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Domination and Independence on Square Chessboard

Abstract- *In this paper, new idea for the problems of independence and domination on chessboard is introduced. Two classical chessboard problems of independence and domination on square chessboard with square cells of size n are determined for some cases when two different types of pieces are used together. For independence, fixed number of the first type of pieces is placed on the board with maximum number of pieces of the second type together. For domination, fixed number of pieces of first type is placed on the board with minimum number of pieces of the second type. The pieces which are used together in this paper are: kings with rooks, kings with bishops, and rooks with bishops.*

Keywords- *Domination, Independence, Square chessboard, Kings, Bishops and Rooks.*

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1. Introduction

In the chessboard there are six kinds of pieces. Let "P" be a piece of any kind on the chessboard. There are two classical chessboard problems; one of them is by placing a maximum number of pieces of a single kind, such that each piece does not attack other pieces. This problem is called independence problem, and the number of pieces that satisfies this criterion is the independence number. Independence number for a "P" kind is denoted by $\beta(P)$. The other problem is by placing a minimum number of a single kind "P", such that all unoccupied positions are under attack by at least one of the placed pieces. This problem is called domination problem and the number of pieces that satisfies this criterion is called domination number of "P" and denoted by $\gamma(P)$. Previous studies were concerned with domination and independence problems with a single kind of pieces only, while our current study is concerned in the same problems but with two kinds of pieces at a time. This study can be used in game theory or any similar life problems.

For a square chessboard $(n \times n)$ the independence and domination numbers are determined for Rook "R", Bishop "B" and King "K". They proved that $\gamma(R) = n$, $(B) = n$, $\gamma(K) = \left\lfloor \frac{n+2}{3} \right\rfloor^2$, also, $\beta(R) = n$, $\beta(B) = 2n - 2$ and $\beta(K) = \left\lfloor \frac{n+2}{2} \right\rfloor^2$ (see [1], [2] and [3]).

In [4], JoeMaio and William proved that $\gamma(R) = \min\{m, n\}$ and $\beta(R) = \min\{m, n\}$for $m \times n$ Toroidal chessboard.

In [5], the minimum number of rooks that can dominate all squares of the STC is determined.

In [6], the triangular hexagon board, in which the cells are hexagons and the board is a triangle is considered. Bishops attack in straight lines through the vertices of their cells, rooks attack along straight lines through the centers of the edges of their cells, and queens have both attacks. The only general upper bound they are able to give on the independence number of the queens graph is by the rooks bound, which is $\left\lfloor \frac{2n+1}{2} \right\rfloor$ for all n. For $n = 3, 4, 6, 7, 13, 16, 19, 25, 31,$ they found that $\beta = \left\lfloor \frac{2n+1}{2} \right\rfloor - 1$, and for the other $n \leq 31$, $\beta = \left\lfloor \frac{2n+1}{2} \right\rfloor$.

In [7],[8],[9],[10] and [11] the independence and domination in Rhombus chessboard, isosceles triangular chessboard and cubic chessboard with square cells are determined.

In this paper, we apply the meaning of the two classical problems on square chessboard for two different types of pieces that have been chosen. The pieces of a first type are placed on the chessboard, and their number is fixed and then the domination or independence number of the other type is determined. Let n_P be the number of pieces of the same kind (P) which are chosen to have a fixed number. N_P refers to the number of the cells which are attacked by one piece (P) in addition to the cell that is occupied by the piece (P).

2 The Chessboard

The chessboard in this work is a square chessboard of size n with square cells. Three types of pieces, rooks R, bishops B and kings K are used with their usual moving or attacking. Let the number of cells (squares) in a side be the length of that side. To simplify the form of our results , the matrix form is used where r_i denote the i^{th} row measured from above to down, $i = 1, 2, \ldots, n$ and let c_i denote the j^{th} column measured from left to right, $j = 1, 2, \ldots, n$. Let the cell (square) of i^{th} row and j^{th} column is denoted by $s_{i,j}$, $i = 1, 2, \ldots n$, and $j = 1, 2, \ldots n$.

Theorem 2.1 [8] In isosceles triangular chessboard of size n the domination number of king pieces (γ(K)) is given by

$$\gamma(K) = \begin{cases} \frac{\left\lceil\frac{2n-1}{3}\right\rceil}{4}\left(\left\lceil\frac{2n-1}{3}\right\rceil + 2\right), \text{if } \left\lceil\frac{2n-1}{3}\right\rceil \text{ is even} \\ \frac{\left(\left\lceil\frac{2n-1}{3}\right\rceil+1\right)^2}{4}, \text{ if } \left\lceil\frac{2n-1}{3}\right\rceil \text{ is odd} \end{cases}$$

(1)

(see Figure 1(a); $n = 6$) .

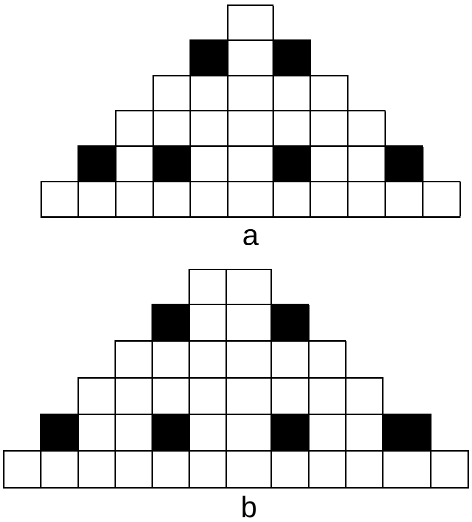


Figure 1

This result is used for determining domination in the isosceles semi triangular chessboard of size n. In this chessboard, there are two equal sides and every i^{th} row contain $2i$ cells as shown in Figure 1(b), $n = 6$

Proposition 2.2. In isosceles semi triangular chessboard of size n the domination number of king pieces (γ(K)) is given by

$$\gamma(K) = \begin{cases} \frac{\left\lceil\frac{2n}{3}\right\rceil}{4}\left(\left\lceil\frac{2n}{3}\right\rceil + 2\right), if \left\lceil\frac{2n}{3}\right\rceil \text{ is even} \\ \frac{\left(\left\lceil\frac{2n}{3}\right\rceil+1\right)^2}{4}, if \left\lceil\frac{2n}{3}\right\rceil \text{ is odd} \end{cases}$$

(2)

3 Domination and Independence of K pieces with a fixed number of R pieces.

In the square chessboard in this work let (n_r) be the number of rook pieces and let $\gamma(K, n_r)$ and $\beta(K, n_r)$ be the domination number and independence number of king pieces (K) with a fixed number of Rook pieces (R) respectively.

Theorem 3.1. The domination number of king pieces with fixed pieces(n_r) of rooks in a square chessboard of size n is given by

$$\gamma(K, n_r) = \left\lceil\frac{n - n_r}{3}\right\rceil^2$$

Proof. If the K pieces are distributed in the chessboard by placing the R pieces in the cells $s_{i,i}$, $i = 1,2, \ldots, n-1$ in order to keep the minimum number of domination (any other placing make a partition on the chessboard) then the maximum of N_r of these pieces is gotten, and a square chessboard of length $n - n_r$ of cells which are not attacked by R pieces. (see Figure 2(b); $n = 9$). Since, from [5] the domination number of king in a square chessboard size n is $\gamma(K) = \left\lfloor\frac{n+2}{3}\right\rfloor^2$ wich eequal to $\left\lceil\frac{n}{3}\right\rceil^2$. Hence, $\gamma(K, n_r) = \left\lceil\frac{n-n_r}{3}\right\rceil^2$

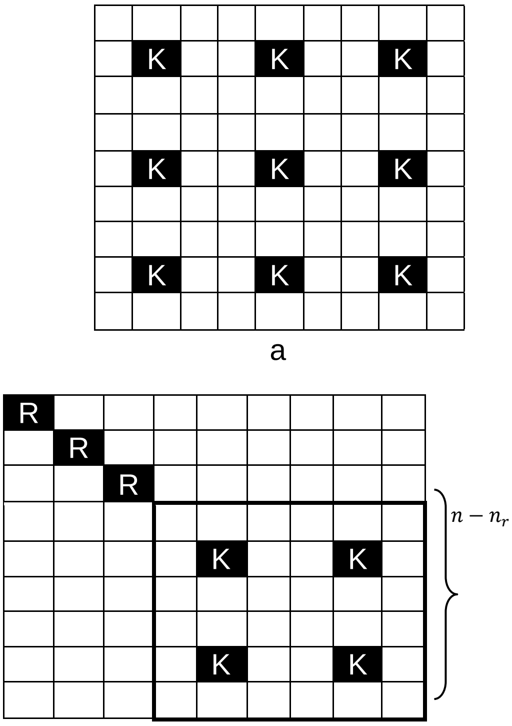


Figure 2

To prove each of the following theorems we must refer to

(1) The remaining P pieces in any step of the proof which we shall denoted by z equal to the difference between n_r and the maximum number n_r of the previous step.

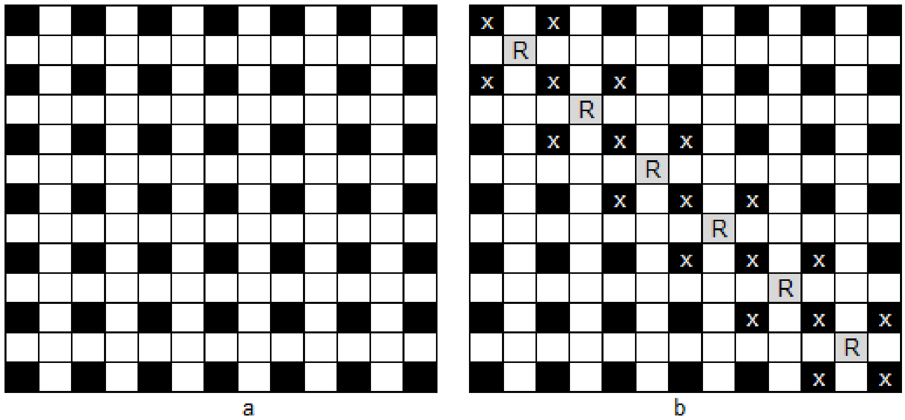


Figure 3

(2) The black cells which appear in our figures represent the places of K pieces.

Theorem 3.2. For $n \geq 12$ the independence number of K pieces with a fixed number n_r of R pieces, where $1 \leq n_r \leq \left\lfloor \frac{n}{2} \right\rfloor$ is given by

$$\beta(K, n_r) = \begin{cases} \left\lceil \frac{n}{2} \right\rceil^2 - (3n_r + 1) & , if \quad n \equiv 1,3 \ (mod\ 4) \\ \left(\frac{n}{2}\right)^2 - (3n_r - 2) & , if \quad n \equiv 0,2 \ (mod\ 4) \end{cases}$$

Proof. The independence number of the king piece on square chessboard is $\beta(K) = \left\lceil \frac{n}{2} \right\rceil^2$ according to [6], where the independence distribution of K pieces is as shown in Figure 3(a); where $n = 13$ (shaded cells). The idea is to distribute the pieces of R such that, it attack a minimum number of K pieces to keep a maximum number of K pieces on the chessboard, and no K piece attacks any of R pieces. The column and row for any cell of the K piece contain n pieces of K where $n \equiv 1,3\ (mod\ 4)$, and $n - 1$ pieces of K where $n \equiv 0,2\ (mod\ 4)$. For this idea, we have two cases that depend on the length n of the square chessboard and as follows.

(i) *If* $n \equiv 1,3\ (mod\ 4)$: the suitable cell to place the first R piece is $s_{2,2}$, since the neighborhood is the minimum. In this place the first R piece does not attack any of the K pieces, but there are four K pieces adjacent to it. So we must remove the adjacent K pieces and we denote each cell of these pieces by "x" as shown in Figure 3(b); for $n = 13$.

To take advantage from the first R piece, we place the second R piece in a cell such that $N(R)$ is shared with the most number of K pieces in the neighborhood of the first R piece. Now, if the neighborhood of the second R piece is shared with two K pieces from the neighborhood of the first R piece, then the place of the first and second R pieces are in the same column or the same row and this mean that the two R pieces are not independent. So the most shared K pieces between the neighborhood of the first and second R piece is one K piece, thus a suitable cell to place the second R piece is $s_{4,4}$.

The second R piece is adjacent to other three K pieces, so we must remove these pieces and we denote each cell of the removable king by "x". Continue to place other R pieces in the cells $s_{2i,2i}$, $i = 1,2,..,\left\lfloor \frac{n}{2} \right\rfloor$ in order (see Figure 3(b); $n = 13$). Then we get

$$\beta(K, n_r) = \left\lceil \frac{n}{2} \right\rceil^2 - (3n_r + 1).$$

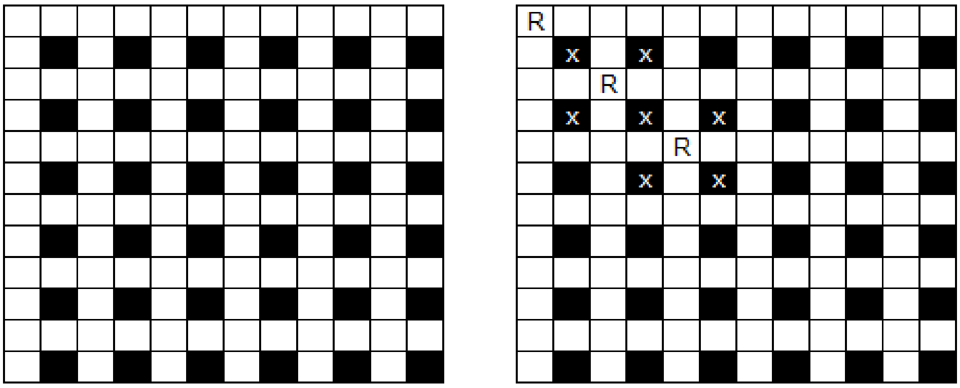
(ii) If $n \equiv 0,2\ (mod\ 4)$: In the same manner in the previous case, the suitable cells to place the R pieces are $s_{2i+,2i+1}$, $i = 0,2,..,\frac{n}{2} - 1$, in order. The first R piece does not attack any K piece and adjacent to one K piece, so this K piece must be removed. The second R piece is adjacent to other three K pieces, so these K pieces must be removed (see Figure 4(a)). Continue to this procedure until $n_r = \frac{n}{2}$. Therefore, $\beta(K, n_r) = \left\lceil \frac{n}{2} \right\rceil^2 - (3n_r - 2)$.

The following example illustrating the above theorem for different values of n_r.

Example 3.3.

1) $n = 13$, $n_r = 6$, implies $\beta(K, 12) = 30$, (see Figure 3).

2) $n = 12$, $n_r = 3$, implies $\beta(K, 8) = 29$ (see Figure 4).



a b

Figure 4

4 Domination and Independence of K pieces with a fixed number of B pieces

Let the domination (Independence) number of K pieces with a fixed number n_b of B pieces are denoted by $\gamma(K, n_b)$ ($\beta(K, n_b)$).

Theorem 4.1. The domination number of K pieces with fixed B pieces n_b in a square chessboard is given as follows.

For $n > 4$, let $m = \frac{2\left(\lceil\frac{n}{2}\rceil - \lceil\frac{n_b}{2}\rceil\right)}{3}$, if

$$\mu_1 = \begin{cases} \frac{\lceil m\rceil}{4}(\lceil m\rceil + 2) & , if\ \lceil m\rceil\ is\ even \\ \frac{(\lceil m\rceil+1)^2}{4} & , if\ \lceil m\rceil\ is\ odd \end{cases},$$

$$\mu_2 = \begin{cases} \frac{\lceil m-\frac{1}{3}\rceil}{4}\left(\lceil m-\frac{1}{3}\rceil + 2\right) & , if\ \lceil m-\frac{1}{3}\rceil\ is\ even \\ \frac{\left(\lceil m-\frac{1}{3}\rceil+1\right)^2}{4} & , if\ \lceil m-\frac{1}{3}\rceil\ is\ odd \end{cases},$$

$$\mu_3 = \begin{cases} \frac{\lceil m+\frac{1}{3}\rceil}{4}\left(\lceil m+\frac{1}{3}\rceil + 2\right) & , if\ \lceil m+\frac{1}{3}\rceil\ is\ even \\ \frac{\left(\lceil m+\frac{1}{3}\rceil+1\right)^2}{4} & , if\ \lceil m+\frac{1}{3}\rceil\ is\ odd \end{cases},$$

$$\mu_4 = \begin{cases} \frac{\lceil m-1\rceil}{4}(\lceil m-1\rceil + 2) & , if\ \lceil m-1\rceil\ is\ even \\ \frac{(\lceil m-1\rceil+1)^2}{4} & , if\ \lceil m-1\rceil\ is\ odd \end{cases}$$

and

$$\mu_5 = \begin{cases} \frac{\lceil m-\frac{2}{3}\rceil}{4}\left(\lceil m-\frac{2}{3}\rceil + 2\right) & , if\ \lceil m-\frac{2}{3}\rceil\ is\ even \\ \frac{\left(\lceil m-\frac{2}{3}\rceil+1\right)^2}{4} & , if\ \lceil m-\frac{2}{3}\rceil\ is\ odd \end{cases}, \text{ then}$$

$$\gamma(K, n_b) = \begin{cases} 4\mu_2 & , if\ n\ and\ n_b\ are\ odd \\ 2\mu_5 + \mu_2 + \mu_4 & , if\ n\ is\ odd\ and\ n_b\ is\ even \\ 2\mu_2 + 2\mu_3 & , if\ n\ is\ even\ and\ n_b\ is\ odd \\ 2\mu_2 + \mu_1 + \mu_5 & , if\ n\ and\ n_b\ are\ even \end{cases}$$

.

Proof. In each cases below $1 \le n_b \le n1$.

Case 1) If n is odd: There are two cases that depend on n_b as follows.

(i) If n_b is odd, then n_b pieces of B are placed in the middle of the column $c_{\lceil\frac{n}{2}\rceil}$, since these cells give the maximum of N_b, so that the minimum number of K pieces is gotten. These pieces of K are distributed on the cells which are not attacked. These cells from four isosceles triangular chessboard with same size $\left(\lceil\frac{n}{2}\rceil - \lceil\frac{n_b}{2}\rceil\right)$ in the square chessboard as shown in Figure 8(a); $n = 11$. Using equations (1) and (2) in section 2, the result is gotten.

(ii) If n_b is even, again the n_b pieces of B are placed in the middle of the column $c_{\lceil\frac{n}{2}\rceil}$, and the cells which are not attacked by these pieces form four shapes. Two of these shapes form two semi isosceles triangular chessboard with the size $\left(\lceil\frac{n}{2}\rceil - \lfloor\frac{n_b}{2}\rfloor\right)$, and the other two form two isosceles triangular chessboard with different sizes $\left(\lceil\frac{n}{2}\rceil - \lfloor\frac{n_b}{2}\rfloor\right)$ and $\left(\lceil\frac{n}{2}\rceil - \lfloor\frac{n_b}{2}\rfloor + 1\right)$, as shown in Figure 8(b); $n = 11$. By using equations (1) and (2), the result is obtained.

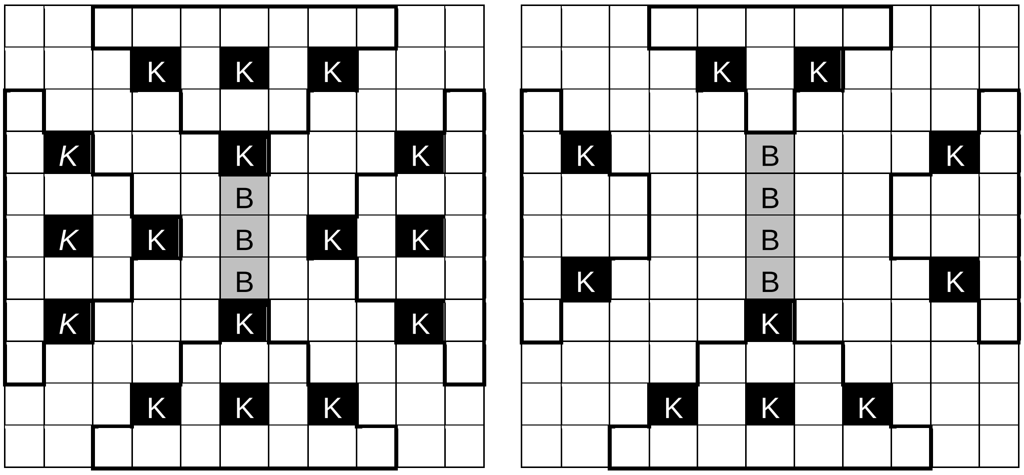


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Figure 8

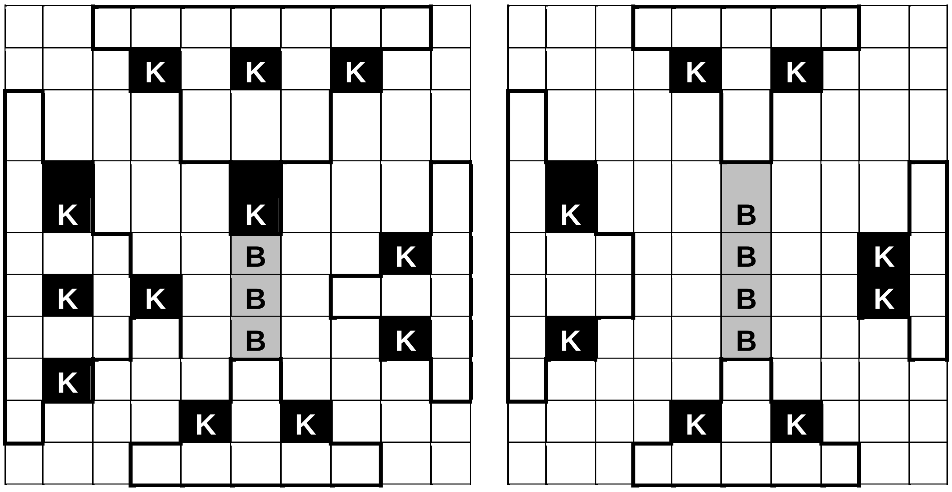

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Figure 9

Case 2) If n is even: again there are two cases depend on the value of n_b as follows.

(iii) If n_b is odd, then n_b pieces of B in the middle of column $c_{\frac{n}{2}+1}$, are placed. So, the maximal of N_b and the minimum number of K pieces are obtained. These K pieces are distributed on the cells which are not attacked by n_b pieces of B. The cells which are not attacked by n_b pieces of B form four isosceles triangular chessboards, two of them are of size $\left(\frac{n}{2}-\left\lceil\frac{n_\mathrm{b}}{2}\right\rceil+1\right)$ and the others are of size $\left(\frac{n}{2}-\left\lceil\frac{n_\mathrm{b}}{2}\right\rceil\right)$ as shown in Figure 9(a). By using the equations (1) and (2), we get the result.

(iv) If n_b is even, again we place the n_b pieces of B in the middle of the column $\mathrm{c}_{\frac{n}{2}+1}$, the cells not attacked by these pieces form four shapes. Two of these shapes are isosceles triangular chessboard with size $\left(\frac{n}{2}-\left\lceil\frac{n_\mathrm{b}}{2}\right\rceil\right)$, and the other two are isosceles semi triangular chessboard one of them with size $\left(\frac{n}{2}-\left\lceil\frac{n_\mathrm{b}}{2}\right\rceil\right)$ and the other with size $\left(\frac{n}{2}-\left\lceil\frac{n_\mathrm{b}}{2}\right\rceil-1\right)$ as shown in Figure 9(b). Using the equations (1) and (2) the result is obtained.

The following example illustrating the above theorem for different values of n and n_b.

Example 4.2.

1) $n = 11$, $n_b = 3$, then $\gamma(K,3) = 16$, (see Figure 8(a))

2) $n = 11$, $n_b = 4$, then $\gamma(K,4) = 10$, (see Figure 8(b))

3) $n = 10$, $n_b = 3$, then $\gamma(K,3) = 12$, (see Figure 9(a))

4) $n = 10$, $n_b = 4$, then $\gamma(K,4) = 8$, (see Figure 9(b))

Theorem 4.3 The independence of K pieces with a fixed number n_{b} of B pieces is given by the following:

(i) If n is odd, then $\beta(K, n_b) = \begin{cases} \left\lceil \frac{n}{2} \right\rceil^2 - (n_b + 1) & \text{, if } 1 \le n_{\mathrm{r}} \le \left\lfloor \frac{n}{2} \right\rfloor \\ \left\lceil \frac{n}{2} \right\rceil^2 - (n_b + 2) & , if \ \left\lfloor \frac{n}{2} \right\rfloor < n_r \le n - 1 \end{cases}$

(ii) If n is even, then $\beta(K, n_b) = \begin{cases} \left(\frac{n}{2}\right)^2 - n_b & \text{, if } 1 \le n_{\mathrm{r}} \le \frac{n}{2} \\ \left(\frac{n}{2}\right)^2 - (n_b + 1) & , if \ \frac{n}{2} < n_r \le n - 1 \end{cases}$

Proof. There are two cases that depend on the value of n as follows.

(i) If n is odd: two successive cases that depend on n_b are obtained as follows.

(a) If $1 \le n_b \le \left\lfloor \frac{n}{2} \right\rfloor$, the distribution of the K pieces is as before in Section 3, and a cell is looked for pleasing one B piece such that the minimum number of K pieces is attacked by it. The suitable cells of this idea are one of $s_{n,2j}$; $j = 1,2, \dots, \left\lfloor \frac{n}{2} \right\rfloor$, where no K pieces are attacked. For the first B piece there are two adjacent of K pieces to this piece, and there is one K piece adjacent to the second B piece. So, the adjacent K pieces must be removed. Continue with the same manner for the other B pieces until reaching cell $s_{n,2\left\lfloor \frac{n}{2} \right\rfloor}$ as shown in Figure 10(a); $n = 11$. According to [6], where $\beta(K) = \left\lceil \frac{n}{2} \right\rceil^2$, hence, $\beta(K, n_b) = \left\lceil \frac{n}{2} \right\rceil^2 - (n_b + 1)$.

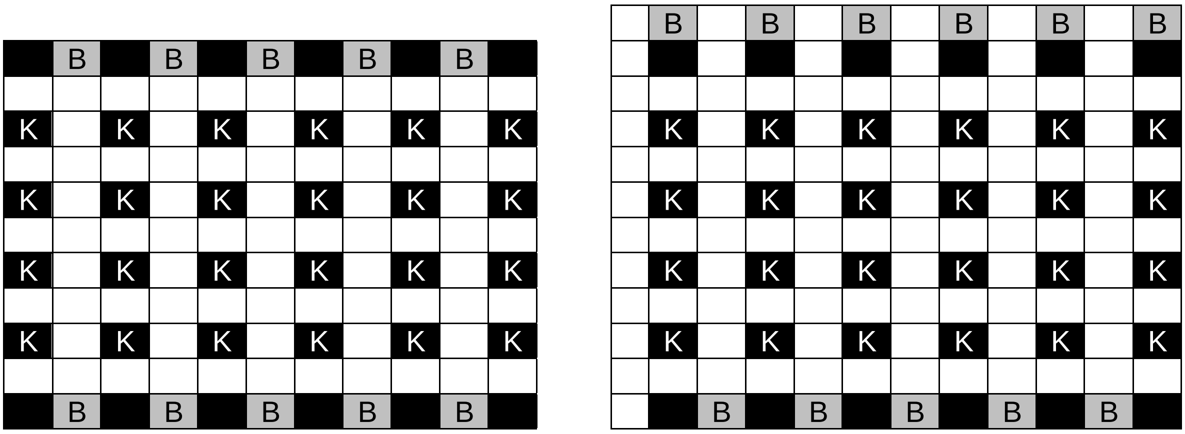
(b) $\left\lfloor \frac{n}{2} \right\rfloor < n_r \le n - 1$, by placing the remaining z pieces in the cells $s_{1,2j}$; $j = 1,2, \dots, \left\lfloor \frac{n}{2} \right\rfloor$ in order (see Figure 10; $n = 11$), as in (i), we get $\beta(K, n_{\mathrm{b}}) = \left\lceil \frac{n}{2} \right\rceil^2 - (n_{\mathrm{b}} + 2)$.

(ii) If n is even: Again we have two successive cases depend on n_b as follows:

(a) If $1 \le n_b \le \frac{n}{2}$, the distribution of the K pieces is as before in Section 3. Now, the next step is looking for a cell to place one B piece such that the minimum number of K pieces is attacked by it. The suitable cell of this idea is one of the cells $s_{1,2j}$; $j = 1,2, \dots, \frac{n}{2}$ in order. There is one K piece adjacent to the first B piece; therefore removing of this piece is necessarily. By continuing with other bishop pieces as the same manner (see in Figure 10(b); $n = 12$).

Thus, $\beta(K, n_b) = \left\lceil \frac{n}{2} \right\rceil^2 - n_b$.

(b) If $\frac{n}{2} < n_b \le n - 1$, by placing the remaining z pieces B in the cells $s_{n,3+2j}$; $j = 0,1, \dots, \frac{n}{2} - 2$ in order. There are two K pieces adjacent to the first B piece, so we must remove these pieces. There is one adjacent K piece to the second B piece, so we must remove this K piece. We continue with the same manner for the other B pieces until reaching the cell $s_{n,n-1}$ as shown in Figure 10(b); $n = 12$. Hence, we get $\beta(K, n_b) = \left\lceil \frac{n}{2} \right\rceil^2 - (n_b + 1)$.



a b

Figure 10

Note 4.4. When $n - 1 < n_r \le 2n - 3$, it is not easy to find general formula for $\beta(K, n_b)$.

The following example illustrating the above theorem for different values of n and n_b.

Example 4.5.

1) $n = 11$, $n_{\mathrm{b}} = 10$, then $\beta(K, 10) = 24$, (see Figure 10(a))

2) $n = 12$, $n_{\mathrm{b}} = 11$, then $\beta(K, 11) = 24$, (see Figure 10(b))

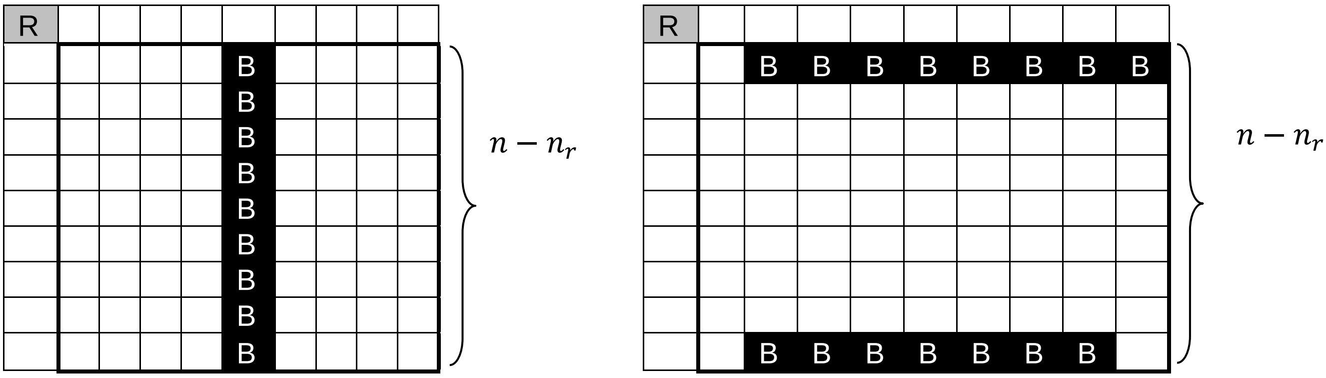


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Figure 11

5 Domination and Independence of B pieces with a fixed number of R pieces.

We denote the domination (Independence) number of B pieces with a fixed number n_r of R pieces by $\gamma(B, n_r)$ ($\beta(B, n_r)$).

Theorem 5.1. The domination (independence) of B pieces $\gamma(B, n_r)$ ($\beta(B, n_r)$) with a fixed number n_r of R pieces is given by the following:

(i) $\gamma(B, n_r) = n - n_r$

(ii) $\beta(B, n_r) = 2(n - n_r) - 3$

Proof. The idea is to place n_r pieces of R and then distributing the B pieces to get the domination (independence) number of the B pieces together with a fixed number n_r.

(i) We look for a cell to place the first R piece such that we obtain the maximum of N_R. Therefore we can distribute minimum number of B pieces such that they are not attacked by the first R piece.

The suitable cells for $(n-1)$ R pieces distribution are the main diagonal of the chessboard $s_{i,i}$, $i = 1, .., r$ in order. The vacuum cells which are not attacked by R pieces form a square chessboard of length $n - n_{\mathrm{r}}$ as shown in Figure 11(a); n =10. We know that $\gamma(B) = n$ (see [6]), where n is the size of square chessboard. Therefore by our distribution we get $\gamma(B, n_r) = n - n_r$.

(ii) Similarly as in (i) we place the R pieces in the main diagonal of square chessboard $s_{i,i}$, $i = 1, .., r$ in order. The cells which are not attacked by these pieces form as square chessboard of length $n - n_{\mathrm{r}}$. We know that $\beta(B) = 2n - 2$ (see [6]) where n is the size of the square chessboard. So we distribute $2(n - n_{\mathrm{r}})$ -2 in the chessboard of size $n - n_{\mathrm{r}}$, but we must remove the B piece from the main diagonal, since it is attacked by the R pieces (see Figure 11 (b), 11(c); n = 10). Thus we get $\beta(B, n_r) = 2(n - n_r) - 3$.

The following example illustrating the above theorem for different values of n_r.

Example 5.2.

1) $n = 10$, $n_r = 1$, then $\gamma(B, 1) = 9$, (see Figure 11(a)).

2) $n = 10$, $n_r = 1$, then $\beta(B, 1) = 15$, (see Figure 11(b)).

Open problems for two different types of pieces

Find the general formula of each of the following numbers

$\gamma(R, n_k)$, $\gamma(B, n_k)$, $\gamma(B, n_r)$, $\beta(R, n_k)$, $\beta(B, n_k)$ *and* $\beta(B, n_r)$.

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