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Solving the Bessel's Functions using the Sadik Transform

Wisam Jalil Kareem
wiisamtweej@gmail.com

Abstract: In this study, we discover that the sadik transform of Bessel functions is crucial for resolving numerous issues and equations. The goal of this study is to use the Sadik integral transform to solve Bessel functions, and I will use it to solve Bessel relations. Heat equations, wave equations, Laplace equations, and . equations are just a few of the equations that can be solved in cylindrical or spherical coordinates with then by of Bessel function.

Keywords: Sadik Transform, Bessel function, Inverse Sadik Transform.

حل دوال بيسل باستعمال تحويلساديك التكاملي

م.م وسام جليل كريم
wiisamtweej@gmail.com
المديرية العامة لتربية النجف الاشرف

ملخص البحث :
في هذه الدراسة، نُبيّن أن لتحويل ساديك لدوال بيسل أهميةً كبيرة في معالجة العديد من المسائل والمعادلات الرياضية. ويهدف هذا البحث إلى توظيف تحويل ساديك التكاملي في حل دوال بيسل، بالإضافة إلى استخدامه في إيجاد حلول لعلاقات بيسل المختلفة. كما تُعد دوال بيسل من الأدوات الرياضية المهمة في حل العديد من المعادلات التفاضلية الجزئية، مثل معادلات الحرارة، والمعادلات الموجية، ومعادلات لابلاس، وغيرها من المعادلات التي تُصاغ بالإحداثيات الأسطوانية أو الكروية.
الكلمات المفتاحية: تحويل ساديك، دوال بيسل، معكوس تحويل ساديك

1.Introduction:

Bessel function are now essential for solving problems in the fields of engineering, mathematical physics, atomic physics, acoustics, radio physics, nuclear physics, fluid mechanics, heat transfer, vibrations, hydrodynamics, stress analysis, and nuclear reactor flux distribution. The nonnegative integer is represented by the Bessel function of order [2, 3, 8].It is also clear how crucial integral transformations are for precisely solving differential equations. Numerous integral transformations have been proposed by researchers, each of which addresses a problem in life sometimes social and economic, sometimes engineering, physical, medical, or astronomical with the goal of finding solutions using less complicated, more technical, and simpler approaches. It is thought that the authentic transformation is easier to use and more universal. [10,11, 12,13, 14].

$$J_{n(t)}=\frac{t^n}{2^{n!}}\left\{1-\frac{t^n}{2(2n+2)}+\frac{t^n}{2.4(2n+2)(2n+4)}-\frac{t^n}{2.4.6(2n+2)(2n+4)(2n+6)}+\cdots\right\}.(1)$$

For $n=0$,The zero order Bessel function is represented by. $J_{0(t)}$ The following infinite power series is used to define.

$$J_{0(t)}=\left\{1-\frac{t^2}{2^2}+\frac{t^4}{2^2 4^2}-\frac{t^6}{2^2 4^2 6^2}+\cdots\right\} \quad ...(2)$$

For $n=1$, The following infinite power series defines Bessel function of order one, symbolizes by$J_{1(t)}$.

$$J_{1(t)}=\left\{\frac{t}{2}-\frac{t^3}{2^3.4}+\frac{t^5}{2^2 4^2.6}-\cdots\right\} \quad ...(3)$$

For $n = 2$, The following infinite power series defines Bessel function of order one, symbolizes by $J_{2(t)}$.

$$J_{2(t)}=\left\{\frac{t^2}{2.4}-\frac{t^4}{2^2.4.6}+\frac{t^6}{2^2 4^2.6.8}-\cdots\right\} \quad ...(4)$$

2. SOME USEFUL PROPERTIES OF Sadik TRANSFORM:

Sadik Transform (S – Transform)

Integral transforms methods (L - Transform, Mo - Transform, M - Transform, S - Transform ,……… etc) are practical mathematical tools that can be used to solve complex science and engineering problems that can be expressed in terms of integral and differential equations.

Definition: [4,9] The Sadik Transform of the function $f(\gamma)$, $\gamma \geq 0$ can be is defined

$$S\{f(\gamma)\} = \frac{1}{v^{\beta}} \int_0^{\infty} f(\gamma) e^{-\gamma v^{\alpha}}\, d\gamma,$$

when v is a complex variable and $\alpha \neq 0$, β are any actual numbers. The Sadik ransform operator is denoted by the letter S.

It should be noted that the Sadik transform of the function $f(\gamma)$ for $\gamma \geq 0$ exists if $f(\gamma)$is piecewise continuous and of exponential order; these are the only necessary conditions for the existence of Sadik transforms of the function $f(\gamma)$.

Convolution Theorem the *Sadik* Transform

To solve integral equation in any transform we should use the convolution theorem it as we have observed in solving the integral equations by previous transform. Let the sadik Transform of the functions $f_1(\gamma)$ and $f_2(\gamma)$ is $S\{f_1(\gamma)\}$ and $S\{f_2(\gamma)\}$ respectively, then:

$$S\{f_1(\gamma) * f_2(\gamma)\} = v^{\beta}\, S\{f_1(x)\} \cdot S\{f_2(\gamma)\} \text{ [6] .}$$

Inverse of S – Transform [4]

If $S\{f(\gamma)\}$ is the Sadik Transforms of the mfunction $f(\gamma)$, the $f(\gamma)$ is known as the inverse of the Sadik Transform and has the following mathematical definition.

$$f(\gamma) = S^{-1}\{S\{f(\gamma)\}\},$$

Such that operator S^{-1} is called the inverse S - Transform operator and for sure it have that linearity property: $S^{-1}\left\{a\, S\{f_1(\gamma)\} + b\, S\{f_2(\gamma)\}\right\}$

$$= a\, S^{-1}\{S\{f_1(\gamma)\}\} + b\, S^{-1}\{S\{f_2(\gamma)\}\}$$

$$= a\, f_1(\gamma) + b\, f_2(\gamma).$$

Table : (*S*adik –Transform)for some elementary functions

S.N	ʄ (ɤ)	Sʄ (ɤ)
1	1	$\frac{1}{v^{\alpha+\beta}}$
2	ɤ	$\frac{1}{v^{2\alpha+\beta}}$
3	$ɤ^2$	$\frac{2!}{v^{3\alpha+\beta}}$
4	$ɤ^n, n \in N$	$\frac{n!}{v^{(n+1)\alpha+\beta}}$
5	$e^{aɤ}$	$\frac{1}{v^{\beta}(v^{\alpha}-a)}$
6	$\sin aɤ$	$\frac{a}{v^{\beta}(v^{2\alpha}+a^2)}$
7	$\cos aɤ$	$\frac{v^{\alpha}}{v^{\beta}(v^{2\alpha}+a^2)}$
8	$\sinh aɤ$	$\frac{a}{v^{\beta}(v^{2\alpha}-a^2)}$
9	$\cosh aɤ$	$\frac{v^{\alpha}}{v^{\beta}(v^{2\alpha}-a^2)}$

2.Dualities between Transforms

We provide examples using φ as a symbol for the other transform result to demonstrate the importance of these connections for the integral transforms that are discussed. This section will discuss the dualities between the L-Transform and several well-known transforms, such as the M-Transform, S-Transform, Mo-Transform, and γ-Transform, as well as between.

1- *Mahgoub – Sadik Duality*

*M*ahgoubTransform of then is function $ʄ(ɤ)$, since $ɤ \geq 0$ such that define as:

$$M\{ʄ(ɤ)\} = v \int_0^{\infty} ʄ(ɤ)\, e^{-vɤ}\, dɤ ,$$

and *S*adik Transforms of then function $ʄ(ɤ)$, since $ɤ \geq 0$ symbolizes by $S\{ʄ(ɤ)\}$ and can be defined as:

$$S\{ʄ(ɤ)\} = \frac{1}{v^{\beta}} \int_0^{\infty} ʄ(ɤ)\, e^{-ɤv^{\alpha}}\, dɤ ,$$

Now, let $v = u^{\alpha}$ then

$$M\{ʄ(ɤ)\} = v \,.\int_0^{\infty} ʄ(ɤ)\,.e^{-u^{\alpha}ɤ}\, dɤ = v^{\beta+1}\left[\frac{1}{v^{\beta}} \int_0^{\infty} ʄ(ɤ)\, ɤ\, dɤ\right]$$

$$= v^{\beta+1}\,.\varphi(u^{\alpha},\beta)\,.$$

On the other hand: Let $r = v^{\alpha}$, then

$$S\{f(\gamma)\} = \frac{1}{v^\beta}\int_0^\infty f(\gamma)\, e^{-\gamma v^\alpha}\, d\gamma = \frac{1}{v^\beta\, v}\{v\int_0^\infty f(\gamma)\, e^{-xr}\, d\gamma\}$$

$$= \frac{1}{v^\beta\, v}\{\varphi(r)\}$$

Thus $S\{f(\gamma)\} = \frac{1}{v^\beta\, v}\{\varphi(r)\}$.

2- Sadik – Mohand Duality

Sadik Transforms of then function $f(\gamma)$, since $x \geq 0$ denoted by $S\{f(\gamma)\}$ and can be they know:

$$S\{f(\gamma)\} = \frac{1}{v^\beta}\int_0^\infty f(\gamma) e^{-\gamma v^\alpha} d\gamma,$$

With mohand Transform:

$$Mo\{f(\gamma)\} = v^2\int_0^\infty f(\gamma)\, e^{-v\gamma}\, d\gamma = \varphi(v).$$

Now let $v^\alpha = u$, then we have:

$$S\{f(\gamma)\} = \frac{1}{v^\beta}\int_0^\infty f(\gamma) e^{-\gamma v^\alpha} d\gamma = \frac{1}{v^\beta\ v^2}\{v^2\int_0^\infty f(\gamma)\,.\,e^{-u\gamma}\, d\gamma\}$$

$$= \frac{1}{v^\beta\ v^2}\,.\,\varphi(u)$$

Thus

$S\{f(x)\} = \frac{1}{v^\beta v^2}\ .\,\varphi(u)$. On the other hand:

Let $v = r^\alpha$, then:

$$Mo\{f(\gamma)\} = v^2\int_0^\infty f(\gamma)\, e^{-v\gamma}\, d\gamma = v^2\, v^\beta\left\{\frac{1}{v^\beta}\int_0^\infty f(\gamma)\, e^{-\gamma r^\alpha}\ d\gamma\right\}$$

$$= v^2 v^\beta\, \varphi(r^\alpha, \beta)$$

Thus

$$Mo\{f(\gamma)\} = v^2 v^\beta\, \varphi(r^\alpha, \beta)\,.$$

Example 1 Solve the following integral equation using sadik integral transforms:

$$u\big((x)\big) = \cos x + \sin x - \int_0^x u(t)\, dt$$

Solution:

since $S\{u(x)\} = \frac{1}{v^\beta} \cdot \{\varphi(v)\}$

$S\{u(x)\} = \frac{1}{v^\beta} \cdot \frac{v}{1+v^2}$

now let $v = r^\alpha$

$S\{u(x)\} = \frac{1}{v^\beta} \cdot \frac{r^\alpha}{1+r^{2\alpha}}$

$S\{u(x)\} = \frac{1}{(r^\alpha)^\beta} \cdot \frac{r^\alpha}{1+r^{2\alpha}}$

$\therefore u(x) = S^{-1}\left\{\frac{1}{(r^\alpha)^\beta} \cdot \frac{r^\alpha}{1+r^{2\alpha}}\right\}$

$u(x) = \cos x$

since $S\{u(x)\} = \frac{1}{v^\beta} \cdot \{\varphi(v)\}$

$S\{u(x)\} = \frac{1}{v^\beta} \cdot \frac{v}{1+v^2}$

now let $v = r^\alpha$

$S\{u(x)\} = \frac{1}{v^\beta} \cdot \frac{r^\alpha}{1+r^{2\alpha}}$

$S\{u(x)\} = \frac{1}{(r^\alpha)^\beta} \cdot \frac{r^\alpha}{1+r^{2\alpha}}$

$\therefore u(x) = S^{-1}\left\{\frac{1}{(r^\alpha)^\beta} \cdot \frac{r^\alpha}{1+r^{2\alpha}}\right\}$

$u(x) = \cos x$

since $S\{u(x)\} = \frac{1}{v^\beta} \cdot \{\varphi(v)\}$

$S\{u(x)\} = \frac{1}{v^\beta} \cdot \frac{v}{1+v^2}$

now let $v = r^\alpha$

$S\{u(x)\} = \frac{1}{v^\beta} \cdot \frac{r^\alpha}{1+r^{2\alpha}}$

$S\{u(x)\} = \frac{1}{(r^\alpha)^\beta} \cdot \frac{r^\alpha}{1+r^{2\alpha}}$

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$$\therefore u(x) = S^{-1}\left\{\frac{1}{(r^{\alpha})^{\beta}} \cdot \frac{r^{\alpha}}{1+r^{2\alpha}}\right\}$$

$$u(x) = \cos x$$

since $S\{u(x)\} = \frac{1}{v^{\beta}} \cdot \{\varphi(v)\}$

$$S\{u(x)\} = \frac{1}{v^{\beta}} \cdot \frac{v}{1+v^{2}}$$

now let $v = r^{\alpha}$

$$S\{u(x)\} = \frac{1}{v^{\beta}} \cdot \frac{r^{\alpha}}{1+r^{2\alpha}}$$

$$S\{u(x)\} = \frac{1}{(r^{\alpha})^{\beta}} \cdot \frac{r^{\alpha}}{1+r^{2\alpha}}$$

$$\therefore u(x) = S^{-1}\left\{\frac{1}{(r^{\alpha})^{\beta}} \cdot \frac{r^{\alpha}}{1+r^{2\alpha}}\right\}$$

$$u(x) = \cos x$$

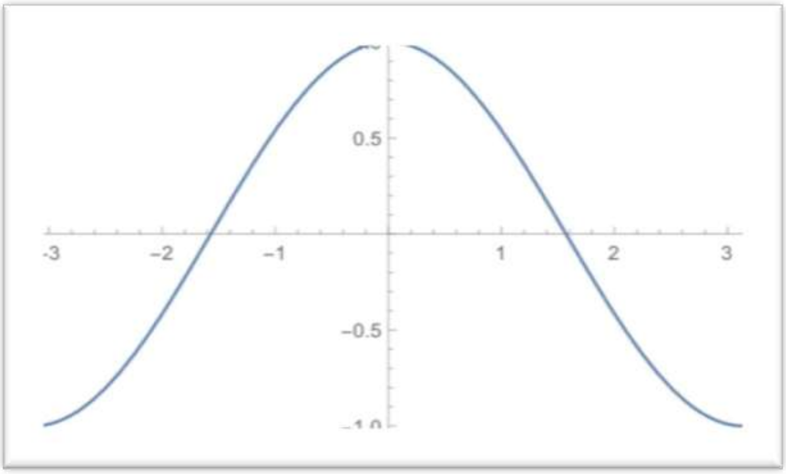

Fig.1: $\mathrm{u}(\mathrm{x}) = \cos x$

3. Relation between ($J_{0(t)}$) and ($J_{1(t)}$)[3,5]:

$$\frac{d}{dt}J_{0(t)} = -J_{1(t)} \qquad \ldots(5)$$

4. Relation between ($J_{0(t)}$) and ($J_{2(t)}$)[5 ,3]:

$J_{2(t)} = J_{0(t)} + 2J_0(t)^{``}$ …(6)

5.**sadik transform to solve Bessel function:**

5.1 sadik transform to solve Bessel function of zero

order : We obtain by applying the sadik transform to both sides of equation (2).

$$J_{0(t)}=\left\{1-\frac{t^2}{2^2}+\frac{t^4}{2^2 4^2}-\cdots\right\}$$

$=\left\{\frac{1}{v^{\alpha+\beta}}-\frac{2!}{2^2 v^{3\alpha+\beta}}+\frac{4!}{2^2 4^2 v^{5\alpha+\beta}}-\cdots\right\}$

$=\left\{\frac{1}{v^{\alpha+\beta}}-\frac{1}{2\,v^{3\alpha+\beta}}+\frac{24}{4.16v^{5\alpha+\beta}}-\cdots\right\}$

$=\frac{1}{v^{\alpha+\beta}}\{1-\frac{1}{2\,v^{2\alpha+\beta}}+\frac{3}{8\,v^{4\alpha+\beta}}\}$ …(7)

5.2 sadik transform to solve Bessel function of order one :

Equation (3) can be solved by applying the Sadik transform to both sides.

$J_{1(t)}=\left\{\frac{t}{2}-\frac{t^3}{2^3.4}+\frac{t^5}{2^2 4^2.6}-\cdots\right\}$

$=\left\{\frac{2}{2v^{\alpha+\beta}}-\frac{3!.}{2^3.4v^{4\alpha+\beta}}+\frac{5!.}{2^2 4^2.6v^{6\alpha+\beta}}-\cdots\right\}$

$=\left\{\frac{1}{v^{\alpha+\beta}}-\frac{3}{16v^{4\alpha+\beta}}+\frac{5}{16v6^{\alpha+\beta}}-\cdots\right\}$ …(8)

5.3sadik transform to solve Bessel's function of order two :

Equation (4) can be solved by applying the Sadik transform to both sides

$$J_{2(t)}=\left\{\frac{t^2}{2.4}-\frac{t^4}{2^2.4.6}+\frac{t^6}{2^2 4^2.6.8}-\cdots\right\}$$

$=\left\{\frac{2!}{8v^{\alpha+\beta}}-\frac{4!.}{96v^{\alpha+\beta}}+\frac{6!}{3072v^{\alpha+\beta}}-\cdots\right\}$ $=\left\{\frac{v^3}{4v3^{\alpha+\beta}}-\frac{v^5}{4v5^{\alpha+\beta}}+\frac{120.v^7}{5v^{7\alpha+\beta}}-\cdots\right\}$
…(9)

6. Conclusion

Here the author exhibits a relation between sadik and Bessels function The rest of the transformation

which we can use to solve a differential equation with variable and constant coefficients.

Like Bessel we may apply the process to solve partial differential equation . We can also solve many complex physical and mathematical phenomena, including the movement of liquids and gases, waves, heat distribution, and digital signals.

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