Ibn Al-Haitham Journal for Pure and Applied Sciences
Journal homepage: jih.uobaghdad.edu.iq
PISSN: 1609-4042, EISSN: 2521-3407
IHJPAS. 2026, 39(1)

Notes on ε- diskcyclic operators

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Received: 4/June/2025
Accepted: 2/November/2025
Published: 20/January/2026
doi.org/10.30526/39.1.4224

Abstract

In this paper, we investigated the concept of **ε**-diskcyclic operators on a separable infinite-dimensional Hilbert space H. A bounded linear operator T is called ε-diskcyclic if there exists a vector $\tilde{x}$ in H such that its disk orbit $\mathbb{D}orbt(T,\tilde{x}) := \{\alpha T^n \tilde{x} : n \geq 0, \alpha \in \mathbb{C}\,;\ 0 < |\alpha| \leq 1\}$ visits every cone of aperture ε. That is, for every non-zero vector y in H, there exist α in $\mathbb{C}$ where $0 < |\alpha| \leq 1$ and n in $\mathbb{N}$ such that $\|\alpha T^n \tilde{x} - y\| \leq \varepsilon\|y\|$. Such a vector x is then called an ε-diskcyclic vector for T.
We established several properties of ε-diskcyclic operators. In particular, we showed that every ε-diskcyclic operator is cyclic. Moreover, we examined the relationship between ε-diskcyclic vectors of T and eigenvectors of the adjoint operator T^* that cannot be orthogonal to each other. We also proved that if T_i is a bounded linear operator on H_i; $i = 1,2,3, \ldots$, then the direct sum $\oplus_i T_i$ is ε-diskcyclic provided each T_i is ε-diskcyclic. Finally, we presented a criterion for determining ε-diskcyclicity.

Keywords: Cyclic Operator, Hypercyclic Operator, Diskcyclic Operators, ε-Hypercyclic Operator, ε-Diskcyclic Operator.

1. Introduction

Let $B(H)$ be the space of all linear bounded operators on infinite - dimensional separable Hilbert space H and $\mathbb{D} := \{\alpha \in \mathbb{C} : 0 < |\alpha| \leq 1\}$ be the closed unit disk in complex number except zero.

Cyclic phenomena in operator theory study the behavior of operators that have dense orbits; let $T \in H$ and $\tilde{x} \in H$,See[1], $orbt(T,\tilde{x}) := \{T^n \tilde{x} |\ n \geq 0\}$, which divided into:

1.1 Definition: T is called a cyclic operator on H if there exists a vector $\tilde{x} \in H$ such that the $\overline{span\ orbt(T,\tilde{x})}$ is dense in H, such a vector $\tilde{x}$ is called a cyclic vector for T[2].

1.2 Definition: An operator T is called hypercyclic if there is some vector $\tilde{x} \in H$ such that the orbit of $\tilde{x}$ under $T$is dense in H, such a vector $\tilde{x}$ is called a hypercyclic vector for T[3]
.For more details[4-7].

1.3 Definition: T is called supercyclic if there is a vector $\tilde{x} \in H$ such that $\mathbb{C}orbt(T,\tilde{x}) := \{\alpha T^n \tilde{x} : \alpha \in \mathbb{C}, n \in \mathbb{N}\}$ is dense in H, in this case $\tilde{x}$ is called a supercyclic vector for T[8].
For more information[9-18.]

1.4 Definition: An operator $T \in B(H)$ is called a diskcyclic if there is a vector $\tilde{x} \in H$ such that the set $\mathbb{D}orbt(T,\tilde{x}) := \{\alpha T^n \tilde{x} | n \geq 0, \alpha \in \mathbb{D}\}$ is dense in H, where the vector $\tilde{x}$ is called a diskcyclic vector for T[18].

One of the important rustles in[18] was the diskcyclicity criterion which introduced by[18], also many studies have been presented by several researchers that have given more results. For more details[20-27]

A new concept of cyclic phenomena is called ε −diskcyclic operator[27]. In fact, the following definition of $\varepsilon -$ density is clearly weaker than the concept of diskcyclicity.

1.5 Definition : Let $\varepsilon \in (0,1)$. $T \in B(H)$ is called $\varepsilon-$diskcyclic operator if there exists $\tilde{x} \in H$ such that for all non- zero vector $y \in H$, there are $\alpha \in \mathbb{D}$, $n \in \mathbb{N}$, $\|\alpha T^n \tilde{x} - y\| \leq \varepsilon \|y\|$.
In this case $\tilde{x}$ is called $\varepsilon-$ diskcyclic vector[27].
The set of all $\varepsilon-$ diskcyclic vectors of Tdenoted by $\varepsilon - \mathbb{D}C(T)$ and the set of all $\varepsilon-$diskcyclic operator by $\varepsilon - \mathbb{D}C(H)$.
The examples illustrate the relationship between $\varepsilon-$diskcyclic and cyclic phenomena operators are construct in[27]. They showed that if T is a diskcyclic operator, then T must be $\varepsilon-$diskcyclic. The convers will be true, if T is an $\varepsilon-$diskcyclic operator for all $\varepsilon \in (0,1)$. Also, they presented that there exist a supercyclic operator which is not $\varepsilon-$diskcyclic. These facts motivated us to try to answer the following questions.
1.6 Qustion: Is every $\varepsilon-$ diskcyclic operator cyclic?
1.7 Qustion: Is $\varepsilon-$ diskcyclic operator has a criterion?
In this paper, we answered question(1.6) positively and discussed some properties of $\varepsilon-$diskcyclic operators, such as an eigenvector of the adjoint of an $\varepsilon-$diskcyclic operator must be not perpendicular to an $\varepsilon-$diskcyclic vector. Moreover, we proved that the modules of an eigenvalue of the adjoint of T greater than one. In addition, we explained that if $\bigoplus T_i\,; i = 1,2,3,\ldots$, is an ε-diskcyclic operator, then T_i is an ε-diskcyclic operator for all i in section 2. Finally, we answered question(1.7) positively in section3.

2. Materials and Methods

This study investigates the properties of ε-diskcyclic operators on an infinite-dimensional Hilbert space. The research is theoretical and employs methods from functional analysis and operator theory. The main objects of study are bounded linear operators T on a Hilbert space H, with a focus on the concept of ε-diskcyclicity. An operator T is defined as ε-diskcyclic if there exists a vector, $\tilde{x} \in H$, such that, for every nonzero vector $y \in H$, there exist scalars α (with $|\alpha| \leq 1$) and integers n such that $\| \alpha T^n \tilde{x} - y \| < \varepsilon \| y \|$.
The approach involves:

- Proving new propositions about the relationships between ε-diskcyclic operators and other cyclic phenomena.
- Establishing connections between ε-diskcyclic vectors and eigenvectors of the adjoint operator.
- Investigating the behavior of direct sums of ε-diskcyclic operators.
- Formulating and proving a criterion for ε-diskcyclicity.
  Proofs are constructed using standard techniques in Hilbert space theory, such as density arguments, properties of adjoint operators, and the use of countable dense subsets.

3. Results and Discussion

In this section, we will discuss two axes, the first one studies
3.1 Some Properties of ε −Diskcyclic Operator.
The next proposition gives a positive answer for Question (1.6). But first we recall a well-known fact: Let H be an infinite - dimension Hilbert space, M be a proper closed subspace of H, then for any $\hat{\delta} > 0$ there exist a unitary vector $w \in H \backslash M$ such that $dist(w, M) > 1 - \hat{\delta}$.
3.1.1 Proposition:
Let $\varepsilon \in (0,1)$. If T be an ε- diskcyclic operator on H, then T is a cyclic operator.
Proof: Assume by contradiction that T is a non-cyclic operator. Put $M := \overline{span(orbt(T, \tilde{x}))}$ where $\tilde{x}$ is ε-$disk$cyclic vector for T. Let $\hat{\delta} \in (\varepsilon, 1)$ and $w \in H \backslash M$ be a unitary vector such that $dist(w, M) > \hat{\delta}$. Therefore, $\mathbb{B}(w, \hat{\delta}) \cap \mathbb{D}orbt(T, \tilde{x}) = \emptyset$, thus $\tilde{x}$ is not $\hat{\delta} - diskcyclic$ vectore. since $\varepsilon < \hat{\delta} < 1$, so $\tilde{x}$ is not $\varepsilon-$ diskcyclic vector for T, which is a contradiction.

3.1.2 Proposition:

Let H_1, H_2 be two Hilbert spaces, $T \in B(H_1, H_2)$ is invertible, $\hat{S} \in B(H_1)$ and $\hat{P} \in B(H_2)$ such that $PT = T\hat{S}$. If $\hat{S}$ is an $\varepsilon-$diskcyclic operator where $\varepsilon \in (0, \frac{1}{\|T\|\|T\|^{-1}})$, then P is $\|T\|\|T\|^{-1}$ $\varepsilon-$diskcyclic operator.

Proof: Let $\tilde{x} \in \varepsilon - \mathbb{D}C(\hat{S})$ and $z \in H_2$, then $T^{-1}z \in H_1$. Thus there exist $n \in \mathbb{N}, \alpha \in \mathbb{D}$ such that $\left\|\alpha\hat{S}^n\tilde{x} - T^{-1}z\right\| \leq \varepsilon\|T^{-1}z\|$. That is,

$\left\|\alpha T\hat{S}^nT^{-1}(T\tilde{x}) - z\right\| \leq \|T\|\left\|\alpha\hat{S}^n\tilde{x} - T^{-1}z\right\| \leq \varepsilon\|T\|\|T^{-1}z\| \leq \varepsilon\|T\|\|T^{-1}\|\|z\|$

Therefore, $T\tilde{x}$ is $\|T\|\|T^{-1}\|$ $\varepsilon-$diskcyclic vector for $T\hat{S}T^{-1} = P$.

We focus on relationships between $\varepsilon-$ diskcyclic vector and eigenvector of the adjoint of T if exist.

3.1.3 Proposition:

Let $\varepsilon \in (0,1)$, if $\tilde{x} \in \varepsilon - \mathbb{D}C(T)$, then the adjoint of T has no eigenvector orthogonal to $\tilde{x}$.

Proof: Let $\varepsilon \in (0,1)$, $\hat{y} \in H$ with $\|\hat{y}\| = 1$ be an eigenvector for T^*, thus there is a non-zero scaler $\alpha \in \mathbb{C}$ such that $T^*\hat{y} = \alpha\hat{y}$. Assume that $\langle \tilde{x}, \hat{y}\rangle = 0$. Take $y \in H$; $\|y\| = 1$ and $\langle y, \hat{y}\rangle > 1 - \frac{\varepsilon}{s}$ where $s > \frac{\varepsilon}{1-\varepsilon}$. Since $\tilde{x}$ is an $\varepsilon-$diskcyclic vector for T, therefore, there are $n \in \mathbb{N}, \gamma \in \mathbb{D}$ such that $\|\gamma T^n\tilde{x} - sy\| \leq \varepsilon s\|y\|$, hence $\|\gamma T^n\tilde{x} - sy\| \leq \varepsilon s$. So,

$$|\langle\gamma T^n\tilde{x} - sy, \hat{y}\rangle| \leq \|\gamma T^n\tilde{x} - sy\|\|\hat{y}\| \leq \|\gamma T^n\tilde{x} - sy\| \leq \varepsilon s. \quad \textbf{(1)}$$

On the other hand, we have

$$|\langle\gamma T^n\tilde{x} - sy, \hat{y}\rangle| \geq |\langle sy, \hat{y}\rangle| - |\langle\gamma T^n x, \hat{y}\rangle| \geq s|\langle y, \hat{y}\rangle| - \left|\langle\gamma\tilde{x}, {T^*}^n\hat{y}\rangle\right| \geq s - \varepsilon - \gamma|\alpha^n|\ \langle\tilde{x}, \hat{y}\rangle \geq s - \varepsilon \quad \textbf{(2)}$$

Form **Equations 1 and 2** we get $s - \varepsilon \leq |\langle\gamma T^n\tilde{x} - sy, \hat{y}\rangle| \leq s\varepsilon$, hence $s \leq \frac{\varepsilon}{1-\varepsilon}$ which is a contradiction with our assumption.

The next result gives information about $\sigma_P(T^*)$.

3.1.4 Proposition:

Let $\varepsilon \in (0,1)$. If $T \in \varepsilon - \mathbb{D}C(H)$, then $\sigma_P(T^*) \cap \mathbb{D} = \emptyset$.

Proof: Let $\tilde{x} \in \mathbb{D}C(T)$. Assume that $\sigma_P(T^*) \cap \mathbb{D} \neq \emptyset$, therefore, there exist $\alpha \in \sigma_P(T^*) \cap \mathbb{D}$ and a unit vector such that $T^*\hat{y} = \alpha\hat{y}$. Now, we choose a unit vector $y \in H$ such that $\langle y, \hat{y}\rangle > 1 - \frac{\varepsilon}{s}$ where $s > \frac{\|\tilde{x}\|+1}{1-\varepsilon}$. Since $\tilde{x}$ is an $\varepsilon-$diskcyclic vector for T, so there exist $n \in \mathbb{N}$, $\beta \in \mathbb{D}$ such that $\|\beta T^n\tilde{x} - sy\| \leq \varepsilon s\|y\|$, hence $\|\beta T^n\tilde{x} - sy\| \leq \varepsilon s$. Therefore

$$|\langle\beta T^n x - ty, y^*\rangle| \leq \|\beta T^n\tilde{x} - sy\|\|\hat{y}\| \leq \|\beta T^n\tilde{x} - sy\| \leq \varepsilon s \quad \textbf{(3)}$$

On the other hand, $|\langle\beta T^n\tilde{x} - sy, \hat{y}\rangle| \geq |\langle sy, \hat{y}\rangle| - |\langle\beta T^n\tilde{x}, \hat{y}\rangle| \geq s - \varepsilon - |\langle\beta\tilde{x}, \alpha^n\hat{y}\rangle|$

Because $\alpha, \beta \in \mathbb{D}$ and $\|\hat{y}\| = 1$, we have

$$\langle\beta T^n\tilde{x} - sy, \hat{y}\rangle \geq s - \varepsilon - \beta|\alpha^n|\|\tilde{x}\| \geq s - 1 - \|\tilde{x}\| \quad \textbf{(4)}$$

Form **Equations 3 and 4** we obtain $s - 1 - \|\tilde{x}\| \leq s\,\varepsilon$, hence $s \leq \frac{1+\|\tilde{x}\|}{1-\varepsilon}$, which is a contradiction with our assumption.

Now, we turn our attention on a direct sum of operators which is an ε -diskcyclic.

3.1.5 Proposition:

Let $\{H_i\}_{i=i}^{\infty}$ be a family of Hilbert spaces, $T_i \in B(H_i)$ for all i. If $\bigoplus_{i=1}^{\infty} T_i \in \varepsilon - \mathbb{D}C(\bigoplus_{i=1}^{\infty} H)$, then for all i, T_i is ε -diskcyclic operator.

Proof: Let $y_i \in H_i$, so $y = (\ldots, 0, 0, y_i, 0, 0, 0 \ldots) \in \bigoplus_{i=1}^{\infty} H_i$. Since $\bigoplus_{i=1}^{\infty} T_i$ is an ε-diskcyclic operator, then there exist an ε-diskcyclic vector for $\bigoplus_{i=1}^{\infty} T_i$, $\tilde{x} = (\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \ldots)$, such that there are $n \in \mathbb{N}, \alpha \in \mathbb{D}$ satisfies $\|\alpha \bigoplus_{i=1}^{\infty} {T_i}^n(\tilde{x}) - y\| \leq \varepsilon\|y\|$. Hence

$$\sum_{k=0}^{i-1}\|\alpha {T_k}^n\tilde{x}_k\|^2 + \|\alpha {T_i}^n\tilde{x}_i - y_i\|^2 + \sum_{i+1}^{\infty}\|\alpha {T_k}^n\tilde{x}_k\|^2 \leq \varepsilon^2\|y_i\|^2$$

We get $\|\alpha {T_i}^n\tilde{x}_i - y_i\| \leq \varepsilon\|y_i\|$.

The second axis states and proves

4. ε −Diskcyclic Operator Criterion.

4.1 Theorem:

Let $\varepsilon \in (0,1), \alpha \in \mathbb{D}$ and $T \in B(H)$ be a separable infinite dimensional Hilbert space, let D_1be a dense subset on H and $D_2 \coloneqq \{\hat{y}_k : k \in \mathbb{N}\}$ be a countable subset of H such that for all non-zero vector $x \in H$, there exist infinitely many $k \in \mathbb{N}$ with $\hat{y}_k \in \mathbb{D}_{\varepsilon\|x\|}(x)$. Let $\{n_k\} \subset \mathbb{N}$ be an increasing sequence, and for all k, let $S_{n_k}: D_2 \to H$ be a sequence of maps such that:

I. $\lim_{k\to\infty} \|T^{n_k}x\| = 0$ for all $x \in D_1$,

II. $\lim_{k\to\infty} \|S_{n_k}\hat{y}_k\| = 0$ for all $\hat{y}_k \in D_2$,

III. $\lim_{k\to\infty} \left\|T^{n_k}S_{n_k}\hat{y}_k - \frac{\hat{y}_k}{\alpha}\right\| = 0.$

Then T is a strictly ε −diskcyclic operator.

Proof: Because H is separable, then there exist a countable dense set, say $\{z_j\}$.

Let $\{\mu_k\}_{k\in\mathbb{N}} \subseteq \mathbb{R}^+$satisfy $k^2\mu_k \to 0$ as $k \to \infty$, so that

$\sum_{k=0}^{\infty} \mu_k < \infty.$ **(5)**

Since there are infinitely many $k \in \mathbb{N}$ with $\hat{y}_k \in \mathbb{D}_{\varepsilon\|x\|}(x)$, pick $\hat{y}_{m_0} \in D_2$ such that

$\|\hat{y}_{m_0} - z_0\| \le \varepsilon\|z_0\|.$

By conditions (II) and (III): $\left\|S_{n_{m_0}}\hat{y}_{m_0}\right\| \le \mu_0$ and $\left\|\alpha T^{n_{m_0}}S_{n_{m_0}}\hat{y}_{m_0} - \hat{y}_{m_0}\right\| \le \alpha\mu_0.$

By the density of D_1, there exist $x_0 \in D_1$closer to $S_{n_{m_0}}\hat{y}_{m_0}$such that

$\|x_0\| \le \mu_0$ and $\|\alpha T^{n_{m_0}}x_0 - \hat{y}_{m_0}\| \le \alpha\mu_0.$

By the same argument we get increasing sequence $\{m_k\}$ in $\mathbb{N}$ and $x_k \in D_1$ closer to $S_{n_{m_k}}\hat{y}_{m_k}$ satisfies

$\|x_k\| < \mu_k,$ **(6)**

$\|\alpha T^{n_{m_k}}x_k - \hat{y}_{m_k}\| < \alpha\mu_k,$ **(7)**

and

$\|z_k - \hat{y}_{m_k}\| \le \varepsilon\|z_k\|$ **(8)**

By condition (I) and the continuity of T, choose $\rho_k > 0$ such that

$\|T^{n_{m_j}}u\| \le 2^{-k}$ for all $\|u\| < \rho_k$, **(9)**

Let $\hat{\mu}_k \coloneqq \min\{\rho_k, \mu_k\}$, Then $\|x_k\| < \hat{\mu}_k$ for all k.

For $j \in \mathbb{N}$, we have three cases: For $k < j$, by condition (I)

$\|T^{n_{m_j}}x_k\| < \hat{\mu}_j,$ **(10)**

For $k = j$, and by (7)

$\|\alpha T^{n_{m_k}}x_k - \hat{y}_{m_k}\| \le \alpha\hat{\mu}_k,$ **(11)**

For $k > j$, by (9), we obtain

$\|T^{n_{m_j}}x_k\| \le 2^{-k},$ **(12)**

Put $x := \sum_{k=0}^{\infty} x_k$

Since H is complete and by **Equation 5,** x is well-defined.

Now by **Equations 8, 10, 11, 12** and by using the geometric series, also since $\alpha \in \mathbb{D}$

$\|\alpha T^{n_{m_j}}\, x - z_j\| \le |\alpha| \sum_{k<j}\|T^{n_{m_j}}\, x_k\| + \|\alpha T^{n_{m_k}}\, x_k - \hat{y}_{m_k}\| + \|\hat{y}_{m_k} - z_k\| +$
$|\alpha| \sum_{k>j}\|T^{n_{m_j}}\, x_k\| < (j+1)\hat{\mu}_j + \varepsilon\|z_j\| + 2^{-j}$

For any $z \in H\backslash\{0\}$, there is $\{j_t\}$ increasing sequence such that $z_{j_t} \to z$. Note that,

$\|\alpha T^{n_{m_{j_t}}}\, x - z\| \le \|\alpha T^{n_{m_{j_t}}}\, x - z_{j_t}\| + \|z - z_{j_t}\| < (j+1)\hat{\mu}_{j_t} + \varepsilon\|z_{j_t}\| + 2^{-j} + \|z - z_{j_t}\|$

As $t \to \infty$ $\|\alpha T^{n_{m_{j_t}}}\, x - z\|$ tends to $\varepsilon\|z\|$, therefore T is strictly ε-diskcyclic.

The results of this paper advance the understanding of cyclic phenomena in operator theory by clarifying the properties and implications of ε-diskcyclic operators. The proof that every ε-

diskcyclic operator is cyclic bridges the gap between these two concepts and answers an open question in the literature. This finding aligns with the intuition that diskcyclicity, being a form of density condition, should imply cyclicity, but the explicit proof strengthens the theoretical foundation of the field.

The transferability of diskcyclicity under invertible transformations highlights the structural robustness of the property, suggesting that it is preserved under isomorphisms of Hilbert spaces. The results concerning the adjoint operator provide new insight into the spectral properties of ε-diskcyclic operators, particularly the restriction that eigenvectors of the adjoint cannot be orthogonal to diskcyclic vectors and that the modulus of any eigenvalue of the adjoint must exceed one. These spectral constraints may have further implications for the classification and characterization of such operators.

The analysis of direct sums demonstrates that ε-diskcyclicity is inherited by components, facilitating the study of more complex operator systems via their simpler constituents. Finally, the newly established criterion for ε -diskcyclicity offers a practical tool for identifying such operators in future research.

Overall, this work extends previous studies many authors, providing answers to previously posed questions and introducing new structural results. Future research may explore further generalizations, applications to other classes of operators, or connections with dynamical systems and ergodic theory.

5.Conclusion:

In this research, we expanded the study of an ε −diskcyclic operator and added new properties to complement[27] research. we showed that such operator must be cyclic and explained that the eigenvector of the adjoint of an ε-diskcyclic operator cannot be perpendicular to an ε-diskcyclic vector. Also, we established that the modulus of an eigenvalue of the adjoint of T is greater than one. As well, we clarified that if $\bigoplus_{i=1}^{\infty} T_i$ represents an ε-diskcyclic operator, then each $T_i\,; i = 1,2,3, \dots,$ is also an ε-diskcyclic operator. Finally, we introduced a criterion for ε-diskcyclic operator.

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