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New Transmuted Rayleigh-Weibull Distribution: Structure and Properties

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Abstract

In this research, the new distribution is built depending on the Rayleigh-Weibull distribution, which is called a new transmuted Rayleigh-Weibull distribution (T R W D). The distribution was built by adding the conversion parameter ß, |ß| ≤1, which contributes to an increase in the flexibility of distribution. At first, the probability cumulative function, the probability density function, the hazard function, and the survival function were found by relying on the transformation rules, and the shapes for these main functions will be discussed at (x⇒∞) and (x⇒0). In addition to this, prove the statistical properties of this distribution, which encompass the median, mean, and moments about the origin. Consequently, by studying the moments of the origin, they found the variance and the first moment, skewness, and kurtosis. In addition, a table will be made to show these properties at different parameter values. Also, it will be found to have a characteristic function, a factorial generating function, a mean time to failure, a quantile function, and the median.

Keywords: Rayleigh-Weibull Distribution, Survival function, Mode Moments about the origin, Quantile function.

1. Introduction

Many researchers are building new statistical distributions by adding a new parameter to make the distribution more flexible in handling data[1]. Because the current distributions are restricted distributions and these restrictions make them insufficient in modeling diverse data, many scientists have resorted to integrating or adding a new parameter to these distributions in order to be more flexible[2,3]. The first to introduce the idea of transmuted distributions were researchers:

In 2007, researchers created the family of transmuted distributions in their scientific paper[4,5]. After this discovery, many scientists have introduced new distributions, for example: a transfer for Rayleigh distribution[6], and transfer for Gumbel distribution[7], and transfer for modified Weibull distribution[8], a transfer for gamma-gompertz distribution[9], a transfer for Lomax distribution[10], a transfer for Probability distributions[11], a transfer for inverse exponential distribution[12], and a transfer for Ishita distribution[13]. Most of the researchers, after finding the new distributions, found the characteristics of the distribution, the statistical function fees; for example: found properties of lindley-exponential (9), found properties of the Beta inverse Rayleigh distribution[14], found properties of the gamma-normal distribution[15], found properties of family of generalized gamma distribution[16], found properties of the extension of the exponentiated Rayleigh distribution[17], found properties of the new exponentiated generalized linear exponential distribution[18], found properties of exponentiated generalized weibull exponential distribution[19], found properties of the generalized extended exponential-Weibull distribution[20], found properties of the exponentiated gamma exponential distribution[21], found properties of complementary exponentiated exponential geometric lifetime distribution[22], found properties of exponentiated exponential distribution[23], and found properties of generalized exponential Rayleigh distribution[24,25]. In this article, the new distribution is dependent on the Rayleigh-Weibull distribution in its construction. Authorized distribution came as a result of mixing the survival function of the Rayleigh distribution with one parameter and the survival function of the Weibull distribution with one parameter as well. The result was the Rayleigh-Weibull distribution, which we relied on in constructing the new distribution. This is done by adding a conversion parameter ß, |ß| ≤1. A new distribution, which is termed the new transmuted Rayleigh-Weibull distribution, and its properties and the graphs of the statistical functions for this distribution, and the shapes of both the probability density function, and the function of instantaneous failure rate will be discussed.

2. The New Transmuted Rayleigh-Weibull Distribution

$$Z(x) = (1+ß)H(x) - ß\big(H(x)\big)^2$$

Where $H(x)$ Is C.D.F. of the Rayleigh-Weibull Distribution, and $ß \in [-1,1]$.

$$Z(x) = (1+ß)\left(1-e^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}\right) - ß\left(1-e^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}\right)^2$$

Where ω is the scale parameter of Rayleigh distribution and ϑ is the shape parameter of Weibull distribution

$$Z(x) = \left(1+ß-ß\left(1-e^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}\right)\right)\left(1-e^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}\right) = \left(1+ß-ß+ße^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}\right)\left(1-e^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}\right)$$

$$Z(x) = \left(1+ße^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}\right)\left(1-e^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}\right)$$

$$Z(x) = \left(1-e^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}+ße^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}-ße^{-2\left(\frac{\omega}{2}x^2+x^\vartheta\right)}\right)$$

$$Z(x) = 1-\left(ß-1-ße^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}\right)e^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)} \quad ,|ß| \le 1, x>0, \vartheta, \omega>0, \qquad \textbf{(1)}$$

The p.d.f of Transmuted Rayleigh -Weibull Distribution is found by:

$$z(x) = h(x)\big(1+ß-2ßH(x)\big)$$

Where $h(x)$ *is p.d.f of* Rayleigh -Weibull Distribution

$$z(x) = (\omega x+\vartheta x^{\vartheta-1})e^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}\left(1+ß-2ß\left(1-e^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}\right)\right)$$

$$z(x) = (\omega x+\vartheta x^{\vartheta-1})e^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}\left(1-ß+2ße^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}\right) \quad ,|ß| \le 1, x>0, \vartheta, \omega>0, \qquad \textbf{(2)}$$

The survival function is defined as follows:

$$S(x) = 1-Z(x) = \left(ß-1-ße^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}\right)e^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)} \quad ,|ß| \le 1, x>0, \vartheta, \omega>0, \qquad \textbf{(3)}$$

the hazard function can be found by:

$$\mathrm{h}(x) = \frac{z(x)}{s(x)} = \frac{(\omega x+\vartheta x^{\vartheta-1})e^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}\left(1-ß+2ße^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}\right)}{\left(ß-1-ße^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}\right)e^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}}$$

$$= \frac{(\omega x+\vartheta x^{\vartheta-1})\left(1-ß+2ße^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}\right)}{\left(ß-1-ße^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}\right)} \quad ,|ß| \le 1, x>0, \vartheta, \omega>0, \qquad \textbf{(4)}$$

In a special case when $ß = 0$ we get Rayleigh -Weibull distribution
and when $ß = 0 = \vartheta$, we get Weibull distribution
and when $ß = 0, \vartheta = 1$ we get exponential-Rayleigh distribution
and when $ß = 1$ we get Rayleigh-Weibull distribution

2.1. The shape of the probability density function of the New Transmuted Rayleigh -Weibull Distribution

Knowing the shape of a New Transmuted Rayleigh -Weibull Distribution helps us understand the drain and oncoming of distribution functions in cooperation with data. To realize this mathematically, especially through the limit values of the probability density functions when(x⇨0 &x⇨∞)

$$\lim_{x\to 0} z(x) = \lim_{x\to 0}\left[(\omega x+\vartheta x^{\vartheta-1})e^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}\left(1-ß+2ße^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}\right)\right]$$

$$=\left[(\omega(0)+\vartheta(0)^{\vartheta-1})e^{-\left(\frac{\omega}{2}(0)^2+(0)^\vartheta\right)}\left(1-ß+2ße^{-\left(\frac{\omega}{2}(0)^2+(0)^\vartheta\right)}\right)\right]$$

$$=1+ß$$

$$lim_{x\to\infty}\, z(x) = lim_{x\to\infty}\left[\left(\omega x e^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}+\vartheta x^{\vartheta-1}e^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}\right)\left(1-ß+2ße^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}\right)\right]$$

$$=\lim_{x\to\infty}\left[\left(\omega(1-ß)xe^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}+2ß\omega xe^{-2\left(\frac{\omega}{2}x^2+x^\vartheta\right)}+\vartheta(1-ß)x^{\vartheta-1}e^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}\right.\right.$$
$$\left.\left.+2\vartheta ßx^{\vartheta-1}e^{-2\left(\frac{\omega}{2}x^2+x^\vartheta\right)}\right)\right]=0$$

2.2. The shape of hazard function of the New Transmuted Rayleigh -Weibull Distribution

$$\lim_{x\to 0} h(x)=\frac{(\omega x+\vartheta x^{\vartheta-1})\left(1-ß+2ße^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}\right)}{\left(ß-1-ße^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}\right)}$$

$$=\frac{(\omega(0)+\vartheta(0)^{\vartheta-1})\left(1-ß+2ße^{-\left(\frac{\omega}{2}(0)^2+(0)^\vartheta\right)}\right)}{\left(ß-1-ße^{-\left(\frac{\omega}{2}(0)^2+(0)^\vartheta\right)}\right)}=-\frac{0}{1}=0$$

$$\lim_{x\to\infty} h(x)=\lim_{x\to\infty}\frac{(\omega x+\vartheta x^{\vartheta-1})\left(1-ß+2ße^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}\right)}{\left(ß-1-ße^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}\right)}$$

$$=\lim_{x\to\infty}\left[\frac{\omega(1-ß)x}{ß-1-ße^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}}+\frac{2ß\omega x}{e^{\left(\frac{\omega}{2}x^2+x^\vartheta\right)}\left(ß-1-ße^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}\right)}\right.$$
$$\left.+\frac{\vartheta(1-ß)}{x^{1-\vartheta}\left(ß-1-ße^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}\right)}+\frac{2\vartheta ß}{x^{1-\vartheta}e^{\left(\frac{\omega}{2}x^2+x^\vartheta\right)}\left(ß-1-ße^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}\right)}\right]$$

$$\lim_{x\to\infty} h(x)=0$$

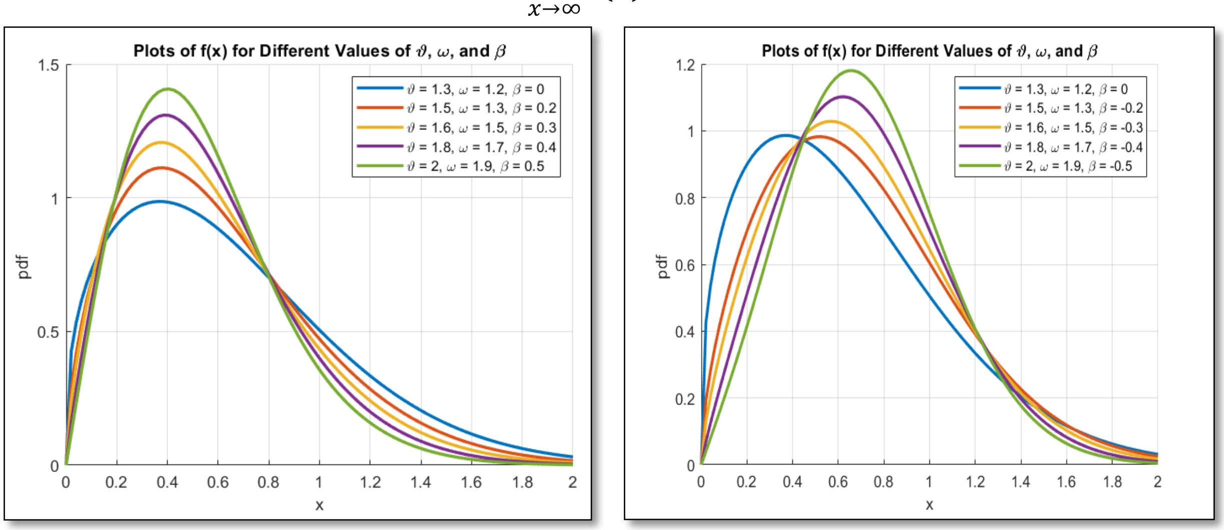

Figure 1. The p.d.f for new Transmuted Rayleigh -Weibull Distribution with different values of ß,ϑ,ω.

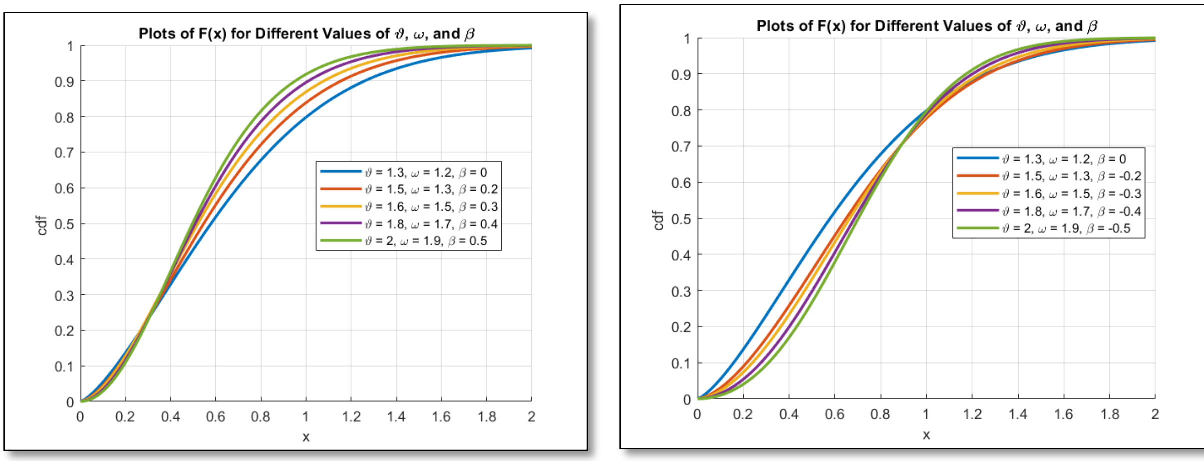

Figure 2. The c.d.f for new Transmuted Rayleigh -Weibull Distribution with various values of ß,ϑ,ω**.**

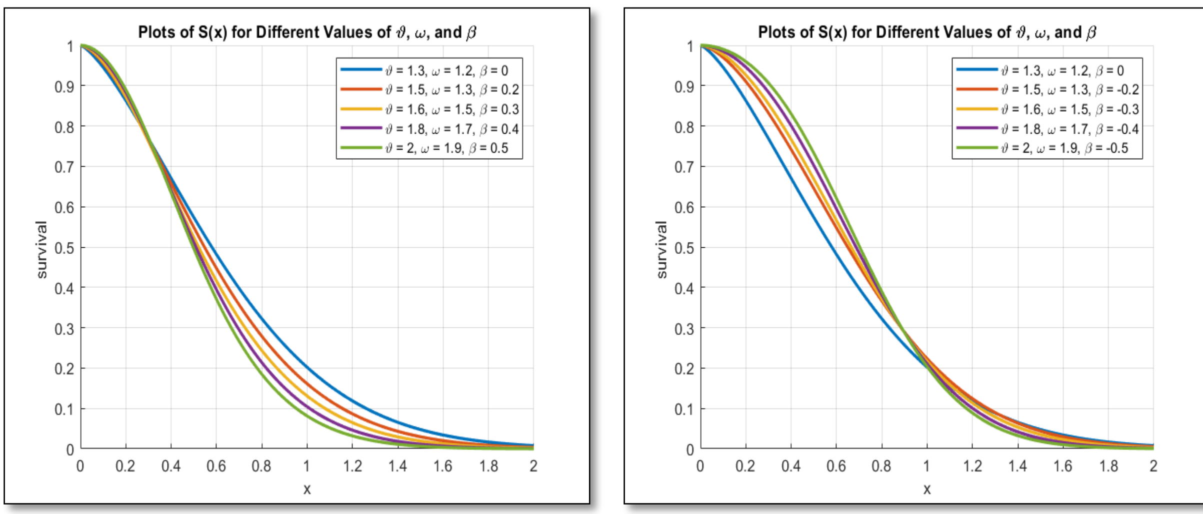

Figure 3. The S(x) for new Transmuted Rayleigh -Weibull Distribution with variant values of ß,ϑ,ω.

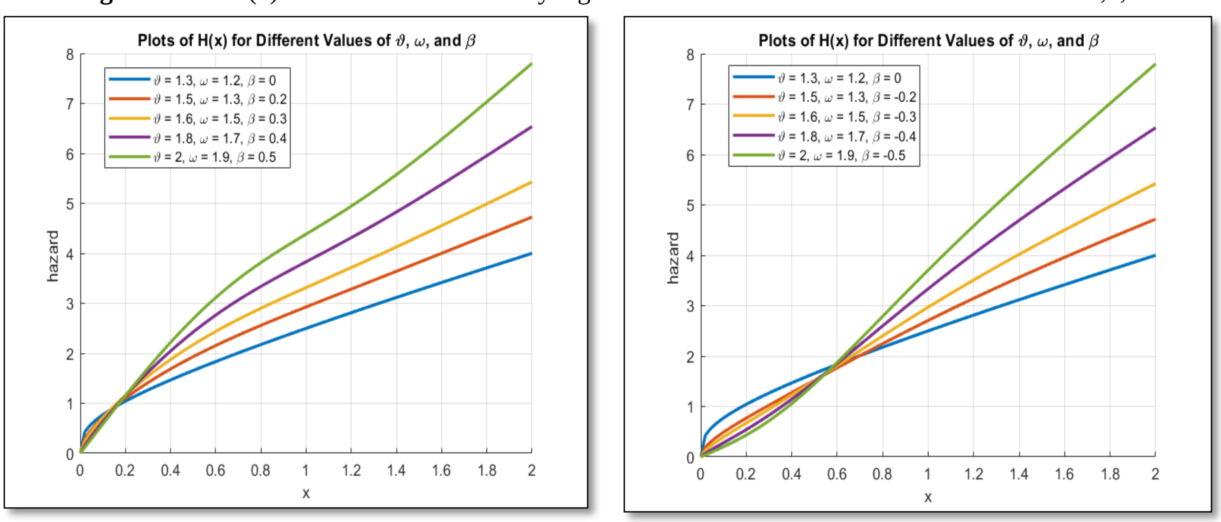

Figure 4. The h(x) for new Transmuted Rayleigh -Weibull Distribution with dissimilar values of ß,ϑ,and ω

3. Mathematical and statistical properties of new distribution

3.1. The Mode

$$z(x) = (\omega x + \vartheta x^{\vartheta-1}) \mathrm{e}^{-\left(\frac{\omega}{2}x^2 + x^{\vartheta}\right)} \left(1 - ß + 2\,ß\mathrm{e}^{-\left(\frac{\omega}{2}x^2 + x^{\vartheta}\right)}\right)$$

$$\frac{\partial z(x)}{\partial x}=\left[(\omega x+\vartheta x^{\vartheta-1})e^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}*\left(-2\,ß(\omega x\ +\vartheta x^{\vartheta-1})e^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}\right)\right]$$
$$+\left[\left(1-\ ß+2\,ße^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}\right)\right.$$
$$*\left((\omega x+\vartheta x^{\vartheta-1})*\left(-(\omega x\ +\vartheta x^{\vartheta-1})e^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}\right)+e^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}\right.$$
$$\left.\left.*\left(\omega+\vartheta(\vartheta-1)x^{\vartheta-2}\right)\right)\right]$$

$$=\left[e^{-2\left(\frac{\omega}{2}x^2+x^\vartheta\right)}(\omega x+\vartheta x^{\vartheta-1})*\left(-2\,ß(\omega x\ +\vartheta x^{\vartheta-1})\right)\right]$$
$$+\left[e^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}\left(1-\ ß+2\,ße^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}\right)\right.$$
$$\left.*\left((\omega x+\vartheta x^{\vartheta-1})*\left(-(\omega x\ +\vartheta x^{\vartheta-1})\right)+e^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}*\left(\omega+\vartheta(\vartheta-1)x^{\vartheta-2}\right)\right)\right]$$

$$=e^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}\left[e^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}(\omega x+\vartheta x^{\vartheta-1})*\left(-2\,ß(\omega x\ +\vartheta x^{\vartheta-1})\right)+\right.$$
$$\left.\left(\left(1-\ ß+2\,ße^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}\right)*\left(\left(-(\omega x+\vartheta x^{\vartheta-1})^2\right)+\left(\omega+\vartheta(\vartheta-1)x^{\vartheta-2}\right)\right)\right)\right]$$

$\frac{\partial z(x)}{\partial x}=0$, Since $e^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}\neq 0$, it is clear that the exponential function is not equal to zero because each of the following $0>\vartheta,\omega,x<\infty$

$$\left[e^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}(\omega x+\vartheta x^{\vartheta-1})*\left(-2\,ß(\omega x\ +\vartheta x^{\vartheta-1})\right)\right.$$
$$\left.+\left(\left(1-\ ß+2\,ße^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}\right)*\left(\left(-(\omega x\ +\vartheta x^{\vartheta-1})^2\right)+\left(\omega+\vartheta(\vartheta-1)x^{\vartheta-2}\right)\right)\right)\right]$$
$$=0$$

In the mode property, find the value of x after differentiating the p.d.f, and the resulting equation after differentiation is a non-linear equation. The value of x cannot be found by ordinary methods, but it can be found by numerical mathematical methods and by using programs such as MATLAB.

3.2 .Moments about the origin

$$M_r'(x)=\int_0^\infty x^r\,z(x)dx=\int_0^\infty x^r\,(\omega x+\vartheta x^{\vartheta-1})e^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}\left(1-\ ß+2\,ße^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}\right)dx$$

Since $e^{-x^\vartheta}=\sum_{i=0}^{\infty}\frac{(-1)^i}{i!}x^{i\vartheta}$ **(5)**

$e^{-2x^\vartheta}=\sum_{j=0}^{\infty}\frac{(-1)^j2^j}{j!}x^{j\vartheta}$ **(6)**

Since $z(x)=(1-\ ß)(\omega x+\vartheta x^{\vartheta-1})e^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}+2\,ß(\omega x+\vartheta x^{\vartheta-1})e^{-2\left(\frac{\omega}{2}x^2+x^\vartheta\right)}$

$$z(x)=(1-\ ß)\sum_{i=0}^{\infty}\frac{(-1)^i}{i!}\left(\omega x^{i\vartheta+1}+\vartheta x^{i\vartheta+\vartheta-1}\right)e^{-\left(\frac{\omega}{2}x^2\right)}+\ ß\sum_{j=0}^{\infty}\frac{(-1)^j2^{j+1}}{j!}(\omega x^{j\vartheta+1}$$
$$+\vartheta x^{j\vartheta+\vartheta-1})e^{-\omega x^2}$$

$$M_r'(x)=\int_0^\infty x^r\left((1-\ ß)\sum_{i=0}^{\infty}\frac{(-1)^i}{i!}\left(\omega x^{i\vartheta+1}+\vartheta x^{i\vartheta+\vartheta-1}\right)e^{-\left(\frac{\omega}{2}x^2\right)}+\ ß\sum_{j=0}^{\infty}\frac{(-1)^j2^{j+1}}{j!}(\omega x^{j\vartheta+1}+\right.$$
$$\left.\vartheta x^{j\vartheta+\vartheta-1})e^{-\omega x^2}\right)dx$$

$=(1-ß)\sum_{i=0}^{\infty}\frac{(-1)^i}{i!}\int_0^{\infty}\left(\left(\omega x^{r+i\vartheta+1}+\vartheta x^{r+\vartheta(i+1)-1}\right)e^{-\left(\frac{\omega}{2}x^2\right)}\right)dx+$

$ß\sum_{j=0}^{\infty}\frac{(-1)^j2^{j+1}}{j!}\int_0^{\infty}((\omega x^{r+j\vartheta+1}+\vartheta x^{r+\vartheta(j+1)-1})e^{-\omega x^2})dx$

Let $\frac{\omega}{2}x^2=r_1\Rightarrow x^2=\frac{2}{\omega}r_1\Rightarrow x=\left(\frac{2}{\omega}r_1\right)^{\frac{1}{2}}\Rightarrow dx=\left(\frac{2}{\omega}r_1\right)^{-\frac{1}{2}}\frac{1}{\omega}dr_1$ **(7)**

And $\omega x^2=r_2\Rightarrow x^2=\frac{1}{\omega}r_2\Rightarrow x=\left(\frac{1}{\omega}r_2\right)^{\frac{1}{2}}\Rightarrow dx=\left(\frac{1}{\omega}r_2\right)^{-\frac{1}{2}}\frac{1}{2\omega}dr_2$ **(8)**

$$M_r'(x)=(1-ß)\sum_{i=0}^{\infty}\frac{(-1)^i}{i!}\left[\int_0^{\infty}\left(\omega\left(\frac{2}{\omega}r_1\right)^{\frac{r+i\vartheta+1}{2}}\right)e^{-r_1}\left(\frac{2}{\omega}r_1\right)^{-\frac{1}{2}}\frac{1}{\omega}dr_1\right.$$
$$\left.+\int_0^{\infty}\vartheta\left(\frac{2}{\omega}r_1\right)^{\frac{r+\vartheta(i+1)-1}{2}}e^{-r_1}\left(\frac{2}{\omega}r_1\right)^{-\frac{1}{2}}\frac{1}{\omega}dr_1\right]+ß\sum_{j=0}^{\infty}\frac{(-1)^j2^{j+1}}{j!}$$

$\left[\int_0^{\infty}\left(\frac{1}{\omega}r_2\right)^{\frac{r+j\vartheta+1}{2}}e^{-r_2}\frac{1}{2}\left(\frac{1}{\omega}r_2\right)^{-\frac{1}{2}}\frac{1}{\omega}dr_2+\int_0^{\infty}\vartheta\left(\frac{1}{\omega}r_2\right)^{\frac{r+\vartheta(j+1)-1}{2}}e^{-r_2}\frac{1}{2}\left(\frac{1}{\omega}r_2\right)^{-\frac{1}{2}}\frac{1}{\omega}dr_2\right]$

$=(1-$

$ß)\sum_{i=0}^{\infty}\frac{(-1)^i}{i!}\left[\left(\frac{2}{\omega}\right)^{\frac{r+i\vartheta+1-1}{2}}\int_0^{\infty}\left((r_1)^{\frac{r+i\vartheta+1-1}{2}}\right)e^{-r_1}dr_1+\right.$

$\left.\frac{\vartheta}{\omega}\left(\frac{2}{\omega}\right)^{\frac{r+\vartheta(i+1)-1-1}{2}}\int_0^{\infty}(r_1)^{\frac{r+\vartheta(i+1)}{2}-1}e^{-r_1}dr_1\right]+ß\sum_{j=0}^{\infty}\frac{(-1)^j2^{j+1-1}}{j!}$

$$\left[\left(\frac{1}{\omega}\right)^{\frac{r+j\vartheta+1-1}{2}}\int_0^{\infty}(r_2)^{\frac{r+j\vartheta+1-1}{2}}e^{-r_2}dr_2+\vartheta\left(\frac{1}{\omega}\right)^{\frac{r+\vartheta(j+1)-1-1}{2}}\int_0^{\infty}(r_2)^{\frac{r+\vartheta(j+1)}{2}-1}e^{-r_2}dr_2\right]$$

since $\Gamma(z)=\int_0^{\infty}x^{z-1}e^{-x}dx$

$M_r'(x)=(1-ß)\sum_{i=0}^{\infty}\frac{(-1)^i}{i!}\left[\left(\frac{2}{\omega}\right)^{\frac{r+i\vartheta}{2}}\Gamma\left(\frac{r+i\vartheta}{2}+1\right)+\frac{\vartheta}{\omega}\left(\frac{2}{\omega}\right)^{\frac{r+\vartheta(i+1)-2}{2}}\Gamma\left(\frac{r+\vartheta(i+1)}{2}\right)\right]+$

$$ß\sum_{j=0}^{\infty}\frac{(-1)^j2^j}{j!}\left[\left(\frac{1}{\omega}\right)^{\frac{r+j\vartheta}{2}}\Gamma\left(\frac{r+j\vartheta}{2}+1\right)+\vartheta\left(\frac{1}{\omega}\right)^{\frac{r+\vartheta(j+1)}{2}}\Gamma\left(\frac{r+\vartheta(j+1)}{2}\right)\right]$$

Now. we can find the mean and variance

$E(x)=M_1'(x)=(1-ß)\sum_{i=0}^{\infty}\frac{(-1)^i}{i!}\left[\left(\frac{2}{\omega}\right)^{\frac{1+i\vartheta}{2}}\Gamma\left(\frac{1+i\vartheta}{2}+1\right)+\frac{\vartheta}{\omega}\left(\frac{2}{\omega}\right)^{\frac{1+\vartheta(i+1)-2}{2}}\Gamma\left(\frac{1+\vartheta(i+1)}{2}\right)\right]+$

$$ß\sum_{j=0}^{\infty}\frac{(-1)^j2^j}{j!}\left[\left(\frac{1}{\omega}\right)^{\frac{1+j\vartheta}{2}}\Gamma\left(\frac{1+j\vartheta}{2}+1\right)+\vartheta\left(\frac{1}{\omega}\right)^{\frac{1+\vartheta(j+1)}{2}}\Gamma\left(\frac{1+\vartheta(j+1)}{2}\right)\right]$$

$E(x^2)=M_2'(x)=(1-ß)\sum_{i=0}^{\infty}\frac{(-1)^i}{i!}\left[\left(\frac{2}{\omega}\right)^{\frac{2+i\vartheta}{2}}\Gamma\left(\frac{r+i\vartheta}{2}+1\right)+\frac{\vartheta}{\omega}\left(\frac{2}{\omega}\right)^{\frac{2+\vartheta(i+1)-2}{2}}\Gamma\left(\frac{2+\vartheta(i+1)}{2}\right)\right]+$

$$ß\sum_{j=0}^{\infty}\frac{(-1)^j2^j}{j!}\left[\left(\frac{1}{\omega}\right)^{\frac{2+j\vartheta}{2}}\Gamma\left(\frac{2+j\vartheta}{2}+1\right)+\vartheta\left(\frac{1}{\omega}\right)^{\frac{2+\vartheta(j+1)}{2}}\Gamma\left(\frac{2+\vartheta(j+1)}{2}\right)\right]$$

$var(x)=M_2'(x)-\left(M_1'(x)\right)^2$

3.3. Coefficients of Skewness and Kurtosis

Relying on the moment, the coefficients of skewness $(C.S)$ and kurtosis$(C.K)$ can be found by the following:

$$M_3'(x) = (1- ß)\sum_{i=0}^{\infty}\frac{(-1)^i}{i!}\left[\left(\frac{2}{\omega}\right)^{\frac{3+i\vartheta}{2}}\Gamma\left(\frac{3+i\vartheta}{2}+1\right)+\frac{\vartheta}{\omega}\left(\frac{2}{\omega}\right)^{\frac{3+\vartheta(i+1)-2}{2}}\Gamma\left(\frac{3+\vartheta(i+1)}{2}\right)\right]+$$

$$ß\sum_{j=0}^{\infty}\frac{(-1)^j 2^j}{j!}\left[\left(\frac{1}{\omega}\right)^{\frac{3+j\vartheta}{2}}\Gamma\left(\frac{3+j\vartheta}{2}+1\right)+\vartheta\left(\frac{1}{\omega}\right)^{\frac{3+\vartheta(j+1)}{2}}\Gamma\left(\frac{3+\vartheta(j+1)}{2}\right)\right]$$

$$\mathrm{C.S} = \frac{\mathrm{E}(x^3)}{(\mathrm{E}(x^2))^{\frac{3}{2}}}$$

$$M_4'(x) = (1- ß)\sum_{i=0}^{\infty}\frac{(-1)^i}{i!}\left[\left(\frac{2}{\omega}\right)^{\frac{4+i\vartheta}{2}}\Gamma\left(\frac{4+i\vartheta}{2}+1\right)+\frac{\vartheta}{\omega}\left(\frac{2}{\omega}\right)^{\frac{4+\vartheta(i+1)-2}{2}}\Gamma\left(\frac{4+\vartheta(i+1)}{2}\right)\right]+$$

$$ß\sum_{j=0}^{\infty}\frac{(-1)^j 2^j}{j!}\left[\left(\frac{1}{\omega}\right)^{\frac{4+j\vartheta}{2}}\Gamma\left(\frac{4+j\vartheta}{2}+1\right)+\vartheta\left(\frac{1}{\omega}\right)^{\frac{4+\vartheta(j+1)}{2}}\Gamma\left(\frac{4+\vartheta(j+1)}{2}\right)\right]$$

$$\mathrm{C.K} = \frac{\mathrm{M}_4'(\mathrm{x})}{(\mathrm{M}_2'(\mathrm{x}))^2} - 3$$

Table 1. The first – fourth moments, kurtosis, skewness, and variance, for a new Transmuted Rayleigh -Weibull distribution

ω	**ϑ**	**ß**	μ_1'	μ_2'	μ_3'	μ_4'	**K**	**S**	**var**
0.5	1.5	0.2	0.4566	0.4470	0.5783	0.8893	1.4510	1.9351	0.2385
	1.5	-0.2	0.5660	0.5984	0.8068	1.2721	0.5524	1.7429	0.2780
0.3	2	0.02	0.4105	0.3928	0.4780	0.6770	1.3883	1.9419	0.2243
	2	-0.02	0.4205	0.4048	0.4944	0.7016	1.2809	1.9192	0.2280
0.1	1.3	0.2	0.3742	0.4787	0.7540	1.3653	2.9570	2.2762	0.3387
	1.3	-0.2	0.4959	0.6614	1.0688	1.9691	1.5011	1.9870	0.4155

For the estimated data, the skewness coefficient is positive because the distribution, as shown in the diagram, is skewed to the right. The values of the specified features determine the flatness. For numbers greater than three, the distribution is flattened; for values less than three, it is flattened. The distribution flattens for values less than three.

3.4.Mean Time to Failure

We can find this property by: MTTF=$\int_0^{\infty} s(x)dx$=$\int_0^{\infty}\left(ß-1-ße^{-\left(\frac{\omega}{2}x^2+x^{\vartheta}\right)}\right)e^{-\left(\frac{\omega}{2}x^2+x^{\vartheta}\right)}dx$

=$(ß-1)\int_0^{\infty} e^{-\left(\frac{\omega}{2}x^2+x^{\vartheta}\right)}dx - ß\int_0^{\infty} e^{-(\omega x^2+2x^{\vartheta})}dx$

Based on **Equations 5, 6**, then

$$\text{MTTF}=(ß-1)\sum_{l=0}^{\infty}\frac{(-1)^l}{l!}\int_0^{\infty} x^{l\vartheta}e^{-\frac{\omega}{2}x^2}dx - ß\sum_{j=0}^{\infty}\frac{(-1)^j}{j!}2^j\int_0^{\infty} x^{j\vartheta}e^{-\omega x^2}dx$$

Using **Hypothesis 7** and **8**, we get the following:

$$\text{MTTF}=(ß-1)\sum_{l=0}^{\infty}\frac{(-1)^l}{l!}\int_0^{\infty}\left(\frac{2}{\omega}y_1\right)^{\frac{l\vartheta}{2}}e^{-y_1}\left(\frac{2}{\omega}y_1\right)^{-\frac{1}{2}}\frac{1}{\omega}dy_1$$

$$-ß\sum_{j=0}^{\infty}\frac{(-1)^j}{j!}2^j\int_0^{\infty}\left(\frac{1}{\omega}y_2\right)^{\frac{j\vartheta}{2}}e^{-y_2}\frac{1}{2}\left(\frac{1}{\omega}y_2\right)^{-\frac{1}{2}}\frac{1}{\omega}dy_2$$

$$=(ß-1)\sum_{l=0}^{\infty}\frac{(-1)^l}{l!}\left(\frac{2}{\omega}\right)^{\frac{l\vartheta-1}{2}}\frac{1}{\omega}\int_0^{\infty}(y_1)^{\frac{l\vartheta-1}{2}}e^{-y_1}dy_1 - ß\sum_{j=0}^{\infty}\frac{(-1)^j}{j!}2^{j-1}\left(\frac{1}{\omega}\right)^{\frac{j\vartheta}{2}}\int_0^{\infty}(y_2)^{\frac{j\vartheta-1}{2}}e^{-y_2}dy_2$$

since $\Gamma(z) = \int_0^\infty x^{z-1} e^{-x}\, dx$

MTTF=$(ß-1)\sum_{l=0}^{\infty} \frac{(-1)^l}{l!} (\frac{2}{\omega})^{\frac{l\vartheta-1}{2}} \frac{1}{\omega}\, \Gamma\left(\frac{l\vartheta+1}{2}\right) - ß \sum_{j=0}^{\infty} \frac{(-1)^j}{j!} 2^{j-1} (\frac{1}{\omega})^{\frac{j\vartheta}{2}} \Gamma\left(\frac{j\vartheta+1}{2}\right)$

3.5.Characteristic function

$$\phi_x(it) = E\left(e^{itx}\right) = \int_0^\infty e^{itx}\ z(x)\, dx$$

$$= \int_0^\infty e^{itx}\, (\omega x + \vartheta x^{\vartheta-1}) e^{-\left(\frac{\omega}{2}x^2 + x^\vartheta\right)} \left(1 - ß + 2\, ße^{-\left(\frac{\omega}{2}x^2 + x^\vartheta\right)}\right) dx$$

$$=(1-ß) \int_0^\infty (\omega x + \vartheta x^{\vartheta-1}) e^{-\left(\frac{\omega}{2}x^2 + x^\vartheta - itx\right)} dx + 2ß \int_0^\infty (\omega x + \vartheta x^{\vartheta-1}) e^{-2\left(\frac{\omega}{2}x^2 + x^\vartheta - itx\right)}\, dx$$

By using the expansion of z(x) in **Equations 5, 6** and $e^{itx} = \sum_{r=0}^{\infty} \frac{(it)^r}{r!} x^r$ then

$$\phi_x(it) = \sum_{l=r=0}^{\infty} \frac{(-1)^l}{l!r!} (it)^r \left[(1-ß) \int_0^\infty (\omega x + \vartheta x^{\vartheta-1}) x^{l\vartheta+r} e^{-\left(\frac{\omega}{2}x^2\right)} dx\right] +$$
$$ß \sum_{j=r=0}^{\infty} \frac{(-1)^j}{j!r!} (it)^r 2^{j+1} \left[\int_0^\infty (\omega x + \vartheta x^{\vartheta-1}) x^{j\vartheta+r} e^{-2\left(\frac{\omega}{2}x^2\right)}\, dx\right]$$

$$= \sum_{l=r=0}^{\infty} \frac{(-1)^l}{l!\, r!} (it)^r \left[(1-ß) \int_0^\infty (\omega x^{l\vartheta+r+1} + \vartheta x^{l\vartheta+r+\vartheta-1}) e^{-\frac{\omega}{2}x^2}\, dx\right]$$
$$+ ß \sum_{j=r=0}^{\infty} \frac{(-1)^j}{j!\, r!} (it)^r 2^{j+1} \left[\int_0^\infty (\omega x^{j\vartheta+r+1} + \vartheta x^{j\vartheta+r+\vartheta-1}) e^{-\omega x^2}\, dx\right]$$

Let $\sum_{l=r=0}^{\infty} \frac{(-1)^l}{l!r!} (it)^r = \Psi_1$, and $ß \sum_{j=r=0}^{\infty} \frac{(-1)^j}{j!r!} (it)^r 2^{j+1} = \Psi_2$

$$\phi_x(it) = \Psi_1 \left[(1-ß)\left(\omega \int_0^\infty x^{l\vartheta+r+1} e^{-\frac{\omega}{2}x^2} dx + \vartheta \int_0^\infty x^{l\vartheta+r+\vartheta-1} e^{-\frac{\omega}{2}x^2} dx\right)\right]$$
$$+ \Psi_2 \left[\omega \int_0^\infty x^{j\vartheta+r+1} e^{-\omega x^2} dx + \vartheta \int_0^\infty x^{j\vartheta+r+\vartheta-1} e^{-\omega x^2} dx\right]$$

Now use **Equations 7 and 8**

$$\phi_x(it) = \Psi_1 \left[(1-ß)\left(\omega \int_0^\infty \left(\frac{2y_1}{\omega}\right)^{\frac{l\vartheta+r+1}{2}} e^{-y_1} \left(\frac{2y_1}{\omega}\right)^{-\frac{1}{2}} \frac{1}{\omega} dy_1 + \vartheta \int_0^\infty \left(\frac{2y_1}{\omega}\right)^{\frac{l\vartheta+r+\vartheta-1}{2}} e^{-y_1} dy_1\right)\right]$$
$$+\Psi_2 \left[\left(\omega \int_0^\infty \left(\frac{y_2}{\omega}\right)^{\frac{j\vartheta+r+1}{2}} e^{-y_2} \frac{1}{2} \left(\frac{y_2}{\omega}\right)^{-\frac{1}{2}} \frac{1}{\omega} dy_2\right.\right.$$
$$\left.\left. + \vartheta \int_0^\infty \left(\frac{y_2}{\omega}\right)^{\frac{j\vartheta+r+\vartheta-1}{2}} e^{-y_2} \frac{1}{2} \left(\frac{y_2}{\omega}\right)^{-\frac{1}{2}} \frac{1}{\omega} dy_2\right)\right]$$

$$= \Psi_1 \left[(1-ß)\left(\left(\frac{2}{\omega}\right)^{\frac{l\vartheta+r}{2}} \int_0^\infty (y_1)^{\frac{l\vartheta+r}{2}} e^{-y_1} dy_1 + \vartheta \left(\frac{2}{\omega}\right)^{\frac{l\vartheta+r+\vartheta-2}{2}} \int_0^\infty (y_1)^{\frac{l\vartheta+r+\vartheta-2}{2}} e^{-y_1} dy_1\right)\right]$$
$$+\Psi_2 \left[\left(\frac{1}{2}\left(\frac{1}{\omega}\right)^{\frac{j\vartheta+r}{2}} \int_0^\infty (y_2)^{\frac{j\vartheta+r}{2}} e^{-y_2} dy_2 + \frac{\vartheta}{2\omega} \left(\frac{1}{\omega}\right)^{\frac{\vartheta+j\vartheta+r-2}{2}} \int_0^\infty (y_2)^{\frac{\vartheta+j\vartheta+r-2}{2}} e^{-y_2} dy_2\right)\right]$$

since $\Gamma(z) = \int_0^\infty x^{z-1} e^{-x}\, dx$

$$\phi_x(it)=\Psi_1\left[(1-ß)\left(\left(\frac{2}{ω}\right)^{\frac{l\vartheta+r}{2}}\Gamma\left(\frac{l\vartheta+r}{2}+1\right)+\vartheta\left(\frac{2}{ω}\right)^{\frac{\vartheta+l\vartheta+r-2}{2}}\Gamma\left(\frac{l\vartheta+r+\vartheta}{2}\right)\right)\right]$$
$$+\Psi_2\left[\left(\frac{1}{2}\left(\frac{1}{ω}\right)^{\frac{j\vartheta+r}{2}}\Gamma\left(\frac{j\vartheta+r}{2}+1\right)+\frac{\vartheta}{2ω}\left(\frac{1}{ω}\right)^{\frac{\vartheta+j\vartheta+r-2}{2}}\Gamma\left(\frac{\vartheta+j\vartheta+r}{2}\right)\right)\right]$$

4.Moment Generating Function

$$M_x(t)=E(e^{tx})=\int_0^\infty e^{tx}\ z(x)\,dx$$
$$=\int_0^\infty e^{tx}\left[(ωx+\vartheta x^{\vartheta-1})e^{-\left(\frac{ω}{2}x^2+x^\vartheta\right)}\left(1-ß+2ße^{-\left(\frac{ω}{2}x^2+x^\vartheta\right)}\right)\right]dx$$
$$=\int_0^\infty\left[(ωx+\vartheta x^{\vartheta-1})e^{-\left(\frac{ω}{2}x^2+x^\vartheta-tx\right)}\left(1-ß+2ße^{-\left(\frac{ω}{2}x^2+x^\vartheta-tx\right)}\right)\right]dx$$
$$=(1-ß)\int_0^\infty(ωx+\vartheta x^{\vartheta-1})e^{-\left(\frac{ω}{2}x^2+x^\vartheta-tx\right)}dx+2ß\int_0^\infty(ωx+\vartheta x^{\vartheta-1})e^{-2\left(\frac{ω}{2}x^2+x^\vartheta-tx\right)}dx$$

By using **Equation 5, 6** and $e^{tx}=\sum_{m=0}^\infty\frac{(t)^m}{m!}x^m$, then

$$M_x(t)=\left[\sum_{l=m=0}^\infty\frac{(-1)^l}{l!m!}(t)^m(1-ß)\int_0^\infty(ωx+\vartheta x^{\vartheta-1})x^{l\vartheta+m}e^{-\left(\frac{ω}{2}x^2\right)}dx+\right.$$
$$\left.2^{j+1}ß\sum_{j=m=0}^\infty\frac{(-1)^j}{j!m!}(t)^m\,(ωx+\vartheta x^{\vartheta-1})x^{j\vartheta+m}e^{-2\left(\frac{ω}{2}x^2\right)}dx\right]$$
$$M_x(t)=\sum_{l=m=0}^\infty\frac{(-1)^l}{l!m!}(t)^m(1-ß)=\Psi_1$$
$$2^{j+1}ß\sum_{j=m=0}^\infty\frac{(-1)^j}{j!\,m!}(t)^m=\Psi_2$$

Then $M_x(t)=\Psi_1\left[\left(ω\int_0^\infty x^{l\vartheta+m+1}e^{-\frac{ω}{2}x^2}dx+\vartheta\int_0^\infty x^{l\vartheta+m+\vartheta-1}e^{-\frac{ω}{2}x^2}dx\right)\right]$

$$+\Psi_2\left[\left(ω\int_0^\infty x^{j\vartheta+m+1}e^{-ωx^2}dx+\vartheta\int_0^\infty x^{j\vartheta+m+\vartheta-1}e^{-ωx^2}dx\right)\right]$$

Depending on the two **Equations 7** and **8**, then

$$M_x(t)=\Psi_1\left[\left(ω\int_0^\infty\left(\frac{2y_1}{ω}\right)^{\frac{l\vartheta+m+1}{2}}e^{-y_1}\left(\frac{2y_1}{ω}\right)^{-\frac{1}{2}}\frac{1}{ω}dy_1+\vartheta\int_0^\infty\left(\frac{2y_1}{ω}\right)^{\frac{l\vartheta+m+\vartheta-1}{2}}e^{-y_1}\left(\frac{2y_1}{ω}\right)^{-\frac{1}{2}}\frac{1}{ω}dy_1\right)\right]$$
$$+\Psi_2\left[\left(ω\int_0^\infty\left(\frac{y_2}{ω}\right)^{\frac{j\vartheta+m+1}{2}}e^{-y_2}\frac{1}{2}\left(\frac{y_2}{ω}\right)^{-\frac{1}{2}}\frac{1}{ω}dy_2\right.\right.$$
$$\left.\left.+\vartheta\int_0^\infty\left(\frac{y_2}{ω}\right)^{\frac{j\vartheta+m+\vartheta-1}{2}}e^{-y_2}\frac{1}{2}\left(\frac{y_2}{ω}\right)^{-\frac{1}{2}}\frac{1}{ω}dy_2\right)\right]$$
$$=\Psi_1\left[\left(\left(\frac{2}{ω}\right)^{\frac{l\vartheta+m}{2}}\int_0^\infty(y_1)^{\frac{l\vartheta+m}{2}}e^{-y_1}dy_1+\vartheta\left(\frac{2}{ω}\right)^{\frac{l\vartheta+m+\vartheta-2}{2}}\int_0^\infty(y_1)^{\frac{l\vartheta+m+\vartheta-2}{2}}e^{-y_1}dy_1\right)\right]$$
$$+\Psi_2\left[\left(\frac{1}{2}\left(\frac{1}{ω}\right)^{\frac{j\vartheta+m}{2}}\int_0^\infty(y_2)^{\frac{j\vartheta+m}{2}}e^{-y_2}dy_2+\frac{\vartheta}{2ω}\left(\frac{1}{ω}\right)^{\frac{\vartheta+j\vartheta+m-2}{2}}\int_0^\infty(y_2)^{\frac{\vartheta+j\vartheta+m-2}{2}}e^{-y_2}dy_2\right)\right]$$

since $\Gamma(z)=\int_0^\infty x^{z-1}e^{-x}\,dx$

$$M_x(t)=\Psi_1\left[\left(\left(\frac{2}{\omega}\right)^{\frac{l\vartheta+m}{2}}\ \Gamma\left(\frac{l\vartheta+m}{2}+1\right)+\vartheta\left(\frac{2}{\omega}\right)^{\frac{\vartheta+l\vartheta+m-2}{2}}\ \Gamma\left(\frac{l\vartheta+m+\vartheta}{2}\right)\right)\right]$$
$$+\Psi_2\left[\left(\frac{1}{2}\left(\frac{1}{\omega}\right)^{\frac{j\vartheta+m}{2}}\ \Gamma\left(\frac{j\vartheta+m}{2}+1\right)+\frac{\vartheta}{2\omega}\left(\frac{1}{\omega}\right)^{\frac{\vartheta+j\vartheta+m-2}{2}}\ \Gamma\left(\frac{\vartheta+j\vartheta+m}{2}\right)\right)\right]$$

4.1.Factorial Moments Generating Function

$$\mathcal{M}_x(t)=E(e^{tx})=\int_0^\infty t^x\ z(x)\,dx=\int_0^\infty e^{x\ln t}\,z(x)\,dx$$

$$\mathcal{M}_x(t)=\int_0^\infty e^{x\ln t}\left[(\omega x+\vartheta x^{\vartheta-1})e^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}\left(1-\ ß+2\,ß e^{-\left(\frac{\omega}{2}x^2+x^\vartheta\right)}\right)\right]dx$$
$$=\int_0^\infty\left[(\omega x+\vartheta x^{\vartheta-1})e^{e^{-\left(\frac{\omega}{2}x^2+x^\vartheta-x\ln t\right)}}\left(1-\ ß+2\,ß e^{-2\left(\frac{\omega}{2}x^2+x^\vartheta-x\ln t\right)}\right)\right]dx$$
$$=(1-\ ß)\int_0^\infty(\omega x+\vartheta x^{\vartheta-1})e^{-\left(\frac{\omega}{2}x^2+x^\vartheta-x\ln t\right)}dx+2ß\int_0^\infty(\omega x+\vartheta x^{\vartheta-1})e^{-2\left(\frac{\omega}{2}x^2+x^\vartheta-x\ln t\right)}\,dx$$

By using the **Equations 5, 6** and $e^{x\ln t}=\sum_{m=0}^{\infty}\frac{(\ln t)^m}{m!}x^m$

$$M_x(t)=\Big[\sum_{l=m=0}^{\infty}\frac{(-1)^l}{l!m!}(\ln t)^m(1-\ ß)\int_0^\infty(\omega x+\vartheta x^{\vartheta-1})x^{l\vartheta+m}e^{-\left(\frac{\omega}{2}x^2\right)}dx+$$
$$2^{j+1}ß\sum_{j=m=0}^{\infty}\frac{(-1)^j}{j!m!}(\ln t)^m\ (\omega x+\vartheta x^{\vartheta-1})x^{j\vartheta+m}e^{-2\left(\frac{\omega}{2}x^2\right)}dx\Big]$$

Let $\sum_{l=m=0}^{\infty}\frac{(-1)^l}{l!m!}(\ln t)^m(1-\ ß)=\Psi_{l,m}$, and $2^{j+1}ß\sum_{j=m=0}^{\infty}\frac{(-1)^j}{j!m!}(\ln t)^m=\Psi_{j,m}$

$$M_x(t)=\Psi_{l,m}\left[\left(\omega\int_0^\infty x^{l\vartheta+m+1}e^{-\frac{\omega}{2}x^2}dx+\vartheta\int_0^\infty x^{l\vartheta+m+\vartheta-1}e^{-\frac{\omega}{2}x^2}dx\right)\right]$$
$$+\Psi_{j,m}\left[\left(\omega\int_0^\infty x^{j\vartheta+m+1}e^{-\omega x^2}dx+\vartheta\int_0^\infty x^{j\vartheta+m+\vartheta-1}e^{-\omega x^2}dx\right)\right]$$

Depending on the **Hypotheses 7 and 8**

$$M_x(t)=\Psi_{l,m}\left[\left(\omega\int_0^\infty\left(\frac{2y_1}{\omega}\right)^{\frac{l\vartheta+m+1}{2}}e^{-y_1}\left(\frac{2y_1}{\omega}\right)^{-\frac{1}{2}}\frac{1}{\omega}dy_1+\vartheta\int_0^\infty\left(\frac{2y_1}{\omega}\right)^{\frac{l\vartheta+m+\vartheta-1}{2}}e^{-y_1}\left(\frac{2y_1}{\omega}\right)^{-\frac{1}{2}}\frac{1}{\omega}dy_1\right)\right]$$
$$+\Psi_{j,m}\left[\left(\omega\int_0^\infty\left(\frac{y_2}{\omega}\right)^{\frac{j\vartheta+m+1}{2}}e^{-y_2}\frac{1}{2}\left(\frac{y_2}{\omega}\right)^{-\frac{1}{2}}\frac{1}{\omega}dy_2\right.\right.$$
$$\left.\left.+\vartheta\int_0^\infty\left(\frac{y_2}{\omega}\right)^{\frac{j\vartheta+m+\vartheta-1}{2}}e^{-y_2}\frac{1}{2}\left(\frac{y_2}{\omega}\right)^{-\frac{1}{2}}\frac{1}{\omega}dy_2\right)\right]$$

$$=\Psi_{l,m}\left[\left(\left(\frac{2}{\omega}\right)^{\frac{l\vartheta+m}{2}}\int_0^\infty(y_1)^{\frac{l\vartheta+m}{2}}e^{-y_1}dy_1+\vartheta\left(\frac{2}{\omega}\right)^{\frac{l\vartheta+m+\vartheta-2}{2}}\int_0^\infty(y_1)^{\frac{l\vartheta+m+\vartheta-2}{2}}e^{-y_1}dy_1\right)\right]$$
$$+\Psi_{j,m}\left[\left(\frac{1}{2}\left(\frac{1}{\omega}\right)^{\frac{j\vartheta+m}{2}}\int_0^\infty(y_2)^{\frac{j\vartheta+m}{2}}e^{-y_2}dy_2+\frac{\vartheta}{2\omega}\left(\frac{1}{\omega}\right)^{\frac{\vartheta+j\vartheta+m-2}{2}}\int_0^\infty(y_2)^{\frac{\vartheta+j\vartheta+m-2}{2}}e^{-y_2}dy_2\right)\right]$$

since $\Gamma(z)=\int_0^\infty x^{z-1}e^{-x}\,dx$

$$M_x(t)=\Psi_{l,m}\left[\left(\left(\frac{2}{\omega}\right)^{\frac{l\vartheta+m}{2}}\Gamma\left(\frac{l\vartheta+m}{2}+1\right)+\vartheta\left(\frac{2}{\omega}\right)^{\frac{\vartheta+l\vartheta+m-2}{2}}\Gamma\left(\frac{l\vartheta+m+\vartheta}{2}\right)\right)\right]$$

$$+\Psi_{j,m}\left[\left(\frac{1}{2}\left(\frac{1}{\omega}\right)^{\frac{j\vartheta+m}{2}}\Gamma\left(\frac{j\vartheta+m}{2}+1\right)+\frac{\vartheta}{2\omega}\left(\frac{1}{\omega}\right)^{\frac{\vartheta+j\vartheta+m-2}{2}}\Gamma\left(\frac{\vartheta+j\vartheta+m}{2}\right)\right)\right]$$

4.2.Quantile Function

This function is defined by: $Z(x)=u \quad ; u\sim u(0,1) ; x=Z^{-1}(u)$

$$u=\left(1+ße^{-\left(\frac{\omega}{2}x^2+x^{\vartheta}\right)}\right)\left(1-e^{-\left(\frac{\omega}{2}x^2+x^{\vartheta}\right)}\right)$$

$let\ q=e^{-\left(\frac{\omega}{2}x^2+x^{\vartheta}\right)}$, $u=(1+ßq)(1-q)$

$u=1-q+ßq-ßq^2$, $u=1-(1-ß)q-ßq^2$, $ßq^2+(1-ß)q+(u-1)=0$

$$q=\frac{(ß-1)\pm\sqrt{(1-ß)^2-4ß(u-1)}}{2ß}$$

$$e^{-\left(\frac{\omega}{2}x^2+x^{\vartheta}\right)}=\frac{(ß-1)\pm\sqrt{(1-ß)^2-4ß(u-1)}}{2ß}$$

$$\frac{\omega}{2}x^2+x^{\vartheta}+ln\left(\frac{(ß-1)\pm\sqrt{(1-ß)^2-4ß(u-1)}}{2ß}\right)=0$$

If $\vartheta=1$

$$x=\frac{-1\pm\sqrt{1-2ßln\left(\frac{(ß-1)\pm\sqrt{(1-ß)^2-4ß(u-1)}}{2ß}\right)}}{ß}$$

If $\vartheta=2$

$$\frac{\omega}{2}x^2+x^2+ln\left(\frac{(ß-1)\pm\sqrt{(1-ß)^2-4ß(u-1)}}{2ß}\right)=0$$

$$\left(\frac{\omega}{2}+1\right)x^2+ln\left(\frac{(ß-1)\pm\sqrt{(1-ß)^2-4ß(u-1)}}{2ß}\right)=0$$

$$x^2=\frac{-ln\left(\frac{(ß-1)\pm\sqrt{(1-ß)^2-4ß(u-1)}}{2ß}\right)}{\left(\frac{\omega}{2}+1\right)}$$

$$x=\pm\sqrt{\frac{-ln\left(\frac{(ß-1)\pm\sqrt{(1-ß)^2-4ß(u-1)}}{2ß}\right)}{\left(\frac{\omega}{2}+1\right)}}$$

Since x>0, we ignore the negative value of x . When $u=\frac{1}{2}$, we get the Median

$$Median=\sqrt{\frac{-ln\left(\frac{(ß-1)\pm\sqrt{(1-ß)^2+2ß}}{2ß}\right)}{\left(\frac{\omega}{2}+1\right)}} \qquad (9)$$

5.Conclusion

In this article, a new distribution was introduced based on a transformation rule, and it is built depending on the Rayleigh-Weibull distribution by adding a new transformation parameter ß, |ß| ≤1. This modification resulted in a more flexible distribution for handling data, which is named the new transmuted Rayleigh-Weibull distribution. The shapes of some statistical functions were discussed, and some important statistical properties are proffered, including the kurtosis, skewness, variance, and moments for different parameters. In the near future, a practical application will be made for the new distribution, and the parameters of it will also be estimated.

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