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A State-Space Approach to Bitcoin Price Dynamics: Evidence from Dynamic Linear Trend Models

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Abstract: This study examines the effectiveness of a parsimonious state-space framework for modeling and forecasting the weekly Bitcoin opening price amid strong trends, regime shifts, and time-varying volatility. A Dynamic Linear Trend (DLT) model is employed, in which Bitcoin prices are decomposed into latent level and slope components that evolve stochastically over time, allowing the model to adapt flexibly to changing market conditions. Stationarity is assessed using first differencing and the Augmented Dickey–Fuller test, supporting an integration order of one and motivating the chosen specification. Model estimation and inference are carried out using the Kalman filter, enabling recursive state updating and probabilistic forecasting. Competing model dimensions are evaluated using the Akaike and Bayesian Information Criteria across training and testing samples. The results indicate that the simplest specification (k = 3) consistently minimizes both criteria, suggesting that additional parameters do not provide sufficient improvements in model fit. Short-horizon forecasts point to a broadly flat to mildly declining price trajectory over a 12-week horizon, while widening predictive intervals reflect increasing uncertainty. Overall, the findings demonstrate that Dynamic Linear Models offer an interpretable and statistically coherent approach to short-term Bitcoin price forecasting, delivering well-calibrated uncertainty estimates while avoiding over-parameterization.

Keywords: Bitcoin price forecasting, Dynamic Linear Model, State-space models, Time-series analysis

نهج مساحة الحالة لتحليل ديناميكيات أسعار البيتكوين: أدلة من نماذج الاتجاه الخطي الديناميكي

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المستخلص: تبحث هذه الدراسة في فعالية إطار نمذجة مقتصد قائم على نماذج فضاء الحالة في تمثيل والتنبؤ بسعر افتتاح عملة البيتكوين الأسبوعي، في ظل وجود اتجاهات قوية، وتحولات في الأنظمة، وتقلبات متغيرة بمرور الزمن. وقد تم اعتماد نموذج الاتجاه الخطي الديناميكي(Dynamic Linear Trend – DLT) ، حيث تُفكَّك أسعار البيتكوين إلى مكونات كامنة تمثل المستوى والميل، وتتطور عشوائيًا عبر الزمن، بما يسمح للنموذج بالتكيف المرن مع تغير ظروف السوق.
تم تقييم خصائص السكون باستخدام الفروق الأولى واختبار ديكي–فولر المعزَّز، مما يدعم رتبة تكامل تساوي واحدًا ويبرر المواصفة المعتمدة للنموذج. أُجريت عملية تقدير النموذج والاستدلال الإحصائي باستخدام مرشح كالمان، الأمر الذي يتيح التحديث التكراري للحالات والتنبؤ الاحتمالي. كما تم تقييم أبعاد النماذج المتنافسة بالاعتماد على معياري أكايكي والبيزي للمعلومات عبر عينات التدريب والاختبار. وتشير النتائج إلى أن أبسط مواصفة للنموذج (k = 3) تحقق أدنى قيم لكلا المعيارين بشكل متسق، مما يدل على أن إضافة معلمات أخرى لا تحقق تحسنًا كافيًا في جودة الملاءمة. وتشير التنبؤات قصيرة الأجل إلى مسار سعري يتسم بالاستقرار النسبي أو الانخفاض الطفيف على مدى أفق زمني يبلغ ١٢ أسبوعًا، في حين تعكس اتساع فترات التنبؤ تزايد درجة عدم اليقين. وبوجه عام، تُبرز النتائج أن النماذج الخطية الديناميكية توفر إطارًا تفسيريًا متماسكًا إحصائيًا للتنبؤ قصير الأجل بأسعار البيتكوين، مع تقديم تقديرات عدم يقين معايرة بدقة وتجنب الإفراط في المعلمات.

الكلمات المفتاحية: التنبؤ بأسعار البيتكوين، النموذج الخطي الديناميكي، نماذج فضاء الحالة، تحليل السلاسل الزمنية.

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Introduction

The rapid expansion of cryptocurrency markets has positioned Bitcoin as a prominent digital asset whose price dynamics attract substantial interest from investors, policymakers, and researchers. Since its inception, Bitcoin has exhibited extreme volatility, pronounced long-term trends, abrupt regime shifts, and periods of heightened uncertainty, distinguishing it from traditional financial assets. These characteristics pose significant challenges for statistical modeling and forecasting, particularly when conventional linear time-series models rely on assumptions of stationarity and constant parameters that are often violated in cryptocurrency data.
Empirical evidence consistently shows that Bitcoin prices are dominated by stochastic trends and persistent shocks, with variance that evolves over time rather than remaining constant. As a result, classical approaches such as static regression models or fixed-parameter ARIMA specifications may struggle to adapt to structural changes and evolving market conditions. Although machine learning and deep learning techniques have recently gained popularity in Bitcoin price prediction, they often suffer from limited interpretability, high computational complexity, and the risk of overfitting, especially in short-horizon forecasting contexts (Jenifel et al., 2024; Martjanov et al., 2023).
State-space models provide an alternative and theoretically coherent framework for analyzing such non-stationary and evolving processes. By decomposing observed prices into latent components—such as level and trend—that evolve stochastically over time, these models allow parameters to adapt naturally to changing market regimes. Within this class, Dynamic Linear Models (DLMs) offer a parsimonious yet flexible representation of time-series dynamics, combining interpretability with full probabilistic inference through the Kalman filter (West & Harrison, 1997; Durbin & Koopman, 2012).
This study applies a Dynamic Linear Trend (DLT) model to the weekly Bitcoin open price, motivated by the series' strong upward drift, absence of clear seasonality, and evidence of near-integration. By explicitly modeling a time-varying level and slope, the proposed framework aims to capture the dominant trend-driven behavior of Bitcoin prices while quantifying forecast uncertainty in a transparent manner. Model adequacy and parsimony are evaluated using information criteria, enabling a systematic assessment of whether additional complexity yields meaningful improvements in fit.
By focusing on a simple yet robust state-space specification, this paper contributes to the growing literature on cryptocurrency forecasting by demonstrating that Dynamic Linear Models can deliver coherent short-term forecasts with calibrated uncertainty, while avoiding the opacity and over-

parameterization often associated with more complex nonlinear or machine learning approaches (Harvey, 1990; Prado & West, 2010; Catania & Grassi, 2017).

1st: Problem statement

This study investigates whether a parsimonious state-space model can adequately characterize and forecast the weekly Bitcoin open price given its trend-dominated dynamics, regime shifts, and heteroskedastic variance. We pose three related questions: (i) does first-differencing support near-integration (d=1) and remove low-frequency drift; (ii) can a Dynamic Linear Trend (DLT) capture a time-varying level/slope and deliver calibrated 12-week probabilistic forecasts; and (iii) what model dimension kkk balances fit and parsimony by AIC/BIC in both training and test splits? The aim is practical: obtain short-horizon forecasts with honest uncertainty while avoiding over-parameterization.

2nd: Literature Reviews

M. Geetha Jenifel,R.Anita Jasmine,D Umanandhini (2024), The paper does not specifically address dynamic linear models for Bitcoin. Instead, it focuses on comparing various machine learning models, including Linear Regression, for predicting Bitcoin prices, evaluating their performance through metrics like MAE, MSE, and R-squared[1].
Dmytro Martjanov,Yaroslav Vyklyuk,Mariya Fleychuk (2023) ,The paper utilizes econometric estimation tools and machine learning models to analyze Bitcoin's market dynamics, employing time series decomposition and lagged shifts of financial indicators to enhance forecasting accuracy, achieving an absolute deviation of $9.5 in short-term forecasts[2]
Leopoldo Catania,Stefano Grassi (2017), The paper develops a dynamic model for Bitcoin that incorporates long-memory, asymmetries in volatility, and time-varying skewness and kurtosis, enhancing predictions for volatility, density, and quantiles, crucial for asset allocation and risk management in cryptocurrency markets.[3]
María Teresa Barrio López, Andrea King-Domínguez, Luis Améstica-Rivas (2023),The study employs a dynamic approach combining multiple linear regression and neural networks to predict Bitcoin prices, optimizing variable selection and data usage to adapt to market volatility, ultimately achieving an 88% prediction accuracy.[4]
Melike Bildirici, Yasemen Uçan, Ramazan Tekercioğlu (2024), The paper does not specifically address dynamic linear models for Bitcoin. Instead, it focuses on hybrid models combining Lie methods and LSTM networks, demonstrating their effectiveness in predicting Bitcoin returns amidst chaos, entropy, and complexity.[5]
Rajan Lal Karna,Bishal Babu Rajbanshi,Pradeepta Mishra(2024) ,The research employs dynamic linear models, specifically ARIMA, alongside deep learning techniques like LSTM and GRU, to analyze and predict Bitcoin prices. This approach enhances prediction accuracy and uncovers underlying patterns in Bitcoin's price fluctuations.[6]
Ersın Sünbül,Hamide ÖZYÜREK(2024),The paper evaluates linear forecasting methods, specifically the Autoregressive Integrated Moving Average (ARIMA), for Bitcoin price predictions. However, it does not specifically address dynamic linear models, focusing instead on ARIMA and Multilayer Perceptron Neural Networks.[7]
Maged Farouk,Nashwa Shaker,Diaa Salama Abdelminaam,Omnia Elrashidy,Lana Mandour,Malak Mesbah,Jana Walid,Mariam Ahmed,Rawan Attia,Nouran Ahmed,Reda Elazab(2024) , The paper does not specifically address dynamic linear models for Bitcoin. However, it discusses challenges in machine learning for Bitcoin price prediction, including model complexities and dynamic adaptation, which may relate to dynamic modeling approaches in financial forecasting.[8]

3rd: Methodology

1. Introduction

Dynamic linear models Statistical analysis of time series data is usually faced with the fact that we have only one realization of a process whose properties might not be fully understood. We need to assume that some distributional properties of the process that generate the observations do not change with time. In linear trend analysis, for example, we assume that there is an underlying change in the background mean that stays approximately constant over time. Dynamic regression avoids this by explicitly allowing temporal variability in the regression coefficients and by letting some of the system properties to change in time. Furthermore, the use of unobservable state variables allows direct modelling of the processes that are driving the observed variability, such as seasonal variation or external forcing, and we can explicitly allow some modelling error.
Dynamic regression can be formulated in very general terms by using a state space representation of the observations and the hidden state of the system. With sequential definition of the processes, having conditional dependence only on the previous time step, the classical recursive Kalman filter algorithms can be used to estimate the model states given the observations. When the operators involved in the definition of the system are linear we have so called dynamic linear model (DLM). A basic model for time series in geodetic or more general environmental applications consists of four elements: a slowly varying background level, a seasonal component, external forcing from known processes modelled by proxy variables, and stochastic noise. The noise component might contain an autoregressive structure to account for temporally correlated model residuals. As we see, the basic components have some physical justification and we might be interested in their contribution to the overall variability and their temporal changes. These components are hidden in the sense that we do not observe them directly and each individual component is masked by various other sources of variability in the observations. Below, we briefly describe the use of dynamic linear models in time series analysis. The examples deal with univariate time series, i.e. the observation at a singe time instance is a scalar, but the framework and the computer code can handle multivariate data, too. All the model equations are written in way that support multivariate observations. In the presented applications we are mostly interested in extracting the components related to the trends and using these to infer about their magnitude and the uncertainties involved. However, these models might not be so good for produce predictions about the behavior of the system in the future, although understanding the system is a first step to be able to make predictions. The use of DLMs in time series analysis is well documented in statistical literature, but they might go by different terminology and notation. In Harvey (1991) they are called structural time series, Durbin and Koopman (2012) uses the state space approach, and the acronym DLM is used in Petris et al (2009).

2. Stationary and Non-Stationary Time Series

A time series is a sequence of observations recorded over time. An essential step in time-series analysis is determining whether the series is stationary or non-stationary, as this strongly affects modeling, inference, and forecasting.

A. Stationary Time Series

A time series is stationary if its statistical properties do not change over time. In the commonly used weak (second-order) sense, stationarity requires:

- a constant mean,
- a constant variance,
- a covariance structure that depends only on the time lag, not on the specific time period.

Stationary series fluctuate around a stable level and exhibit consistent variability. A special case is the white-noise process, which has zero mean, constant variance, and no autocorrelation at non-zero lags. Stationary processes are fundamental because many classical models (AR, MA, ARMA) are valid only under stationarity assumptions.

B. Non-Stationary Time Series

A non-stationary time series has statistical properties that change over time. Common features include:

- trends in the mean,
- changing variance (heteroskedasticity),
- persistent shocks whose effects do not disappear over time.

Most economic and social time series are non-stationary. Two main types are distinguished:

- **Trend-stationary (TS) processes**, where non-stationarity arises from a deterministic trend that can be removed by detrending.
- **Difference-stationary (DS) processes**, where non-stationarity is stochastic and stationarity is achieved by differencing the series.

(1) Achieving Stationarity

Non-stationary series can often be transformed into stationary ones using:

- logarithmic or Box–Cox transformations (to stabilize variance),
- differencing (to remove stochastic trends),
- detrending (to remove deterministic trends).

Once stationarity is achieved, standard time-series models and forecasting techniques can be reliably applied.

(2) Importance

Stationarity is crucial because it ensures stable relationships over time, valid parameter estimation, and meaningful forecasts. Identifying and correcting non-stationarity is therefore a foundational step in time-series analysis.

C. Augmented Dickey-Fuller test

The ADF test checks whether a time series has a unit root.

- Null hypothesis (H_0): The series has a unit root → it is non-stationary.
- Alternative hypothesis (H_1): The series does not have a unit root → it is stationary.

The ADF regression equation is:

$$\Delta y_t = \alpha + \beta t + \gamma y_{t-1} + \sum_{i=1}^{p} \delta_i \Delta y_{t-1} + \varepsilon_t \qquad (1)$$

Where

$\Delta y_t = y_t - y_{t-1}$ is the first difference of the series.

t is a time trend (optional).

p is the number of lagged differences included to remove autocorrelation.

γ is the key coefficient: if $\gamma = 0$, the series has a unit root.

3. What is a Dynamic Linear Model (DLM)?

A Dynamic Linear Model is a type of state-space model used for time series analysis. Its core idea is that the observed data is a function of some underlying, unobserved "state" that evolves over time according to a probabilistic law.

The Mathematical Framework: Two Equations

Every DLM is defined by two sets of equations:

A. The Observation Equation

$$y_t = F_t \theta_t + v_t \ , \qquad v_t \sim N(0, V_t) \qquad (2)$$

where

y_t: The observed value at time t. This represents the data actually collected from the system or process under study.

θ_t : The *state vector* at time t. It contains the latent (unobserved) quantities that characterize the underlying structure of the process, such as level, trend, or seasonal effects

F_t : The *observation (design) matrix*. This matrix defines how the latent state vector maps to the observed data. It selects and weights the relevant components of $\theta_t$that contribute to the observation at time t

v_t: The *observation error* (measurement noise). It captures random deviations between the observed value and the systematic component $F_t\theta_t$

$v_t \sim N(0, V_t)$: The observation error is assumed to follow a normal distribution with mean zero and variance V_t, reflecting unbiased measurement error with known (or estimable) variability.

B. The State Evolution (System) Equation

$\theta_t = G_t\theta_t + w_t$, $w_t \sim N(0, W_t)$ (3)

Where

θ_t: The state vector at time t. It represents the latent characteristics of the process, such as level, trend, or seasonal components.

G_t: The *state evolution (transition) matrix*. This matrix governs how the state vector evolves from time $t - 1$to time t. It encodes the assumed dynamics of the system, for example, persistence of a level or propagation of a trend.

ω_t: The *system (process) noise*. This term captures random disturbances or structural changes affecting the state itself, such as unanticipated shocks or model imperfections.

$\omega_t \sim \mathcal{N}(0, W_t)$: The system noise is assumed to be normally distributed with mean zero and covariance matrix W_t, reflecting unbiased random variation in the state evolution.

(1) Key Assumptions

- The observation noise sequence $\{v_t\}$and the system noise sequence $\{\omega_t\}$are mutually independent.
- Both noise sequences are independent across time.
- The noise terms are independent of the initial state θ_0.

Together with the observation equation, this evolution equation completes the state-space representation of a Dynamic Linear Model, providing a probabilistic description of both the hidden state dynamics and the data-generating process.

(2) The Kalman Filter: The Engine of the Dynamic Linear Model

In a Dynamic Linear Model, the true state vector $\theta_t$is unobserved. The **Kalman Filter** provides an optimal recursive algorithm (under Gaussian assumptions) for sequentially estimating this hidden state as new data become available. It proceeds through two fundamental steps at each time point t.

(A) Forecast Step (Prediction)

Given all observations up to time $t - 1$, denoted $y_{1:t-1}$, the filter predicts both the next state and the corresponding observation.

State prediction

$\theta_t \mid y_{1:t-1} \sim \mathcal{N}(a_t, R_t)$ (4)

where

- $a_t = G_t m_{t-1}$is the prior mean of the state,
- $R_t = G_t C_{t-1} G_t^\top + W_t$is the prior covariance matrix.

Observation prediction

$y_t \mid y_{1:t-1} \sim \mathcal{N}(f_t, Q_t)$ (5)

where

$f_t = F_t a_t$is the one-step-ahead forecast mean,

$Q_t = F_t R_t F_t^\top + V_t$is the forecast variance.

This step propagates uncertainty forward in time according to the assumed system dynamics.

(B) Update Step (Filtering)

Once the new observation y_t is observed, the filter updates its belief about the current state by combining the forecast with the new information.
State update

$$\theta_t \mid y_{1:t} \sim \mathcal{N}(m_t, C_t) \quad (6)$$

where
$m_t = a_t + K_t(y_t - f_t)$ is the posterior mean,
$C_t = R_t - K_t Q_t K_t^{\top}$ is the posterior covariance,
$K_t = R_t F_t^{\top} Q_t^{-1}$ is the Kalman gain, which determines how much weight is given to the new observation relative to the prior forecast.
Key Properties

- The Kalman Filter is recursive, requiring only the previous posterior distribution and the current observation.
- It is computationally efficient, making it suitable for real-time and large-scale applications.
- Under linearity and Gaussian noise assumptions, it yields the minimum mean square error (MMSE) estimator of the state.

Together, the forecast and update steps provide a complete sequential inference mechanism for Dynamic Linear Models.

(3) Common Types of Dynamic Linear Models (DLMs)

The flexibility of Dynamic Linear Models lies in the specification of the observation matrix F_t and the state evolution matrix G_t. By appropriately choosing these components, a wide range of stochastic processes can be modeled within a unified state-space framework.

(A) Local Level Model

The local level model is the simplest form of a DLM. The state consists of a single component representing the underlying level of the series.

Model specification

- Observation equation:

$$y_t = \theta_t + \nu_t \quad (7)$$

- State evolution equation:

$$\theta_t = \theta_{t-1} + \omega_t \quad (8)$$

- Here, $F_t = 1$ and $G_t = 1$.

Interpretation

- The state follows a random walk, allowing the level to evolve gradually over time.
- Variability in ν_t reflects measurement noise, while variability in ω_t controls how rapidly the level can change.

Use case

Suitable for relatively stable processes whose mean changes slowly, such as environmental measurements or economic indicators with gradual drift.

(B) Linear Growth Model (Local Level + Trend)

The linear growth model extends the local level model by incorporating a stochastic trend component.
State vector

$$\theta_t = \begin{bmatrix} \text{level}_t \\ \text{trend}_t \end{bmatrix} \tag{9}$$

Model specification

- Observation equation:

$$y_t = [1 \quad 0]\, \theta_t + v_t \tag{10}$$

- State evolution equations:

$$\begin{aligned} \text{level}_t &= \text{level}_{t-1} + \text{trend}_{t-1} + \omega_t^{(1)}, \\ \text{trend}_t &= \text{trend}_{t-1} + \omega_t^{(2)}. \end{aligned} \tag{11}$$

Evolution matrix

$$G = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \tag{12}$$

Interpretation

- he level evolves according to the previous level and trend.
- The trend itself is allowed to change stochastically over time.

Use case

- Appropriate for series exhibiting persistent growth or decline with evolving slope, such as sales figures, economic output, or financial time series.

(C) Seasonal Dynamic Linear Model

Seasonality can be incorporated into a DLM by augmenting the state vector with seasonal components.

Example (quarterly seasonality, $s = 4$)

$$\theta_t = [\text{level,trend}, \text{seasonal}_1, \text{seasonal}_2, \text{seasonal}_3]'$$

- Key feature

Seasonal effects are typically constrained so that their sum over a full cycle equals zero, ensuring identifiability and preventing confounding with the level component.

- Interpretation

Seasonal states evolve over time, allowing seasonal patterns to change gradually rather than remaining fixed.

- Use case

Time series with recurring seasonal behavior, such as quarterly economic indicators, monthly demand data, or climatological series.

4. Advantages of Dynamic Linear Models (DLMs)

Dynamic Linear Models offer several important theoretical and practical advantages that make them particularly suitable for modern time-series analysis and forecasting.

A. Handling Missing Data

DLMs handle missing observations in a natural and principled way. When an observation $y_t$is unavailable, the Kalman filter simply omits the update step and propagates the state forward using the evolution equation. No ad hoc imputation is required, and uncertainty is automatically increased to reflect the lack of new information.

B. Recursive and Computationally Efficient Processing

DLMs process observations sequentially, updating the state estimate one data point at a time. This recursive structure makes them computationally efficient and well suited for:

- online learning,
- real-time monitoring,
- streaming data applications.

The computational cost grows linearly with time, rather than with the full data history.

C. High Modeling Flexibility

DLMs provide a unified and extensible framework that encompasses many classical time-series models as special cases, including:

- exponential smoothing models,
- regression models with time-varying coefficients,
- local level and trend models,
- seasonal and structural time-series models,
- approximations to ARIMA models.

By modifying the state vector and the system matrices $F_t$and G_t, complex dynamics can be modeled in a transparent and interpretable manner.

D. Full Uncertainty Quantification

Unlike purely deterministic or point-estimation methods, DLMs yield full posterior distributions for both:

- the latent states,
- future observations.

As a result, they naturally provide credible intervals (confidence intervals) for state estimates and forecasts, allowing principled assessment of uncertainty and risk.

5. Disadvantages of Dynamic Linear Models (DLMs)

Despite their flexibility and strong theoretical foundations, Dynamic Linear Models have several important limitations that should be considered when applying them in practice.

A. Linearity and Gaussian Assumptions

Classical DLMs rely on linear state and observation equations and assume normally distributed system and observation noise. These assumptions may be restrictive for many real-world processes exhibiting:

- non-linear dynamics,
- heavy-tailed or skewed noise distributions,
- abrupt regime changes.

Although extensions such as non-linear state-space models and particle filtering methods exist, they are computationally more demanding and theoretically more complex.

B. Model Specification and Subjectivity

The analyst must specify:

- the structure of the observation and evolution matrices $F_t$and G_t,
- the covariance matrices V_t(observation noise) and W_t(system noise).

These choices often depend on prior knowledge or modeling judgment and can substantially influence the results. Misspecification may lead to biased state estimates or poor forecasting performance, particularly when the underlying system dynamics are not well understood.

C. Computational Cost in High Dimensions

While the Kalman filter is efficient for low- to moderate-dimensional state vectors, computational complexity increases rapidly with state dimension due to matrix multiplications and inversions. In very high-dimensional systems, this can result in:

- increased computation time,
- numerical instability,
- memory constraints.

In such cases, dimension reduction techniques or approximate filtering methods may be required.

6. Forecasting in Time Series Analysis

Forecasting in time series analysis refers to the process of predicting future values of a variable based solely on its historical behavior. In economic and financial applications, forecasting aims to capture systematic patterns such as trends, volatility clustering, and structural changes over time.
Formally, if $\{\boldsymbol{y_t}\}$denotes a time series observed up to time $\boldsymbol{T}$, forecasting involves estimating:

$$\hat{\boldsymbol{y}}_{\boldsymbol{T+h}} = \boldsymbol{E}(\boldsymbol{y}_{\boldsymbol{T+h}} \mid \boldsymbol{\mathcal{F}_T}) \tag{13}$$

where $\boldsymbol{\mathcal{F}_T}$represents the information set available at time $\boldsymbol{T}$, and $\boldsymbol{h}$is the forecast horizon.

A. Role of AIC and BIC in Forecasting

In time-series forecasting, the accuracy of predictions depends critically on selecting an appropriate model. The Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) are widely used model selection criteria that balance:

- Goodness of fit (via the likelihood function)
- Model complexity (number of parameters)

They do not directly forecast future values, but they determine which model is most suitable for producing reliable forecasts.

B. Akaike Information Criterion (AIC)

The AIC is defined as:

$$\text{AIC} = -2\ln(L) + 2k \tag{14}$$

where:
L is the maximized likelihood of the model,
k is the number of estimated parameters.

(1) Interpretation:

- AIC rewards better fit but penalizes excessive complexity.
- Among competing models, the one with the lowest AIC is preferred.
- AIC is particularly effective for forecasting purposes, as it focuses on minimizing information loss.

(2) In Time Series Forecasting:

- Frequently used to select ARIMA and GARCH orders.
- Tends to favor **more flexible models**, which may improve short-term forecasts.

C. Bayesian Information Criterion (BIC)

The BIC is defined as:

$$\text{BIC} = -2\ln(L) + k\ln(n) \tag{15}$$

where:
n is the sample size.

(1) Interpretation:

- BIC imposes a stronger penalty for model complexity than AIC.
- As sample size increases, complex models are penalized more heavily.
- The model with the lowest BIC is preferred.

(2) In Time Series Forecasting:

- BIC favors **parsimonious models**.
- Often preferred when the objective is **structural interpretation** rather than short-term prediction.
- Helps avoid overfitting, especially in small or moderate samples.

4th: Results and Discussions

The raw series displays a pronounced upward drift punctuated by corrections: an early-2023 drawdown to a trough, followed by a multi-phase rally with intermittent pullbacks and an acceleration into 2025 before a slight easing near the forecast start. No clear weekly seasonality is evident; instead, the dynamics look trend-dominated with regime changes and heteroskedastic variance. This structure motivates the modeling choices shown above: differencing to address near-integration, and a DLT state-space formulation to track a time-varying level and slope while quantifying forecast uncertainty through the model's probabilistic state evolution.

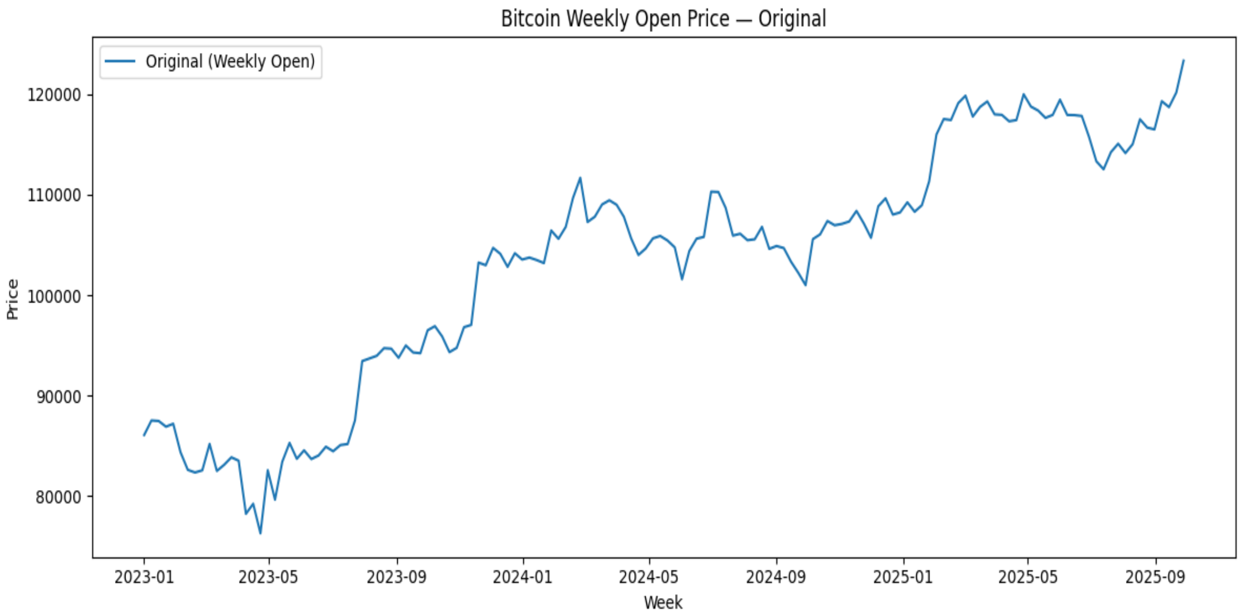

Figure (1): presents the raw data of weekly Bitcoin open price

The figure below shows the weekly first difference of the Bitcoin open price, a standard transformation to induce stationarity by removing the low-frequency trend component. The series is centred near zero with pronounced bursts of large positive and negative changes, indicating volatility clustering and heavy-tailed innovations typical of crypto assets. Visually, there is no obvious deterministic seasonality at the weekly cadence, and the differenced path looks mean-reverting, supporting an integration order of one (d=1). The wide spikes (both upward and downward) suggest conditional heteroskedasticity; inference and forecasting may therefore benefit from robust errors or volatility.

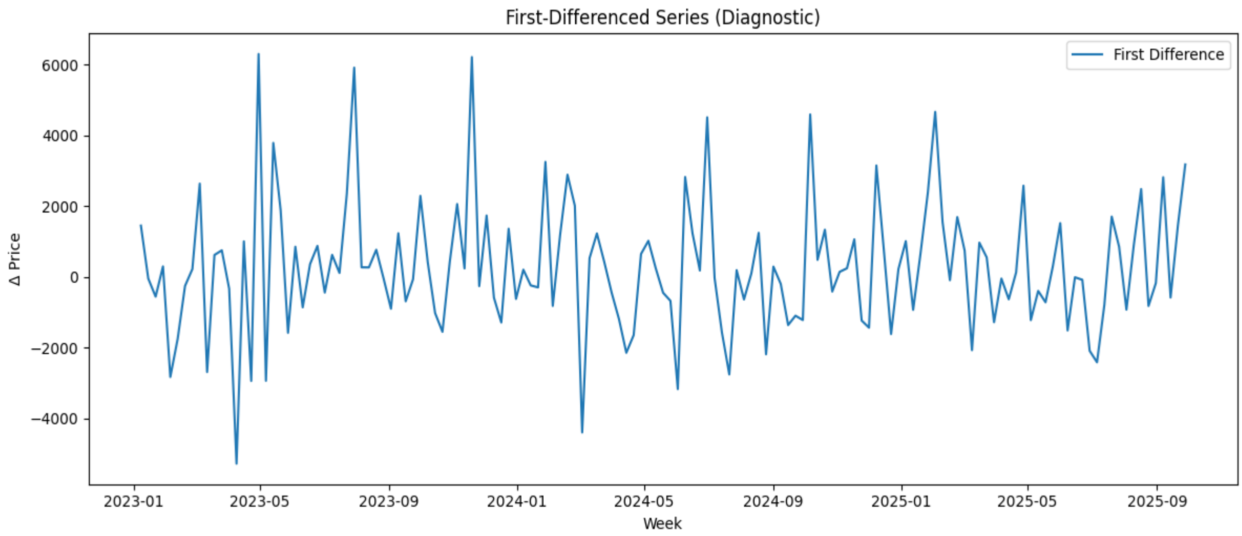

Figure (2): shows the first non-difference of the weekly dataset

Figure 3 demonstrates using a Dynamic Linear Trend (DLT) specification, the fitted state space model extrapolates a short-term path (orange) that gently mean-reverts from the recent high and then flattens, while the shaded 95% interval widens with horizon reflecting increasing parameter and process uncertainty. The central track sits around 111–113 thousand, but the predictive fan spans roughly from 98 thousand to 122 thousand by the end of the window, emphasizing that trend uncertainty dominates short-run point predictions. The downward inflection at the forecast origin indicates that the model has smoothed the most recent downturn in history and projects a soft correction rather than continuation of the late upswing, as shown in Figure 2.

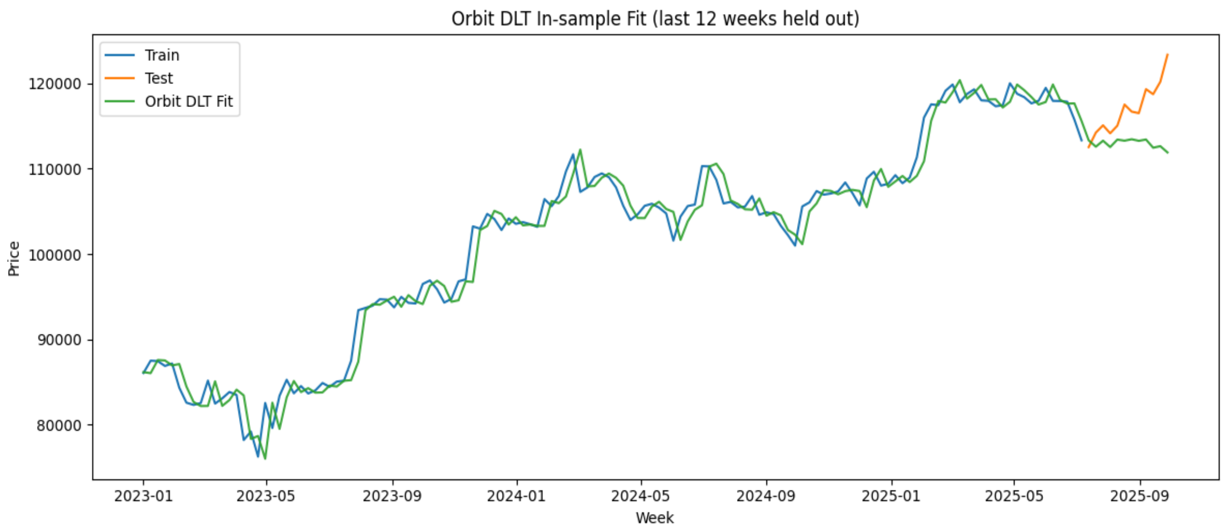

Figure (3)

Table (1)

Dataset	k	n	AIC	BIC
Training	3	132	2380.35	2389
	4		2382.35	2393.88
	5		2384.35	2398.76
Testing	3	12	244.9	246.41
	4		246.95	248.89
	5		248.95	251.38

Sum up the table, both AIC and BIC are **lowest at** k = 3 in training (2380.35, and 2389) and testing (244.90, and 246.41) respectively. Moving to k=4, 5 increases AIC by approximately 2, and BIC by *ln*(n) per step, essentially just the penalties showing no likelihood gain from extra parameters.

Summing to the below figure, the raw series displays a pronounced upward drift punctuated by corrections: an early-2023 drawdown to a trough, followed by a multi-phase rally with intermittent pullbacks and an acceleration into 2025 before a slight easing near the forecast start. No clear weekly seasonality is evident; instead, the dynamics look trend-dominated with regime changes and heteroskedastic variance. This structure motivates the modeling choices shown above: differencing to address near-integration, and a DLT state-space formulation to track a time-varying level and slope while quantifying forecast uncertainty through the model's probabilistic state evolution.

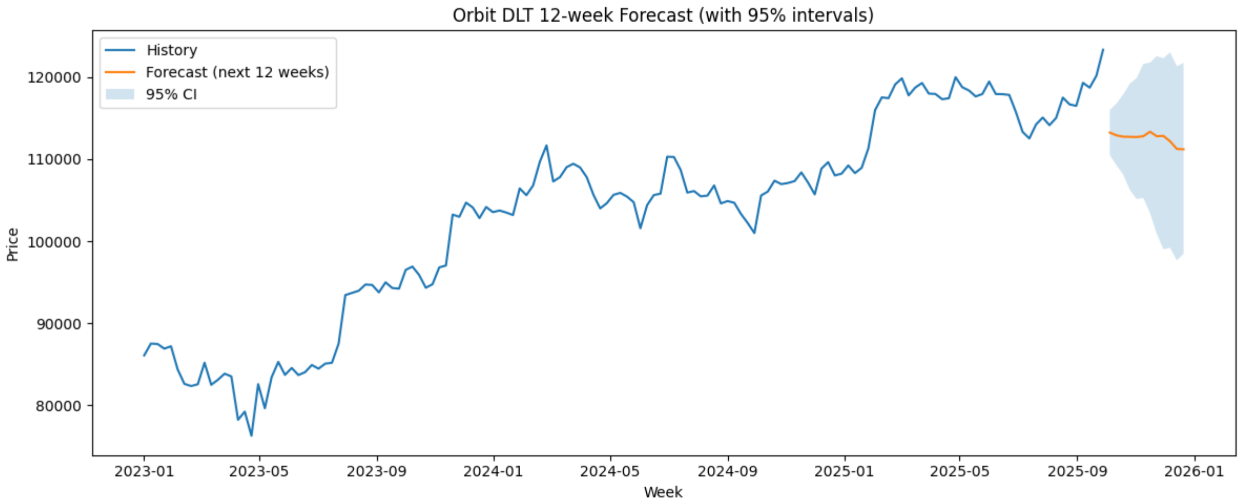

Figure (4):

Table (2)

Weeks	Forecasted Values	Confidence Interval	
		lower_95	upper_95
10/5/2025	113237.10	110495.73	115985.52
10/12/2025	112902.07	109248.02	116777.59
10/19/2025	112738.31	108122.63	117890.72
10/26/2025	112723.86	106236.08	119202.52
11/2/2025	112697.97	105134.50	119891.47
11/9/2025	112798.08	105267.49	121603.16
11/16/2025	113344.08	103437.71	121767.01
11/23/2025	112796.26	100921.33	122551.23
11/30/2025	112833.98	99001.72	122303.92
12/7/2025	112170.31	99189.70	123015.74
12/14/2025	111247.30	97654.74	121333.51
12/21/2025	111199.44	98459.92	121737.76

The above table reports the 12-week outlook is broadly flat with a slight drift down from 113.2 thousand (Oct 5) to 111.2 thousand (Dec 21), about **−1.8%.** Uncertainty widens over time (95% predictive bands grow from ±2.7 thousand to ±11.6 thousand), and downside risk increases, with the lower bound dipping **below 100 thousand** by late November.

Conclusions

Empirically, the raw series shows a pronounced upward drift punctuated by corrections, with no clear weekly seasonality; first differences appear mean-reverting, motivating an I(1) treatment. A DLT specification tracks the evolving level or slope and yields a flat-to-mildly declining 12-week path (−1.8%), with prediction bands that widen appropriately with horizon consistent with growing state and parameter uncertainty. Model selection via AIC or BIC on train/test partitions favors the simplest specification (k = 3), indicating that additional parameters do not improve likelihood enough to offset complexity penalties.

Limitations

Findings rest on a single modeling family (DLT) without explicit volatility dynamics; heavy-tailed shocks and conditional heteroskedasticity can make Gaussian state-space intervals optimistic. The evaluation window is short and untested under structural breaks typical of crypto markets; exogenous information (on-chain, macro, derivatives) was not incorporated.

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