Sumer university Journal of Administrative and Economic Sciences (JAES)
ISSN 3078-6525
مجلة كلية الإدارة والاقتصاد – جامعة سومر المجلد (2) - العدد (1)

Compare Cramér–von Mises and Partial Estimation Methods for Weibull–Compertz Distribution

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----Abstract

This paper presents a new probabilistic model New Weibull Compertz Distribution (NWGD). It goes beyond the Compertz distribution in the New Weibull-X family and introduces an additional shape parameter, 6. The three-parameter form (α λ, b,) obtained is more flexible and more resilient to the data fitting in skewed, heavy tailed and lifetime data applications. Two estimation methods are obtained to the unknown parameters of NWGD. The first one is the partial fractional estimator (PM). The second one is the Cramer von Mises estimator (CVM). The analysis of the stability, accuracy and sensitivity of both procedures in small and moderate sample regimes at different parameter values and random seeds are analyzed. A Monte Carlo simulation experiment compares the two estimators at the sample sizes n = 10, 25, 50 and 100. The bias is used to measure performance and mean squared error (MSE). The outcomes suggest that PM and CVM are both consistent but CVM generally has lower bias and MSE; particularly with increased n. The most noticeable increase is at n = 100 as CVM has the lowest overall MSE of all settings to be tested. The paper recommends that PM and CVM should be benchmarked with maximum likelihood and method of moments to check their efficiency. Pareto and Campbell distributions are readily estimated using the same ideas as the estimation performed on distributions.

Keyword: New Weibull–Compertz Distribution; Cramér–von Mises Estimation; Partial Estimation Method; Monte Carlo Simulation; Bias; Mean Squared Error (MSE); Statistical Modeling

1. Introduction

Probability distributions are essential to statistical modeling and data analysis because they allow mathematical models of uncertainty and variability in phenomena of real life. Classical distributions are typically unable to reflect complicated data features like skewness, heavy tails or different hazard rate

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patterns. Consequently, more recent work in statistical theory has been on building up families of distribution by the inclusion of new parameters in existing models. Such extensions add more flexibility to the model as well as better goodness-of-fit in a wide range of application scenarios such as engineering, economics, reliability analysis, and survival studies.

In these terms, the Compertz distribution has gained some interest because of its mathematical simplicity and applicability, but is limited in flexibility, thus limiting its performance in modelling more complex datasets. The New Weibull-X family of distributions is an attempt to overcome this shortcoming, with systematic construction of more flexible models by augmenting the parameters. Based on this strategy, the current study suggests a new extension of the Compertz distribution, which is called the New Weibull-Compertz Distribution (NWGD). The proposed distribution can fit a broader set of data behaviors with an addition of shape parameter, and fits well than the original distribution.

The correct estimation of the parameters of any new statistical distribution is an important point in the development of the new statistical distribution. Conventional types of estimations like the maximum likelihood estimation and method of moments might face some computational challenges or can be inefficient especially when the sample size is small or medium. Consequently, other estimation methods are required to attain strong and sound estimates of parameters. Two estimation methods, which are the partial (fractional) estimation method and the Cramer von Mises (CVM) estimation method are used and compared to estimate the parameters of the proposed NWGD in this paper.

The issue that this study seeks to solve is the inflexibility of the current probability models and estimation tools to effectively capture the complex data that is available, which compelled the creation of a new distribution and a better and more effective method of estimating the parameter with higher accuracy and reliability.

This study is expected to produce a new flexible probability distribution modeled on the New Weibull-X family and explore effective means of estimating the parameters of the proposed model. A number of distinct objectives are followed in order to achieve this goal. The following are some of the objectives of the research: (i) developing the New Weppel–Computerz Distribution and establishing its basic statistical properties; (ii) estimating the distribution parameters through the use of partial and Cramer-Mises estimation methods; (iii) the performance of the estimation methods through application of the Monte Carlo simulation based on the bias and mean square error criteria at varying sample sizes; and (iv) the application of the derived estimation methods to other related distributions e.g. the Pareto and Campbell distributions.

A research was included in (2022) a research paper submitted by (Ali, Sajid, Jehan Ara, and Ismail Shah, 2022). Exponentially Modified Gaussian Distribution (EMG) distribution is the product of the Gaussian and exponential distribution which is defined by asymmetric tails. It has a lot of flexibility, and thus it is applicable in various fields of science. The paper explores the making of EMG parameters estimation process through studying eleven classical and goodness-of-fit based methods using simulation and a real cancer cell detecting dataset [1].

Hassan, Amal S., and Ahmed M. Abdelghaffar, (2025) their study concerns the estimation of the stress-strength reliability measure(R=P(Y<X)) in a step-stress partially accelerated Life test. In the case of independent Gompertz stress and strength variables, the maximum likelihood, Bayesian as well as E-Bayesian methods are used, which are supported by simulation and real data analysis [2]. This study adds to the distribution theory by offering a versatile model and delivery of useful estimation tactics in both the development of theoretical concepts and practical statistical analysis.

2. Compertz Distribution

A generalized probability distribution that is commonly applied in survival analysis, reliability engineering, actuarial science etc. It is also especially useful in the modeling of lifetimes with a growing hazard rate, therefore suitable in the operation of aging and wear-out. It has a two-positive parameter distribution, consisting of a shape parameter and a scale parameter that determine the characteristics of the behavior of the failure rate with time.

Among the key characteristics of the Gompertz distribution one can note exponentially growing hazard function that represents the realistic assumption that the risk of failure increases as the time goes. The property has resulted in widespread use in demographic analysis, modeling in human mortality, medical studies and testing reliability of mechanical and electronic components. The Gompertz model is commonly applied to explain the survival of patients in biomedical research, especially cancer biomedical research and age-associated research.

Statistically, the Gompertz distribution is mathematically tractable and flexible in that it can be expressed in closed form with the probability density, cumulative distribution and hazard functions. Estimation of a parameter is usually done through methods like maximum likelihood estimation, Bayesian estimation and moment based methods. In real-life cases, the Gompertz distribution may be easier to model compared to conventional exponential or Weibull distributions since it may model skewed lifetime data and the failure rate is an increasing function.

The probability density function is [3]:-

$$g(x) = b\lambda e^{\lambda+bx-\lambda\ bx} \quad \dots (1)$$

The cumulative distribution function is:-

$$G(x) = 1 - e^{-\lambda(e^{bx}-1)} \quad \dots (2)$$

In real-life cases, the Gompertz distribution may be easier to model compared to conventional exponential or Weibull distributions since it may model skewed lifetime data and the failure rate (t) is an increasing function.

$$h(x) = \frac{f(x)}{R(x)} \quad \dots (3)$$

$$R(x) = 1 - F(x)$$

$$h(x) = \frac{b\lambda e^{\lambda+bx-\lambda e^{bx}}}{1-\left[1-e^{-\lambda(e^{bx}-1)}\right]}$$

$$h(x) = b\lambda e^{bx} \quad \dots (4)$$

$$H(x) = \int_0^x h(u)\, d(u)$$

$$H(x) = \int_0^x b\lambda e^{bu} d(u)$$

$$H(x) = \lambda e^{bx} - \lambda \quad \dots (5)$$

1.2. New Weibull- x family

The New Weibull-X (NW-X) family of distributions is a flexible class of continuous probability distributions generated by transforming a baseline distribution through the Weibull function. This family was proposed to enhance the modeling capability of classical distributions by introducing additional shape parameters that allow greater control over skewness, kurtosis, and tail behavior. This family has a cumulative distribution function (cdf) and a probability density function (pdf), as shown below [4][9].

$$F(x, \alpha, \varepsilon) = 1 - e^{-\left[\frac{H(x,\varepsilon)}{1-G(x,\varepsilon)}\right]^{\alpha}} \quad \dots (6)$$

$\alpha, \varepsilon > 0, x\epsilon R$

$$f(x, \alpha, \varepsilon) = \frac{\alpha\, g(x,\varepsilon) H(x,\varepsilon)^{\alpha-1}[1 + H(x,\varepsilon)]}{[1 - G(x,\varepsilon)]^{\alpha+1}}\, e^{-\left[\frac{H(x,\varepsilon)}{1-G(x,\varepsilon)}\right]^{\alpha}} \quad \dots (7)$$

$\alpha, \varepsilon > 0, x\epsilon R$

2.2. New Weibull- Gompertz Distribution

The New Weibull–Gompertz (NWG) distribution is a flexible lifetime distribution derived by applying the New Weibull-X family transformation to the Gompertz distribution. It combines the advantages of both the Weibull and Gompertz distributions, offering enhanced flexibility in modeling lifetime data, particularly when hazard rates exhibit complex behaviors such as increasing, decreasing,

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or bathtub shapes. By substituting equations (1), (2), and (5) into equation (7), we obtain the probability density function (pdf) for the new distribution (NWGD), as shown below [5].

$$f(x,\alpha,\lambda,b)=\frac{\alpha b\lambda e^{bx}[\lambda e^{bx}-\lambda]^{\alpha-1}\left[1+\left(\lambda e^{bx}-\lambda\right)\right]}{e^{-\lambda\alpha(e^{bx}-1)}} \quad \dots(8)$$

3. Estimation Methods

Estimation methods are techniques used to determine the unknown parameters of a probability distribution based on observed data.

1.3. Cramér–von Mises (CVM) method

The Cramér–von Mises (CVM) method is a goodness-of-fit approach used to estimate the parameters of a probability distribution by minimizing the discrepancy between the empirical cumulative distribution function (ECDF) and the theoretical cumulative distribution function (CDF).[6][11]

$$T=\frac{1}{12n}+\sum_{i=1}^{n}\left[F(x_i)-\frac{2i-1}{2n}\right]^2 \quad \dots(9)$$

$$T=\frac{1}{12n}+\sum_{i=1}^{n}\left[\left[1-e^{-\frac{(\lambda e^{bx}-\lambda)^{\alpha}}{e^{-\lambda\alpha(e^{bx}-1)}}}\right]-\frac{2i-1}{2n}\right]^2 \quad \dots(10)$$

Taking the partial derivative with respect to the parameter (α)

$$\frac{\partial T}{\partial \alpha}=2\sum_{i=1}^{n}\left[\left[1-e^{-\frac{(\lambda e^{bx}-\lambda)^{\alpha}}{e^{-\lambda\alpha(e^{bx}-1)}}}\right]-\frac{2i-1}{2n}\right]\left[\frac{e^{-\lambda\alpha(e^{bx}-1)}\left(\lambda e^{bx}-\lambda\right)ln\left(\lambda e^{bx}-\lambda\right)+[\lambda e^{bx}-\lambda]^{\alpha}\lambda\left(e^{bx}-1\right)e^{-\lambda\alpha(e^{bx}-1)}}{e^{-2\lambda\alpha(e^{bx}-1)}}\right]e^{-\frac{(\lambda e^{bx}-\lambda)^{\alpha}}{e^{-\lambda\alpha(e^{bx}-1)}}} \quad \dots(11)$$

Taking the partial derivative with respect to the parameter (λ)

$$\frac{\partial T}{\partial \lambda}=2\sum_{i=1}^{n}\left[\left[1-e^{-\frac{(\lambda e^{bx}-\lambda)^{\alpha}}{e^{-\lambda\alpha(e^{bx}-1)}}}\right]-\frac{2i-1}{2n}\right]\left[\frac{e^{-\lambda\alpha(e^{bx}-1)}\alpha\left(\lambda e^{bx}-\lambda\right)^{\alpha-1}\left(e^{bx}-1\right)+\left(\lambda e^{bx}-\lambda\right)^{\alpha}\alpha\left(e^{bx}-1\right)e^{-\lambda\alpha(e^{bx}-1)}}{e^{-2\lambda\alpha(e^{bx}-1)}}\right]e^{-\frac{(\lambda e^{bx}-\lambda)^{\alpha}}{e^{-\lambda\alpha(e^{bx}-1)}}} \quad \dots(12)$$

Taking the partial derivative with respect to the parameter (b)

$$\frac{\partial T}{\partial b} = 2\sum_{i=1}^{n}\left[\left[1-e^{-\frac{(\lambda e^{bx}-\lambda)^{\alpha}}{e^{-\lambda\alpha(e^{bx}-1)}}}\right] - \frac{2i-1}{2n}\right]\left[\frac{e^{-\lambda\alpha(e^{bx}-1)}\alpha(\lambda e^{bx}-\lambda)^{\alpha-1}\lambda x e^{bx}+(\lambda e^{bx}-\lambda)^{\alpha}\lambda\alpha x e^{bx}e^{-\lambda\alpha(e^{bx}-1)}}{e^{-2\lambda\alpha(e^{bx}-1)}}e^{-\frac{(\lambda e^{bx}-\lambda)^{\alpha}}{e^{-\lambda\alpha(e^{bx}-1)}}}\right] \quad \dots(13)$$

2.3. Partial Estimation Method

The Partial Estimation Method (sometimes called the Fractional Estimation Method) is an estimation technique used to estimate parameters of complex distributions when classical methods like maximum likelihood or method of moments are difficult to apply [7][10].

$$F(x_i) = 1-e^{-\frac{(\lambda e^{bx}-\lambda)^{\alpha}}{e^{-\lambda\alpha(e^{bx}-1)}}} \quad \dots(14)$$

$$R_i = \frac{i-\frac{3}{8}}{n+\frac{1}{4}}$$

$$F(x_i) = R_i$$

$$R_i = 1-e^{-\frac{(\lambda e^{bx}-\lambda)^{\alpha}}{e^{-\lambda\alpha(e^{bx}-1)}}} \quad \dots(15)$$

By taking the square and the sum and setting them equal to zero, we get

$$Pc = \sum_{i=1}^{n}\left[\left[1-e^{-\frac{(\lambda e^{bx}-\lambda)^{\alpha}}{e^{-\lambda\alpha(e^{bx}-1)}}}\right]-R_i\right]^2 \quad \dots(16)$$

Taking the partial derivative with respect to the parameter (α)

$$\frac{\partial Pc}{\partial \alpha} = 2\sum_{i=1}^{n}\left[\left[1-e^{-\frac{(\lambda e^{bx}-\lambda)^{\alpha}}{e^{-\lambda\alpha(e^{bx}-1)}}}\right] - R_i\right]\left[\frac{e^{-\lambda\alpha(e^{bx}-1)}(\lambda e^{bx}-\lambda)ln(\lambda e^{bx}-\lambda)+[\lambda e^{bx}-\lambda]^{\alpha}\lambda(e^{bx}-1)e^{-\lambda\alpha(e^{bx}-1)}}{e^{-2\lambda\alpha(e^{bx}-1)}}\right]e^{-\frac{(\lambda e^{bx}-\lambda)^{\alpha}}{e^{-\lambda\alpha(e^{bx}-1)}}} \quad \dots(17)$$

Taking the partial derivative with respect to the parameter (λ)

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$$\frac{\partial Pc}{\partial \lambda} = 2\sum_{i=1}^{n}\left[\left[1-e^{-\frac{(\lambda e^{bx}-\lambda)^{\alpha}}{e^{-\lambda\alpha(e^{bx}-1)}}}\right] - R_i\right]\left[\frac{e^{-\lambda\alpha(e^{bx}-1)}\alpha(\lambda e^{bx}-\lambda)^{\alpha-1}(e^{bx}-1)+[\lambda e^{bx}-\lambda]^{\alpha}\alpha(e^{bx}-1)e^{-\lambda\alpha(e^{bx}-1)}}{e^{-2\lambda\alpha(e^{bx}-1)}}\right]e^{-\frac{(\lambda e^{bx}-\lambda)^{\alpha}}{e^{-\lambda\alpha(e^{bx}-1)}}} \dots (18)$$

Taking the partial derivative with respect to the parameter (b)

$$\frac{\partial Pc}{\partial b} = 2\sum_{i=1}^{n}\left[\left[1-e^{-\frac{(\lambda e^{bx}-\lambda)^{\alpha}}{e^{-\lambda\ (e^{bx}-1)}}}\right] - R_i\right]\left[\frac{e^{-\lambda\alpha(e^{bx}-1)\alpha(\lambda e^{bx}-\lambda)^{\alpha-1}\lambda x e^{bx}+(\lambda e^{bx}-\lambda)^{\alpha}}\lambda\alpha x e^{bx}e^{-\lambda\ (e^{bx}-1)}}{e^{-2\lambda\alpha(e^{bx}-1)}}\right]e^{-\frac{(\lambda e^{bx}-\lambda)^{\alpha}}{e^{-\lambda\alpha(e^{bx}-1)}}} \dots (19)$$

4. Comparison criteria

When multiple estimation methods are applied to estimate the parameters of a distribution, it is important to assess and compare their performance. This is typically done using statistical comparison criteria, which measure the accuracy, bias, and variability of the estimators. Common criteria include:

1.4. Bias

The bias of an estimator measures the difference between the expected value of the estimator and the true parameter value:[8]

$$Bias(\hat{\vartheta}) = E(\hat{\vartheta}) - \vartheta \dots (20)$$

2.4. MSE

The Mean Squared Error combines variance and bias to provide an overall measure of estimator accuracy:[8]

$$MSE(\hat{\vartheta}) = \sum_{i=1}^{k}[\hat{\vartheta}-\vartheta]^2/k \dots (21)$$

5. Experiential Results

o evaluate the performance of different parameter estimation methods for the New Weibull–Gompertz distribution, a Monte Carlo simulation experiment was conducted. The study aimed to compare the accuracy and efficiency of multiple estimation techniques under varying sample sizes and parameter settings.

Table (1) the simulation results

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	method	CVM	bm	CVM	bm	CVM	bm	CVM	bm
	n	10		25		50		100	
es	α	1.10075459	1.11674872	0.90599165	0.88686673	1.0868133	1.07955694	1.24200103	1.24077462
	λ	2.909198	2.90582633	2.5482041	2.55120004	2.48007799	2.48001143	2.50011292	2.49988273
	b	3.88094249	3.87746321	3.55444528	3.55708727	3.47761228	3.4776099	3.50200889	3.50171686
bias	α	-0.39924541	-0.38325128	-0.59400835	-0.61313327	-0.4131867	-0.42044306	-0.25799897	-0.25922538
	λ	0.409198	0.40582633	0.0482041	0.05120004	-0.01992201	-0.01998857	0.00011292	-0.00011727
	b	0.38094249	0.37746321	0.05444528	0.05708727	-0.02238772	-0.0223901	0.00200889	0.00171686
mse	α	2.66442442	3.18211907	1.10569652	1.20511139	0.48544231	0.48657181	0.14872102	0.14970357
	λ	0.47074651	0.46171601	0.08917913	0.08961364	0.01981451	0.01991695	0.00278277	0.00279027
	b	0.86635824	0.86910968	0.14462611	0.14680264	0.03233703	0.03246767	0.00486719	0.00487535

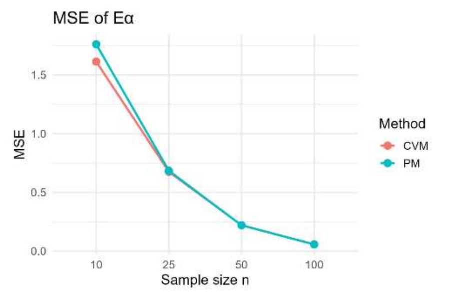
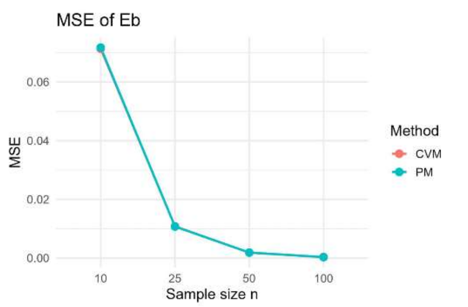
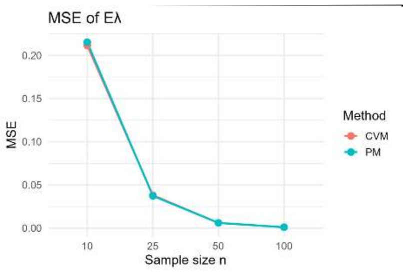


Fig (1) the MSE for (α)



Fig(2) the MSE for (λ)

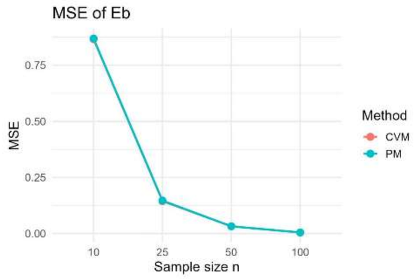


Fig(3) the MSE for (b)

From previous results the best estimation method was (CVM) with minimum MSE was (0.056895129) for (α),(0.001038872) for (λ) and (0.000334524) for (b) each case for (n=100)

Table (2) the simulation results

	method	CVM	bm	CVM	bm	CVM	bm	CVM	Bm
	n	10		25		50		100	
Es	α	1.15618294	1.14451187	1.09901096	1.09037813	1.23678193	1.23246964	1.35170251	1.35075736
	λ	2.68679024	2.68168265	2.47531399	2.47470161	2.48398688	2.48369863	2.49773403	2.49760768
	b	1.57306167	1.57171481	1.48858632	1.48781214	1.49194725	1.49177794	1.49915943	1.4990919
Bias	α	-0.34381706	-0.35548813	-0.40098904	-0.40962187	-0.26321807	-0.26753036	-0.14829749	-0.14924264
	λ	0.18679024	0.18168265	-0.02468601	-0.02529839	-0.01601312	-0.01630137	-0.00226597	-0.00239232
	b	0.07306167	0.07171481	-0.01141368	-0.01218786	-0.00805275	-0.00822206	-0.00084057	-0.0009081
Mse	α	1.61355368	1.76119281	0.67542858	0.68438238	0.22072696	0.21995816	0.05689513	0.05729238
	λ	0.21148164	0.21513596	0.03811373	0.03735928	0.00604041	0.00607054	0.00103887	0.00104133
	b	0.07131308	0.0717238	0.01082437	0.01074477	0.00186933	0.00187601	0.00033452	0.00033496



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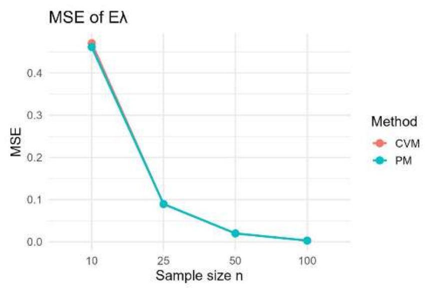
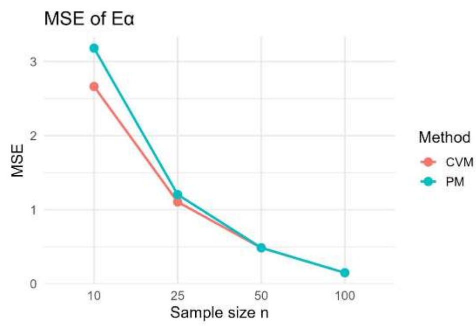
Fig(4) the MSE for (α)



Fig(5) the MSE for (λ)



Fig(6) the MSE for (b)

From previous results the best estimation method was (CVM) with minimum MSE was (0.05689513) for (α),(0.00103887) for (λ) and (0.00033452) for (b) each case for (n=100)

6. Conclusions and Suggestions

1.6. Conclusions

1. The paper has effectively presented the New Weibull-Gompertz (NWG) taxonomy, comprising of both the versatility of the Weibull-X family and Gompertz distribution that are appropriate in the analysis of complicated lifetime and reliability data.
2. Some of the ways to estimate the parameters of the NWG distribution were explored, such as Cramer von Mises (CVM), Partial estimations (PM).
3. The results of simulation experiments indicated that the CVM method has always yielded the best parameter estimates with least bias and least mean squared error (MSE) especially when the sample size (n = 100) is large.
4. The Partial Estimation Method was effective in small samples and provided reasonable preliminary estimates at a lower computational cost.

2.6. Suggestions for Future Research

1. Apply the NWG distribution to a wider range of real-life datasets in engineering, medicine, and actuarial studies to further validate its practical performance.
2. Explore hybrid or computationally advanced estimation methods, such as MCMC-based Bayesian approaches, to improve accuracy for highly skewed or censored data.
3. Investigate goodness-of-fit measures and model selection criteria (AIC, BIC, CVM, AD) to determine the optimal estimation method for different sample sizes and applications.
4. Extend the NWG model to multivariate or regression frameworks to analyze dependent lifetime data or incorporate covariates.
5. Consider stress–strength reliability analysis under more complex accelerated life testing designs to evaluate the robustness of estimation methods in practical reliability experiments.
6. Develop software packages or tools for easy implementation of the NWG distribution and its estimation methods to facilitate research and industrial applications.

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