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Constrained Reliability Allocation for Complex Systems Using an Enhanced Harmony Search Optimization Approach

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Abstract:

This paper focuses on calculating the allocation and optimal reliability distribution for each component of a complex system by estimating a polynomial reliability function using the shortest path method and applying matrices to find the most successful path for the device. Most studies, with their development, rely on increasing closed-loop functions to link cost and its direct relationship to reliability that is, a direct relationship between reliability and cost, meaning that the cost of a component increases with its reliability. However, these relationships can sometimes be complex, ambiguous, or difficult to construct. In this study, the reliability function of the complex system was extracted using MATLAB version R2020a, and the correlation between cost and reliability was examined across several datasets. In addition to determining the overall cost and reliability of the system, the Harmonic Search (HSA) algorithm was used to determine the cost and reliability of each individual component. The performance of this algorithm was optimized, and an optimal reliability distribution was achieved, by applying the Behavior model (cost function).

Keywords: Harmony search method, cost reliability model, reliability allocation, and complex systems.

Introduction

In order to ascertain the efficiency and stability of a complex, composite system, the dependability of the system was examined using mathematical techniques (Abed, Sulaiman, & Hassan, 2019) (Dhillon, 1999). Short paths through connectivity matrices were used to assess the system's dependability. In order to determine minimal paths that represent the critical paths in the system, all potential paths were derived using Boolean algebra, and superfluous nodes were eliminated (Abd Alhasan Hameed Saleh & Hassan, 2023) (Abdullah & Hassan, 2020) (Abdullah & Hassan, 2021) (Yousif, Ameer, & Sulaiman, 2025). The reliability function of the composite system was ascertained in order to evaluate its safety. This research tackles the mathematical issue of optimal dependability allocation across system components, despite its historical roots in the study of networks. The goal of this work is to reduce overall expenses while boosting system lifetime and dependability by optimizing the reliability levels of each component based on the significance of each component's location within the system (Govil, 1983) (Sulaiman et al., 2023). Based on their position and impact on the system design, these components require the highest reliability allocations possible in order to contribute to increased overall dependability. Optimizing mechanical, electrical, and missile systems presents many challenges for engineers (Abdullah & Hassan, 2020) (Govil, 1983) (Sulaiman, Ali, & Hassan, 2023). The reliability and distribution of complex systems are the main topics of the study, and they are directly related. Size, weight, and other technical and technological factors can be used to describe system cost. Two key criteria affect component reliability: the model needs to be cost-based before input element validation. Depending on the features of the system, the suggested cost factor needs may be changed. This allows engineers to evaluate component cost allocations and prepare each system component to fulfill minimum reliability requirements. Since simple systems can cause major issues when applied to larger, more complex systems, the model must also consider a mathematical analysis of the system's overall reliability. The Harmonic Search Algorithm (HSA), a powerful technique for resolving optimization problems in complex systems, was used to produce the results. Additionally, a behavior model (cost function) was used in the cost computation to guarantee an accurate representation of the cost-reliability connection.

Allocation of reliability and Complex Network optimization

Think about a sophisticated system that has elements linked to dependability. (Kuo, Way, & Zuo, 2003) (Sulaiman, Ali, & Hassan, 2023). We make use of the following notes:

$C_i(R_i)$ = element *i* cost;
$0 \le R_i \le 1$ = dependability of the component;
R_s = dependability of the system;
$[C(R_1, \dots, R_n) = \sum_{i=1}^{n} a_i c_i(R_i)]$ is the entire cost of the system;
in which (a_i) is greater than 0;

RG stands for system reliability goal.

There are many possible outcomes because of the system's modular design and the functions that each component performs. The same operational capacity can be achieved by using a variety of system components, each with varying degrees of dependability. Making it feasible for the system to distribute resources across all or specific components in an optimal manner is the ultimate goal. These problems are fundamental topics in nonlinear programming (Sulaiman, Ali, & Hassan, 2023) (Sulaiman et al., 2025) (Yousif, Ameer, & Sulaiman, 2025). Even though the linked constraint and cost functions of the system do not follow a linear relationship, they can be carefully studied and analyzed.

$Minimized\ C(R_i, \dots, R_i) = \sum_{i=1}^{n} a_i C_i(R_i),\ a_i > 0,$

Subject to:

$$R_s \geq R_G$$
$$0 \leq R_i < 1, in\ which\ i = 1, \dots, n \quad (1)$$

Let $C_i(R_i)$ satisfy certain constraints (Yousif, Ameer, & Sulaiman, 2025), and let the partial cost function be reasonable. These are positive, differentiated functions that increase from$\left[\Longrightarrow \frac{dC_i}{dR_i} \geq 0\right]$.

The fact that its derivatives are comparable $\frac{d^2 C_i}{dR_i^2} \geq 0$ are consistently raised ,

i, e. $\frac{d^2 C_i}{dR_i^2} \geq 0$.

is the Euclidean convexity's component costs function. $[C_i(R_i)]$. The prior design aims to achieve an all-out framework cost base, and RG reduces the system reliability constraint (Sulaiman et al., 2025).

implementation in a complex system

To estimate the complex system, we need to transform it into a more manageable network, just like we might build a parallel network out of a collection of items. The following describes the reliability of n-component parallel and series networks:

$$R_s = \prod_{i=1}^{n} R_i \quad (2)$$
$$R_s = 1 - \prod_{i=1}^{n} (1 - R_i) \quad (3)$$

In this instance, (R_i) is the dependability of component I , and (R_N) is the reliability network (Govil, 1983) (Kuo, Way, & Zuo, 2003). Each complex network's reliability with p minimum paths will be compared. provided by equations (1) and (2).

$$R_s = 1 - \prod_{z=1}^{p} \left(1 - \prod_{j=\alpha}^{\omega} R_j\right) \quad (4)$$

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In a minimal path z, the index of the first component is represented by α, while the index of the final component is represented by ω.
The dependability of the complex network in Fig. 1 below can be ascertained using the equation (3).

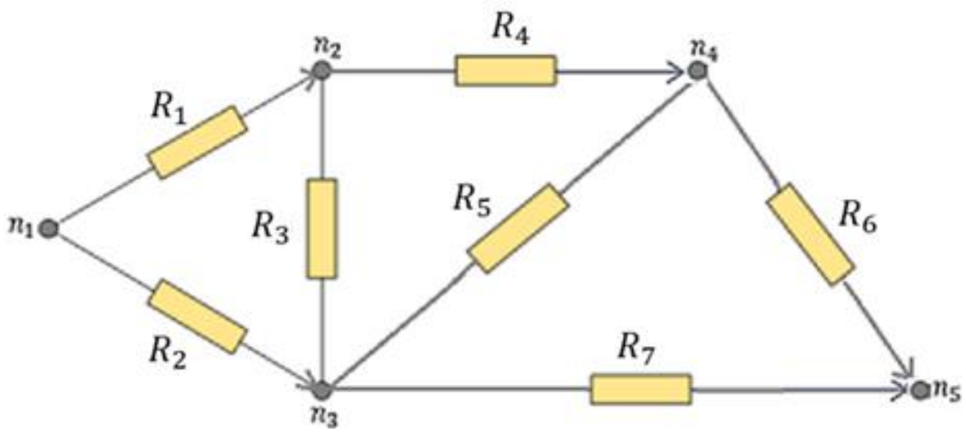

Figure 1. Complex Network

The sets:

$$S = \{\{x_1x_4x_6\},\{x_1x_3x_5x_6\},\{x_1x_4x_5x_7\},\{x_1x_3x_7\},\{x_2x_3x_4x_6\},\{x_2x_5x_6\}, \{x_2x_7\}\ \ \{x_2x_3x_4x_5x_7\},\}.$$

$$R_s = [1-[1-p_r(x_1x_4x_6)]\times[1-p_r(x_1x_3x_5x_6)]\times[1-p_r(x_1x_4x_5x_7)] \times[1-p_r(x_1x_3x_7)]\times[1-p_r(x_2x_3x_4x_6)]\times[1-p_r(x_2x_5x_6)] \times[1-p_r(x_2x_7)]\times[1-p_r(x_2x_3x_4x_5x_7)]] \qquad (5)$$

Note: When the i −If that component is successful, then ($R_i = 1$), and when it fails, then ($R_i = 0$) $\forall\ i = 1,2,\cdots,7$, these lead to:

$[\ R_i^n = R_i]$.

Applying the aforementioned note to equation (5) yields the following polynomial.

$$R_s = R_1R_2 + R_6R_7 + R_2R_3R_4 + R_4R_5R_7 + R_1R_3R_5R_7 + R_2R_3R_5R_6 + R_1R_2R_3R_4R_5R_6 + 2R_1R_2R_3R_4R_5R_7 + R_1R_2R_3R_4R_6R_7 + 2R_1R_2R_3R_5R_6R_7 + R_1R_2R_4R_5R_6R_7 + R_1R_3R_4R_5R_6R_7 + 2R_2R_3R_4R_5R_6R_7 - R_1R_2R_3R_4 - R_1R_2R_6R_7 - R_4R_5R_6R_7 - R_1R_2R_3R_5R_6 - R_1R_2R_3R_5R_7 - R_1R_2R_4R_5R_7 - R_2R_3R_4R_5R_6 - R_2R_3R_4R_5R_7 - R_1R_3R_5R_6R_7 - R_2R_3R_4R_6R_7 - R_1R_3R_4R_5R_7 - R_2R_3R_5R_6R_7 - 3R_1R_2R_3R_4R_5R_6R_7$$

Harmony Search Algorithm (HSA)

Optimization algorithms are a useful tool to overcome the challenges presented by complex problems that are difficult to tackle with traditional methods. The technique of musical improvisation, in which musicians attempt to produce the best possible harmony between musical notes, served as the model for the well-known harmony search algorithm (HSA). The goal of the HSA is to achieve the best value of the objective function by determining the ideal values for the decision variables. In order to arrive at the optimal global solution, it gradually updates the solutions. This study uses Boolean algebra and connection matrices to determine the primary paths and their minimum number in order to assess the reliability of a complex system. It also considers improving the distribution of dependability across system components according to their respective relevance in order to extend the system's life and reduce overall expenses. Optimization is essential to achieving a balance between cost and dependability since the location of each component and its impact on overall performance dictate how resources are distributed.

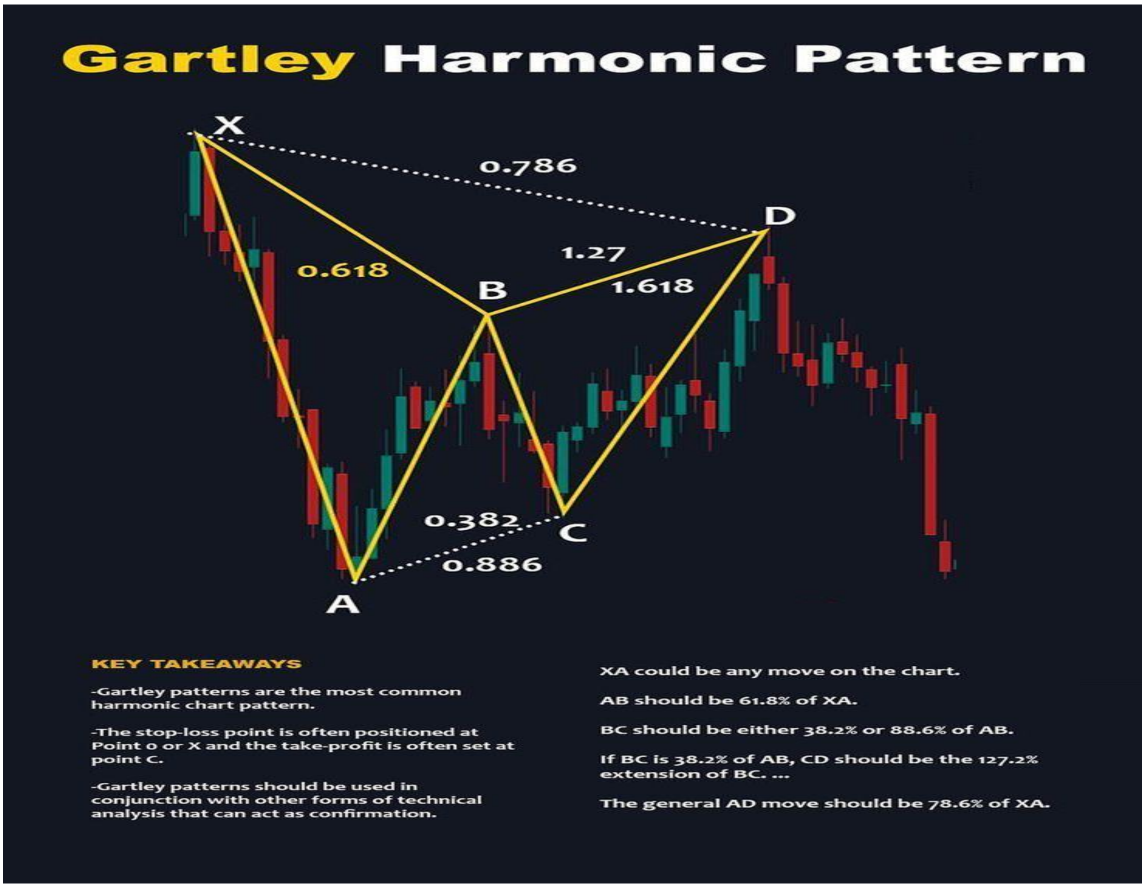

Figure 2**:** Gartley Harmonic pattern

Implementation of HS Algorithem

The Harmony Search (HS) approach uses the symbol "harmony" to suggest a workable solution, with each decision variable representing a unique musical note (Abd Alhasan Hameed Saleh & Hassan, 2023) (Abdullah & Hassan, 2021). One component of the HS method is a harmonic memory (HM), which stores a set number of harmonics (N). If the objective is to maximize or reduce a fitness function (f) while accounting for d choice factors, the optimization equation can be expressed as follows.

Max. (or Min.) $f(x_1, x_2, \ldots, x_n)$

The fitness function is denoted by f in the given context, the decision variable is represented by [x_i ($i = 1, 2, \ldots, n$)], and the dimension f problem is denoted by d. The following actions must be taken in order to apply the Harmony Search algorithm for optimization:

1. Creating a harmonious memory.
2. Creating new harmony through improvisation.
3. Using Harmony Memory to add or remove new harmonies.
4. Until the specified termination condition is met, steps 2 and 3 are repeated repeatedly.
5. After completing the pausing requirement, go to Step 6.
6. Since HM has the best harmony, it is the best result.

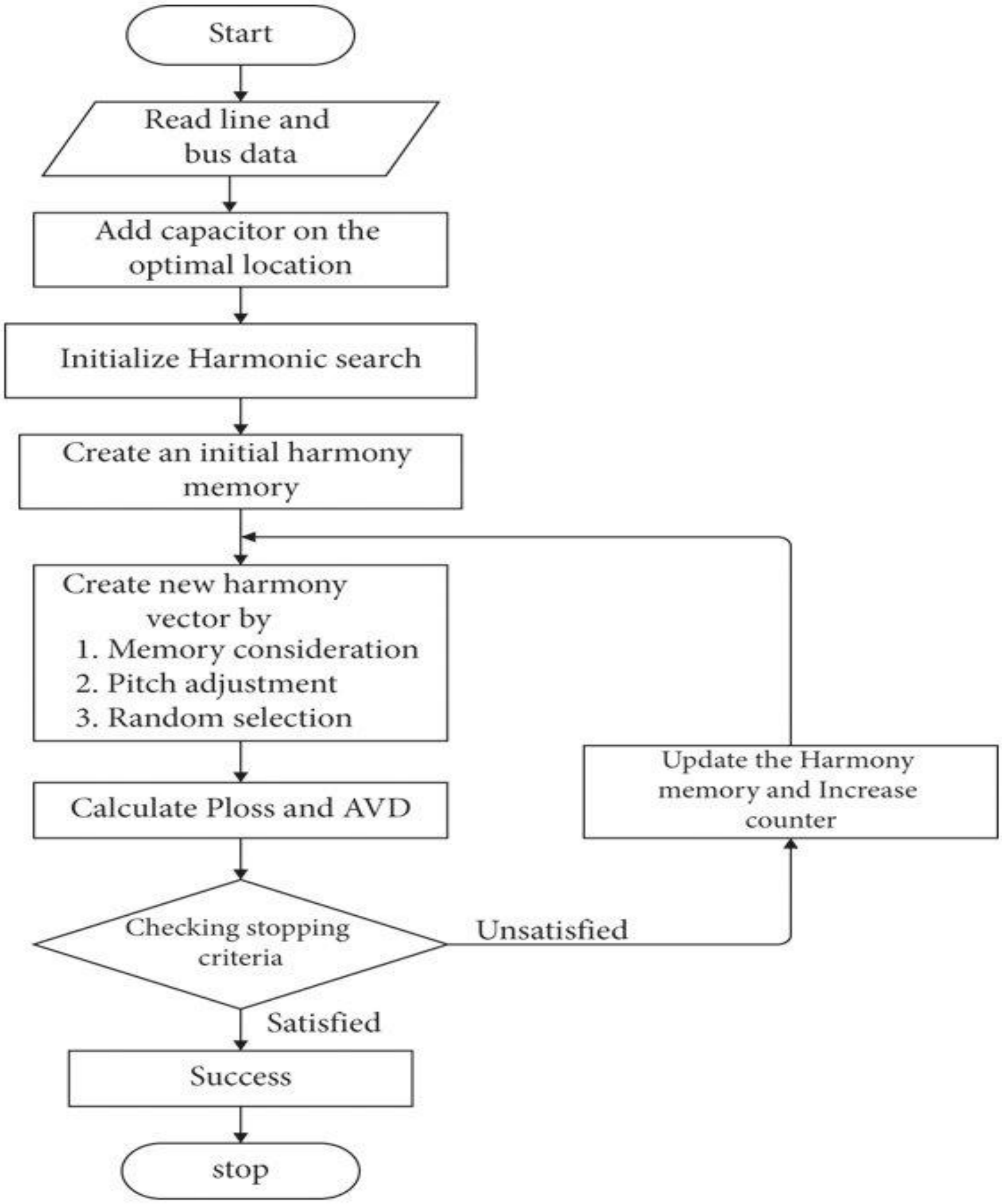

Figure 3. Flow chart of Harmony Search Algorithm (HSA)

Materials and Methods

A. Materials

The following tools and resources were employed in this study:
theory of network reliability.
a model of the logarithmic cost function.
The combinatorial search algorithm (HSA) is one such algorithm.
MATLAB R2020a is a computer and simulation program.
Complex networks were simulated using minimum path sets and connection matrices.

B. Methods

A mathematical model was developed for the complicated system network in order to optimize cost savings and dependability allocation.
The optimization task was described as a nonlinear programming problem with constraints on reliability and cost.
The HSA approach was employed to ascertain the best reliability distribution among system components.
Reliability performance was assessed using traditional formulas for parallel and serial systems.
The simulations and outcomes were produced using MATLAB.

Behavior model (cost function)

Let $[0 \le R_i < 1, i = 1,2,\dots,n]$ and a_i, b_i, are constants. The most important aspect of expenditure is its exponential behavior. It was proposed as (Sulaiman, Ali, & Hassan, 2023).

$$(R_i) = a_i e^{\left(\frac{b_i}{1-R_i}\right)}, a_i > 0, b_i > 0, \quad i = 1,2,\dots,n$$

The optimization problem become

$$Minimize \ \ C(R_i,\dots,R_i) = \sum_{i=1}^{n} a_i \ e^{\left(\frac{b_i}{1-R_i}\right)}, i = 1,2,\dots,n.$$

Subject to :

$$R_s \ge R_G$$
$$0 \le R_i < 1, i = 1,2,\dots,n$$

Using the Harmonic Search (HS) algorithm to optimize the reliability allocation

the best allocation for each system component was obtained, along with the calculation of the total cost, the cost of each component, and the overall system allocation. A direct relationship was found between cost and allocation, as shown in the results in the table below.

Table 1: HSA with an applied cost function for the best dependability allocation.

Components	HSA	COST
R_1	0.99	598.9235
R_2	0.99	598. 9235
R_3	0.95	276.3698
R_4	0.91	189.3699
R_5	0.95	276.3698
R_6	0.98	487.4787
R_7	0.98	487. 4787
R_{system}	0.99	2427.4352e+04

Conclusion

In order to optimize and enhance the total cost and reliability of a complex network, this study used analytical engineering approaches to design the dependability of each system component. To achieve optimal efficiency in the reliability of complex systems, a nonlinear programming model including a cost function and operational constraints relating to labor and resources was used to address the system optimization problem. The Harmonic Search (HSA) method was used to solve the reliability allocation problem. The findings demonstrated that the behavioral model (cost function), which was explained in the paragraphs before this one, produced an ideal value for total reliability, Rs = 0.99, which represented the system's best performance. When the system's components were compared, components (1, 2) had the highest cost and allocation ratio, followed by components (6, 7), (3, 5), and (4), which had the lowest allocation. The sophisticated system's overall cost was determined and is displayed in the tables above. This discrepancy results from the placement of these elements within the complex system design and their direct impact on overall reliability. The proposed model is important because it can manage complex mathematical research in a flexible and efficient way, which makes it helpful for the design and analysis of large engineering systems that need to balance reliability and cost.

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تخصيص الموثوقية المقيد للأنظمة المعقدة باستخدام منهجية محسّنة للبحث التوافقي

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مستخلص البحث:
تركز هذه الورقة البحثية على حساب تخصيص وتوزيع الموثوقية الأمثل لكل مكون من مكونات النظام المعقد، وذلك من خلال تقدير دالة موثوقية متعددة الحدود باستخدام طريقة أقصر مسار، وتطبيق المصفوفات لإيجاد المسار الأمثل للجهاز. تعتمد معظم الدراسات، في تطورها، على دوال الحلقة المغلقة المتزايدة لربط التكلفة وعلاقتها المباشرة بالموثوقية، أي العلاقة المباشرة بين الموثوقية والتكلفة، بمعنى أن تكلفة المكون تزداد مع زيادة موثوقيته. ومع ذلك، قد تكون هذه العلاقات معقدة أو غامضة أو يصعب تحديدها في بعض الأحيان. في هذه الدراسة، تم استخراج دالة الموثوقية للنظام المعقد باستخدام برنامج MATLAB الإصدار R2020a، ودُرست العلاقة بين التكلفة والموثوقية عبر عدة مجموعات بيانات. بالإضافة إلى تحديد التكلفة الإجمالية وموثوقية النظام، استُخدمت خوارزمية البحث التوافقي (HSA) لتحديد تكلفة وموثوقية كل مكون على حدة. جرى تحسين أداء هذه الخوارزمية، وحُقق توزيع أمثل للموثوقية، بتطبيق نموذج السلوك (دالة التكلفة).
الكلمات المفتاحية: طريقة البحث التوافقي، نموذج التكلفة والموثوقية، الموثوقية.