



Study of Nuclear Properties in the Exotic Mirror Nuclei ${}^8\text{B}$ and ${}^8\text{Li}$

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Abstract

The ground-state properties like the nuclear densities and the root mean square radii of exotic mirror nuclei ${}^8\text{B}$ and ${}^8\text{Li}$ have been investigated using the Skyrme Hartree Fock and Symmetrized Woods Saxon calculations. Additionally, several nuclear observables have been investigated, including longitudinal elastic electron scattering form factors, binding energies, Coulomb displacement energies, magnetic dipole moments, and electric quadrupole moments. The results of the evaluation are contrasted with the experimental data that is currently available. It found that a common feature of the neutron and matter densities for the above-selected exotic nuclei is the long tail behavior. We assumed that both ${}^8\text{Li}$ and ${}^8\text{B}$ have a structure of the core nuclei ${}^7\text{Be}$ and ${}^7\text{Li}$ plus a valence. It found that the structure of the valence one-neutron of ${}^8\text{Li}$ and one-proton ${}^8\text{B}$ is a pure $1p_{1/2}$ orbit. The elastic charge form factors of the above selected exotic nuclei are evaluated using the plane wave Born approximation and compared with those of their stable isotope ${}^7\text{Li}$ and ${}^{10}\text{B}$.

Keywords: Mirror nuclei, Symmetrized Woods-Saxon potential.

1. Introduction

The investigation of short-lived nuclei far from the β -stability line has become a hot point in nuclear physics due to their unusual structural features and their role in advancing our understanding of nuclear forces and exotic ^{1, 2}. This has led to the discovery of neutron halos in light neutron-rich nuclei such as ${}^6\text{He}$, ${}^{11}\text{Li}$, and ${}^{14}\text{Be}$. These halos result from weakly bound valence nucleons occupying low angular momentum states ($l = 0$ or 1), allowing their wave functions to extend significantly. A key characteristic of halo nuclei is the presence of a low-density tail in their matter distribution at large radial distances ^{3, 4}. The fragment momentum distribution, total reaction cross-section Coulomb dissociation, and nuclear quadrupole are all experimental methods used in nuclear physics to extract information about nuclear structure, especially for exotic or unstable nuclei. These methods can help identify the properties of nuclear halos and skins, as well as determine other nuclear properties like size and shape ^{5, 6}. Additional information on the nuclear structure can be obtained by the proton elastic scattering at intermediate energies. It gives insights into both the nuclear-matter density distribution and the nuclear-matter radius. This technique is well-established for the study of stable nuclei and may also be used to investigate unstable nuclei when applied to inverse kinetics with radiation beams ⁷. One of the classic problems in nuclear structure physics is to understand the displacement energies of nuclei. The displacement energy is the binding energy difference between mirror nuclei (those with the same mass number A but with the proton Z and neutron N numbers interchanged). If the nuclear force is charge symmetric, then this binding energy difference can be related to the well-understood Coulomb interaction between the protons ⁸. In addition, a rather interesting feature of studying the mirror nuclei is that the neutron skin thickness can be found through the knowledge of proton radii alone from the mirror pairs. It has recently been proposed that the difference between the charge radii of mirror nuclei is proportional to the neutron skin ⁹.

The ground-state properties, including nuclear densities and rms radii, of exotic mirror nuclei ${}^8\text{B}$ and ${}^8\text{Li}$ were investigated using Skyrme-Hartree-Fock (SHF) and symmetrized Woods-Saxon (SWS) models. Their longitudinal and transverse elastic form factors ($|F_L(q)|^2$), as well as electric quadrupole (Q) and magnetic dipole (μ) moments, were also studied.

2. Materials and Methods

The nucleon density of exotic nuclei is given by the following expression ²:

$$\rho_m(r) = \rho_c(r) + \rho_v(r) \quad (1)$$

Where $\rho_c(r)$ (core density) and $\rho_h(r)$ (halo density) are expressed as ²:

$$\rho_c(r) = \frac{1}{4\pi} \sum_{n\ell j} N_c^{n\ell j} |R_{n\ell j}(r)|^2 \quad (2)$$

$$\rho_h(r) = \frac{1}{4\pi} N_h^{n\ell j} |R_{n\ell j}(r)|^2 \quad (3)$$

Where $R_{n\ell j}(r)$ and $N^{n\ell j}$ denote radial wave function and occupation number of the orbit $n\ell j$, respectively.

The $R_{n\ell j}(r)$ was obtained by solving the radial Schrödinger equation with the SWS potential ²:

$$\frac{d^2 R_{n\ell j}(r)}{dr^2} + \frac{2m}{\hbar^2} \left[\varepsilon_{n\ell j} - V(r) - \frac{\hbar^2 \ell(\ell+1)}{2m r^2} \right] R_{n\ell j}(r) = 0 \quad (4)$$

Where $\varepsilon_{n\ell j}$ is the single-particle binding energy and the $V(r)$ is the core potential given as ²:

$$V(r) = V_0(r) + V_{so}(r) \mathbf{L} \cdot \mathbf{S} + V_c(r) \quad (5)$$

$V_0(r)$ is the central potential takes the SWS from ¹⁰:

$$V_0(r) = -V_0 \frac{1 + \sinh(R_0/a_0)}{\cosh(r/a_0) + \cosh(R_0/a_0)} \quad (6)$$

$V_{so}(r)$ is spin orbit potential ²:

$$V_{so}(r) = V_{so} \frac{1}{r} \left[\frac{d}{dr} \frac{1}{(1 + e^{(r-R_{so})/a_{so}})} \right] \quad (7)$$

$V_c(r)$ (for protons only) is the Coulomb potential ¹¹:

$$V_c(r) = \begin{cases} \frac{Ze^2}{r} & \text{for } r > R_c \\ \frac{Ze^2}{R_c} \left[\frac{3}{2} - \frac{r^2}{2R_c^2} \right] & \text{for } r \leq R_c \end{cases} \quad (8)$$

and $V_c(r) = 0$ for neutrons.

The Skyrme force are given by ¹:

$$V_{Skyrme} = \sum_{i < j} V_{ij} = t_0(1 + x_0 P_\sigma) \delta(\vec{r}) + \frac{t_1}{2} (1 + x_1 P_\sigma) [\delta(\vec{r}) \vec{k}^2 + \vec{k}'^2 \delta(\vec{r})] + t_2 (1 + x_2 P_\sigma) \vec{k}' \cdot \delta(\vec{r}) \vec{k} \\ + \frac{1}{6} t_3 (1 + x_3 P_\sigma) \rho^\alpha(\vec{R}) \delta(\vec{r}) + it_4 \vec{k}' \cdot \delta(\vec{r}) (\vec{\sigma}_i + \vec{\sigma}_j) \\ \times \vec{k}, \quad (9)$$

where P_σ , $\vec{\sigma}$, $\delta(\vec{r})$ and \vec{k} are the space exchange operator, Pauli spin matrix vector, delta function pairing force, and the relative momentum, respectively and t_0 , t_1 , t_2 , t_3 , x_0 , x_1 , x_2 , x_3 , W_0 and γ are the parameters of the Skyrme force.

The charge, as well as the densities of protons or neutrons with the range of the Skyrme HF methodology, are obtained as ¹²:

$$\rho_g(\vec{r}) = \sum_{\beta \in g} w_\beta \psi_\beta^\dagger(\vec{r}) \psi_\beta(\vec{r}), \quad g = n, p, ch \quad (10)$$

The coefficient ψ_β denotes the occupation probability of the single-particle quantum state β . It reflects the likelihood that the state is occupied by a nucleon (proton or neutron).

The nuclear charge distribution ($\rho_{ch}(r)$) was obtained using the folding relation ²:

$$\rho_{ch}(r) = \int \rho_p(r') f_p(r' - r) dr', \quad (11)$$

Where $\rho_p(r)$ is the proton density and f_p the intrinsic charge distribution of a proton.

Where f_p takes the following form of Gaussian ²:

$$f_p(r) = \frac{1}{(\sqrt{\pi}a_p)^3} e^{(-r^2/a_p^2)} \quad (12)$$

a_p is a range parameter in the Gaussian function used to model the proton's charge distribution.

The core (R_c), matter (R_m), proton (R_p) and neutron (R_n) RMS radii are obtained by as follow ¹³:

$$R_g = \langle r_g^2 \rangle^{1/2} = \left[\frac{\int r^2 \rho_g(r) dr}{\int \rho_g(r) dr} \right]^{1/2} \quad g = c, m, n, p, ch \quad (13)$$

The proton and neutron skin thicknesses are, respectively ¹⁴:

$$\Delta R_p = R_p(Z, N) - R_n(Z, N) \quad (14)$$

$$\Delta R_n = R_n(Z, N) - R_p(Z, N) \quad (15)$$

We have in mirror nuclei with assumed that perfect charge symmetry ¹⁴:

$$R_n(Z, N) = R_p(Z, N) \quad (16)$$

Then, the difference of the proton radii of the mirror pair is ¹⁴:

$$\Delta R_{mirror} = R_p(N, Z) - R_p(Z, N) \quad (17)$$

In plane the wave Born approximation (PWBA), the elastic charge form factor ($F_{ch}(q)$) is given by ¹⁵:

$$F_{ch}(q) = \frac{4\pi}{Z} \int_0^\infty \rho_{ch}(r) j_0(qr) r^2 dr, \quad (18)$$

Where $j_0(qr)$ denotes the Bessel function and q the momentum transfer.

The longitudinal, transverse electric and transverse magnetic form factors between nuclear states J_i and J_f are given by ¹⁶:

$$|F_J^\eta(q)|^2 = \frac{4\pi}{Z^2(2J_i + 1)} \sum_J |\langle J_f || \hat{T}_J^\eta(q) || J_i \rangle|^2 \quad (19)$$

Where η refers to the type (longitudinal, transverse magnetic or transverse electric).

The Q in terms of the E2 operator is defined as ¹⁷:

$$Q = \sqrt{\frac{16\pi}{5}} \begin{pmatrix} J & 1 & J \\ -J & 0 & J \end{pmatrix} \sum_{t_z} |\langle J || \hat{O}(E2)_{t_z} || J \rangle|^2 e_{t_z} \quad (20)$$

The μ in terms of the M1 operator is defined as ¹⁸:

$$\mu = \sqrt{\frac{4\pi}{3}} \begin{pmatrix} J & 1 & J \\ -J & 0 & J \end{pmatrix} \sum_{t_z} |\langle J || \hat{O}(M1)_{t_z} || J \rangle|^2 \mu_N \quad (21)$$

Where μ_N is the nuclear magneton.

3. Results

In this study, we have been calculated the nuclear proton, neutron and matter densities, the corresponding rms radii, the elastic longitudinal ($|F_L(q)|^2$) and transverse ($|F_T(q)|^2$) form factors along with the electric and magnetic dipole moments for exotic mirror nuclei ⁸B ($S_{1p}=0.136$ MeV, $\tau_{1/2}=770$ ms) and ⁸Li ($S_{1n}=2.032$ MeV, $\tau_{1/2}=839.4$ ms) ^{19,20} using the SHF and SWS calculations. The LNS5 Skyrme parameterization has been used within the SHF calculations in this work. The values of the LNS5 parameterization employed in our calculations are $t_0 = -2194.776$, $t_1 = 482.518$,

$t_2 = 138.137$, $t_3 = 10784.169$, $x_0 = 0.134$, $x_1 = -0.097$, $x_2 = -1.399$, $x_3 = 0.171$, $W_0 = 105.674$, $\gamma = 0.16667$ ²¹. A core ${}^7\text{Be}$ ($J^\pi, T=3/2^-, 1/2$) with configuration $\{(1s_{1/2})^4, (1p_{3/2})^3\}$ plus a proton structure is assumed for ${}^8\text{B}$ ($J^\pi, T=2^+, 1$) and a core ${}^7\text{Li}$ ($J^\pi, T=3/2^-, 1/2$) with configuration $\{(1s_{1/2})^4, (1p_{3/2})^3\}$ with a neutron structure is assumed for ${}^8\text{Li}$ ($J^\pi, T=2^+, 1$). The valence proton in ${}^8\text{B}$ (valence neutron in ${}^8\text{Li}$) was assumed to be a pure $1p_{1/2}$ orbit.

Table 1. The SWS parameters ²²

Nuclei	V_0 (MeV)		V_{so} (MeV)	$a_0 = a_{so}$ (fm)	$r_0 = r_{so}$ (fm)	r_c (fm)
	Core	Valence				
${}^8\text{B}$	60.491	39.64	6.0	0.548	1.458	1.511
${}^8\text{Li}$	60.491	37.02	6.0	0.485	1.486	1.511
${}^7\text{Li}$	49.378		6.0	0.411	1.321	1.533
${}^{10}\text{B}$	48.606		6.0	0.494	1.307	1.477

Table 2. The calculated ε together with that of Ref. ²² as well as experimental data ²⁰

Nuclei	$n\ell_j$	proton			neutron		
		ε_{cal} (MeV)	ε (MeV) ²²	$\varepsilon_{exp.}$ (MeV) ²⁰	ε_{cal} (MeV)	ε (MeV) ²²	$\varepsilon_{exp.}$ (MeV) ²⁰
${}^8\text{B}$	$1s_{1/2}$	-33.322	-33.322	----	-35.798	-35.798	----
	$1p_{3/2}$	-16.592	-16.592	----	-18.819	-18.819	----
	$1p_{1/2}$	-0.136	----	-0.136	----	----	----
${}^8\text{Li}$	$1s_{1/2}$	-36.161	-36.161	----	-36.267	-36.267	----
	$1p_{3/2}$	-19.340	-19.340	----	-19.757	-19.757	----
	$1p_{1/2}$	----	----	----	-2.032	----	-2.032

Table 3. The calculated R_c and R_m rms radii and experimental ones

Nuclei	R_c			R_m		
	SHF	SWS	Exp. ^{23,24}	SHF	SWS	Exp. ^{24,25}
${}^8\text{B}$	2.26	2.27	2.31±0.02	2.55	2.55	2.55±0.08
${}^8\text{Li}$	2.27	2.27	2.33±0.02	2.50	2.50	2.50±0.06

Table 4. The calculated R_p and R_n rms radii and experimental ones

Nuclei	R_p			R_n		
	SHF	SWS	Exp. ^{23,24}	SHF	SWS	Exp. ^{24,25}
${}^8\text{B}$	2.74	2.73	2.76±0.04	2.20	2.18	2.16±0.03
${}^8\text{Li}$	2.20	2.18	2.16±0.04	2.66	2.66	2.66±0.11

Table 5. Calculated ΔR_p , ΔR_n and ΔR_{mirror} of ${}^8\text{B}$ and ${}^8\text{Li}$

Nucleus	ΔR_p		Mirror	ΔR_n		ΔR_{mirror}	
	SHF	SWS		SHF	SWS	SHF	SWS
${}^8\text{B}$	0.54	0.55	${}^8\text{Li}$	0.46	0.48	-0.54	-0.55

Table 6. Calculated and experimental results of Q moment.

Nuclei	PMOM	PJP	CKII	PJT	Exp. ²⁶⁻²⁸
${}^8\text{B}$	3.67	4.39	4.20	4.51	6.30±0.05
${}^{10}\text{B}$	9.08	9.44	9.12	9.40	8.47±0.06
${}^8\text{Li}$	2.60	2.95	2.64	3.14	3.17±0.02
${}^7\text{Li}$	-3.86	-3.80	-3.85	-3.78	-3.70±0.08

Table 7. Calculated and experimental results of μ moment.

Nuclei	PMOM	PJP	CKII	PJT	Exp. ^{26,29,30}
${}^8\text{B}$	1.019	1.225	1.298	1.176	1.03±0.003
${}^{10}\text{B}$	1.822	1.836	1.811	1.830	1.80±0.006
${}^8\text{Li}$	1.611	1.451	1.377	1.495	1.65±0.002
${}^7\text{Li}$	3.26	3.21	3.235	3.206	3.25±0.002

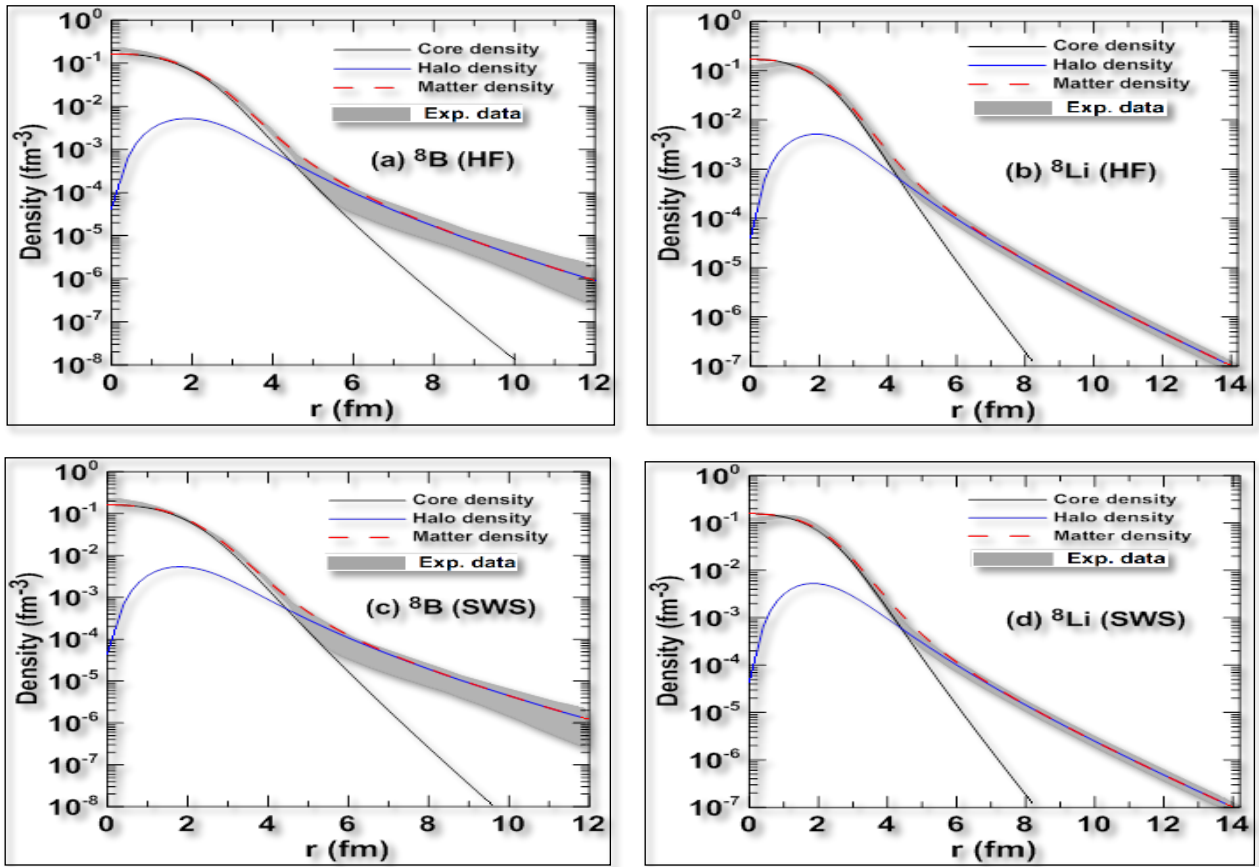


Figure 1. The core, halo and matter density distributions of exotic mirror nuclei ^8B and ^8Li .

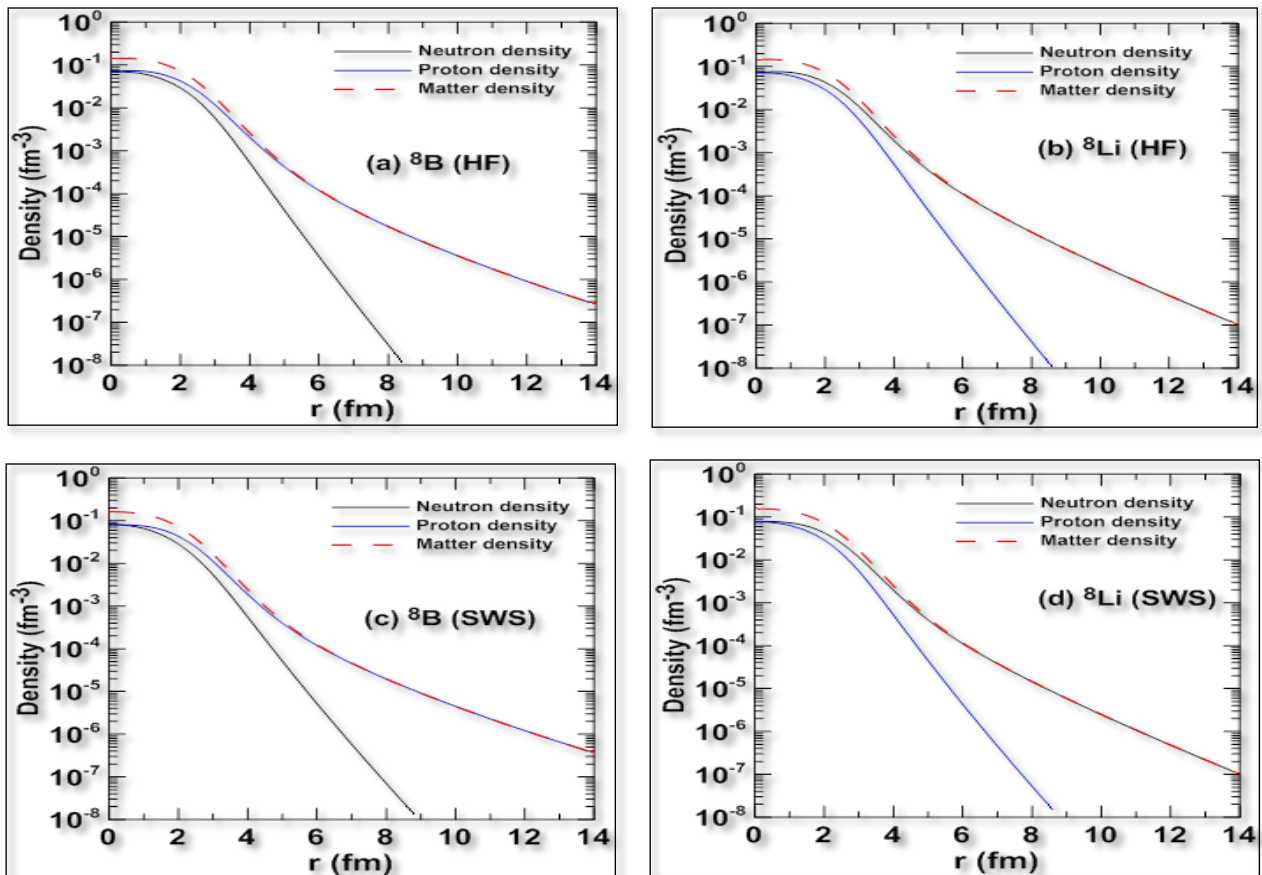


Figure 2. The proton, neutron and matter density distributions of exotic mirror nuclei ^8B and ^8Li .

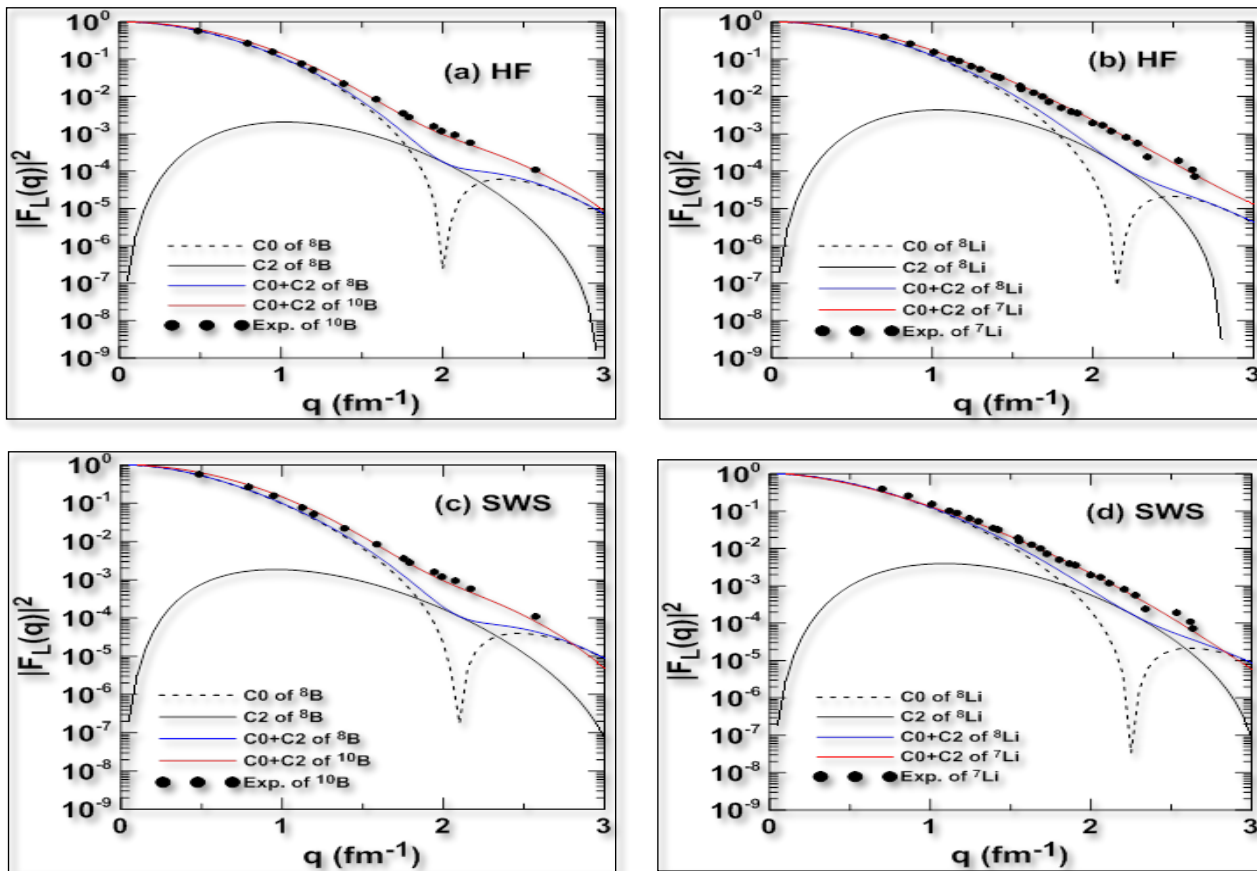


Figure 3. The longitudinal form factors of the isotopes $^{10,8}\text{B}$ and $^{7,8}\text{Li}$.

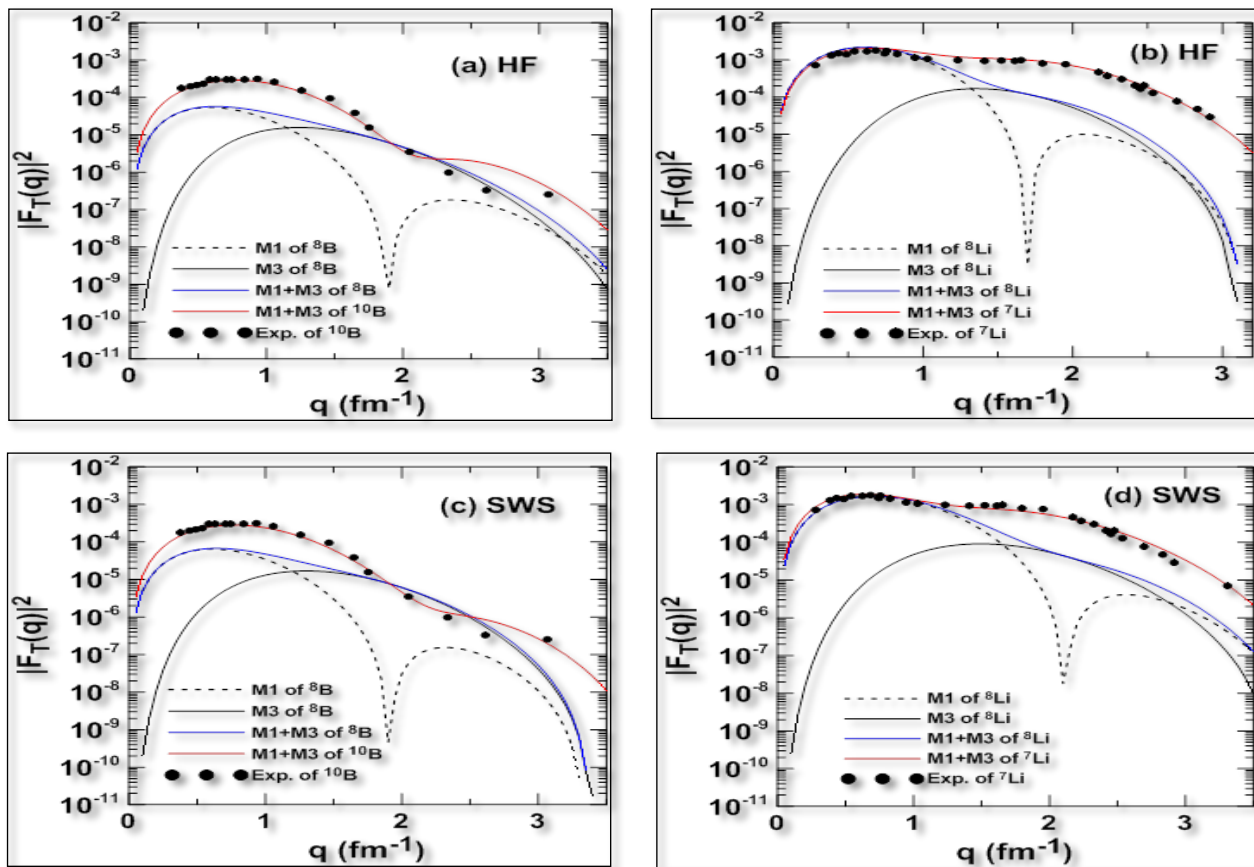


Figure 4. The transverse form factors of the isotopes $^{10,8}\text{B}$ and $^{7,8}\text{Li}$.

4. Discussion

The values of the SWS parameters utilized in the present calculations for the selected nuclei and showed in **Table 1**. The potential depth for neutrons (V_0^n) and protons (V_0^p) in core nuclei has been used the default of the NushellX@MSU program²², where the V_0^n for valence neutron and other parameters, provide the experimental ε of the last neutron as well as the matter rms radius of the halo nuclei.

The calculated $\varepsilon_{n\ell j}$ for proton and neutron is listed in **Table 2**. Due to the absence of the available experimental single particle energies for core nuclei, the experimental data is given for the last proton and neutron only.

Tables 3 and 4 present the core (R_c), matter (R_m), proton (R_p) and neutron (R_n) rms radii (in fm) of selected mirror nuclei obtained by the SHF and SWS calculations. The corresponding experiment rms radii²³⁻²⁵ are also given in these tables for comparison purposes. From these tables, we noted that the calculated results of our present study agree reasonably within the quoted error with the experimental results.

Table 5 shows the calculated proton skin of halo nucleus ${}^8\text{B}$ and the neutron-skin of its mirror nucleus ${}^8\text{Li}$ as well as the difference of proton radii for these mirror pair obtained by the SHF and SWS calculations. It can be seen from Table 5 that the proton skin of the proton-rich nucleus ${}^8\text{B}$ is larger than the neutron skin of its mirror neutron-rich nucleus ${}^8\text{Li}$. This is attributed to the Coulomb repulsion of protons.

The calculated Q and μ moments for ${}^{10,8}\text{B}$ and ${}^{7,8}\text{Li}$ isotopes are presented in **Tables 6 and 7**, respectively, together with the experimental data²⁶⁻³⁰. These calculations were performed using the p-model space with different interactions labeled as PMOM, CKII, PJP and PJT. The effective charges of the NuShellX@MSU code ($e_p^{eff} = 1.5 e$, $e_n^{eff} = 0.5e$) are used to calculate the quadrupole moment. Free-nucleon g factors are used for calculating the dipole moments. From **Table 6** it can be seen that the calculated quadrupole moment for ${}^8\text{B}$ obtained by all interactions are underestimates the measured value (6.30 ± 0.05). The sign is correctly reproduced. For ${}^{10}\text{B}$ all calculations overestimate the measured value (8.47 ± 0.06) with the correct sign. Calculations of ${}^8\text{Li}$ with PJT interaction give the value of the quadrupole moment (3.17) which is in a very good agreement with the experimental value (3.14 ± 0.02) in sign and magnitude. The experimental value of ${}^7\text{Li}$ (-3.70 ± 0.08) is very well reproduced by PJT interaction (-3.78) within its error. Calculations predict a negative sign as the measured value. The measured quadrupole moment for ${}^8\text{B}$ is about twice that of the ${}^8\text{Li}$. This large enhancement of the quadrupole moment of ${}^8\text{B}$ indicates a large deformation in comparison with that of ${}^8\text{Li}$, its mirror nucleus. It can be shown from **Table 7**, that the measured magnetic moment for ${}^8\text{B}$ (1.03 ± 0.003) is very well reproduced by PMOM interaction (1.019) in sign and magnitude. For ${}^{10}\text{B}$, all calculations are in an excellent agreement with the measured value (1.80 ± 0.006) within the experimental error. Calculations predict a positive sign as the measured value. The experimental value (1.65 ± 0.002) of ${}^8\text{Li}$ is very good reproduced by the PMOM interaction (1.611), while it is underestimated with other interactions. The sign is correctly reproduced. The magnetic moment for ${}^7\text{Li}$ (3.25 ± 0.004) is very well explained by all calculations.

In **Figure 1** we present the calculated matter densities obtained by both the SHF (left part) and SWS (right part) of the mirror nuclei ${}^8\text{B}$ [**Figures 1(a) and 1(c)**] and ${}^8\text{Li}$ [**Figs. 1(b) and 1(d)**] and compared them with experimental densities (grey area). The experimental data of ${}^8\text{B}$ and ${}^8\text{Li}$ are taken from Refs.¹ and ³¹ correspondingly. In these figures the black and blue curves represent the calculated results of the core and halo densities, respectively. For both mirror nuclei a good description of the experimental matter densities is attained with both calculations which exhibit an extended halo. In ${}^8\text{B}$, the proton rich nucleus, the halo density extends further outward, indicating a more pronounced "proton halo." This is due to the very low binding energy of the valence proton in ${}^8\text{B}$ ($S_{1p} = 0.136$ MeV), which allows its wavefunction to spread significantly outside the nuclear core. In contrast, ${}^8\text{Li}$, which is neutron-rich, shows a less extended tail for the neutron halo, consistent with its higher neutron separation energy ($S_{1n} = 2.032$ MeV).

Figure 2 depict the neutron, proton and matter densities displayed as black, blue and dashed-red distributions, respectively for exotic ${}^8\text{B}$ - ${}^8\text{Li}$ mirror nuclei. It is clearly shown from the left panel of these figures that a long tail exists in the proton density distributions of ${}^8\text{B}$, which supports the proton halo structure for these nuclei. Moreover, the difference between the proton and neutron rms radii ($R_p - R_n$) of ${}^8\text{B}$ are 0.54 fm obtained by SHF and SWS calculations, respectively. This difference gives an extra support for the proton halo structure of ${}^8\text{B}$. On the contrary, it can be seen from the right panel of **Figure 3** that the long-tail performance is clearly noticed in the neutron densities of ${}^8\text{Li}$, which supports the neutron halo structure for these nuclei. In addition, the difference between the neutron and proton rms radii ($R_n - R_p$) of ${}^8\text{Li}$ are 0.46 and 0.48 fm obtained by SHF and SWS calculations, respectively. This difference gives an extra support for the neutron halo structure of ${}^8\text{Li}$. The total longitudinal charge form factors (C_0+C_2) for ${}^{10,8}\text{B}$ and ${}^{7,8}\text{Li}$, isotopes calculated by the SHF and SWS methods are displayed in **Figure 3** along with the experimental charge form factors (dotted symbols) of the stable isotopes ${}^{10}\text{B}$ ³² and ${}^7\text{Li}$ ³³. Therein, the form factors C_0+C_2 of unstable (halo) and stable isotopes are portrayed by the blue and red curves, respectively. The C_0 and C_2 components of unstable nuclei form factors are given by the dashed and black curves, respectively. In these figures the top panel correspond to nuclei pairs ${}^{10,8}\text{B}$ and ${}^{7,8}\text{Li}$. Calculations of the OBDM elements are performed with p -model space using PMOM interaction which are carried out via the NuShellX@MSU code²². The level of agreement between the results of our calculations for stable nuclei ${}^{10}\text{B}$ and ${}^7\text{Li}$ with the experimental data is very good. Although the number of the protons within a given pair of isotopes is the same, the charge form factors are quite different from each other. One can see from **Figures 3a** and **3c** that the form factors of ${}^8\text{B}$ decreases faster than that of ${}^{10}\text{B}$ with increasing momentum transfer. This change is due to the influence of the charge density of the outer proton in ${}^8\text{B}$. As with the longitudinal form factors of ${}^8\text{B}$, the addition of neutron to ${}^8\text{Li}$ pull the charge density out and thus the form factors decrease with momentum transfer as shown in **Figures 3b** and **3d**.

Figure 4 compare the total transverse elastic form factors ($M1+M3$) of ${}^{10,8}\text{B}$ (left panel) and ${}^{7,8}\text{Li}$ (right panel) isotopes calculated using the SHF and SWS methods. The blue and red curves refer to $M1+M3$ of unstable and stable isotopes, respectively. While the $M1$ and $M3$ components for unstable ${}^8\text{Li}$ and ${}^8\text{B}$ nuclei are given by the dashed and black curves, respectively. For comparison the experimental charge form factors of stable isotopes ${}^{10}\text{B}$ ³⁴ and ${}^7\text{Li}$ ³³ are given by dotted symbols. The agreement of theoretical results with the experimental data for the stable isotopes ${}^{10}\text{B}$ and ${}^7\text{Li}$ is very well. The effect of the neutron (proton) halo on this form factor for ${}^8\text{Li}$ (${}^8\text{B}$) is just as dramatic as that on the longitudinal one. It can be seen from these figures; the blue curve decreases faster than the red curve with increasing momentum transfer. As with the longitudinal form factor, that decrease is due to the influence of the charge density of the outer proton in ${}^8\text{B}$ and due to the coupling of the extra neutron to the ${}^7\text{Li}$ core.

5. Conclusion

The ground-state properties like the nuclear densities and the rms radii for exotic neutron-rich nuclei ${}^8\text{B}$ and ${}^8\text{Li}$ have been investigated in the framework of the SHF and SWS calculations. The evaluated results are compared with available experimental data. This study draws the following conclusions:

It is found that both the SHF and SWS calculations are capable of providing theoretical predictions on the structure of exotic nuclei (considered in this study) and provide a satisfactory description of experimental data.

The halo structure of the above exotic nuclei is emphasized through exhibiting the long tail performance in their calculated matter density distributions, where this performance is considered a distinctive feature of halo nuclei.

The elastic $|F_L(q)|^2$ and $|F_T(q)|^2$ form factors for the ${}^{10,8}\text{B}$ and ${}^{7,8}\text{Li}$ isotopes have also been studied. Calculations of the OBDM elements were performed with p -model space using the PMOM

interaction, which was carried out via the NuShellX@MSU code. It was found that, the $|F_L(q)|^2$ and $|F_T(q)|^2$ form factors of exotic nuclei decreased faster than those of stable isotopes with increasing q . The calculated results of the nuclear quadrupole (Q) and magnetic dipole (μ) moments using the shell model calculations within the effective interactions show a reasonable agreement with the experimental values.

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Conflict of Interest

The author declares no conflicts of interest.

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Ethical Clearance

As a theoretical study with no human or animal subjects, ethical approval was unnecessary.

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