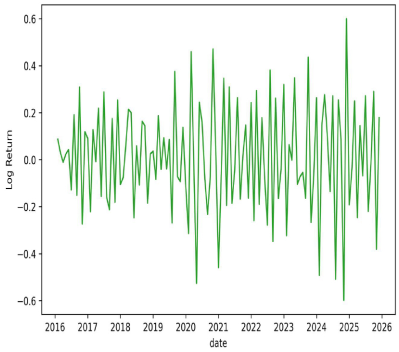

Fig. 6. Logarithmic returns of withd.
Source: Prepared by the researchers.

Table 3. ADF unit root test results at level.

Variable	ADF_Statistic	p_value	Lags_Used	Conclusion
y (cic)	−0.46598	0.8985	1	NON-STATIONARY
x1 (dep)	−2.04169	0.2686	4	NON-STATIONARY
x2 (withd)	−1.24213	0.6552	4	NON-STATIONARY

Source: Prepared by the researchers.

Table 4. ADF unit root test results at logarithmic return transformation.

Variable	ADF_Statistic	p_value	Lags_Used	Conclusion
$r_t y$ (rcic)	−7.11529	0	0	STATIONARY
$r_t x_1$ (rdep)	−8.06451	0	3	STATIONARY
$r_t x_2$ (rwithd)	−10.6807	0	3	STATIONARY

Source: Prepared by the researchers.

hypothesis i.e. the series is stationary; otherwise, the series is non-stationary.

Table 3, presents Augmented Dickey-Fuller (ADF) unit root test results for assessing series stationarity. The high p-values and statistically insignificant test statistics lead to failure to reject the null hypothesis of a unit root, confirming that all three variables, currency in circulation (y), banks' deposits (x1), and withdrawals (x2), are non-stationary at level and require transformation (e.g., first differencing) to achieve stationarity prior to modeling.

Table 4, presents the Augmented Dickey-Fuller (ADF) test results for the logarithmic return series. The highly negative test statistics and p-values close to zero (< 0.05) lead to the rejection of the unit root null hypothesis. This confirms that all transformed variables, $r_{y,t}$ (return of currency in circulation), $r_{x1,t}$ (return of banks' deposits), and $r_{x2,t}$ (return of withdrawals), are stationary and suitable for subsequent modeling.

4.2. Model selection criteria and estimation ARDL model

Table 5, presents model selection criteria for the Autoregressive Distributed Lag (ARDL) specification. The (2,1,1) structure, featuring two lags for the dependent variable (CIC) and one lag for each independent variable (dep and withd), was selected based on the minimization of BIC, reflecting an optimal balance between goodness-of-fit and parsimony in model specification. Consistent with the conclusions of Wang and Liu (2006) favoring the BIC criterion over the AIC criterion.

Table 5. Optimal lag order selection.

Rank	cic_p	dep_q	withd_q	AIC	BIC	HQIC
1	2	1	1	–734.06	–710.52	–724.53
2	1	0	0	–724.52	–708.83	–718.17
3	2	0	1	–728.64	–707.71	–720.17
4	2	1	3	–736.18	–707.41	–724.53
5	4	2	2	–740.90	–706.90	–727.14
6	4	3	3	–746.02	–706.79	–730.14
7	2	1	2	–732.83	–706.68	–722.25
8	4	3	2	–742.99	–706.38	–728.17
9	2	0	0	–724.46	–706.15	–717.05
10	1	1	0	–724.43	–706.12	–717.02

Source: Prepared by the researchers.

Table 6. ARDL (2,1,1) model estimation results.

Variable	Coefficient	Std_Error	t_Statistic	p_value
β_0	0.000108	0.001269	0.09	0.9323
λ	0.000005	0.000019	0.28	0.7776
β_{11}	0.56	0.0972	5.8	0.0000
β_{12}	0.37	0.0955	3.8	0.0002
β_{20}	–0.05	0.0023	–21.6	0.0000
β_{21}	–0.01	0.0053	–2.8	0.0067
β_{30}	0.07	0.0033	21.8	0.0000
β_{31}	0.03	0.0077	3.7	0.0003

Source: Prepared by the researchers.

Table 6. That was discovered in this research shows the approximated coefficients of the ARDL (2,1,1) specification. The constant term as well as the time-trend coefficient are both statistically insignificant, which shows that the dynamics of currency circulation is mostly caused by monetary processes, not by autonomous temporal changes. The presence of a statistically significant positive short run persistence can be seen in lagged currency holdings in circulation. Deposits exhibit strong negative contemporaneous and lagged effects, which have an economic meaning of liquidity absorption that drains cash out of the circulation over time. The impact of withdrawals is significantly positive and contemporaneous and lagged, which highlights the role of the Central Bank as the main liquidity source injector by government disbursements (as an example, the salaries of public workers), thus directly increasing the increase in the amount of money in the market. All the dynamic effects are statistically significant at the one-percent level.

The estimation results of the GARCH (1,1) model indicate that the coefficient $\alpha_1 = 0.52$ is statistically significant, while $\beta_1 = 0.03$ is not significant, with the stability condition satisfied ($\alpha_1 + \beta_1 = 0.55 < 1$). Furthermore, the GED shape parameter $\nu = 1.37$ is statistically significant, confirming the presence of fat tails. The adoption of this model contributed to improving forecasting accuracy by reducing the values of the comparison criteria (RMSE, MAE) both in-sample and out-of-sample.

The BIGRU model was implemented as a bidirectional recurrent architecture with two hidden layers (16 and 8 units) and a sequence length of 6, using tanh and sigmoid activations within the GRU cells, and trained with the Adam optimizer (learning rate = 0.01) over a maximum of 200 epochs with a batch size of 16. A dropout rate of 0.2 and early stopping (patience = 20) were applied to prevent overfitting, while 20% of the training data was used for validation to monitor performance and ensure generalization.

4.3. *Diagnostic tests*

Table 7, presents the results of the tests demonstrate that the model residual does not follow a normal distribution (Jarque Bera test), but do not exhibit any autocorrelation (Ljung Box test). However, ARCH-LM test indicates high conditional heteroskedasticity (ARCH effects) and this reason is why GARCH-type volatility modelling is necessary to provide the opportunity to accommodate the time-varying variance structure that is embedded in the residual series.

Table 7. Diagnostic test results for model residuals.

Test	Statistic	p_value	Conclusion
Jarque-Bera (Normality)	22.21	0.0000	Non-Normal
Ljung-Box (Autocorr)	13.29	0.2079	No AC
ARCH-LM (Heterosk)	24.25	0.0070	ARCH Effect

Source: Prepared by the researchers.

4.4. *Model forecasting performance comparison*

After estimating individual and hybrid models, we measured the performance of forecasting using three complementary error measures: RMSE, MAE and MAPE. The three metrics collectively evaluate the quality of prediction, sensitivity to outliers, and the relative percentage error, thus making it easier to conduct an extensive and statistically sound model choice. Smaller metric values indicate better predictive ability and the triple hybrid architecture (ARDL -GARCH -BIGRU) would theoretically outperform nested models because it is capable of simultaneously capturing linear dynamics, volatility clusters, and nonlinear temporal dependencies.

Table 8 presents results for combining a linear (ARDL) model, a neural network (BIGRU) model, and variance (GARCH/Sigma) modeling within an additive framework. The models are fitted by Ordinary Least Squares (OLS), which is the most effective approach. The analysis shows that the Triple hybrid model outperforms others in in-sample forecasting, followed by the bivariate hybrid ARDL-BIGRU-OLS. This suggests that the hybrid format improves understanding of data dynamics,

Table 8. In-Sample (Training) forecasting performance of individual and hybrid models.

Models	RMSE	MAE	MAPE
ARDL_GARCH_BIGRU_OLS	0.00496	0.00379	267.7
ARDL_BIGRU_OLS_HYBRID	0.00508	0.00374	184.2
ARDL_BIGRU_HYBRID	0.00517	0.00379	232.5
ARDL_GARCH_OLS_HYBRID	0.00559	0.00421	221.5
ARDL	0.00575	0.00417	159.1
ARDL_GARCH_ONE_STEP	0.00575	0.00417	159.1
ARDL_GARCH_HYBRID	0.00575	0.00416	199.0
ARDL_GARCH_BIGRU_HYBRID	0.00584	0.00432	307.8
BIGRU	0.01364	0.01043	702.5

Source: Prepared by the researchers.

Table 9. Out-of-Sample (Testing) forecasting performance of individual and hybrid models.

Models	RMSE	MAE	MAPE
ARDL_GARCH_BIGRU_HYBRID	0.00253	0.00209	39.5
ARDL_GARCH_HYBRID	0.00378	0.00338	62.1
ARDL_BIGRU_HYBRID	0.00393	0.00365	80.3
ARDL	0.00406	0.00369	67.9
ARDL_GARCH_ONE_STEP	0.00406	0.00369	67.9
ARDL_BIGRU_OLS_HYBRID	0.00468	0.00436	98.5
ARDL_GARCH_OLS_HYBRID	0.00468	0.00445	86.5
ARDL_GARCH_BIGRU_OLS	0.00516	0.00478	109.6
BIGRU	0.01098	0.00935	148.3

Source: Prepared by the researchers.

leading to higher accuracy. This superiority results from methodological differences between the two estimation paradigms. In the Triple hybrid model, forecasts are generated additively by directly adding the linear ARDL output, BIGRU nonlinear predictions, and the GARCH-derived conditional variance.

(σ_t) being used only to form confidence intervals and improve the representation of uncertainty. The Triple hybrid model, OLS, is on the other hand a model that takes into account all the elements within one linear regression equation that is estimated using training data thus, converting σ_t) into an explanatory variable that has an effect on the point forecast and not just an expression of uncertainty. In this arrangement, statistical re-weighting of the three constituents is allowed according to their actual explanatory power and this results in better accuracy measures.

Fig. 7 compares the visual data on actual yield values (Actual Return) with the predictions from linear and hybrid models over the years 2016–2025. A significant similarity between the hybrid model paths, particularly the paths based on the ordinary least squares, and the empirical data path is observed, and it is consistent with the fact that the hybrid specifications are much better in capturing pronounced volatility and irregularities in currency-in-circulation returns with greater accuracy than the independent models. This empirical data promotes the accuracy measures mentioned above.

Table 9 presents the triple hybrid model ARDL GARCH BIGRU HYBRID with a direct additive formulation, which demonstrates the best results in the out-of-sample test, with the lowest forecast error of (RMSE = 0.00253, MAE = 0.00209 and MAPE = 39.5). These statistics provide strong evidence of the model's ability to capture the underlying dynamics and forecast future observations with high accuracy. In comparison, the hybrids based on ordinary least squares (OLS), which had dominated the in-sample analysis, displayed significantly reduced predictive power on the hold-out data. The resulting performance difference strongly suggests overfitting, since the high in-sample values may have been due to the model's tendency to fit random idiosyncratic noise rather than structural regularities. It is important to note that the additive hybrid has strong statistical rigor due to its seamless

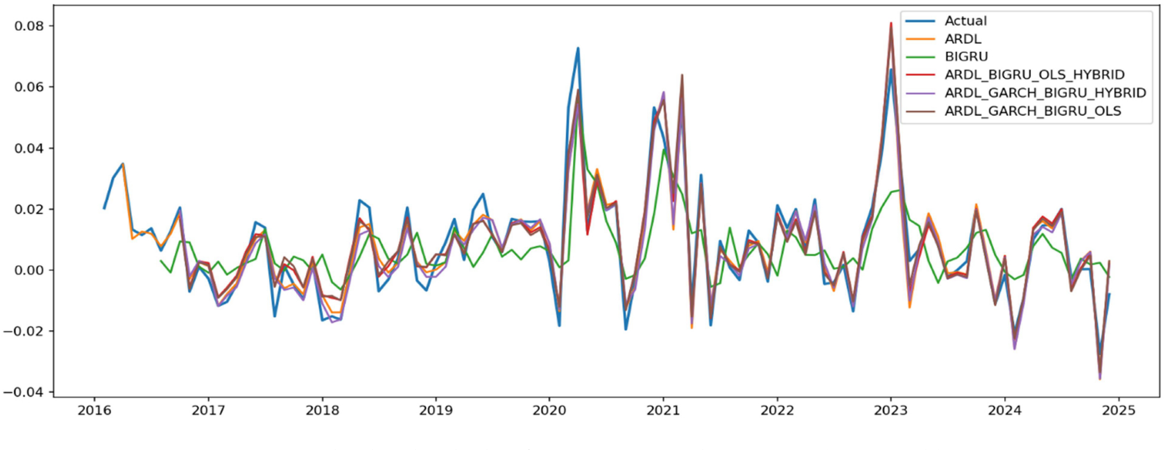

Fig. 7. Actual and predicted values for currency in circulation returns (train sample).

Source: Prepared by the researchers.

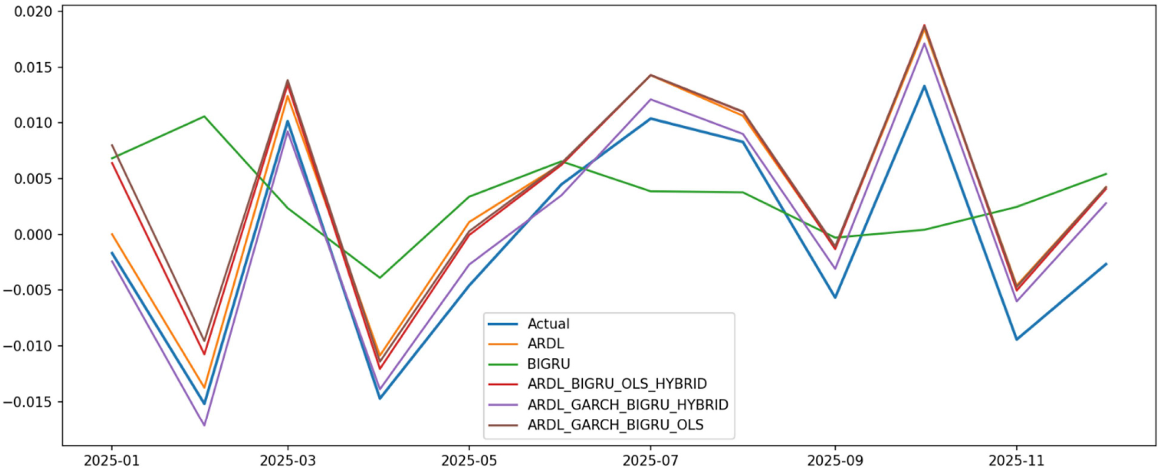

Fig. 8. Actual and forecasted values of currency in circulation returns (test sample).
Source: Prepared by the researchers.

integration of three modeling philosophies: the linear ARDL component, the volatility-capturing GARCH specification, and the bidirectional gated recurrent unit (BIGRU), which provides nonlinear learning. This type of architecture avoids the rigidity of traditional re-estimation processes, thereby enabling the model to generalize more effectively to new observations. We have therefore chosen the additive ARDL_GARCH_BIGRU_HYBRID to be the key predictive tool in our research. It offers the best trade-off between in-sample accuracy and out-of-sample fidelity, while mitigating the risks of over-parameterization and overfitting.

Fig. 8, presents a comparison between out of sample forecasting performance among the competing models. The difference between the real series (Actual) and the chosen triple hybrid model is remarkable, highlights its ability to capture abrupt changes, peaks, and troughs with a very high degree of precision. On the other hand, some alternative models deviate visibly from the actual path, particularly during periods of high volatility. This visual evidence confirms the previous quantitative results, demonstrating the superior predictive ability and robustness of the hybrid model in forecasting future movements.

4.5. *Forecasting out of sample test*

Based on the comparative analysis, the triple-hybrid model (ARDL-GARCH-BIGRU) was selected as the best specification for predicting, due to its ability to model linear dynamics, volatility clustering, and nonlinear time-series relations in currency circulation data. Because the ARDL regression structure requires input of future values of the independent variables (X1, X2) to predict the dependent variable (Y) over three months, a two-step procedural structure was applied using a custom-written program:

Step 1: Independent variables forecasting (Dynamic forecasting) https://abdulrazaq1995.pythonanywhere.com/

- logarithmic returns were initially made of each independent variable series to achieve statistical stationarity.
- The three statistical specifications for each variable evaluated by the program were AutoRegressive Moving Average (ARMA; models autocorrelations), Generalized Autoregressive Conditional Heteroskedasticity (GARCH; models time-varying volatility), and the combined ARMA-GARCH.
- The best specification for each variable was selected separately based on the lowest RMSE, ensuring the best inputs were used in the final forecasting equation.

Step 2: Final forecast generation.

- The future values of the independent variables (calculated in Step 1) were input into the best hybrid model (ARDL-GARCH-BIGRU).
- The final forecasts of currency circulation were produced and reported in their original data format (Level Form), thereby making them easier for economists to interpret and use in policy decision-making.

Table 10 presents the ultimate projections of currency in circulation for the first quarter of 2026, based on the selected triple hybrid model. The outcomes show a continuous positive trend, with the estimated

Table 10. Three month ahead out-of-sample forecasts of currency in circulation **(Million IQD)**.

Month	January 2026	February 2026	March 2026
Lower Bound CI 95%	96,305,828	96,860,582	97,362,644
Point Forecast of currency in circulation (cic)	100,460,953	101,188,886	101,858,705
Upper Bound CI 95%	104,795,352	105,710,604	106,562,386

Source: Prepared by the researchers.

number increasing from 100.5 trillion dinars in January to 101.9 trillion dinars in March. Moreover, the 95percent confidence interval defines the expected range of variation between 96.3 and 104.8 trillion dinars in January and 97.4 and 106.6 trillion dinars in March. The interval estimates provided to the Central Bank give it a robust planning framework that considers both the lower and upper limits of currency in circulation, enhancing liquidity management efficiency and ensuring liquidity requirements are met during peak cash demands in the event of liquidity shocks. This allows the Central Bank to plan cash issuance schedules, adjust liquidity injections, and prepare for peak cash demand periods based on the upper and lower bounds of the forecast intervals.

5. Discussion

Economically, the results have significant implications for financial inclusion. Bank deposits have a statistically significant negative influence on funds in circulation, reflecting liquidity absorption through formal banking channels. This suggests that policies promoting bank deposits and formal financial intermediation can help manage excess liquidity and support financial stability. Conversely, withdrawals have a strong positive effect on currency in circulation, highlighting a persistent reliance on cash-based transactions and revealing structural barriers to deeper financial inclusion. Policymakers should address these barriers by encouraging digital payments, increasing trust in formal institutions, and broadening access to banking services to reduce excessive reliance on cash and promote more inclusive financial systems.

In rent-based economies like Iraq, where government spending and oil payments are the primary concerns, changes in liquidity directly affect currency circulation through injections from government disbursements. In line with this, the relationship between deposits, withdrawals, and currency in circulation not only quantifies the dynamics of monetary policy but also reflects liquidity management and the Central Bank of Iraq's role in regulating and supplying the required amounts of Iraqi dinar in the domestic economy. This discrepancy may be attributed to the structural limitations of financial inclusion in rent-based economies. The superiority of the triple hybrid can be attributed to structural complementarity: ARDL models the short-run linear corrections, GARCH models the time-varying, and BIGRU models the nonlinear temporal adaptations, which cannot be modeled using the traditional econometric models. Despite these promising results, several limitations must be noted. The fairly limited sample size of 120 monthly observations may violate the short-run stability assumption. Moreover, the large financial magnitude, measured in trillions of Iraqi dinars, required a logarithmic transformation to achieve numerical stability in both the GARCH and BIGRU estimation procedures.

Overall, the hybrid framework proposed could deliver better forecasting performance and provide a quantitative tool for evaluating the dynamics of liquidity and its nexus with financial inclusion policies. This framework has practical value for the Central Bank of Iraq in coordinating liquidity management, promoting financial inclusion, and gradually shifting the economic model toward a less-cash-based, more digitally tied financial system.

6. Conclusion

In conclusion, this research provides important evidence regarding the dynamics and predictability of currency in circulation in Iraq using an integrated ARDL–GARCH–BIGRU hybrid modeling framework. The findings demonstrate that the proposed hybrid approach successfully captures both simple linear adjustments and complex nonlinear patterns, allowing for more accurate forecasting of future currency values. The results further reveal that increased bank deposits reduce currency in circulation in the streets through liquidity absorption, whereas withdrawals increase cash holdings, indicating the continued dominance of cash-based transactions and limited financial deepening.

However, it is important to recognize some limitations of the study, such as the relatively small sample size of 120 monthly observations, the exclusion of key macroeconomic variables such as inflation, exchange rates, oil prices, and interest rates, and the absence of structural break or political shock analysis, which may limit the scope of the findings. Addressing these limitations is a priority for future research.

Specifically, future studies should investigate the inclusion of additional macroeconomic determinants, longer time horizons, structural break testing, and alternative hybrid modeling approaches to improve forecasting robustness and interpretability. Moreover, a deeper investigation into the potential role of Central Bank Digital Currencies (CBDCs) is essential for a more complete understanding of currency circulation dynamics, as digital currencies may enhance financial inclusion, reduce dependence on physical cash, increase transaction transparency, lower costs, and improve monetary policy transmission. Such analysis will help in developing more effective solutions for transitioning from a cash-dominated economy to a digitally integrated financial system.

Expanding on this work will allow researchers to fill existing gaps between econometric forecasting, artificial intelligence modeling, and financial inclusion policy and ensure that the findings can be applied more broadly in practical monetary management and real-world policy settings.

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Conflict of interest

The second author is a member of the journal's editorial board but was not involved in the review or decision process for this manuscript.

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Data availability

The data that support the findings of this research are available from the Central Bank of Iraq, but restrictions apply to the availability of these data, which were used under license for the current research, and so are not publicly available. Data are however available from the authors upon request and with permission of the Central Bank of Iraq (official correspondence Ref. No. 2617, dated 18 September 2025).

Authors' declaration

- We hereby confirm that all the Figures and Tables in the manuscript are ours, and all results were prepared by the authors using the Python programming language.
- Ethical Clearance: The Research Was Approved by The Local Ethical Committee in The University of Baghdad.
- No animal studies are present in the manuscript.

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