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RESEARCH ARTICLE

A Power Log-logistic Modified Weibull Distribution: Model, Properties and a Simulation

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ABSTRACT

This paper deals with a new model called the power log-logistic modified Weibull distribution (PLLoGMWD) using the power transformation method, which can be regarded as an extension of the distribution introduced by Broderick O. Oluyede et al. Several significant statistical properties of the power log-logistic modified Weibull distribution are computed, including reliability, hazard rate functions, reverse hazard function, rth moment, characteristic function, Renyi entropy, conditional moments, and the reliability stress strength model. The distribution function, density function, hazard rate function and reliability of the new model have several interesting properties, which are illustrated in the graphs of each function. The estimation of model parameters is justified using the generalized method of moments, accompanied by a Monte Carlo simulation study to test the suitability of the proposed distribution. A comprehensive numerical simulation was conducted using nine different scenarios and two sample sizes, n = 100 and n = 500. It was observed that the performance of the estimators, especially for large samples, was less biased and more stable, ranging between –0.432 and 0.573. It also showed a significant decrease in mean square error values, ranging from 0.073 to 1.534, and mean absolute square error values, ranging from 0.298 to 1.654, offering the accuracy and robustness of the proposed model. Confidence intervals for the unknown parameters were calculated, ranging from 1.427 to 4.964. The results revealed in terms of bias, mean estimate, confidence intervals, mean absolute errors, and mean squared errors to demonstrate its future applicability in some real- life situations, especially, in the economic fields.

Keywords: Generalized method of moments estimation, Log-logistic modified Weibull distribution, Power transformation, Renyi entropy, Stress-strength reliability

Introduction

In recent years, the need has emerged to provide more versatile and flexible distributions that can be used to study data. Among the most principal of these distributions are those formed using the power transformation approach. For example, Ghitany et al. proposed the two-parameter power Lindley distribution, the properties of which are discussed.[1] Rady et al. introduced the power Lomax (POLO) distribution, which they used for remission times in bladder cancer data.[2] Gemeay et al.[3] recently proposed a two-parameter distribution, the power unit inverse Lindley (PUILD), in which some uncertainty measures were calculated for, mainly through an incomplete gamma function. Murat Bulut et al.[4] introduced the alpha power Lomax (APL) distribution. Mikhall Nikulin and Firoozeh Haghigh presented the power generalized Weibull family.[5] This family has been shown to be suitable for modeling data indicating nonmonotone hazard rates and can be used in survival analysis and reliability studies. In the study by Riad et al.,[6] the Bilal distribution was originally developed to model lifetime data, and two different COVID-19 data sets were used to illustrate its applications for accurately modeling real-life data sets. Gemeay et al.[7] have proposed a new bounded model, known as the

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power Lambert uniform distribution, which demonstrates the construction of a new quantile regression model based on the new Unit distribution. Some actuarial measures have also been derived mathematically for the model. In addition, Elif Yildirim et al.[8] contributed to the definition of a novel distribution, the Power unit Burr-XII distribution, and investigated its statistical properties. They also compared its performance with other distributions. Furthermore, it can be used to model skewed data. Ahmed M. Gemeay et al.[9] studied the power Mira distribution and calculated basic risk metrics to enhance its applicability in actuarial and financial domains.[9] More details can be explored in references.[10–13] The question here lies in the increasing demand for extended distributions capable of simulating complex data. This study aims to develop a continuous distribution generated by a power transformation that can be used in economics, has a symmetric density function, and is quite easy to handle statistical properties. The proposed distribution has several advantages over the Weibull and Log-Logistic distributions. While the Weibull distribution is more flexible in modeling failure rate behavior, and the Log-Logistic distribution is better suited for heavy-tailed data, Renny's entropy analysis has demonstrated that the proposed distribution exhibits a wider range of uncertainty values, providing greater flexibility in representing different probability patterns.

The log-logistic modified Weibull distribution was originally introduced for modeling lifetime data by Broderick O. Oluyede et al.[14] and has the following cumulative distribution function (CDF) and probability density function (PDF).

$$F_{\mathrm{LLoGMW}}\ (x) = 1 - \left[1 + \left(\frac{x}{s}\right)^{c}\right]^{-1} \exp\left(-\alpha\, x^{\beta}\, e^{\lambda\, x}\right);\ \ x \geq 0; s, c, \alpha, \beta > 0 \text{ and } \lambda \geq 0. \tag{1}$$

$$f_{\mathrm{LLoGMW}}\,(x) = e^{-\alpha x^{\beta} e^{\lambda x}} \left[1 + \left(\frac{x}{s}\right)^{c}\right]^{-1} \left[\alpha\, x^{\beta-1}\ \ e^{\lambda x}\ \ (\beta + \lambda x\ ) + \frac{c x^{c-1}}{(s^{c} + x^{c})}\right]. \tag{2}$$

This paper is organized into two parts: the first presents the definition of the PLLoGMW distribution, focusing on its corresponding reliability statistics and exploring some of its statistical properties. It also provides the generalized method of moments for estimating model parameters. The second part evaluates the model's performance using Monte Carlo simulations for nine different scenarios.

Materials and methods

Power Log-logistic modified Weibull (PLLoGMW) distribution

Here we introduce a new extension of the log-logistic modified Weibull distribution, considering the power transformation,[15] $Y = X^{\frac{1}{a}}$, where the random variable X is said to follow the log-logistic modified Weibull distribution with parameters $s, c, \alpha, \beta > 0$ *and* $\lambda \geq 0$.

Suppose Y is represented to random variable for a power log-logistic modified Weibull distribution. Then, the CDF and PDF of the PLLoGMWD are obtained as follows:

$$G\,(y) = 1 - \left[1 + \left(\frac{y^{a}}{s}\right)^{c}\right]^{-1} \exp\left(-\alpha y^{a\beta} e^{\lambda y^{a}}\right), \tag{3}$$

For $a, \beta,\ c,\ \alpha,\ s\ > 0,\ \lambda \geq 0$ and $y \geq 0$,

$$g\,(y) = a\ e^{-\alpha y^{a\beta} e^{\lambda y^{a}}} \left\{y \left[1 + \left(\frac{y^{a}}{s}\right)^{c}\right]\right\}^{-1} \cdot \left[\alpha\ y^{a\beta}\ e^{\lambda y^{a}}\,(\beta + \lambda y^{a}) + \frac{c y^{ac}}{(s^{c} + y^{ac})}\right], \tag{4}$$

It is clear that the Weibull distribution, the log-logistic distribution, the exponential Weibull distribution, and the modified exponential distribution are particular cases of the corresponding expressions (3) and (4) for $(\lambda = c = 0)$, $(\alpha = 0, c = 1)$, $(\beta = 1, \lambda = 0, c = 0)$ and $(a = \beta = 1)$ respectively.

So, the Reliability $Ʈ_1(y)$, hazard rate functions $Ʈ_2(y)$, cumulative hazard function $Ʈ_3(y)$ and reverse hazard function $Ʈ_4(y)$ can easily be written respectively as[16]

$$Ʈ_1\,(y) = 1 - G\,(y) = \left(1 + \left(\frac{y^{a}}{s}\right)^{c}\right)^{-1} \exp\left(-\alpha y^{a\beta} e^{\lambda y^{a}}\right), \tag{5}$$

$$\text{Ʈ}_2(y) = \frac{g(y)}{\text{Ʈ}_1(y)} = \left\{ a\,\alpha\, y^{a\beta-1} e^{\lambda y^a} (\beta + \lambda y^a) + \frac{ac}{y} \cdot \left(\frac{y^a}{s}\right)^c \left[1 + \left(\frac{y^a}{s}\right)^c\right]^{-1} \right\}, \tag{6}$$

$$\text{Ʈ}_3(y) = -\ln(1 - G(y)) = \ln\left(1 + \left(\frac{y^a}{s}\right)^c\right) + \alpha y^{a\beta} e^{\lambda y^a}, \tag{7}$$

$$\text{Ʈ}_4(y) = \frac{g(y)}{G(y)} = \left\{1 - \left[1 + \left(\frac{y^a}{s}\right)^c\right]^{-1} e^{-\alpha y^{a\beta} e^{\lambda y^a}}\right\}^{-1} . \, a \exp\left(-\alpha y^{a\beta} e^{\lambda y^a}\right) . \left\{y\left[1 + \left(\frac{y^a}{s}\right)^c\right]\right\}^{-1}$$
$$. \left\{\alpha\, y^{a\beta} e^{\lambda y^a} (\beta + \lambda y^a) + \frac{c y^{ac}}{(s^c + y^{ac})}\right\}. \tag{8}$$

The rth moment about origin of PLLoGMWD can be derived as

$$E(y^r) = \int_0^\infty y^r g(y)\, dy = a \int_0^\infty y^{r-1}\, e^{-\alpha y^{a\beta} e^{\lambda y^a}} \left[1 + \left(\frac{y^a}{s}\right)^c\right]^{-1} . \left[\alpha\, y^{a\beta} e^{\lambda y^a} (\beta + \lambda y^a) + \frac{c y^{ac}}{(s^c + y^{ac})}\right] dy.$$

Now since, $e^{-z} = \sum_{k=0}^{\infty} \frac{(-1)^k z^k}{k!}$ with $|z| < 1,\ k > 0$[17] then, will be get $e^{-\alpha y^{a\beta} e^{\lambda y^a}} = \sum_{k=0}^{\infty} \frac{(-1)^k \alpha^k y^{ka\beta}\ e^{k\lambda y^a}}{k!}$,
Then

$$E(x^r) = a \sum_{k=0}^{\infty} \frac{(-1)^k \alpha^k}{k!} \int_0^\infty y^{r+ka\beta-1} .\, e^{k\lambda y^a} . \left[1 + \left(\frac{y^a}{s}\right)^c\right]^{-1} . \left\{\alpha\, y^{a\beta} e^{\lambda y^a} (\beta + \lambda y^a) + \frac{c y^{ac}}{s^c \left[1 + \left(\frac{y^a}{s}\right)^c\right]}\right\} dy,$$

Since $e^z = \sum_{w=0}^{\infty} \frac{z^w}{w!}$,[17] then, will be get $e^{(k+1)\lambda y^a} = \sum_{w=0}^{\infty} \frac{[(k+1)\lambda]^w}{w!} y^{aw}$ and $e^{k\lambda y^a} = \sum_{w=0}^{\infty} \frac{(k\lambda)^w}{w!} y^{aw}$,
Then

$$E(y^r) = \left\{ a\beta \sum_{k=0}^{\infty} \frac{(-1)^k \alpha^{k+1}}{k!} \cdot \sum_{w=0}^{\infty} \frac{[(k+1)\lambda]^w}{w!} \int_0^\infty y^{r+a(k\beta+\beta+w)-1} \left[1 + \left(\frac{y^a}{s}\right)^c\right]^{-1} dy \right.$$
$$+ a\lambda \sum_{k=0}^{\infty} \frac{(-1)^k \alpha^{k+1}}{k!} \cdot \sum_{w=0}^{\infty} \frac{[(k+1)\lambda]^w}{w!} \int_0^\infty y^{r+a(k\beta+\beta+1+w)-1} \left[1 + \left(\frac{y^a}{s}\right)^c\right]^{-1} dy$$
$$\left. + \frac{ac}{s^c} \sum_{k=0}^{\infty} \frac{(-1)^k \alpha^k}{k!} \cdot \sum_{w=0}^{\infty} \frac{(k\lambda)^w}{w!} \int_0^\infty y^{r+a(k\beta+c+w)-1} \left[1 + \left(\frac{y^a}{s}\right)^c\right]^{-2} dy \right\}.$$

Now, by setting $z = [1 + (\frac{y^a}{s})^c]^{-1}$, then $y = s^{\frac{1}{a}} (\frac{1-z}{z})^{\frac{1}{ac}}$ and $dy = \frac{-s^{\frac{1}{a}}}{ca} z^{-2} (\frac{1-z}{z})^{\frac{1-ac}{ac}}$ *dz*, so that

$$E(y^r) = \left\{ \frac{\beta}{c} \sum_{k=0}^{\infty} \frac{(-1)^k \alpha^{k+1}}{k!} \sum_{w=0}^{\infty} \frac{[(k+1)\lambda]^w}{w!} \cdot s^{\left[\frac{r+a(k\beta+\beta+w)}{a}\right]} . \int_0^1 z^{\left[1-\frac{r+a(k\beta+\beta+w)}{ac}\right]-1} \cdot (1-z)^{\left[\frac{r+a(k\beta+\beta+w)}{ac}\right]-1} dz \right.$$
$$+ \frac{\lambda}{c} \sum_{k=0}^{\infty} \frac{(-1)^k \alpha^{k+1}}{k!} \sum_{w=0}^{\infty} \frac{[(k+1)\lambda]^w}{w!} \cdot s^{\left[\frac{r+a(k\beta+\beta+1+w)}{a}\right]} . \int_0^1 z^{\left[1-\frac{r+a(k\beta+\beta+1+w)}{ac}\right]-1} \cdot (1-z)^{\left[\frac{r+a(k\beta+\beta+1+w)}{ac}\right]-1} dz$$
$$\left. + \sum_{k=0}^{\infty} \frac{(-1)^k \alpha^k}{k!} \sum_{w=0}^{\infty} \frac{(k\lambda)^w}{w!} \cdot s^{\left[\frac{r+a(k\beta+w)}{a}\right]} . \int_0^1 z^{\left[1-\frac{r+a(k\beta+w)}{ac}\right]-1} \cdot (1-z)^{\left[1+\frac{r+a(k\beta+w)}{ac}\right]-1} dz \right\}.$$

By using beta function $\text{Ƀ}(a, b) = \int_0^1 t^{a-1}(1-t)^{b-1} dt$, then

$$E(y^r) = \left\{ \frac{\beta}{c} \sum_{k=0}^{\infty} \frac{(-1)^k \alpha^{k+1}}{k!} \sum_{w=0}^{\infty} \frac{[(k+1)\lambda]^w}{w!} \cdot s^{\left[\frac{r+a(k\beta+\beta+w)}{a}\right]} \cdot \text{Ƀ}\left[1 - \frac{r + a(k\beta+\beta+w)}{ac}, \frac{r + a(k\beta+\beta+w)}{ac}\right] \right.$$

$$+\frac{\lambda}{c}\sum_{k=0}^{\infty}\frac{(-1)^k\alpha^{k+1}}{k!}\sum_{w=0}^{\infty}\frac{[(k+1)\lambda]^w}{w!}\cdot s^{\left[\frac{r+a(k\beta+\beta+1+w)}{a}\right]}\cdot B\left[1-\frac{r+a(k\beta+\beta+1+w)}{ac},\frac{r+a(k\beta+\beta+1+w)}{ac}\right]$$

$$+\sum_{k=0}^{\infty}\frac{(-1)^k\alpha^k}{k!}\sum_{w=0}^{\infty}\frac{(k\lambda)^w}{w!}\cdot s^{\left[\frac{r+a(k\beta+w)}{a}\right]}.B-\left[1-\frac{r+a(k\beta+w)}{ac},\ 1+\frac{r+a(k\beta+w)}{ac}\right]\Bigg\}. \quad (9)$$

So, the characteristic function of the PLLoGMW distribution can be obtained as[17]

$$Q_y(t)=E\left(e^{iyt}\right)=E\left(\sum_{r=0}^{\infty}\frac{(it)^r}{r!}y^r\right)=\sum_{r=0}^{\infty}\frac{(it)^r}{r!}E\left(y^r\right),$$

where $E(y^r)$ is the r^{th} moment.

The quantile function y_q of the can be derived as

$$q=P\left(y\le y_q\right)=G\left(Y_q\right)=1-\left[1+\left(\frac{y^a}{s}\right)^c\right]^{-1}e^{-\alpha y^{a\beta}e^{\lambda y^a}},\ 0<q<1,\ Y_q>0,$$

$$\alpha y^{a\beta}e^{\lambda y^a}+\ln\left(1+\left(\frac{y^a}{s}\right)^c\right)+\ln\left(1-G\left(y\right)\right)=0,$$

$$\alpha y^{a\beta}e^{\lambda y^a}+\ln\left(1+\left(\frac{y^a}{s}\right)^c\right)+\ln\left(1-U\right)=0. \quad (10)$$

Stress strength model of power log-logistic modified Weibull (PLLoGMW) distribution

Let's have two independent random variables, say Y: Stress and X: strength, that follow a PLLoGMW distribution with different parameters. The reliability stress strength model of PLLoGMW distribution can be obtained as follows.

$$ss=P\left(Y<X\right)=\int_0^{\infty}G_y\left(x\right)\ g_x\left(x\right)\ dx=E\left(G_y\left(x\right)\right), \quad (11)$$

Where $G_y(x)$ represents cumulative distribution function of the PLLoGMW distribution as in Eq. (3) with parameters $a_1, \beta_1, c_1, \alpha_1, s_1$ and λ_1.

$$ss=E\left(1-\left[1+\left(\frac{X^{a_1}}{s_1}\right)^{c_1}\right]^{-1}e^{-\alpha_1X^{a_1\beta_1}e^{\lambda_1X^{a_1}}}\right)=1-E\left(\left[1+\left(\frac{X^{a_1}}{s_1}\right)^{c_1}\right]^{-1}e^{-\alpha_1X^{a_1\beta_1}e^{\lambda_1X^{a_1}}}\right).$$

Let $I_1=E\left(\left[1+\left(\frac{X^{a_1}}{s_1}\right)^{c_1}\right]^{-1}e^{-\alpha_1X^{a_1\beta_1}e^{\lambda_1X^{a_1}}}\right).$

Since $e^{-z}=\sum_{q=0}^{\infty}\frac{(-1)^qz^q}{q!}$,[17] then, will be get $e^{-\alpha_1X^{a_1\beta_1}e^{\lambda_1X^{a_1}}}=\sum_{q=0}^{\infty}\frac{(-1)^q\alpha_1{}^q}{q!}x^{qa_1\beta_1}e^{q\lambda_1x^{a_1}}$, Then

$$I_1=\sum_{q=0}^{\infty}\frac{(-1)^q\alpha_1{}^q}{q!}\cdot E\left(\left[1+\left(\frac{X^{a_1}}{s_1}\right)^{c_1}\right]^{-1}x^{qa_1\beta_1}e^{q\lambda_1x^{a_1}}\right).$$

Since $e^z=\sum_{p=0}^{\infty}\frac{z^p}{p!}$,[17] we get $e^{q\lambda_1x^{a_1}}=\sum_{p=0}^{\infty}\frac{(q\lambda_1)^p}{p!}x^{pa_1}$, Then

$$I_1=\sum_{q=0}^{\infty}\frac{(-1)^q\alpha_1{}^q}{q!}\sum_{p=0}^{\infty}\frac{(q\lambda_1)^p}{p!}\cdot E\left(\left[1+\left(\frac{X^{a_1}}{s_1}\right)^{c_1}\right]^{-1}\cdot X^{a_1(q\beta_1+p)}\right).$$

Now by using Taylor series expansion[18] of $\frac{1}{1-Z} = \sum_{n=0}^{\infty} Z^n$ So $[1 + (\frac{x^{a_1}}{s_1})^{c_1}]^{-1} = \sum_{n=0}^{\infty} \frac{(-1)^n}{s_1^{nc_1}} x^{na_1c_1}$, Then

$$I_1 = \sum_{q=0}^{\infty}\sum_{p=0}^{\infty}\sum_{n=0}^{\infty} \frac{(-1)^{q+n}\alpha_1{}^q(q\lambda_1)^p}{q!p!\; s_1{}^{nc_1}} \cdot E(X^{a_1(nc_1+q\beta_1+p)}),$$

where

$$E\left(X^{a_1(nc_1+q\beta_1+p)}\right) = \Bigg\{\frac{\beta}{c}\cdot\sum_{k=0}^{\infty}\sum_{w=0}^{\infty}\frac{(-1)^k\alpha^{k+1}[(k+1)\lambda]^w}{k!w!}\cdot s^{\frac{a_1(nc_1+q\beta_1+p)+a(k\beta+\beta+w)}{a}}$$
$$\cdot B\left[1-\frac{a_1(nc_1+q\beta_1+p)+a(k\beta+\beta+w)}{ac},\frac{a_1(nc_1+q\beta_1+p)+a(k\beta+\beta+w)}{ac}\right]$$
$$+\frac{\lambda}{c}\sum_{k=0}^{\infty}\sum_{w=0}^{\infty}\frac{(-1)^k\alpha^{k+1}[(k+1)\lambda]^w}{k!w!}\cdot s^{\frac{a_1(nc_1+q\beta_1+p)+a(k\beta+\beta+1+w)}{a}}\cdot B-\left[1-\frac{a_1(nc_1+q\beta_1+p)+a(k\beta+\beta+1+w)}{ac},\right.$$
$$\left.\frac{a_1(nc_1+q\beta_1+p)+a(k\beta+\beta+1+w)}{ac}\right]+\sum_{k=0}^{\infty}\sum_{w=0}^{\infty}\frac{(-1)^k\alpha^k[k\lambda]^w}{k!w!}\cdot s^{\frac{a_1(nc_1+q\beta_1+p)+a(k\beta+w)}{a}}.$$
$$B-\left[1-\frac{a_1(nc_1+q\beta_1+p)+a(k\beta+w)}{ac},\;1+\frac{a_1(nc_1+q\beta_1+p)+a(k\beta+w)}{ac}\right]\Bigg\}.$$

Then, the stress-strength reliability model for PLLoGMWD is as follows,

$$\begin{aligned} ss = 1 - &\sum_{q=0}^{\infty}\sum_{p=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{q+n}\alpha_1{}^q(q\lambda_1)^p}{q!p!\;s_1{}^{nc_1}}\cdot\Bigg\{\frac{\beta}{c}\cdot\sum_{k=0}^{\infty}\sum_{w=0}^{\infty}\frac{(-1)^k\alpha^{k+1}\left[(k+1)\lambda\right]^w}{k!w!}\cdot s^{\frac{a_1(nc_1+q\beta_1+p)+a(k\beta+\beta+w)}{a}}\\ &\cdot B\left[1-\frac{a_1(nc_1+q\beta_1+p)+a(k\beta+\beta+w)}{ac},\frac{a_1(nc_1+q\beta_1+p)+a(k\beta+\beta+w)}{ac}\right]+\frac{\lambda}{c}\sum_{k=0}^{\infty}\sum_{w=0}^{\infty}\\ &\times\frac{(-1)^k\alpha^{k+1}\left[(k+1)\lambda\right]^w}{k!w!}\cdot s^{\frac{a_1(nc_1+q\beta_1+p)+a(k\beta+\beta+1+w)}{a}}\cdot B\left[1-\frac{a_1(nc_1+q\beta_1+p)+a(k\beta+\beta+1+w)}{ac},\right.\\ &\left.\frac{a_1(nc_1+q\beta_1+p)+a(k\beta+\beta+1+w)}{ac}\right]+\sum_{k=0}^{\infty}\sum_{w=0}^{\infty}\frac{(-1)^k\alpha^k[k\lambda]^w}{k!w!}\cdot s^{\frac{a_1(nc_1+q\beta_1+p)+a(k\beta+w)}{a}}\\ &\cdot B\left[1-\frac{a_1(nc_1+q\beta_1+p)+a(k\beta+w)}{ac},\;1+\frac{a_1(nc_1+q\beta_1+p)+a(k\beta+w)}{ac}\right]\Bigg\}. \end{aligned} \tag{12}$$

Conditional moments

In the context of describing probability distributions, our interest here is to find the conditional moments and the mean residual life time function.

The conditional moments of the PLLoGMWD is given[14]

$$E\left(Y^r/Y>t\right) = \frac{1}{\bar{G}(t)}\int_t^{\infty} y^r g(y)\,dy. \tag{13}$$

Substituting from Eq. (4) in to Eq. (13), then, will be

$$E\left(Y^r/Y>t\right) = \frac{1}{\bar{G}(t)}\int_t^{\infty} y^{r-1}.\;ae^{-\alpha y^{a\beta}e^{\lambda y^a}}\left[1+\left(\frac{y^a}{s}\right)^c\right]^{-1}.\left\{\alpha y^{a\beta}e^{\lambda y^a}\left(\beta+\lambda y^a\right)+\frac{cy^{ac}}{s^c\left[1+\left(\frac{y^a}{s}\right)^c\right]}\right\}dy.$$

Since $e^{-z} = \sum_{k=0}^{\infty}\frac{(-1)^k}{k!}z^k$,[17] then, will be get $e^{-\alpha y^{a\beta}e^{\lambda y^a}} = \sum_{k=0}^{\infty}\frac{(-1)^k\alpha^k}{k!}y^{ka\beta}e^{k\lambda y^a}$. Then

$$E\left(Y^r/Y>t\right) = \frac{1}{\bar{G}(t)}\left\{a\beta.\sum_{k=0}^{\infty}\frac{(-1)^k\alpha^{k+1}}{k!}\int_t^{\infty} y^{r+a(k\beta+\beta)-1}e^{(k+1)\lambda y^a}\left[1+\left(\frac{y^a}{s}\right)^c\right]^{-1}dy\right.$$

$$+ a\lambda \sum_{k=0}^{\infty} \frac{(-1)^k \alpha^{k+1}}{k!} \int_t^{\infty} y^{r+a(k\beta+\beta+1)-1} e^{(k+1)\lambda y^a} \left[1+\left(\frac{y^a}{s}\right)^c\right]^{-1} dy$$

$$+ \frac{ac}{s^c} \sum_{k=0}^{\infty} \frac{(-1)^k \alpha^k}{k!} \int_t^{\infty} y^{r+a(k\beta+c)-1} e^{k\lambda y^a} \left[1+\left(\frac{y^a}{s}\right)^c\right]^{-2} dy \Bigg\},$$

Since $e^z = \sum_{p=0}^{\infty} \frac{z^p}{p!}$. Then[17], will be get $e^{(k+1)\lambda y^a} = \sum_{p=0}^{\infty} \frac{[(k+1)\lambda]^p}{p!} y^{pa}$ and $e^{k\lambda y^a} = \sum_{p=0}^{\infty} \frac{(k\lambda)^p}{p!} y^{pa}$ Then

$$E\left(Y^r/Y>t\right) = \frac{1}{\bar{G}(t)} \Bigg\{ a\beta \sum_{k=0}^{\infty}\sum_{p=0}^{\infty} \frac{(-1)^k \alpha^{k+1} [(k+1)\lambda]^p}{k!p!} \int_t^{\infty} y^{r+a(k\beta+\beta+p)-1} \left[1+\left(\frac{y^a}{s}\right)^c\right]^{-1} dy$$
$$+ a\lambda \sum_{k=0}^{\infty}\sum_{p=0}^{\infty} \frac{(-1)^k \alpha^{k+1} [(k+1)\lambda]^p}{k!p!} \int_t^{\infty} y^{r+a(k\beta+\beta+1+p)-1} \left[1+\left(\frac{y^a}{s}\right)^c\right]^{-1} dy$$
$$+ \frac{ac}{s^c} \sum_{k=0}^{\infty}\sum_{p=0}^{\infty} \frac{(-1)^k \alpha^k [k\lambda]^p}{k!p!} \int_t^{\infty} y^{r+a(k\beta+c+p)-1} \left[1+\left(\frac{y^a}{s}\right)^c\right]^{-2} dy \Bigg\}. \tag{14}$$

Now, by setting $z = [1+(\frac{y^a}{s})^c]^{-1}$, then $y = s^{\frac{1}{a}} (\frac{1-z}{z})^{\frac{1}{ac}}$ and $dy = \frac{-s^{\frac{1}{a}}}{ca} z^{-2} (\frac{1-z}{z})^{\frac{1-ac}{ac}}$ *dz*, so that

$$E\left(Y^r/Y>t\right) = \frac{1}{\bar{G}(t)} \Bigg\{ \frac{\beta}{c} \sum_{k=0}^{\infty}\sum_{p=0}^{\infty} \frac{(-1)^k \alpha^{k+1} [(k+1)\lambda]^p}{k!p!} . s^{\frac{r+a(k\beta+\beta+p)}{a}} \int_0^{\left(1+\left(\frac{t^a}{s}\right)^c\right)^{-1}} z^{\left[1-\frac{r+a(k\beta+\beta+p)}{ac}\right]-1} (1-z)^{\left[\frac{r+a(k\beta+\beta+p)}{ac}\right]-1} dz$$
$$+ \frac{\lambda}{c} \sum_{k=0}^{\infty}\sum_{p=0}^{\infty} \frac{(-1)^k \alpha^{k+1} [(k+1)\lambda]^p}{k!p!} . s^{\frac{r+a(k\beta+\beta+1+p)}{a}} \int_0^{\left(1+\left(\frac{t^a}{s}\right)^c\right)^{-1}} z^{\left[1-\frac{r+a(k\beta+\beta+1+p)}{ac}\right]-1} (1-z)^{\left[\frac{r+a(k\beta+\beta+1+p)}{ac}\right]-1} dz$$
$$+ \sum_{k=0}^{\infty}\sum_{p=0}^{\infty} \frac{(-1)^k \alpha^k [k\lambda]^p}{k!p!} s^{\frac{r+a(k\beta+p)}{a}} \int_0^{\left(1+\left(\frac{t^a}{s}\right)^c\right)^{-1}} z^{\left[1-\frac{r+a(k\beta+p)}{ac}\right]-1} (1-z)^{\left[1+\frac{r+a(k\beta+p)}{ac}\right]-1} dz \Bigg\}.$$

By using $B_y(a,b) = \int_0^y z^{a-1}(1-z)^{b-1} dz$ is the incomplete function, then

$$E(Y^r/Y>t) = \frac{1}{\bar{G}(t)} \Bigg\{ \frac{\beta}{c} \sum_{k=0}^{\infty}\sum_{p=0}^{\infty} \frac{(-1)^k \alpha^{k+1} [(k+1)\lambda]^p}{k!p!} . s^{\frac{r+a(k\beta+\beta+p)}{a}} . B_{\left(1+\left(\frac{t^a}{s}\right)^c\right)^{-1}} \left[1-\frac{r+a(k\beta+\beta+p)}{ac}, \frac{r+a(k\beta+\beta+p)}{ac}\right]$$
$$+ \frac{\lambda}{c} \sum_{k=0}^{\infty}\sum_{p=0}^{\infty} \frac{(-1)^k \alpha^{k+1} [(k+1)\lambda]^p}{k!p!} . s^{\frac{r+a(k\beta+\beta+1+p)}{a}} . B_{\left(1+\left(\frac{t^a}{s}\right)^c\right)^{-1}} \left[1-\frac{r+a(k\beta+\beta+1+p)}{ac}, \frac{r+a(k\beta+\beta+1+p)}{ac}\right]$$
$$+ \sum_{k=0}^{\infty}\sum_{p=0}^{\infty} \frac{(-1)^k \alpha^k [k\lambda]^p}{k!p!} . s^{\frac{r+a(k\beta+p)}{a}} . B_{\left(1+\left(\frac{t^a}{s}\right)^c\right)^{-1}} \left[1-\frac{r+a(k\beta+p)}{ac}, 1+\frac{r+a(k\beta+p)}{ac}\right] \Bigg\} \tag{15}$$

while the Mean residual life time function is given by $E(Y/Y>t) - t$.

Renyi entropy

Renyi entropy is one of the most general concepts in information theory, where it is usually seen as a degree of uncertainty around the state that a physical system can reach. Renyi entropy (I_R) for PLLoGMWD is defined as:[19,20]

$$I_R\ (v) = \frac{1}{1-v}\ \log\left(\int_0^{\infty}[g\,(y)]^{v}dy\right),\ \ v \neq 1,\ \ v > 0. \tag{16}$$

and tends to Shannon entropy as $v \to 1$, Putting $[g(y)]^v = g^v(y)$, therefor

$$g^v\,(y) = \left[a\ e^{-\alpha y^{a\beta} e^{\lambda y^a}}\left[y\left(1+\left(\frac{y^a}{s}\right)^c\right)\right]^{-1}.\left\{\alpha\ y^{a\beta}e^{\lambda y^a}\ (\beta+\lambda y^a)+\frac{cy^{ac}}{s^c}\left[1+\left(\frac{y^a}{s}\right)^c\right]^{-1}\right\}\right]^v.$$

Now by using $(x+y)^n = \sum_{k=0}^{n}\binom{n}{k}x^k y^{n-k}$, then, will be get

$$g^v\,(y) = a^v\sum_{k=0}^{v}\binom{v}{k}\ \alpha^k\left(\frac{c}{s^c}\right)^{v-k} y^{-v}\ e^{-v\alpha y^{a\beta}e^{\lambda y^a}}\ y^{ka\beta}y^{ac(v-k)}e^{k\lambda y^a}\left[1+\left(\frac{y^a}{s}\right)^c\right]^k\left[1+\left(\frac{y^a}{s}\right)^c\right]^{-2v},$$

Since $e^{-z} = \sum_{p=0}^{\infty}\frac{(-1)^p}{p!}z^p$ then[17]

$$g^v\,(y) = a^v\ \sum_{k=0}^{v}\binom{v}{k}\ \alpha^k\left(\frac{c}{s^c}\right)^{v-k}.\sum_{p=0}^{\infty}\frac{(-1)^p}{p!}\ (v\alpha)^p y^{pa\beta+ka\beta+ac(v-k)-v}e^{(p+k)\lambda y^a}\left[1+\left(\frac{y^a}{s}\right)^c\right]^{k-2v},$$

Since $e^z = \sum_{m=0}^{\infty}\frac{z^m}{m!}$, then[17]

$$g^v\,(y) = \sum_{k=0}^{v}\sum_{p=0}^{\infty}\sum_{m=0}^{\infty}\binom{v}{k}\ \frac{(-1)^p\alpha^k a^v}{p!m!}.\left(\frac{c}{s^c}\right)^{(v-k)}(v\alpha)^p[(p+k)\,\lambda]^m y^{pa\beta+ka\beta+ac(v-k)-v+m}\ \left[1+\left(\frac{y^a}{s}\right)^c\right]^{k-2v},$$

Considering

$$\int_0^{\infty} g^v(y)dy = \sum_{k=0}^{v}\sum_{p=0}^{\infty}\sum_{m=0}^{\infty}\binom{v}{k}\ \frac{(-1)^p\alpha^k a^v}{p!m!}.\left(\frac{c}{s^c}\right)^{(v-k)}(v\alpha)^p[(p+k)\,\lambda]^m$$
$$\times\ \int_0^{\infty} y^{pa\beta+ka\beta+ac(v-k)-v+m}\ \left[1+\left(\frac{y^a}{s}\right)^c\right]^{k-2v}\ dy.$$

Setting $I_2 = \int_0^{\infty} y^{pa\beta+ka\beta+ac(v-k)-v+m}\ [1+(\frac{y^a}{s})^c]^{k-2v}dy.$
Let $z = [1+(\frac{y^a}{s})^c]^{-1}$, then $y = s^{\frac{1}{a}}\ (\frac{1-z}{z})^{\frac{1}{ac}}$ and $dy = \frac{-s^{\frac{1}{a}}}{ca}\ z^{-2}\ \ (\frac{1-z}{z})^{\frac{1-ac}{ac}}\ dz$, so that

$$I_2 = \frac{s^{\frac{[pa\beta+ka\beta+ac(v-k)-v+m-1]}{a}}}{ac}\ \int_0^1 z^{\left[\frac{a(cv+k\beta-p\beta)+v-m-1}{ac}\right]-1}\ (1-z\,)^{\left[\frac{pa\beta+ka\beta+ac(v-k)-v+m+1}{ac}\right]-1}\ dz,$$

By using $B(a,b) = \int_0^1 z^{a-1}(1-z)^{b-1}dz$, then

$$I_2 = \frac{s^{\frac{\{pa\beta+ka\beta+ac(v-k)-v+m-1\}}{a}}}{ac}\cdot B\left[\frac{a(cv-k\beta-p\beta)+v-m-1}{ac},\frac{a(p\beta+k\beta+c(v-k))-v+m+1}{ac}\right],$$

so

$$I_R(v) = \frac{1}{1-v}\cdot\log\left\{\sum_{k=0}^{v}\sum_{p=0}^{\infty}\sum_{m=0}^{\infty}\binom{v}{k}\frac{(-1)^p\alpha^k a^v}{p!m!}\cdot\left(\frac{c}{s^c}\right)^{(v-k)}(v\alpha)^p[(p+k)\lambda]^m\cdot\frac{s^{\frac{[a(p\beta+k\beta+c(v-k))-v+m-1]}{a}}}{ac}\right.$$
$$\left.\cdot B\left[\frac{a(cv-k\beta-p\beta)+v-m-1}{ac},\frac{a(p\beta+k\beta+c(v-k))-v+m+1}{ac}\right]\right\}. \tag{17}$$

Probability weighted moments (Pwms)

Greenwood, Landwehr, Matalas, and Willis (1997) explored the idea of probability weighted moments (Pwms) as a method for estimating the parameters of a number of distributions in the statistical literature. The PWMs for PLLoGMWD are defined as follows[21]

$$E\left(y^r G^L(y)\,\bar{G}^M(y)\right)=\int_0^\infty y^r G^L(y)\,\bar{G}^M(y)\;g(y)\;dy=\int_0^\infty y^r\left(1-\bar{G}(y)\right)^L\bar{G}^M(y)\;g(y)\,dy. \tag{18}$$

Using binomial expansion[22] $(1-z)^L=\sum_{j=0}^{\infty}\frac{(-1)^j\Gamma(L+1)}{\Gamma(L+1-j)\Gamma(j+1)}z^j$, then will be get

$$\left(1-\bar{G}(y)\right)^L=\sum_{j=0}^{\infty}\frac{(-1)^j\Gamma(L+1)}{\Gamma(L+1-j)\,\Gamma(j+1)}\left(\bar{G}(y)\right)^j,$$

Then

$$E\left(y^r G^L(y)\,\bar{G}^M(y)\right)=\sum_{j=0}^{\infty}\frac{(-1)^j\Gamma(L+1)}{\Gamma(L+1-j)\,\Gamma(j+1)}\int_0^\infty y^r\left(\bar{G}(y)\right)^{j+M}g(y)\,dy$$
$$=\sum_{j=0}^{\infty}\frac{(-1)^j\Gamma(L+1)}{\Gamma(L+1-j)\,\Gamma(j+1)}\cdot E\left(y^r\left(\bar{G}(y)\right)^{j+M}\right).$$

Let $I_3=E(y^r(\bar{G}(y))^{j+M})$.
Since $G(y)+\bar{G}(y)=1\ \rightarrow\bar{G}(y)=1-G(y)=\left[1+\left(\frac{y^a}{s}\right)^c\right]^{-1}e^{-\alpha y^{a\beta}e^{\lambda y^a}}$, so

$$I_3=E\left(y^r\left(\bar{G}(y)\right)^{j+M}\right)=\int_0^\infty y^r\left[1+\left(\frac{y^a}{s}\right)^c\right]^{-(j+M)}e^{-(j+M)\alpha y^{a\beta}e^{\lambda y^a}}\;g(y)\,dy,$$

$$I_3=a\cdot\int_0^\infty y^{r-1}\left[1+\left(\frac{y^a}{s}\right)^c\right]^{-(j+M+1)}e^{-(j+M+1)\alpha y^{a\beta}e^{\lambda y^a}}\cdot\left\{\alpha y^{a\beta}e^{\lambda y^a}\left(\beta+\lambda y^a\right)+\frac{cy^{ac}}{s^c}\left[1+\left(\frac{y^a}{s}\right)^c\right]^{-1}\right\}dy,$$

Since $e^{-z}=\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}z^k$, then

$$I_3=\sum_{k=0}^{\infty}\frac{(-1)^k(j+m+1)^k\alpha^k}{k!}\cdot a\cdot\int_0^\infty y^{r+ka\beta-1}\left[1+\left(\frac{y^a}{s}\right)^c\right]^{-(j+M+1)}e^{k\lambda y^a}$$
$$\cdot\left\{\alpha y^{a\beta}e^{\lambda y^a}\left(\beta+\lambda y^a\right)+\frac{cy^{ac}}{s^c}\left[1+\left(\frac{y^a}{s}\right)^c\right]^{-1}\right\}dy,$$

Since $e^z=\sum_{p=0}^{\infty}\frac{z^p}{p!}$,[17] then

$$I_3=\left\{\sum_{k=0}^{\infty}\sum_{p=0}^{\infty}\frac{(-1)^k(j+m+1)^k\alpha^{k+1}\left[(k+1)\lambda\right]^p}{k!p!}\cdot a\,\beta\cdot\int_0^\infty y^{r+a(k\beta+\beta+p)-1}\left[1+\left(\frac{y^a}{s}\right)^c\right]^{-(j+M+1)}dy\right.$$
$$+\sum_{k=0}^{\infty}\sum_{p=0}^{\infty}\frac{(-1)^k(j+m+1)^k\alpha^{k+1}\left[(k+1)\lambda\right]^p}{k!p!}\cdot a\lambda\cdot\int_0^\infty y^{r+a(k\beta+\beta+1+p)-1}\left[1+\left(\frac{y^a}{s}\right)^c\right]^{-(j+M+1)}dy$$
$$\left.+\sum_{k=0}^{\infty}\sum_{p=0}^{\infty}\frac{(-1)^k(j+m+1)^k\alpha^k(k\lambda)^p}{k!p!}\cdot\left(\frac{ac}{s^c}\right)\cdot\int_0^\infty y^{r+a(k\beta+c+p)-1}\left[1+\left(\frac{y^a}{s}\right)^c\right]^{-(j+M+2)}dy\right\}.$$

Now, by setting $z = \left[1 + \left(\frac{y^a}{s}\right)^c\right]^{-1}$, then $y = s^{\frac{1}{a}} \left(\frac{1-z}{z}\right)^{\frac{1}{ac}}$ and $dy = \frac{-s^{\frac{1}{a}}}{ca} z^{-2} \left(\frac{1-z}{z}\right)^{\frac{1-ac}{ac}} dz$, so that

$$I_3 = \left\{ \sum_{k=0}^{\infty}\sum_{p=0}^{\infty} \frac{(-1)^k (j+M+1)^k \alpha^{k+1} \left[(k+1)\lambda\right]^p}{k!p!} . \frac{\beta}{c} .s^{\frac{r+a(k\beta+\beta+p)}{a}} \int_0^1 z^{(j+M+1)-\left[\frac{r+a(k\beta+\beta+p)}{ac}\right]-1} (1-z)^{\left[\frac{r+a(k\beta+\beta+p)}{ac}\right]-1} dz \right.$$
$$+ \sum_{k=0}^{\infty}\sum_{p=0}^{\infty} \frac{(-1)^k (j+M+1)^k \alpha^{k+1} \left[(k+1)\lambda\right]^p}{k!p!} . \frac{\lambda}{c} .s^{\frac{r+a(k\beta+\beta+1+p)}{a}} \int_0^1 z^{(j+M+1)-\left[\frac{r+a(k\beta+\beta+1+p)}{ac}\right]-1} (1-z)^{\left[\frac{r+a(k\beta+\beta+1+p)}{ac}\right]-1} dz$$
$$\left. + \sum_{k=0}^{\infty}\sum_{p=0}^{\infty} \frac{(-1)^k (j+M+1)^k \alpha^k [k\lambda]^p}{k!p!} .s^{\frac{r+a(k\beta+p)}{a}} \int_0^1 z^{(j+M+1)+\left[\frac{r+a(k\beta+p)}{ac}\right]-1} (1-z)^{1+\left[\frac{r+a(k\beta+p)}{ac}\right]-1} dz \right\}.$$

By using beta function $B(a,b) = \int_0^1 t^{a-1}(1-t)^{b-1} dt$, then

$$I_3 = \left\{ \sum_{k=0}^{\infty}\sum_{p=0}^{\infty} \frac{(-1)^k (j+M+1)^k \alpha^{k+1} \left[(k+1)\lambda\right]^p}{k!p!} \cdot \frac{\beta}{c} \cdot s^{\frac{r+a(k\beta+\beta+p)}{a}} \right.$$
$$\cdot\ B - \left((j+M+1) - \left[\frac{r+a(k\beta+\beta+p)}{ac}\right], \left[\frac{r+a(k\beta+\beta+p)}{ac}\right]\right)$$
$$+ \sum_{k=0}^{\infty}\sum_{p=0}^{\infty} \frac{(-1)^k (j+M+1)^k \alpha^{k+1} \left[(k+1)\lambda\right]^p}{k!p!} \cdot \frac{\lambda}{c} \cdot s^{\frac{r+a(k\beta+\beta+1+p)}{a}}$$
$$\cdot\ B - \left((j+M+1) - \left[\frac{r+a(k\beta+\beta+1+p)}{ac}\right], \left[\frac{r+a(k\beta+\beta+1+p)}{ac}\right]\right)$$
$$+ \sum_{k=0}^{\infty}\sum_{p=0}^{\infty} \frac{(-1)^k (j+M+1)^k \alpha^k [k\lambda]^p}{k!p!} \cdot s^{\frac{r+a(k\beta+p)}{a}}$$
$$\left. \cdot\ B - \left((j+M+1) - \left[\frac{r+a(k\beta+p)}{ac}\right], 1 + \left[\frac{r+a(k\beta+p)}{ac}\right]\right) \right\},$$

Then

$$Pwms = E\left(y^r G^L(y)\, \bar{G}^M(y)\right)$$
$$= \sum_{j=0}^{\infty}\sum_{k=0}^{\infty} \frac{(-1)^{j+k}\Gamma(L+1)(j+M+1)^k \alpha^k}{k!\Gamma(L+1-j)\Gamma(j+1)} \cdot \left\{ \frac{\alpha\beta}{c} \sum_{p=0}^{\infty} \frac{\left[(k+1)\lambda\right]^p}{p!} .s^{\frac{r+a(k\beta+\beta+p)}{ac}} .B\left((j+M+1) - \left[\frac{r+a(k\beta+\beta+p)}{ac}\right],\right.\right.$$
$$\left.\left[\frac{r+a(k\beta+\beta+p)}{ac}\right]\right) + \frac{\alpha\lambda}{c} \sum_{p=0}^{\infty} \frac{\left[(k+1)\lambda\right]^p}{p!} .s^{\frac{r+a(k\beta+\beta+1+p)}{ac}} .\ B\left((j+M+1) - \left[\frac{r+a(k\beta+\beta+1+p)}{ac}\right],\right.$$
$$\left.\left.\left[\frac{r+a(k\beta+\beta+1+p)}{ac}\right]\right) + \sum_{p=0}^{\infty} \frac{(k\lambda)^p}{p!} s^{\frac{r+a(k\beta+\beta)}{a}} .B\left((j+M+1) - \left[\frac{r+a(k\beta+p)}{ac}\right], 1 + \left[\frac{r+a(k\beta+p)}{ac}\right]\right)\right\} \tag{19}$$

Estimation of parameters of PLLOGMW distribution

This section is devoted to the estimation of unknown parameters of PLLOGMW distribution using the generalized method of moments (GMM). Suppose that Y_n be a vector of random variables sample taken from the PLLOGMW distribution with the pdf given in (4) of size n, θ be a vector of unknown parameters which are to be estimated and $g(Y_n, \theta)$ A vector of expected values calculated from the model.

To detail this procedure, used the traditional method of generalized moments, to see whether more subtle aspects of the define of the objective function might be revealed. Define the objective function $Q_n(\theta)$ as

$$Q_n(\theta) = \frac{1}{n}\sum_{i=1}^{n}\left[Y_i - g(Y_i, \theta)\right]^2. \tag{20}$$

$\hat{a}$, $\hat{\beta}$, $\hat{\alpha}$, $\hat{s}$, $\hat{c}$ and $\hat{\lambda}$ can be obtained by minimizing,

$$\hat{\theta} = \arg\min_{\theta} Q_n(\theta). \tag{21}$$

As a result, numerical optimization techniques such as Monte Carlo simulation will be used to obtain parameter estimates.

Results and discussion

A Monte Carlo simulation study is given to illustrate the performance and accuracy of the (GMM) estimators for the PLLOGMW distribution's. Using random sampling to generate random data from the PLLOGMW distribution, the following default data were used to test for bias, mean estimate, approximate confidence intervals (95%), mean absolute error and MSE values of PLLOGMWD.

set 1: $a = 2,\ \beta = 5,\ \alpha = 0.3,\ s = 0.4,\ c = 1,\ \lambda = 0.1$; set 2: $a = 0.9,\ \beta = 3,\ \alpha = 1.5,\ s = 2,\ c = 4,\ \lambda = 0.5$;

set 3: $a = 1,\ \beta = 6,\ \alpha = 0.9,\ s = 2.5,\ c = 3,\ \lambda = 0.4$; set 4: $a = 1,\ \beta = 2,\ \alpha = 0.5,\ s = 3,\ c = 1.5,\ \lambda = 0.1$;

set 5: $a = 0.9,\ \beta = 3,\ \alpha = 2,\ s = 0.5,\ c = 4,\ \lambda = 0.3$; set 6: $a = 1.9,\ \beta = 6,\ \alpha = 1.2,\ s = 4,\ c = 0.9,\ \lambda = 0.2$;

set 7: $a = 1.5,\ \beta = 2,\ \alpha = 6,\ s = 4,\ c = 0.9,\ \lambda = 0.2$; set 8: $a = 0.3,\ \beta = 4,\ \alpha = 5,\ s = 4,\ c = 2.5,\ \lambda = 0.5$;

set 9: $a = 0.5,\ \beta = 2.5,\ \alpha = 0.8,\ s = 0.5,\ c = 1.2,\ \lambda = 0.4$,

for sample sizes n = 100 and 500. the Iteration was performed q = 100 times for each sample size and for the above sets. The initial values were used for the iteration process to be near to the true parameter values, with random noise. For parameter estimations, we selected λ = 0.1, 0.2, 0.3, 0.4 and 0.5, using optimization methods such as the fmincon approach in Mat lab Program. The following formula are provided to calculate approximate (95%) confidence intervals;

CI $= \left(\hat{\theta} \pm Z_{\frac{\epsilon}{2}} \times \frac{\sigma_{\hat{\theta}}}{\sqrt{q}}\right)$, where $Z_{\frac{\epsilon}{2}} = 1.96$ is used as the 95% upper percentile of the standard normal distribution and $\sigma_{\hat{\theta}}$ standard deviation of estimates. Tables 1 to 9 include the simulation results.

Table 1. Simulation results for set 1.

		set 1($a = 2$, $\beta = 5$, $\alpha = 0.3$, $s = 0.4$, $c = 1$, $\lambda = 0.1$)					
n	Parameter	Mean estimate	Bias	MSE	MAE	CI 95% (Lower)	CI 95% (Upper)
100	a	1.5671	−0.43292	0.43806	0.60896	1.4685	1.6657
100	β	0.72744	−4.2726	18.603	4.2726	0.6112	0.8437
100	α	0.010165	−0.28984	0.084006	0.28984	0.0099	0.0104
100	s	4.917	4.517	20.537	4.517	4.8451	4.9889
100	c	4.3023	3.3023	12.533	3.3029	4.0510	4.5536
100	λ	0.68447	0.58447	0.42432	0.59801	0.6278	0.7411
500	a	1.464	−0.536	0.39081	0.55224	1.4006	1.5274
500	β	1.6359	−3.3641	12.698	3.3641	1.4044	1.8673
500	α	0.010436	−0.28956	0.083856	0.28956	0.0099	0.0110
500	s	4.9004	4.5004	20.411	4.5004	4.8222	4.9785
500	c	3.8526	2.8526	9.6776	2.8526	3.6081	4.0971
500	λ	0.39242	0.29242	0.15818	0.32314	0.3393	0.4455

Table 2. Simulation results for set 2.

		set 2($a = 0.9$, $\beta = 3$, $\alpha = 1.5$, $s = 2$, $c = 4$, $\lambda = 0.5$)					
n	Parameter	Mean estimate	Bias	MSE	MAE	CI 95% (Lower)	CI 95% (Upper)
100	a	1.5588	0.6588	0.64472	0.65916	1.4684	1.6492
100	β	0.70685	–2.2931	5.6078	2.334	0.5904	0.8233
100	α	0.010249	–1.4898	2.2194	1.4898	0.0098	0.0107
100	s	4.9318	2.9318	8.7216	2.9318	4.8617	5.0018
100	c	4.23	0.22997	1.8052	1.2289	3.9692	4.4907
100	λ	0.67076	0.17076	0.10885	0.28724	0.6152	0.7264
500	a	1.4637	0.56373	0.43108	0.57276	1.3974	1.5300
500	β	1.6557	–1.3443	3.1544	1.6542	1.4270	1.8843
500	α	0.01051	–1.4895	2.2186	1.4895	0.009	0.0111
500	s	4.8997	2.8997	8.5229	2.8997	4.8331	4.9664
500	c	3.8498	–0.15018	1.5474	1.1563	3.6066	4.0931
500	λ	0.39097	–0.10903	0.085003	0.24974	0.3377	0.4442

Table 3. Simulation results for set 3.

		set 3($a = 1$, $\beta = 6$, $\alpha = 0.9$, $s = 2.5$, $c = 3$, $\lambda = 0.4$)					
n	Parameter	Mean estimate	Bias	MSE	MAE	CI 95% (Lower)	CI 95% (Upper)
100	a	1.5884	–0.58836	0.59716	0.59472	1.4897	1.6870
100	β	0.70535	–5.2946	28.364	5.2946	0.5920	0.8187
100	α	0.010049	–0.88995	0.79201	0.88995	0.0100	0.0101
100	s	4.9467	2.4467	6.069	2.4467	4.8901	5.0033
100	c	4.2025	1.2025	3.2545	1.7377	3.9376	4.4374
100	λ	0.66209	0.26209	0.1494	0.34185	0.6061	0.7181
500	a	1.4604	0.46041	0.32266	0.477	1.3949	1.5259
500	β	1.6648	–4.3352	20.152	4.3352	1.4352	1.8944
500	α	0.010503	–0.8895	0.79121	0.8895	0.0099	0.0111
500	s	4.8947	2.3947	5.8587	2.3947	4.8254	4.9641
500	c	3.8626	0.8626	2.2577	1.3071	3.6203	4.1050
500	λ	0.3907	–0.0093001	0.073471	0.22838	0.3373	0.4441

Table 4. Simulation results for set 4.

		set 4($a = 1$, $\beta = 2$, $\alpha = 0.5$, $s = 3$, $c = 1.5$, $\lambda = 0.1$)					
n	Parameter	Mean estimate	Bias	MSE	MAE	CI 95% (Lower)	CI 95% (Upper)
100	a	1.5953	1.4953	2.4955	1.4953	1.4949	1.6957
100	β	0.73759	–1.2624	1.9471	1.3482	0.6205	0.8547
100	α	0.013272	–0.48673	0.23792	0.48673	0.0070	0.0196
100	s	4.9278	1.9278	3.8189	1.9278	4.8648	4.9909
100	c	4.1833	2.6833	8.9781	2.7015	3.9207	4.4460
100	λ	0.65925	0.55925	0.39771	0.5728	0.6018	0.7167
500	a	1.4659	1.3659	1.9828	1.3659	1.3985	1.5333
500	β	1.6796	–0.32037	1.4679	1.0104	1.4495	1.9098
500	α	0.013572	–0.48643	0.23717	0.48643	0.0089	0.0182
500	s	4.8542	1.8542	3.6419	1.8568	4.7653	4.9431
500	c	3.8298	2.3298	6.9727	2.3298	3.5849	4.0746
500	λ	0.38423	0.28423	0.15317	0.31581	0.3312	0.4372

Table 5. Simulation results for set 5.

		set 5($a = 0.9$, $\beta = 3$, $\alpha = 2$, $s = 0.5$, $c = 4$, $\lambda = 0.3$)					
n	Parameter	Mean estimate	Bias	MSE	MAE	CI 95% (Lower)	CI 95% (Upper)
100	a	1.5709	0.67088	0.6769	0.67124	1.4771	1.6647
100	β	0.69443	–2.3056	5.6302	2.3472	0.5840	0.8049
100	α	0.010074	–1.9899	3.9598	1.9899	0.0099	0.0102
100	s	4.9461	4.4461	19.854	4.4461	4.8884	5.0039
100	c	4.2059	0.20591	1.8336	1.2412	3.9423	4.4695
100	λ	0.66778	0.36778	0.21708	0.41726	0.6114	0.7241
500	a	1.473	0.573	0.44462	0.58208	1.4058	1.5402
500	β	1.6372	–1.3628	3.2412	1.6846	0.3347	0.4419
500	α	0.010439	–1.9896	3.9584	1.9896	0.0099	0.0110
500	s	4.9204	4.4204	19.648	4.4204	4.8555	4.9853
500	c	3.8426	–0.15735	1.5856	1.1769	3.5965	4.0887
500	λ	0.38829	0.088288	0.08173	0.23478	0.3347	0.4419

Table 6. Simulation results for set 6.

		set 6($a = 1.9$, $\beta = 6$, $\alpha = 1.2$, $s = 4$, $c = 0.9$, $\lambda = 0.2$)					
n	Parameter	Mean estimate	Bias	MSE	MAE	CI 95% (Lower)	CI 95% (Upper)
100	a	1.5814	–0.31857	0.34787	0.53452	1.4837	1.6792
100	β	0.70806	–5.2919	28.327	5.2919	0.5962	0.8199
100	α	0.010046	–1.19	1.416	1.19	0.0100	0.0101
100	s	4.9414	0.94143	0.97391	0.97391	4.8832	4.9996
100	c	4.2873	3.3873	3.3873	3.3873	4.0334	4.5412
100	λ	0.66612	0.46612	0.30463	0.49976	0.6079	0.7243
500	a	1.4673	–0.43272	0.30279	0.47511	1.4003	1.5342
500	β	1.658	–4.342	20.243	4.342	1.4257	1.8902
500	α	0.010458	–1.1895	1.415	1.1895	0.0099	0.0110
500	s	4.9011	0.90109	0.93087	0.94775	4.8332	4.9690
500	c	3.8547	2.9547	10.271	2.9547	3.6102	4.0992
500	λ	0.3861	0.1861	0.10849	0.26215	0.3326	0.4396

Table 7. Simulation results for set 7.

		set 7($a = 1.5$, $\beta = 2$, $\alpha = 6$, $s = 4$, $c = 0.9$, $\lambda = 0.2$)					
n	Parameter	Mean estimate	Bias	MSE	MAE	CI 95% (Lower)	CI 95% (Upper)
100	a	1.5773	0.077274	0.23059	0.34045	1.4839	1.6706
100	β	0.71249	–1.2875	2.0294	1.3945	0.5924	0.8326
100	α	0.010038	–5.99	35.88	5.99	0.0100	0.0101
100	s	4.9444	0.94436	0.97961	0.97677	4.8860	5.0027
100	c	4.1208	3.2208	12.339	3.2208	3.8447	4.3970
100	λ	0.65111	0.45111	0.28739	0.48855	0.5941	0.7082
500	a	1.4675	–0.032513	0.1168	0.27438	1.4005	1.5345
500	β	1.6398	–0.36018	1.5344	1.0541	1.4064	1.8733
500	α	0.01046	–5.9895	35.875	5.9895	0.0099	0.0110
500	s	4.9114	0.91143	0.93831	0.95377	4.8468	4.9761
500	c	3.8469	2.9469	10.235	2.9469	3.6015	4.0922
500	λ	0.39478	0.19478	0.11148	0.26748	0.3414	0.4482

Table 8. Simulation results for set 8.

		set 8($a = 0.3$, $\beta = 4$, $\alpha = 5$, $s = 4$, $c = 2.5$, $\lambda = 0.5$)					
n	Parameter	Mean estimate	Bias	MSE	MAE	CI 95% (Lower)	CI 95% (Upper)
100	a	1.5983	1.2983	1.9257	1.2983	1.5018	1.6948
100	β	0.70686	–3.2931	11.168	3.3129	0.5948	0.8189
100	α	0.01049	–4.9895	24. 895	4.9895	0.0096	0.0114
100	s	4.9339	0.93392	0.9874	0.98466	4.8671	5.0008
100	c	4.2024	1.7024	4.664	1.9845	3.9407	4.4642
100	λ	0.64291	0.14291	0.11202	0.29409	0.5833	0.7025
500	a	1.4631	1.1631	1.4565	1.1631	1.3997	1.5265
500	β	1.6579	–2.3421	6.8083	2.4443	1.4314	1.8845
500	α	0.011713	–4.9883	24.883	4.9883	0.0094	0.0141
500	s	4.874	0.87398	0.95322	0.96053	4.7883	4.9597
500	c	3.8295	1.3295	3.2717	1.4779	3.5879	4.0711
500	λ	0.38898	–0.11102	0.08287	0.24475	0.3367	0.4413

Table 9. Simulation results for set 9.

		set 9($a = 0.5$, $\beta = 2.5$, $\alpha = 0.8$, $s = 0.5$, $c = 1.2$, $\lambda = 0.4$)					
n	Parameter	Mean estimate	Bias	MSE	MAE	CI 95% (Lower)	CI 95% (Upper)
100	a	1.5629	1.0629	1.3938	1.065	1.4616	1.6641
100	β	0.79812	–1.7019	3.5761	1.8636	0.6357	0.9605
100	α	0.01019	–0.78981	0.6238	0.78981	0.0100	0.0104
100	s	4.9226	4.4226	19.664	4.4226	4.8590	4.9863
100	c	4.1135	2.9135	10.593	2.9614	3.8277	4.3993
100	λ	0.69322	0.29322	0.18796	0.372	0.6303	0.7561
500	a	1.4642	0.96416	1.0328	0.96416	1.4009	1.5274
500	β	1.6618	–0.83816	2.0662	1.3089	1.4318	1.8919
500	α	0.011355	–0.78864	0.62201	0.78864	0.0100	0.0127
500	s	4.8778	4.3778	19.335	4.3778	4.7965	4.9591
500	c	3.8372	2.6372	8.4612	2.6372	3.5954	4.0790
500	λ	0.3826	–0. 017395	0.069404	0.22011	0.3308	0.4344

Plots of PDF, CDF, Ʈ1, Ʈ2, Renyi entropy and, Analysis of properties: skewness, kurtosis, and tail behavior of the PLLoGMWD for some parameters values are displayed in Figs. 1 to 6.

Notably, the graph of the hazard function shows an increasing, decreasing, or constant shape. Obviously, the graph of the PDF for the new distribution may be symmetric, increasing, or right-skewed.

The plots of the CDF and the reliability function indicate that the PILLoW distribution has an increasing and monotonically decreasing shape, respectively.

From the simulation results, it can be observed that increasing the sample size improves parameter estimation, decreases bias, MAE and MSE values, and reduces the width of the confidence intervals, thereby strengthening the dependability of the estimation process.

Table 10. Parameter values used for Renyi Entropy analysis.

Sets	a	α	β	λ	s	c
10	3	0.8	1.2	2	2	0.3
11	1	0.5	6	6	3	9
12	8	5	0.6	10	5	4
13	6	3	0.5	2	9	6
14	10	5	1	3	4	5
15	1	4	2	5	6	3

Table 10 displays the values of the six parameters that were used to calculate the Renyi entropy and analyze of properties: skewness, kurtosis, and tail behavior for the PLLoGMW distribution.

Table 11. Renyi Entropy results for the six sets at selected values of v coefficients and Analysis of properties: skewness, kurtosis, and tail behavior of the PLLoGMWD.

R_v	Set 10	Set 11	Set 12	Set 13	Set 14	Set 15
R_0.1	0.029	–2.008	–0.228	0.050	–0.457	–1.450
R_1	0.090	0.001	–9.722	–5.531	–1.261	0.072
R_2.5	0.156	13.981	–3.762	–2.751	–1.501	2.948
R_3	0.100	12.382	–3.588	–2.600	–1.538	2.453
R_4	0.028	10.771	–3.430	–2.467	–1.590	1.948
R_4.5	0.001	10.307	–3.390	–2.434	–1.610	1.801
R_5	–0.022	9.957	–3.362	–2.411	–1.626	1.689
Min Entropy	–1.840	–86.298	–16.008	–13.433	–1.626	–26.985
Max Entropy	2.059	102.062	10.938	9.997	–0.457	29.403
Mean Entropy	0.092	10.340	–3.284	–2.314	–1.410	1.912
Skewenss	–0.355	–0.067	–1.715	–1.088	–1.145	0.407
Skewness Coefficient	–0.3548	–0.0670	–1.7148	–1.0884	–1.1447	0.4071
Kurtosis	–1.141	–0.179	3.660	0.857	1.727	–0.226
Kurtosis Coefficient	–1.1407	–0.1793	3.6595	0.8574	1.7268	–0.2262
Upper tail mean(5%)	0.9133	0.3309	0.8100	0.9264	0.8894	0.2129
Lower tail mean (5%)	0.0194	0.1194	0.4154	0.2821	0.5738	0.0184
Tail ratio	47.1932	2.7717	1.9501	3.2842	1.5500	11.5997
Tail type	Very Heavy Tail	Moderate Tail	Light Tail	Moderate Tail	Light Tail	Very Heavy Tail

Tables 10 and 11 represent the Renyi Entropy values and descriptive property analyses, including skewness, kurtosis, and tail behavior of PLLoGMWD across six different parameter sets. The results of these tables show that set 11 recorded the highest Renyi entropy values across all v-coefficients, with an average of 10.340, indicating that it represents the most dispersed and uncertain distribution. On the other hand, set 12 recorded the lowest Renyi entropy values, with an average of –3.284, reflecting its concentrated nature and high predictability, and the highest skewness (–1.715) and kurtosis (+ 3.660), reflecting the highest degree of model flexibility.

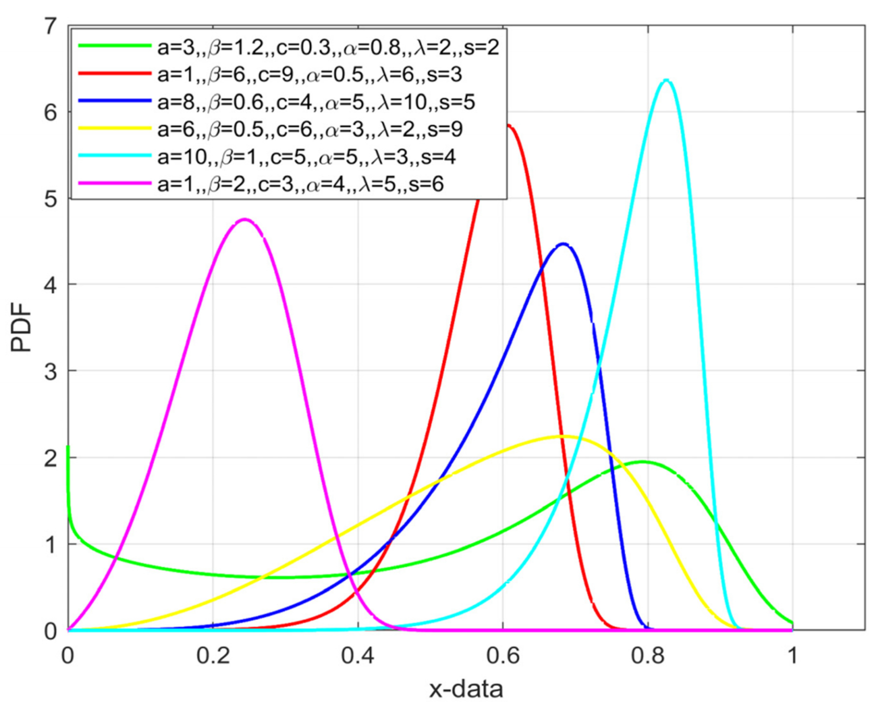

Fig. 1. Probability density function of PLLoGMWD for several values of a, β, c, α, λ and s.

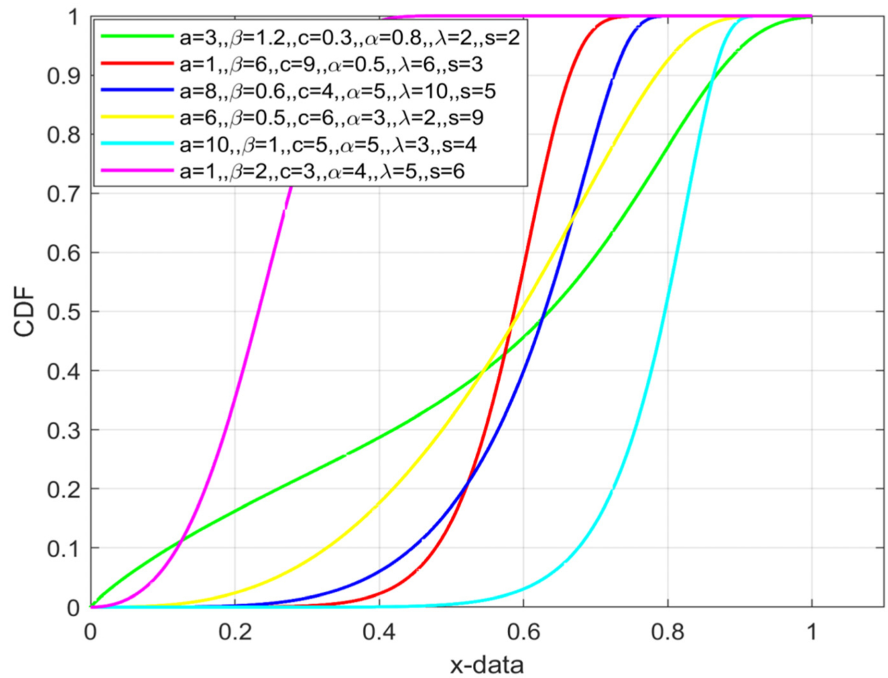

Fig. 2. Cumulative distribution function of PLLoGMWD for several values of a, β, c, α, λ and s.

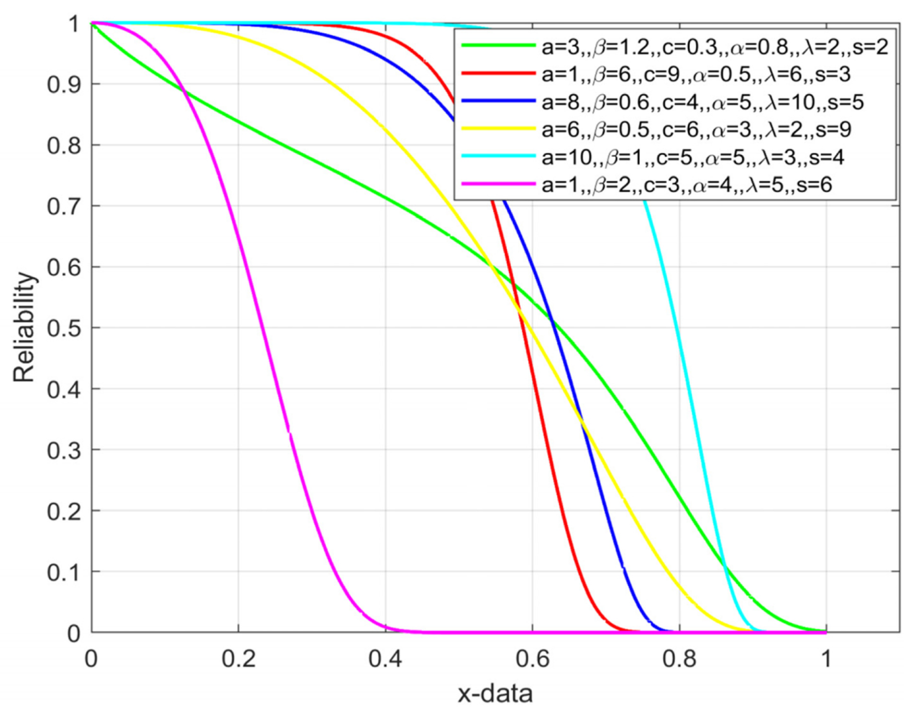

Fig. 3. Reliability function for PLLoGMWD for several values of a, β, c, α, λ and s.

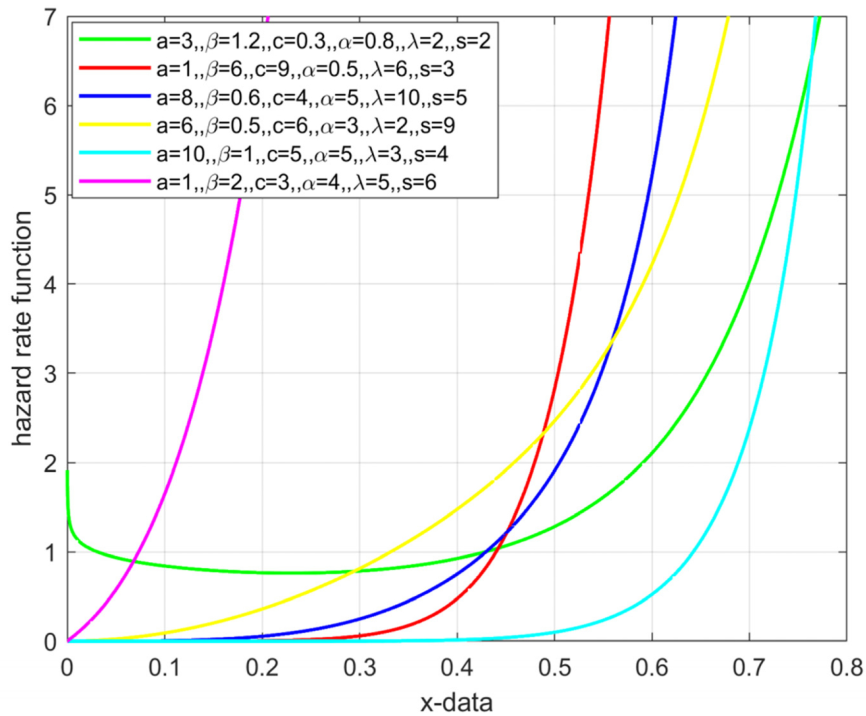

Fig. 4. Hazard rate function for PLLoGMWD for several values of a, β, c, α, λ and s.

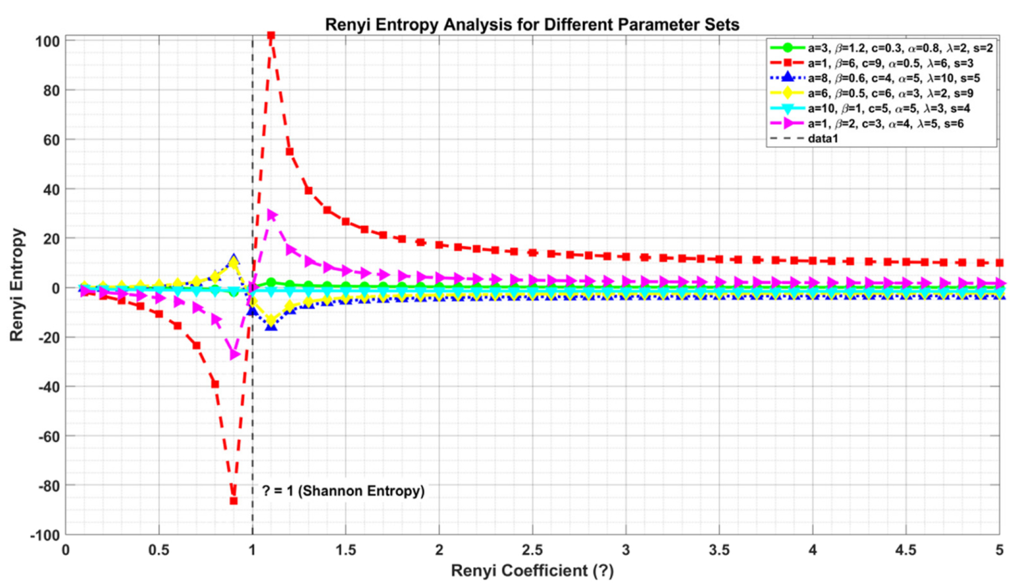

Fig. 5. Renyi Entropy for PLLoGMWD for several values of a, β, c, α, λ and s.

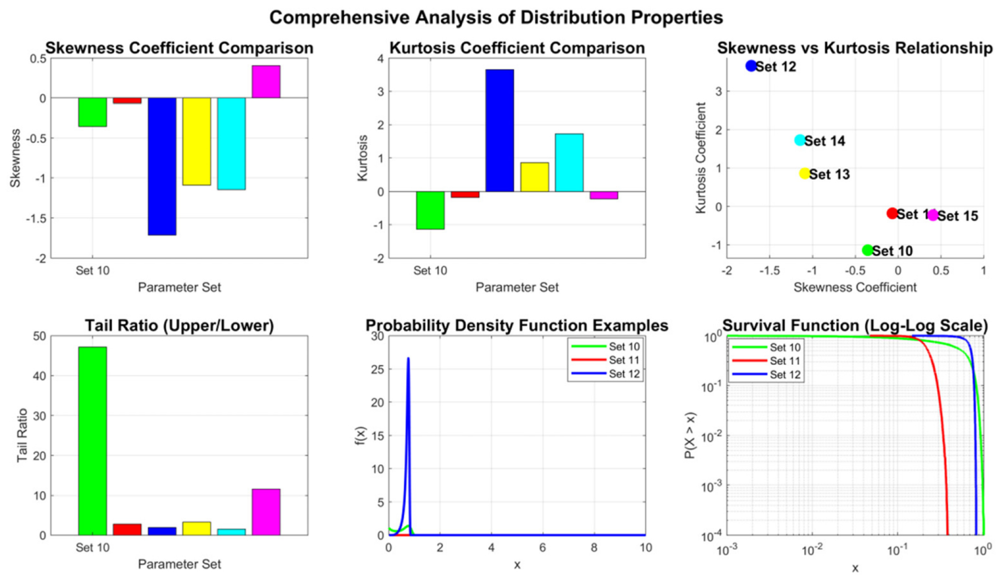

Fig. 6. Comparison of properties: skewness, kurtosis, and tail behavior of the PLLoGMWD for several values of a, β, c, α, λ and s.

Conclusion

In this paper, a new six-parameter distribution is introduced, which is a novel extension of the log-logistic modified Weibull distribution. Some structural properties of this proposed model are derived. The plots show how the added parameter affects the density elasticity of the PLLoGMW. The distribution parameters are estimated using the generalized moments method, and the generalized moments estimators are investigated via Monte Carlo simulations. Dataset analysis is performed across several metrics, which indicates that the PLLoGMWD model possesses a flexible characteristic that makes it suitable for modeling data in economics. Our results suggest that these estimators exhibit good properties as the sample size increases. Furthermore, Simulation results show that the proposed model produces the most consistent and efficient parameter estimates across small and large sample sizes confirming its potential in the fields of modeling, machine learning, and advanced computing. In future studies, the PLLoGMW distribution could be used in a wide range of real-world applications. We can also extend the PLLoGMW distribution to include bivariate contexts and Bayesian inference frameworks.

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Authors' declaration

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are ours. Furthermore, any Figures and images that are not ours have been included with the necessary permission for re-publication, which is attached to the manuscript.
- No animal studies are present in the manuscript.
- No human studies are present in the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee at Directorate General of Education in Holy Karbala.

Data availability

The datasets generated during and analyzed during the current study are available from the corresponding author on reasonable request.

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توزيع ويبل المعدل اللوغاريتمي - اللوجستي ذو القوة : النموذج والخصائص والمحاكاة

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الخلاصة

تتناول هذه الورقة تقديم نموذج جديد يُسمى توزيع ويبل المعدل اللوغاريتمي - اللوجستي ذو القوة باستخدام طريقة تحويل القوى، والذي يُمكن اعتباره امتدادًا للتوزيع الذي قدمه برودريك أولويد وآخرون . تم حساب العديد من الخصائص الإحصائية المهمة لتوزيع ويبل المعدل اللوغاريتمي - اللوجستي ذو القوة ، والتي تتضمن المعولية ، ودالة الخطر، والدالة العكسية للخطر ، والعزم الرائي ، والدالة المميزة ، ورريني انتروبي ، والعزم الشرطي ، ونموذج معوليه الإجهاد - القوة . تتميز دالة التوزيع والكثافة ودالة الخطر ومعوليه النموذج الجديد بالعديد من الخصائص المثيرة للاهتمام موضحة في الرسم البياني لكل دالة. يتم تبرير تقدير معلمات النموذج بطريقة العزوم المعممة، مصحوبة بدراسة محاكاة مونت كارلو لاختبار مدى ملاءمة التوزيع المقترح. تم إجراء محاكاة عددية شاملة باستخدام تسعة سيناريوهات مختلفة وحجمين للعينة n = 100 و n = 500. وقد لوحظ أن أداء المقدرين، وخاصة للعينات الكبيرة، كان أقل تحيزًا وأكثر استقرارًا، ويتراوح بين 0.432 - و 0.573. كما يُظهر أيضًا انخفاضًا كبيرًا في قيم متوسط الخطأ التربيعي، والتي تتراوح بين 0.073 و 1.534، وقيم متوسط الخطأ التربيعي المطلق، والتي تتراوح بين 0.298 و 1.654، مما يوفر دقة ومتانة النموذج المقترح . تم حساب فترات الثقة للمعلمات غير المعروفة وتراوحت من 1.427 إلى 4.964. وقد تم الكشف عن النتائج من حيث التحيز، ومتوسط التقديرات ، وفترات الثقة، ومتوسط الأخطاء المطلقة، ومتوسط الأخطاء التربيعية لإثبات إمكانية تطبيقها مستقبلا في بعض مواقف الحياة الواقعية، وخاصة في المجالات الاقتصادية.

الكلمات المفتاحية: طريقة العزوم المعممة ، توزيع ويبل المعدل اللوغاريتمي - اللوجستي ، تحويل القوى ، ريني انتروبي، معوليه الإجهاد – المتانة .