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RESEARCH ARTICLE

Approximate Solution and Simulation for Fractional Burger Equation with Two Initial Conditions

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ABSTRACT

This paper considers the nonlinear homogeneous fractional Burger's equation as a type of nonlinear fractional partial differential equations (FPDE). Our goal in this paper is to show that an initial value problem (IVP) can be modified with a second initial condition when ($\alpha \in (1, 2]$) as the velocity of the movement, and the obtained solution agrees with the nature of the wave with space and time for the problem. The Caputo fractional derivative is used in all the fractional derivatives. Also, the algorithm of the Laplace transform decomposition method (LTDM) for fractional PDEs is presented. The approximate solution converges to the exact solution in Theorem 1. Also, a numerical simulation is made to confirm the theoretical results. In addition, the solution is displayed graphically for three values of (α) that belong to the interval $(1, 2]$ to study the effects of changing the value of the fractional order derivative on the wave solutions of the time-fractional Burger PDE. The time interval is extended in each graph to check the effect of time on the number and shape of the waves in addition to changing the fractional order. Finally, a comparison of the obtained solutions is made.

Keywords: Burger's equation, Caputo fractional derivative, Initial value problem, Laplace decomposition method, Mittag-Leffler function

Introduction

Fractional calculus is considered a basis for many mathematical applications. Researchers have given a lot of attention to Fractional differential equations (FDE), both ordinary and partial, due to their wide applications in all fields such as science, engineering, electrochemistry, viscoelasticity, and optics.[1] Burgers' fractional partial differential equation (BFPDE) is a fundamental convection–diffusion equation and it is used widely in applied mathematics.[2] It was first introduced by Bateman H in 1915 and he found its steady solutions.[3] Burgers JM studied the equation in 1948 as a type of equation used to describe mathematical models of turbulence.[3] Qazza A introduced the direct power series technique in solving various types of time-FPDEs and systems.[4] Ahmed SA et al. considered the Double Sumudu-Elzaki transform to solve FPDE with boundary conditions.[5] Tarate SA et al. used the new Sumudu T iterative method to find the solution of nonlinear PDEs.[6] Elbadri M used the fractional Laplace transform with the Adomian decomposition method (ADM) to obtain the approximate solutions of Burgers' equation with the Caputo-Katugampola Fractional derivative.[7] Mohammed Mulla MA developed a numerical method for solving nonlinear (FDEs), with the Caputo-Fabrizio fractional operator and the fractional Laplace transform.[8] Javed I applied ADM to obtain approximate solutions for the nonlinear FPDEs.[9] Jassim HK

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and Al-Rkhais HA employed the fractional Sumudu DM to solve the time-FPDEs and system of time-FPDEs.[10] Kadri I et al. used combined Laplace transform (CLT) and ADM to find the solutions of nonlinear time-fractional Burgers' equation.[11] Alquran M, et al. used the Laplace transform (LT) method with the Differential transform (DT) method to solve non-homogeneous linear PDEs.[12] Ali AI, et al. solved a coupled system of nonlinear PDEs by using the power series (PS) technique with fractional order.[13] Jafari H proposed the iterative LTM for solving systems of linear and nonlinear FPDEs.[14] The method of Laplace residual PS is also used to obtain the solution of systems of FPDEs.[15] Sultana M, et al. introduced a new method which is called Aboodh Tamimi Ansari transform to solve systems of linear and nonlinear FPDEs.[16] Magdy E, et al. presented a numerical strategy to find approximate solutions for the nonlinear time fractional Burgers's equation.[17] Youssri YH and Atta AG used a combination of Lucas polynomials basis to solve the time-fractional diffusion equation spectrally.[18]

In this paper, the Laplace transform DM is used to prove the approximate solution of the homogeneous Burger's FPDE. Two initial conditions were included in the equation and were satisfied in the obtained solution. An application of Theorem 1 is introduced for another fractional Burger's type equation. The graph of the wave solutions was illustrated by taking different values for the fractional order (α) of the FPDE and a comparison is made for the obtained results.

This paper includes three sections. Section one introduces some definitions that will be needed in this research. In section two, the Laplace ADM will be proposed. Finally, in section three, the results will be discussed, as well as introducing to the numerical simulation with comparison.

Preliminaries

Definition 1:[19,20] If $f : \mathbb{R}_+ \to \mathbb{R}$ and $n-1 < \alpha < n,\ n \in \mathbb{N}$, then:

$$_*D_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-\tau)^{n-\alpha-1} f^{(n)}(\tau) d\tau,$$

where Γ is the gamma function, called the Caputo fractional derivative of order α, provided it exists.

Definition 2:[21] The one parameter Mittag-Leffler (ML) function is defined by:

$$E_{\alpha}(t) = \sum_{k=0}^{\infty} \frac{t^k}{\Gamma(\alpha k + 1)},\ \alpha > 0,\ t \in \mathbb{C}.$$

The two parameters Mittag-Leffler function is given by

$$E_{\alpha,\beta}(t) = \sum_{k=0}^{\infty} \frac{t^k}{\Gamma(\alpha k + \beta)},\ Re(\alpha) > 0,\ \beta \in \mathbb{C},\ t \in \mathbb{C}.$$

Lemma 1:[22] *The Laplace transform (LT) for Caputo's fractional derivative is given by:*

$$\mathcal{L}_{\alpha}\{D_t^{\alpha} f(t)\} = S^{\alpha} F_{\alpha}(s) - \sum_{k=0}^{n-1} S^{\alpha-k-1} f^{(k)}(0),$$

where $n < \alpha < n+1$ and $\alpha \in \mathbb{N}$. Laplace transform for the fractional PDEs with respect to t is given by:

$$\mathcal{L}_{\alpha}\{D_t^{\alpha} u(x,t)\}\ S^{\alpha} \mathcal{L}_{\alpha}\{u(x,t)\} - \left(S^{\alpha-1}\ u(x,0) + S^{\alpha-2}\ \frac{\partial u(x,0)}{\partial t} + S^{\alpha-3}\ \frac{\partial^2 u(x,0)}{\partial t^2} + \cdots + S^{\alpha-n}\ \frac{\partial^{n-1} u(x,0)}{\partial t^{n-1}}\right).$$

Methodology

In this section, we introduce the Laplace ADM for solving the fraction Burger's PDE. This method is very effective in dealing with nonlinear equations and systems and it helps to simplify the terms for the application of the Laplace transform. Considering the following FPDE in its general form:[11]

$$D_t^{\alpha} u(x,t) + D_x^{\alpha} u(x,t) + R(u(x,t)) + N(u(x,t)) = f(x,t), \quad (1)$$

with the I.C's.

$$\begin{array}{l} u\left(x,0\right)=g_1\left(x\right) \\ u_t\left(x,0\right)=g_2\left(x\right) \end{array},\ x>0,\ t>0,\ 1<\alpha\le 2, \tag{2}$$

Where:
D_t^{α}: linear operator in Caputo's sense of order α with respect to t,
D_x^{α}: highest order linear differential operator with respect to x,
R: a linear term with a lower derivative,
N: a nonlinear term,
$f(x,t)$: non-homogeneous part,
$x,\ t$: the spatial dimension and time, respectively.

Algorithm of laplace transform decomposition method (LTDM) [11]

The following steps will be applied to the IVP in Eqs. (1) and (2):

Step 1: Applying the Caputo Laplace transform [19] to both sides of Eq. (1) with respect to t, yields:

$$\mathcal{L}_\alpha\left\{D_t^{\alpha}u\left(x,t\right)\right\}+\mathcal{L}_\alpha\left\{D_x^{\alpha}u\left(x,t\right)\right\}+\mathcal{L}_\alpha\left\{R\left(u\left(x,t\right)\right)+N\left(u\left(x,t\right)\right)\right\}=\mathcal{L}_\alpha\left\{f\left(x,t\right)\right\}. \tag{3}$$

Using Lemma 1 and Eq. (2), in Eq. (3), yields

$$S^{\alpha}\mathcal{L}_\alpha\{u(x,t)\}-S^{\alpha-1}u(x,0)-S^{\alpha-2}u_t(x,0)+\mathcal{L}_\alpha\{D_x^{\alpha}u(x,t)\}+\mathcal{L}_\alpha\{R(u(x,t))+N(u(x,t))\}=\mathcal{L}_\alpha\{f(x,t)\}. \tag{4}$$

Step 2: Dividing by s^{α} and applying the Caputo inverse LT to Eq. (4), yields:

$$\mathcal{L}_\alpha\{u(x,t)\}-\frac{1}{s}u(x,0)-\frac{1}{S^2}u_t(x,0)+\frac{1}{S^{\alpha}}\mathcal{L}_\alpha\{D_x^{\alpha}u(x,t)\}+\frac{1}{s^{\alpha}}\mathcal{L}_\alpha\{R(u(x,t))+N(u(x,t))\}=\frac{1}{s^{\alpha}}\mathcal{L}_\alpha\{f(x,t)\},$$

$$u(x,t)=\mathcal{L}_\alpha^{-1}\left\{\frac{1}{s}u(x,0)+\frac{1}{S^2}u_t(x,0)+\frac{1}{s^{\alpha}}\mathcal{L}_\alpha\{f(x,t)\}\right\}-\mathcal{L}_\alpha^{-1}\left\{\frac{1}{s^{\alpha}}\mathcal{L}_\alpha\{D_x^{\alpha}u(x,t)\}+\frac{1}{s^{\alpha}}\mathcal{L}_\alpha\{R(u(x,t))+N(u(x,t))\}\right\}. \tag{5}$$

Step 3: Presenting the solution of Eq. (5) as an infinite Series:

$$u(x,t)=\sum_{n=0}^{\infty}u_n(x,t). \tag{6}$$

Decomposing the nonlinear term as follows:

$$N(u(x,t))=\sum_{n=0}^{\infty}A_n, \tag{7}$$

where A_n are the Adomian Polynomials of $u_0,u_1,u_2,\ldots,u_n$, and it will be decomposed as follows:

$$A_n=\frac{1}{n!}\left\{\frac{\partial^n}{\partial\lambda^n}\left(N\left(\sum_{i=0}^{n}\lambda^i u_i\right)\right)\right\}_{\lambda=0}, \tag{8}$$

where $n=0,1,2,3,\ldots$

Substituting Eqs. (6) and (7) in Eq. (5), yields

$$\sum_{n=0}^{\infty}u_n(x,t)=\mathcal{L}_\alpha^{-1}\left\{\frac{1}{s}u(x,0)+\frac{1}{S^2}u_t(x,0)+\frac{1}{s^{\alpha}}\mathcal{L}_\alpha\{f(x,t)\}\right\}-\mathcal{L}_\alpha^{-1}\left\{\frac{1}{S^{\alpha}}\mathcal{L}_\alpha\left\{D_x^{\alpha}\sum_{n=0}^{\infty}u_n(x,t)+R\left(\sum_{n=0}^{\infty}u_n(x,t)\right)+\sum_{n=0}^{\infty}A_n\right\}\right\}. \tag{9}$$

Step 4: Matching both sides of Eq. (9), gives the following iterative algorithm:

$$u_0(x,t) = \mathcal{L}_\alpha^{-1}\left\{\frac{1}{s}u(x,0) + \frac{1}{s^2}u_t(x,0) + \frac{1}{s^\alpha}\mathcal{L}_\alpha\{f(x,t)\}\right\},$$

$$= u(x,0) + t\,u_t(x,0) + \mathcal{L}_\alpha^{-1}\left\{\frac{1}{s^\alpha}\mathcal{L}_\alpha\{f(x,t)\}\right\},$$

$$u_0(x,t) = g_1(x) + tg_2(x) + \mathcal{L}_\alpha^{-1}\left\{\frac{1}{s^\alpha}\mathcal{L}_\alpha\{f(x,t)\}\right\}. \tag{10}$$

$\vdots$

$$u_{n+1}(x,t) = -\mathcal{L}_\alpha^{-1}\left\{\frac{1}{S^\alpha}\mathcal{L}_\alpha\left\{D_x^\alpha\sum_{n=0}^{\infty}u_n(x,t) + R\left(\sum_{n=0}^{\infty}u_n(x,t)\right) + \sum_{n=0}^{\infty}A_n\right\}\right\}. \tag{11}$$

By calculating enough terms of u_n, the solution $u(x,t)$ can be obtained from Eq. (6) which is the approximate analytic solution of Eqs. (1) and (2).

The main results

Laplace transform decomposition method will be used to find an approximate solution for the following initial value problem of the homogeneous Burger's FPDE:

$$\frac{\partial^\alpha u(x,t)}{\partial t^\alpha} - \frac{\partial^2 u(x,t)}{\partial x^2} - 2u(x,t)\frac{\partial u(x,t)}{\partial x} + \frac{\partial u^2(x,t)}{\partial x} = 0, \tag{12}$$

with I.Cs:

$$u(x,0) = \sin x,$$

$$u_t(x,0) = \frac{-1}{\Gamma(\alpha+1)}\sin x,$$

$1 < \alpha \le 2,\ t > 0$ and $x \in \mathbb{R}$ Eq. (12) is the homogeneous fractional Burger's equation.[11]

Theorem 1: *The solution of Eq. (12) is*

$$u(x,t) = \left(E_\alpha(-t) - \frac{t}{\Gamma(\alpha+1)}E_{\alpha,2}(-t)\right)\sin x.$$

Proof: Applying the LT with Caputo sense[19] to both sides of Eq. (12)

$$\mathcal{L}_\alpha\{D_t^\alpha u(x,t)\} - \mathcal{L}_\alpha\left\{\frac{\partial^2 u}{\partial x^2}\right\} + \mathcal{L}_\alpha\left\{-2u(x,t)\frac{\partial u(x,t)}{\partial x} + \frac{\partial(u(x,t))^2}{\partial x}\right\} = 0.$$

$$S^\alpha\mathcal{L}_\alpha\{u(x,t)\} - S^{\alpha-1}u(x,0) - S^{\alpha-2}u_t(x,0) - \mathcal{L}_\alpha\left\{\frac{\partial^2 u}{\partial x^2} + 2u\frac{\partial u}{\partial x} - \frac{\partial u^2}{\partial x}\right\} = 0.$$

Dividing both sides of the above equation by S^α

$$\mathcal{L}_\alpha\{u\} = \frac{1}{S}u(x,0) + \frac{1}{s^2}u_t(x,0) + \frac{1}{s^\alpha}\mathcal{L}_\alpha\left\{\frac{\partial^2 u}{\partial x^2} + 2u\frac{\partial u}{\partial x} - \frac{\partial u^2}{\partial x}\right\} = 0. \tag{13}$$

Let

$$u(x,t) = \sum_{n=0}^{\infty}u_n(x,t). \tag{14}$$

Substituting Eq. (14) in Eq. (13), gives

$$\mathcal{L}_\alpha\left\{\sum_{n=0}^{\infty}u_n(x,t)\right\}=\frac{1}{S}u(x,0)+\frac{1}{s^2}u_t(x,0)+\frac{1}{s^\alpha}\mathcal{L}_\alpha\left\{\frac{\partial^2\sum_{n=0}^{\infty}u_n}{\partial x^2}+2\sum_{n=0}^{\infty}u_n\frac{\partial u_n}{\partial x}-\frac{\partial\sum_{n=0}^{\infty}u_n^2}{\partial x}\right\}. \tag{15}$$

Let

$$A_n=\sum_{n=0}^{\infty}u_n^2(x,t),$$
$$A_0=u_0^2,$$
$$A_1=2u_0\,u_1,$$
$$A_2=2u_0u_2+u_1^2,$$
$$\vdots$$

And

$$B_n=\sum_{n=0}^{\infty}u_n\frac{\partial u_n}{\partial x},$$
$$B_0=u_0u_{0x},$$
$$B_1=u_0u_{1x}+u_1u_{0x},$$
$$B_2=u_0u_{2x}+u_1u_{1x}+u_2u_{0x},$$
$$\vdots$$

Substituting the above terms in Eq. (15) yields:

$$\mathcal{L}_\alpha\left\{\sum_{n=0}^{\infty}u_n(x,t)\right\}=\frac{1}{S}u(x,0)+\frac{1}{s^2}u_t(x,0)+\frac{1}{s^\alpha}\mathcal{L}_\alpha\left\{\frac{\partial^2\sum_{n=0}^{\infty}u_n}{\partial x^2}+2B_n-\frac{\partial A_n}{\partial x}\right\},$$
$$\mathcal{L}_\alpha\left\{\sum_{n=0}^{\infty}u_n\right\}=\frac{1}{s}u(x,0)+\frac{1}{s^2}u_t(x,0)+\frac{1}{s^\alpha}\mathcal{L}_\alpha\left\{\frac{\partial^2\sum_{n=0}^{\infty}u_n}{\partial x^2}-\frac{\partial A_n}{\partial x}+2B_n\right\},$$

Then

$$\mathcal{L}_\alpha\{u_0(x,t)\}=\frac{1}{s}u(x,0)+\frac{1}{s^2}u_t(x,0),$$
$$\vdots$$
$$\mathcal{L}_\alpha\{u_{n+1}\}=\frac{1}{s^\alpha}\mathcal{L}_\alpha\left\{\frac{\partial^2\sum_{n=0}^{\infty}u_n}{\partial x^2}-\frac{\partial A_n}{\partial x}+2B_A\right\}. \tag{16}$$

Now, applying the Caputo inverse LT for both sides of Eq. (16):

$$u_0(x,t)=\mathcal{L}_\alpha^{-1}\left\{\frac{1}{S}u(x,0)+\frac{1}{S^2}u_t(x,0)\right\},$$
$$=u(x,0)+t\;u_t(x,0),$$
$$u_0(x,t)=\sin x-\frac{t}{\Gamma(\alpha+1)}\sin x,$$

$$u_{n+1}(x,t) = \mathcal{L}_\alpha^{-1}\left\{\frac{1}{s^\alpha}\mathcal{L}_\alpha\left\{\frac{\partial^2 u_n}{\partial x^2} - \frac{\partial A_n}{\partial x} + 2B_n\right\}\right\}.$$

For $n = 1$.

$$u_1 = \mathcal{L}_\alpha^{-1}\left\{\frac{1}{s^\alpha}\mathcal{L}_\alpha\left[\frac{\partial^2 u_0}{\partial x^2} - \frac{\partial A_0}{\partial x} + 2B_0\right]\right\},$$

$$= \mathcal{L}_\alpha^{-1}\left\{\frac{1}{S^\alpha}\mathcal{L}_\alpha\left\{\frac{\partial^2 u_0}{\partial x^2} - \frac{\partial u_0^2}{\partial x} + 2u_0 u_x\right\}\right\},$$

$$= \mathcal{L}_\alpha^{-1}\left\{\frac{1}{S^\alpha}\mathcal{L}_\alpha\left\{\frac{\partial^2\left(\sin x - \frac{t}{\Gamma(\alpha+1)}\sin x\right)}{\partial x^2} - \frac{\partial\left(\sin x - \frac{t}{\Gamma(\alpha+1)}\sin x\right)^2}{\partial x}\right.\right.$$

$$\left.\left. +2\left(\sin x - \frac{t}{\Gamma(\alpha+1)}\sin x\right)\left(\sin x - \frac{t}{\Gamma(\alpha+1)}\sin x\right)_x\right\}\right\},$$

$$= \mathcal{L}_\alpha^{-1}\left\{\frac{1}{s^\alpha}\mathcal{L}_\alpha\left\{-\sin x + \frac{t}{\Gamma(\alpha+1)}\sin x - 2\sin x\cos x + \frac{4t}{\Gamma(\alpha+1)}\sin x\cos x - \frac{2t^2}{(\Gamma(\alpha+1))^2}\sin x\cos x\right.\right.$$

$$\left.\left. +2\sin x\cos x - \frac{2t}{\Gamma(\alpha+1)}\sin x\cos x - \frac{2t}{\Gamma(\alpha+1)}\sin x\cos x + \frac{2t^2}{(\Gamma(\alpha+1))^2}\sin x\cos x\right\}\right\},$$

$$= \mathcal{L}_\alpha^{-1}\left\{\frac{1}{S^\alpha}\mathcal{L}_\alpha\left\{-\sin x + \frac{t}{\Gamma(\alpha+1)}\sin x\right\}\right\},$$

$$= \mathcal{L}_\alpha^{-1}\left\{\frac{1}{s^\alpha}\left(\frac{1}{s^2\Gamma(\alpha+1)} - \frac{1}{s}\right)\sin x\right\},$$

$$u_1 = \left(\frac{t^{\alpha+1}}{\Gamma(\alpha+1)\Gamma(\alpha+2)} - \frac{t^\alpha}{\Gamma(\alpha+1)}\right)\sin x,$$

For $n = 2$.

$$u_2 = \mathcal{L}_\alpha^{-1}\left\{\frac{1}{s^\alpha}\mathcal{L}_\alpha\left\{\frac{\partial^2 u_1}{\partial x^2} - \frac{\partial A_1}{\partial x} + 2B_1\right\}\right\},$$

$$= \mathcal{L}_\alpha^{-1}\left\{\frac{1}{s^\alpha}\mathcal{L}_\alpha\left\{\frac{\partial^2 u_1}{\partial x^2} - \frac{\partial(2u_0u_1)}{\partial x} + 2(u_0u_{1x} + u_1u_{0x})\right\}\right\},$$

$$= \mathcal{L}_\alpha^{-1}\left\{\frac{1}{s^\alpha}\mathcal{L}_\alpha\left\{\frac{\partial^2\left(\left(\frac{t}{\Gamma(\alpha+1)\Gamma(\alpha+2)} - \frac{t^\alpha}{\Gamma(\alpha+1)}\right)\sin x\right)}{\partial x^2} - \frac{\partial\left(2\left(1 - \frac{t}{\Gamma(\alpha+1)}\right)\sin x\left(\frac{t^{\alpha+1}}{\Gamma(\alpha+1)\Gamma(\alpha+2)} - \frac{t^\alpha}{\Gamma(\alpha+1)}\right)\sin x\right)}{\partial x}\right.\right.$$

$$+ 2\left(1 - \frac{t}{\Gamma(\alpha+1)}\right)\sin x\left(\left(\frac{t^{\alpha+1}}{\Gamma(\alpha+1)\Gamma(\alpha+2)} - \frac{t^\alpha}{\Gamma(\alpha+1)}\right)\sin x\right)_x$$

$$\left.\left. + \left(\frac{t^{\alpha+1}}{\Gamma(\alpha+1)\Gamma(\alpha+2)} - \frac{t^\alpha}{\Gamma(\alpha+1)}\right)\sin x\left(\left(1 - \frac{t}{\Gamma(\alpha+1)}\right)\sin x\right)_x\right\}\right\},$$

$$= \mathcal{L}_\alpha^{-1}\left\{\frac{1}{s^\alpha}\mathcal{L}_\alpha\left\{\left(\frac{-t^{\alpha+1}}{\Gamma(\alpha+1)\Gamma(\alpha+2)} + \frac{t^\alpha}{\Gamma(\alpha+1)}\right)(-\sin x)\right.\right.$$

$$- 4\left(1 - \frac{t}{\Gamma(\alpha+1)}\right)\left(\frac{t^{\alpha+1}}{\Gamma(\alpha+1)\Gamma(\alpha+2)} - \frac{t^\alpha}{\Gamma(\alpha+1)}\right)\sin x\cos x + 2\left(\left(1 - \frac{t}{\Gamma(\alpha+1)}\right)\left(\frac{t^{\alpha+1}}{\Gamma(\alpha+1)\Gamma(\alpha+2)}\right.\right.$$

$$\left.\left.\left.\left. -\frac{t^\alpha}{\Gamma(\alpha+1)}\right)\sin x\cos x + \left(1 - \frac{t}{\Gamma(\alpha+1)}\right)\left(\frac{t^{\alpha+1}}{\Gamma(\alpha+1)\Gamma(\alpha+2)} - \frac{t^\alpha}{\Gamma(\alpha+1)}\right)\sin x\cos x\right)\right\}\right\},$$

$$= \mathcal{L}_\alpha^{-1}\left\{\frac{1}{s^\alpha}\mathcal{L}_\alpha\left\{\left(\frac{-t^{\alpha+1}}{\Gamma(\alpha+1)\Gamma(\alpha+2)} + \frac{t^\alpha}{\Gamma(\alpha+1)}\right)\sin x\right\}\right\},$$

$$= \mathcal{L}_{\alpha}^{-1}\left\{\frac{1}{s^{\alpha}}\mathcal{L}_{\alpha}\left\{\frac{-t^{\alpha+1}}{\Gamma(\alpha+1)\Gamma(\alpha+2)}\sin x+\frac{t^{\alpha}}{\Gamma(\alpha+1)}\sin x\right\}\right\},$$

$$= \mathcal{L}_{\alpha}^{-1}\left\{\frac{1}{s^{\alpha}}\left\{\frac{-\Gamma(\alpha+2)\sin x}{s^{\alpha+2}\Gamma(\alpha+1)\Gamma(\alpha+2)}+\frac{\Gamma(\alpha+1)\sin x}{s^{\alpha+1}\Gamma(\alpha+1)}\right\}\right\},$$

$$u_2 = \left(\frac{-t^{2\alpha+1}}{\Gamma(\alpha+1)\Gamma(2\alpha+2)}+\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}\right)\sin x.$$

For $n = 3$.

$$u_3 = \mathcal{L}_{\alpha}^{-1}\left\{\frac{1}{s^{\alpha}}\mathcal{L}_{\alpha}\left\{\frac{\partial^2 u_2}{\partial x^2}-\frac{\partial A_2}{\partial x}+2B_2\right\}\right\},$$

$$= \mathcal{L}_{\alpha}^{-1}\left\{\frac{1}{s^{\alpha}}\mathcal{L}_{\alpha}\left\{\frac{\partial^2 u_2}{\partial x^2}-\frac{\partial\left(2u_0u_2+u_1^2\right)}{\partial x}+2\left(u_0u_{2x}+u_1u_{1x}+u_2u_{0x}\right)\right\}\right\},$$

$$= \mathcal{L}_{\alpha}^{-1}\left\{\frac{1}{s^{\alpha}}\mathcal{L}_{\alpha}\left\{\frac{\partial^2\left(-\frac{t^{2\alpha+1}}{\Gamma(\alpha+1)\Gamma(2\alpha+2)}+\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}\right)}{\partial x^2}\sin x-2\left(1-\frac{t}{\Gamma(\alpha+1)}\right)\left(\frac{-t^{2\alpha+1}}{\Gamma(\alpha+1)\Gamma(2\alpha+2)}+\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}\right)(\sin x)^2\right.\right.$$

$$+\left(\left(\left(\frac{t^{\alpha+1}}{\Gamma_{(\alpha+1)}\Gamma_{(\alpha+2)}}-\frac{t^{\alpha}}{\Gamma_{(\alpha+1)}}\right)\sin x\right)^2\right)_x+2\left[\left(1-\frac{t}{\Gamma_{(\alpha+1)}}\right)\sin x\left(\left(\frac{-t^{2\alpha+1}}{\Gamma(\alpha+1)\Gamma(2\alpha+2)}+\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}\right)\sin x\right)_x\right]$$

$$+\left(\left(\frac{t^{\alpha+1}}{\Gamma(\alpha+1)\Gamma(\alpha+2)}-\frac{t^{\alpha}}{\Gamma(\alpha+1)}\right)\sin x\right)\left(\left(\frac{t^{\alpha+1}}{\Gamma(\alpha+1)\Gamma(\alpha+2)}-\frac{t^{\alpha}}{\Gamma(\alpha+1)}\right)\sin x\right)_x$$

$$\left.\left.+\left(\left(\frac{-t^{2\alpha+1}}{\Gamma(\alpha+1)\Gamma(2\alpha+2)}+\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}\right)\sin x\right)\left(\left(1-\frac{t}{\Gamma(\alpha+1)}\right)\sin x\right)_x\right\}\right\},$$

$$= \mathcal{L}_{\alpha}^{-1}\left\{\left(\frac{1}{s^{\alpha}}\mathcal{L}_{\alpha}\left\{\frac{t^{2\alpha+1}}{\Gamma(\alpha+1)\Gamma(2\alpha+1)}-\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}\right)\sin x-4\left(1-\frac{t}{\Gamma(\alpha+1)}\right)\left(\frac{-t^{2\alpha+1}}{\Gamma(\alpha+1)\Gamma(2\alpha+2)}+\frac{t^{2\alpha}}{\Gamma(\alpha+2)}\right)\right.\right.$$

$$\times\sin x\cos x+4\left(1-\frac{t}{\Gamma(\alpha+1)}\right)\left(\frac{-t^{2\alpha+1}}{\Gamma_{(\alpha+1)\Gamma(2\alpha+2)}}+\frac{t^{2\alpha}}{\Gamma(\alpha+2)}\right)\sin x\cos x-2\left(\frac{t^{\alpha+1}}{\Gamma(\alpha+1)\Gamma(\alpha+2)}-\frac{t^{\alpha}}{\Gamma(\alpha+1)}\right)^2$$

$$\left.\times\sin x\cos x+2\left(\frac{t^{\alpha+1}}{\Gamma(\alpha+1)\Gamma(\alpha+2)}-\frac{t^{\alpha}}{\Gamma(\alpha+1)}\right)^2\sin x\cos x\right\},$$

$$= \mathcal{L}_{\alpha}^{-1}\left\{\frac{1}{s^{\alpha}}\mathcal{L}_{\alpha}\left\{\left(\frac{t^{2\alpha+1}}{\Gamma(\alpha+1)\Gamma(2\alpha+1)}-\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}\right)\sin x\right\}\right\},$$

$$= \mathcal{L}_{\alpha}^{-1}\left\{\frac{1}{s^{\alpha}}\left(\frac{\Gamma(2\alpha+2)}{s^{2\alpha+2}\Gamma(\alpha+1)\Gamma(2\alpha+2)}-\frac{\Gamma(2\alpha+1)}{s^{2\alpha+1}\Gamma(2\alpha+1)}\right)\sin x\right\},$$

$$= \mathcal{L}_{\alpha}^{-1}\left\{\left(\frac{1}{s^{3\alpha+2}\Gamma(\alpha+1)}-\frac{1}{s^{3\alpha+1}}\right)\sin x\right\},$$

$$u_3 = \left(\frac{t^{3\alpha+1}}{\Gamma(\alpha+1)\Gamma(3\alpha+2)}-\frac{t^{3\alpha}}{\Gamma(3\alpha+1)}\right)\sin x.$$

Then series of the solution is

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t),$$

$$= u_0(x,t)+u_1(x,t)+u_2(x,t)+u_3(x,t)+\cdots,$$

$$= \sin x - \frac{t}{\Gamma(\alpha+1)}\sin x + \left(\frac{t^{\alpha+1}}{\Gamma(\alpha+1)\Gamma(\alpha+2)} - \frac{t^{\alpha}}{\Gamma(\alpha+1)}\right)\sin x + \left(\frac{-t^{2\alpha+1}}{\Gamma(\alpha+1)\Gamma(2\alpha+2)} + \frac{t^{2\alpha}}{\Gamma(2\alpha+1)}\right)\sin x$$
$$+ \left(\frac{t^{3\alpha+1}}{\Gamma(\alpha+1)\Gamma(3\alpha+2)} - \frac{t^{3\alpha}}{\Gamma(3\alpha+1)}\right)\sin x + \cdots,$$
$$= \left(\left(1 - \frac{t^{\alpha}}{\Gamma(\alpha+1)} + \frac{t^{2\alpha}}{\Gamma(2\alpha+1)} - \frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + \cdots\right) - \frac{t}{\Gamma(\alpha+1)}\left(1 - \frac{t^{\alpha}}{\Gamma(\alpha+2)} + \frac{t^{2\alpha}}{\Gamma(2\alpha+2)}\right.\right.$$
$$\left.\left. - \frac{t^{3\alpha}}{\Gamma(3\alpha+2)} + \cdots\right)\right)\sin x.$$

Satisfying the initial Conditions of Eq. (12):

$$u(x,0) = ((1-0)-0)\sin x = \sin x.$$
$$u_t(x,t) = \left(\left(0 - \frac{\alpha t^{\alpha-1}}{\Gamma(\alpha+1)} + \frac{2\alpha t^{2\alpha-1}}{\Gamma(2\alpha+1)} + \cdots\right) + \left(\frac{-1}{\Gamma(\alpha+1)} + \frac{(\alpha+1)t^{\alpha}}{\Gamma(\alpha+2)} + \cdots\right)\right)\sin x,$$
$$u_t(x,0) = \left((0) + \left(\frac{-1}{\Gamma(\alpha+1)} - 0 + \cdots\right)\right)\sin x,$$
$$u_t(x,0) = \frac{-1}{\Gamma(\alpha+1)}\sin x.$$

The solution can be written as

$$u(x,t) = \left(\sum_{n=0}^{\infty}\frac{(-t)^n}{\Gamma(n\alpha+1)} - \frac{t}{\Gamma(\alpha+1)}\sum_{n=0}^{\infty}\frac{(-t)^n}{\Gamma(n\alpha+2)}\right)\sin x.$$

From Definition 2, the above solution can be represented by the ML function:

$$u(x,t) = \left(E_{\alpha}(-t) - \frac{t}{\Gamma(\alpha+1)}E_{\alpha,2}(-t)\right)\sin x. \quad \blacksquare \tag{17}$$

Application

In this subsection, another fractional Burger's equation[11] will be solved by using the same procedure as in Theorem 1. Consider the homogeneous nonlinear fractional Burger's equation:

$$\frac{\partial^{\alpha}u(x,t)}{\partial t^{\alpha}} + u(x,t)\frac{\partial u(x,t)}{\partial x} - \frac{\partial^2 u(x,t)}{\partial x^2} = 0, \tag{18}$$

with I.Cs:

$$u(x,0) = x,$$
$$u_t(x,0) = -\frac{x}{\Gamma(\alpha+1)},$$

where $1 < \alpha \le 2,\ t > 0$ and $x \in \mathbb{R}$.

Solution: Applying the steps of the LTDM on Eq. (18), yields

$$u_0(x,t) = x - \frac{x\,t}{\Gamma(\alpha+1)} = x\left(1 - \frac{t}{\Gamma(\alpha+1)}\right), \tag{19}$$

$$u_{n+1}(x,t) = \mathcal{L}_{\alpha}^{-1}\left\{\frac{1}{s^{\alpha}}\mathcal{L}_{\alpha}\left\{\frac{\partial^2 u_n}{\partial x^2} - A_n\right\}\right\}. \tag{20}$$

Where

$$u(x,t)=\sum_{n=0}^{\infty}u_n(x,t),$$

and

$$A_n=\sum_{n=0}^{\infty}u_n\frac{\partial u_n}{\partial x},$$

$$A_0=u_0u_{0x},$$

$$A_1=u_0u_{1x}+u_1u_{0x},$$

$$A_2=u_0u_{2x}+u_1u_{1x}+u_2u_{0x}.$$

$$\vdots$$

For $n=1$,

$$u_1=\mathcal{L}_\alpha^{-1}\left\{\frac{1}{s^\alpha}\mathcal{L}_\alpha\left\{\frac{\partial^2u_0}{\partial x^2}-A_0\right\}\right\},$$

$$=\mathcal{L}_\alpha^{-1}\left\{\frac{1}{S^\alpha}\mathcal{L}_\alpha\left\{\frac{\partial^2u_0}{\partial x^2}-u_0u_{0x}\right\}\right\},$$

then,

$$u_1=-x\left(\frac{t^\alpha}{\Gamma(\alpha+1)}-\frac{2t^{\alpha+1}}{\Gamma(\alpha+1)\Gamma(\alpha+2)}+\frac{2t^{\alpha+2}}{\Gamma(\alpha+1)^2\Gamma(\alpha+3)}\right). \tag{21}$$

For $n=2$,

$$u_2=\mathcal{L}_\alpha^{-1}\left\{\frac{1}{s^\alpha}\mathcal{L}_\alpha\left\{\frac{\partial^2u_1}{\partial x^2}-A_1\right\}\right\},$$

$$=\mathcal{L}_\alpha^{-1}\left\{\frac{1}{S^\alpha}\mathcal{L}_\alpha\left\{\frac{\partial^2u_1}{\partial x^2}-(u_0u_{1x}+u_1u_{0x})\right\}\right\},$$

$$u_2=x\left(\frac{2t^{2\alpha}}{\Gamma(2\alpha+1)}-\frac{2(\alpha+4)t^{2\alpha+1}}{\Gamma(\alpha+1)\Gamma(2\alpha+2)}+\frac{4(\alpha+4)t^{2\alpha+2}}{\Gamma(\alpha+1)^2\Gamma(2\alpha+3)}-\frac{4(\alpha+4)t^{2\alpha+3}}{\Gamma(\alpha+1)^3\Gamma(2\alpha+4)}\right). \tag{22}$$

By the same way the rest of the terms can be found and then the series of the approximate solution is:

$$u(x,t)=\sum_{n=0}^{\infty}u_n(x,t),$$

$$=u_0(x,t)+u_1(x,t)+u_2(x,t)+\cdots,$$

$$=x\left(1-\frac{t}{\Gamma(\alpha+1)}\right)-x\left(\frac{t^\alpha}{\Gamma(\alpha+1)}-\frac{2t^{\alpha+1}}{\Gamma(\alpha+1)\Gamma(\alpha+2)}+\frac{2t^{\alpha+2}}{\Gamma(\alpha+1)^2\Gamma(\alpha+3)}\right)$$
$$+x\left(\frac{2t^{2\alpha}}{\Gamma(2\alpha+1)}-\frac{2((\alpha+4))t^{2\alpha+1}}{\Gamma(\alpha+1)\Gamma(2\alpha+2)}+\frac{4(\alpha+4)t^{2\alpha+2}}{\Gamma(\alpha+1)^2\Gamma(2\alpha+3)}-\frac{4(\alpha+4)t^{2\alpha+3}}{\Gamma(\alpha+1)^3\Gamma(2\alpha+4)}\right)+\cdots,$$

$$u(x,t)=x\left(1-\frac{t}{\Gamma(\alpha+1)}-\frac{t^\alpha}{\Gamma(\alpha+1)}+\frac{2t^{\alpha+1}}{\Gamma(\alpha+1)\Gamma(\alpha+2)}-\frac{2t^{\alpha+2}}{\Gamma(\alpha+1)^2\Gamma(\alpha+3)}+\frac{2t^{2\alpha}}{\Gamma(2\alpha+1)}\right.$$

$$-\frac{2(\alpha+4)t^{2\alpha+1}}{\Gamma(\alpha+1)\Gamma(2\alpha+2)}+\frac{4(\alpha+4)t^{2\alpha+2}}{\Gamma(\alpha+1)^2\Gamma(2\alpha+3)}-\frac{4(\alpha+4)t^{2\alpha+3}}{\Gamma(\alpha+1)^3\Gamma(2\alpha+4)}+\cdots\Bigg). \quad (23)$$

Satisfying the initial Conditions of Eq. (18) in Eq. (23):

$$u(x,0)=x(1-0-0+0\ldots)=x,$$

$$u_t(x,t)=x\Bigg(0-\frac{1}{\Gamma(\alpha+1)}-\frac{\alpha t^{\alpha-1}}{\Gamma(\alpha+1)}+\frac{2(\alpha+1)t^{\alpha}}{\Gamma(\alpha+1)\Gamma(\alpha+2)}-\frac{2(\alpha+2)t^{\alpha+1}}{\Gamma(\alpha+1)^2\Gamma(\alpha+3)}-\frac{2(\alpha+4)(2\alpha+1)t^{2\alpha}}{\Gamma(\alpha+1)\Gamma(2\alpha+2)}$$
$$+\frac{4\alpha t^{2\alpha-1}}{\Gamma(2\alpha+1)}+\frac{4(\alpha+4)(2\alpha+2)t^{2\alpha+1}}{\Gamma(\alpha+1)^2\Gamma(2\alpha+3)}-\frac{4(\alpha+4)(2\alpha+3)t^{2\alpha+2}}{\Gamma(\alpha+1)^3\Gamma(2\alpha+4)}+\cdots\Bigg),$$

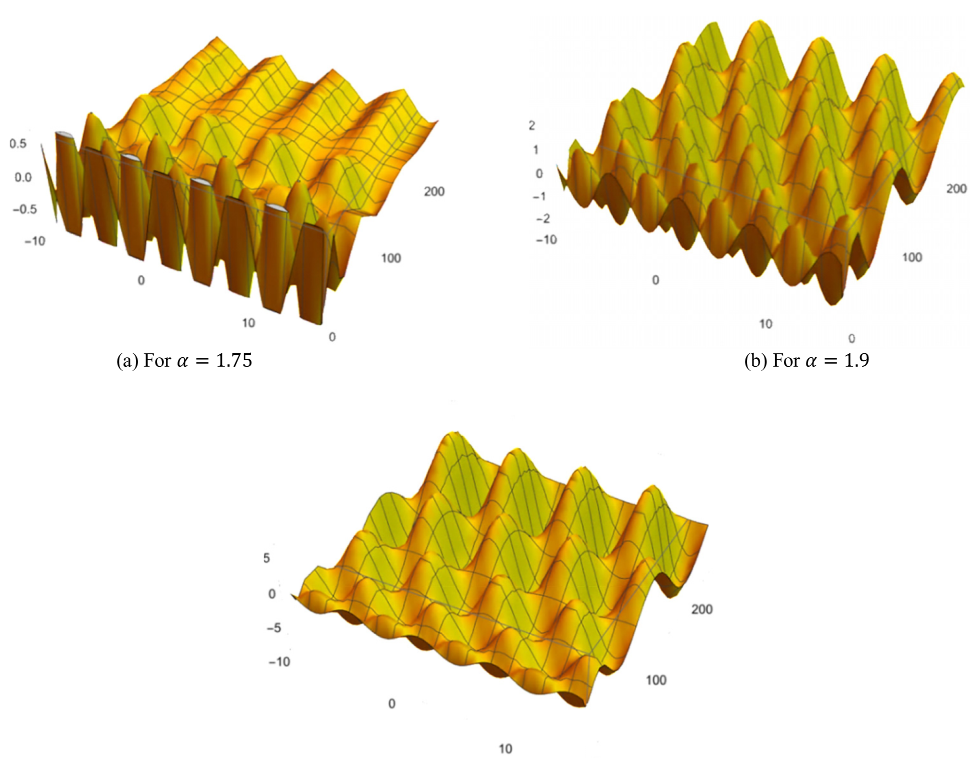
$$u_t(x,0)=x\left(0-\frac{1}{\Gamma(\alpha+1)}-0+0-0-0+\cdots\right)=-\frac{x}{\Gamma(\alpha+1)}.$$

The approximate solution obtained in Eq. (23) satisfies the two given initial conditions.

Results and discussion

Numerical simulation

In this subsection, the approximate solution in Eq. (12) is explained graphically. Figs. 1 to 3 show 3D plots of some of the obtained solutions for different values of $\alpha \in (1, 2]$ by using Mathematica. The figures illustrate the change in the shape and number of waves when the time interval is extended and the spatial dimension is fixed. Fig. 1 represents the solution when $\alpha = 1.75$, 1.9 and 2 within the interval $-10 \le x \le 15$ and $0 \le t \le 250$.

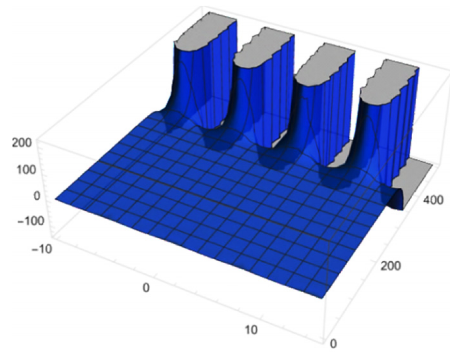


(a) For $\alpha = 1.75$ (b) For $\alpha = 1.9$

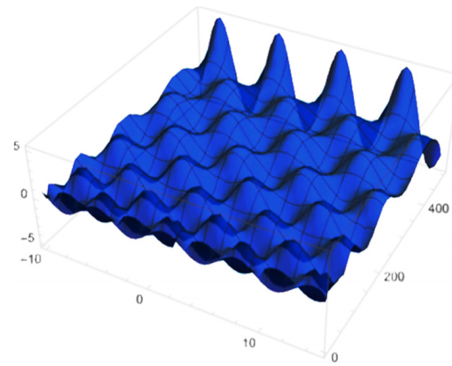
(c) For $\alpha = 2$

Fig. 1. The fractional solutions $u(x, t)$ of Eq. (12) for $-10 \le x \le 15$ and $0 \le t \le 250$.

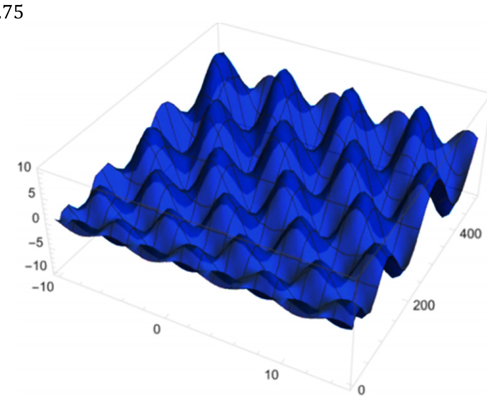
Fig. 2 represents the solution when $\alpha = 1.75,\ 1.9$ and 2 within the interval $-10 \le x \le 15$ and $0 \le t \le 500$.



(a) For $\alpha = 1.75$



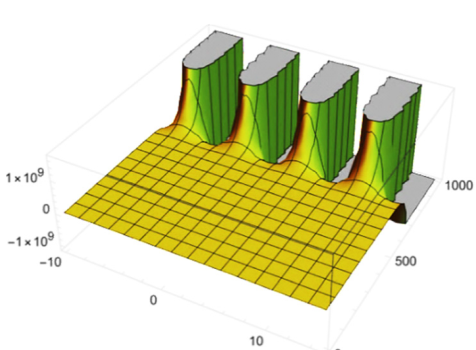
(b) For $\alpha = 1.9$



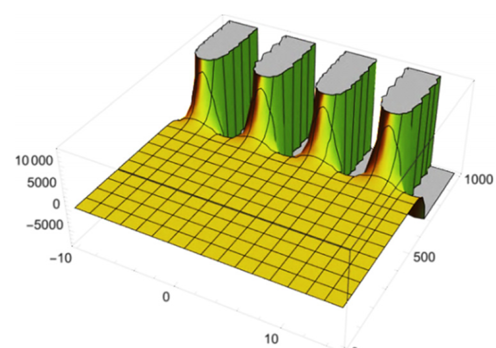
(c) For $\alpha = 2$

Fig. 2. The fractional solutions $u(x,t)$ of Eq. (12) for $-10 \le x \le 15$ and $0 \le t \le 500$.

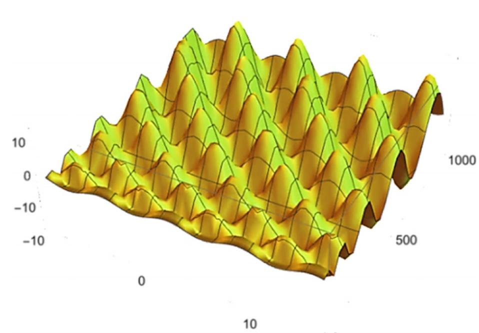
Fig. 3 represents the solution when $\alpha = 1.75,\ 1.9$ and 2 within the interval $-10 \le x \le 15$ and $0 \le t \le 1000$.



(a) For $\alpha = 1.75$



(b) For $\alpha = 1.9$



(c) For $\alpha = 2$

Fig. 3. The fractional solutions $u(x,t)$ of Eq. (12) for $-10 \le x \le 15$ and $0 \le t \le 1000$.

The physical interpretation of the above figures is as follows:

1. The waves are oscillatory and nonperiodic depending on the x-coordinate, time and the fractional order of the derivative.
2. The waves decrease and increase alternatively.
3. The amplitude of the waves grows at first then dampens with time.
4. The largest length of the wave before damping is at $\alpha = 2$.

Comparing the results

In this subsection, the comparison between the solutions obtained in this paper and the solutions in[11] for Eqs. (12) and (23) with Caputo sense. Tables 1 and 2 below show the details of each solution in the two works.

Table 1. Comparison between the solution in Eq. (17) and the solution in Eq. (40) in.[11]

	Equation	Initial Conditions	Interval of α	Method of Solution	Solution
Solution in Eq. (40) in[11]	$\frac{\partial^\alpha u(x,t)}{\partial t^\alpha} - \frac{\partial^2 u(x,t)}{\partial x^2} - 2u(x,t)\frac{\partial u(x,t)}{\partial x} + \frac{\partial u^2(x,t)}{\partial x} = 0$	$u(x,\ 0) = \sin x$	$\alpha \in (0, 1]$	Laplace transform decomposition method	$u(x,t) = \sin x\, E_\alpha(-t)$
Solution in Eq. (17)		$u(x,0) = \sin x$ $u_t(x,0) = \frac{-1}{\Gamma(\alpha+1)} \sin x$	$\alpha \in (1, 2]$		$u(x,t) = (E_\alpha(-t) - \frac{t}{\Gamma(\alpha+1)} E_{\alpha,2}(-t)) \sin x$

Table 2. Comparison between the solution in Eq. (23) and the solution in Eq. (67) in.[11]

	Equation	Initial Conditions	Interval of α	Method of Solution	Solution
Solution in Eq. (67) in[11]	$\frac{\partial^\alpha u(x,t)}{\partial t^\alpha} + u(x,t)\frac{\partial u(x,t)}{\partial x} - \frac{\partial^2 u(x,t)}{\partial x^2} = 0$	$u(x,\ 0) = x$	$\alpha \in (0, 1]$	Laplace transform decomposition method	$u(x,t) = x\left(1 - \frac{t^\alpha}{\Gamma(\alpha+1)} + \frac{2t^{2\alpha}}{\Gamma(2\alpha+1)} - \frac{6t^{3\alpha}}{\Gamma(3\alpha+1)} + \cdots\right).$
Solution in Eq. (23)		$u(x,0) = x$ $u_t(x,0) = \frac{-1}{\Gamma(\alpha+1)} x$	$\alpha \in (1, 2]$		$u(x,t) = x\left(1 - \frac{t}{\Gamma(\alpha+1)} - \frac{t^\alpha}{\Gamma(\alpha+1)} + \frac{2t^{\alpha+1}}{\Gamma(\alpha+1)\Gamma(\alpha+2)} - \frac{2t^{\alpha+2}}{\Gamma(\alpha+1)^2\Gamma(\alpha+3)} - \frac{2(\alpha+4)t^{2\alpha+1}}{\Gamma(\alpha+1)\Gamma(2\alpha+2)} + \frac{2t^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{4(\alpha+4)t^{2\alpha+2}}{\Gamma(\alpha+1)^2\Gamma(2\alpha+3)} - \frac{4(\alpha+4)t^{2\alpha+3}}{\Gamma(\alpha+1)^3\Gamma(2\alpha+4)} + \cdots\right).$

Conclusion

This work introduces a simple idea by suggesting an IVP consisting of the nonlinear homogeneous Burgers' FPDE with two initial conditions and the fractional order (α) belongs to the interval $(1, 2]$. The two initial conditions represent the position and velocity and it was proven that they satisfy the approximate solution, specially the second condition. The obtained solution in Theorem 1 converges to the one parameter and two parameters Mittag-Leffler function. Another Burger's type FPDE is solved by the algorithm of (LTDM) as an application on Theorem 1. The approximate solution was illustrated graphically for three values of the fractional order (α). It can be Concluded from this work that an approximated solution obtained from the suggested IVP may converge to the exact solution as in Theorem 1 (Eq. (17)) or it may be represented as an infinity series without reaching its exact form as in Eq. (23), but in both cases it is a successful and interesting attempt to solve such IVPs. In the future, researchers can extend the initial conditions and take a higher order for the fractional derivative (α) and use another suitable method of solution.

Authors' declaration

- Conflicts of Interest: None.

- We hereby confirm that all the Figures and Tables in the manuscript are ours. Furthermore, any Figures and images that are not ours have been included with the necessary permission for re-publication, which is attached to the manuscript.
- No animal studies are present in the manuscript.
- No human studies are present in the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee at University of Baghdad.

Author's contributions statement

H.E.A.J. and F.A.S. participated in solving the equation and finding the results in this paper. S.N.A. suggested the main idea and analyzed the results. H.E.A.J. and F.A.S. wrote the paper with input from all authors.

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الحل التقريبي والمحاكاة لمعادلة برجر الكسرية مع شرطين ابتدائيين

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الخلاصة

تناول هذه البحث معادلة برجر الكسرية المتجانسة اللاخطية كنوع من المعادلات التفاضلية الجزئية الكسرية اللاخطية (FPDE). هدفنا في هاه البحث هو إظهار أنه يمكن تعديل مسألة قيمة ابتدائية (IVP) بشرط ابتدائي ثانٍ عندما () يمثل سرعة الحركة والحل الناتج يتفق مع طبيعة الموجة بالمكان والزمان للمسألة. استخدمت مشتقة كابوتو الكسرية في جميع المشتقات الكسرية. كماستعرض خوارزمية طريقة تفريق تحويل لابلاس (LTDM) للمعادلات التفاضلية الجزئية الكسرية. يتقارب الحل التقريبي الى الحل التام في مبرهنة 1. كما يتم إجراء محاكاة عددية لتأكيد النتائج النظرية. بالإضافة إلى ذلك، عرض الحل بيانياً لثلاث قيم لـ (α) تنتمي إلى الفترة [1,2] لدراسة تأثيرات تغيير قيمة رتبة المشتقة الكسرية على حلول الموجة لمعادلة برجر الجزئية التفاضلية. يتم تمديد الفترة الزمنية في كل رسم بياني للتحقق من تأثير الوقت على عدد وشكل الموجات بالإضافة إلى تغيير قيمة الرتبة الكسري. وأخيرًا، إجريت مقارنة للحلول الناتجة.

الكلمات المفتاحية: معادلة برجر، مشقة كابوتو الكسرية، مسألة قيمة ابتدائية، طريقة تفريق لابلاس، دالة ميتاج- لفلر.