Significant Properties of Subclass of Meromorphic Univalent Functions with Positive Coefficients by Associated with Linear Operator

Layth S.Ibrahaim & Hayder Kareem Gate

Ministry of Education, The General Directorate For Educational Qadisiyah,

laith.salem.ibrahim@gmail.com , haidarkarime@gmail.com

ليث سالم ابراهيم , حيدر كريم كاطع

وزارة التربية / المديرية العامة للتربية في محافظة القادسية

Abstract

In this study, study subclass $L(\vartheta, \curlyvee, \lambda, m)$ of the punctured unit for meromorphic univalent functions disk U^* which associated with linear operator will be introduced, and will be obtained coefficient estimates, distortion and growth theorems. Also we discuss some results related with δ −neighborhoods and radii of convexity and starlikeness.

Keywords and phrases : Linear operator, Meromorphic univalent function, Neighborhoods ,Distortion and growth theorems with convexity.

1. Introduction

The paper titled "Some Properties for Subclass of Meromorphic Univalent Functions with Positive Coefficients Associated with Linear Operator" focuses on the study of a subclass of meromorphic univalent functions defined in the unit disk associated with a linear operator. The authors investigate coefficient estimates, distortion and growth theorems, as well as results related to neighborhoods and convexity and radii of starlikeness. The paper contributes to the field of meromorphic functions and provides insights into the properties of this specific subclass.

Let T is class of all functions:

$$f(z) = z^{-1} + \sum_{k=1}^{\infty} a_k z^k \ , \ \ (a_k \geq 0 \, , k \in \mathbb{N} = \{1,2,\dots\}) \, , \qquad (1.1)$$

Meromorphic univalent and analytic in the punctured unit disk $\{z \in \mathbb{C}: |z| < 1\}$

The function $f \in T$ is like to be meromorphic starlike function with ρ $(0 \leq \rho < 1)$ if

$$|-Re\left\{\frac{zf'(z)}{f(z)}\right\} > \rho \, .$$

A function $f \in T$ is said to be meromorphic, is convex function of order ρ $(0 \leq \rho < 1)$ if

$$-Re\left\{1 + \frac{zf''(z)}{f'(z)}\right\} > \rho$$

In [7] Tehranchi and Kulkarni introduced the operator $I(m,\lambda): T \longrightarrow T$ by the following infinite series

$$I(m,\lambda)f(z) = z^{-1} + \sum_{k=1}^{\infty} \left(\frac{k+\lambda}{\lambda+1}\right)^m a_k z^k, \qquad (\lambda \geq 0, m \in \mathbb{Z}). \qquad (1.2)$$

Now, defining the class $L(\vartheta, \Upsilon, \lambda, m)$ that building the functions $f \in T$ such that

$$\left|\frac{z\big(I(m,\lambda)f(z)\big)'' + 2\big(I(m,\lambda)f(z)\big)'}{\Upsilon z\big(I(m,\lambda)f(z)\big)'' + [1 + \Upsilon - \vartheta(1-\Upsilon)]\big(I(m,\lambda)f(z)\big)'}\right| < 1 \, , \qquad (1.3)$$

with $0 \leq \vartheta < 1 \, , 0 \leq \Upsilon < 1 \, , \lambda \geq 0, m \in \mathbb{Z}$.

such this type of study was carried out by various authors for another classes such Srivastava et al. [6], Mostafa [2], Reddy et al. [4], Aouf [1], Wanas and Frasin [8].

2. Coefficient Estimates

Theorem 2.1. Suppose that $f \in T$. Then $f \in L(\vartheta, \Upsilon, \lambda, m)$ if and only if

$$\sum_{k=1}^{\infty} \frac{k(k+\lambda)^m[k(1+\Upsilon)-\vartheta(1-\Upsilon)+2]}{(\lambda+1)^m} a_k \leq (1-\vartheta)(1-\Upsilon)\,, \tag{2.1}$$

where $0 \leq \vartheta < 1\,, 0 \leq \Upsilon < 1, \lambda \geq 0, m \in \mathbb{Z}$.

The result is sharp for the function

$$f(z) = z^{-1} + \frac{(1-\vartheta)(1-\Upsilon)(\lambda+1)^m}{k(k+\lambda)^m[k(1+\Upsilon)-\vartheta(1-\Upsilon)+2]} z^k, \qquad (k \geq 1)\,. \tag{2.2}$$

Proof . Suppose that the equation (2.1) is true and $|z| = 1$. Then obtain

$$\left|z\big(I(m,\lambda)f(z)\big)'' + 2\big(I(m,\lambda)f(z)\big)'\right| - \left|\Upsilon z\big(I(m,\lambda)f(z)\big)'' + [1+\Upsilon-\vartheta(1-\Upsilon)]\big(I(m,\lambda)f(z)\big)'\right|$$

$$= \left|\sum_{k=1}^{\infty} k(k+1)\left(\frac{k+\lambda}{\lambda+1}\right)^m a_k z^{k-1}\right||$$

$$- \left|-(1-\vartheta)(1-\Upsilon)z^{-2} + \sum_{k=1}^{\infty} k[1+k\Upsilon-\vartheta(1-\Upsilon)]\left(\frac{k+\lambda}{\lambda+1}\right)^m a_k z^{k-1}\right|$$

$$= \left|\sum_{k=1}^{\infty} k(k+1)\left(\frac{k+\lambda}{\lambda+1}\right)^m a_k z^{k-1}\right|$$

$$- \left|(1-\vartheta)(1-\Upsilon)z^{-2} - \sum_{k=1}^{\infty} k[1+k\Upsilon-\vartheta(1-\Upsilon)]\left(\frac{k+\lambda}{\lambda+1}\right)^m a_k z^{k-1}\right|$$

$$\leq \sum_{k=1}^{\infty} k(k+1)\left(\frac{k+\lambda}{\lambda+1}\right)^m a_k |z|^{k-1} - (1-\vartheta)(1-\Upsilon)|z|^{-2}$$

$$+ \sum_{k=1}^{\infty} k[1+k\Upsilon-\vartheta(1-\Upsilon)]\left(\frac{k+\lambda}{\lambda+1}\right)^m a_k |z|^{k-1}$$

$$= \sum_{k=1}^{\infty} \frac{k(k+\lambda)^m[k(1+\Upsilon)-\vartheta(1-\Upsilon)+2]}{(\lambda+1)^m} a_k - (1-\vartheta)(1-\Upsilon) \leq 0\,,$$

by hypothesis .

Thus , by the principle maximum modulus, $f \in L(\vartheta, \Upsilon, \lambda, m)$.

To show the converse , suppose that $f \in L(\vartheta, \Upsilon, \lambda, m)$. Then from (1.3) , we have

$$\left|\frac{z\big(I(m,\lambda)f(z)\big)'' + 2\big(I(m,\lambda)f(z)\big)'}{\Upsilon z\big(I(m,\lambda)f(z)\big)'' + [1+\Upsilon-\vartheta(1-\Upsilon)]\big(I(m,\lambda)f(z)\big)'}\right|$$

$$= \left| \frac{\sum_{k=1}^{\infty} k(k+1)\left(\frac{k+\lambda}{\lambda+1}\right)^m a_k z^{k-1}}{(1-\vartheta)(1-\Upsilon)z^{-2} - \sum_{k=1}^{\infty} k[1+k\Upsilon-\vartheta(1-\Upsilon)]\left(\frac{k+\lambda}{\lambda+1}\right)^m a_k z^{k-1}} \right| < 1 .$$

Because $Re(z) \leq |z|$ for z ($z \in U$) , then

$$Re\left\{ \frac{\sum_{k=1}^{\infty} k(k+1)\left(\frac{k+\lambda}{\lambda+1}\right)^m a_k z^{k-1}}{(1-\vartheta)(1-\Upsilon)z^{-2} - \sum_{k=1}^{\infty} k[1+k\Upsilon-\vartheta(1-\Upsilon)]\left(\frac{k+\lambda}{\lambda+1}\right)^m a_k z^{k-1}} \right\} < 1 . \qquad (2.3)$$

By choosing the value of z on the real axis which it $\frac{z\left(I(m,\lambda)f(z)\right)''}{\left(I(m,\lambda)f(z)\right)'}$ is real. Depending the clearing the denominator of equation (2.3) and $z \to 1^-$, so we can obtain the equation (2.3) through which it the real values:

$$\sum_{k=1}^{\infty} \frac{k(k+\lambda)^m[k(1+\Upsilon)-\vartheta(1-\Upsilon)+2]}{(\lambda+1)^m} a_k \leq (1-\vartheta)(1-\Upsilon).$$

Sharpness of the results by:

$$f(z) = z^{-1} + \frac{(1-\vartheta)(1-\Upsilon)(\lambda+1)^m}{k(k+\lambda)^m[k(1+\Upsilon)-\vartheta(1-\Upsilon)+2]} z^k, \qquad (k \geq 1) .$$

Corollary 2.1. Let $f \in L(\vartheta, \Upsilon, \lambda, m)$. Then

$$a_k \leq \frac{(1-\vartheta)(1-\Upsilon)(\lambda+1)^m}{k(k+\lambda)^m[k(1+\Upsilon)-\vartheta(1-\Upsilon)+2]} , \qquad (k \geq 1) .$$

Theorem 2.2. The class $L(\vartheta, \Upsilon, \lambda, m)$ is a convex.

Proof . the arbitrary elements of $L(\vartheta, \Upsilon, \lambda, m)$ suppose that f_1 and f_2 is represented it. Then for t $(0 \leq t \leq 1)$, obtain $(1-t)f_1 + tf_2 \in L(\vartheta, \Upsilon, \lambda, m)$. Thus we have

$$(1-t)f_1 + tf_2 = z^{-1} + \sum_{k=1}^{\infty} [(1-t)a_k + tb_k] z^k .$$

Hence

$$\sum_{k=1}^{\infty} \frac{k(k+\lambda)^m[k(1+\Upsilon)-\vartheta(1-\Upsilon)+2]}{(\lambda+1)^m} [(1-t)a_k + tb_k]$$

$$= (1-t) \sum_{k=1}^{\infty} \frac{k(k+\lambda)^m[k(1+\Upsilon)-\vartheta(1-\Upsilon)+2]}{(\lambda+1)^m} a_k$$

$$+t\sum_{k=1}^{\infty}\frac{k(k+\lambda)^m[k(1+\Upsilon)-\vartheta(1-\Upsilon)+2]}{(\lambda+1)^m}b_k$$

$$\leq (1-t)(1-\vartheta)(1-\Upsilon)+t(1-\vartheta)(1-\Upsilon)=(1-\vartheta)(1-\Upsilon).$$

This completes the proof.

3. Distortion and Growth Theorems for the operator $I(m,\lambda)$

Theorem 3.1. If $f\in L(\vartheta,\Upsilon,\lambda,m)$, then

$$\frac{1}{r}-\frac{(1-\vartheta)(1-\Upsilon)}{\Upsilon-\vartheta(1-\Upsilon)+3}r\leq |I(m,\lambda)f(z)|\leq \frac{1}{r}+\frac{(1-\vartheta)(1-\Upsilon)}{\Upsilon-\vartheta(1-\Upsilon)+3}r \quad (|z|=r<1)\,. \tag{3.1}$$

The sharp result for the function are:

$$f(z)=z^{-1}+\frac{(1-\vartheta)(1-\Upsilon)}{\Upsilon-\vartheta(1-\Upsilon)+3}z\,. \tag{3.2}$$

Proof . Let $f\in L(\vartheta,\Upsilon,\lambda,m)$. Then by using 2.1 , obtain:

$$[\Upsilon-\vartheta(1-\Upsilon)+3]\sum_{k=1}^{\infty}a_k\leq\sum_{k=1}^{\infty}\frac{k(k+\lambda)^m[k(1+\Upsilon)-\vartheta(1-\Upsilon)+2]}{(\lambda+1)^m}a_k\leq(1-\vartheta)(1-\Upsilon),$$

or

$$\sum_{k=1}^{\infty}a_k\leq\frac{(1-\vartheta)(1-\Upsilon)}{\Upsilon-\vartheta(1-\Upsilon)+3}\,. \tag{3.3}$$

Hence

$$|I(m,\lambda)f(z)|\leq\frac{1}{|z|}+\sum_{k=1}^{\infty}\left(\frac{k+\lambda}{\lambda+1}\right)^m a_k|z|^k\leq\frac{1}{|z|}+|z|\sum_{k=1}^{\infty}a_k=\frac{1}{r}+r\sum_{k=1}^{\infty}a_k$$

$$\leq\frac{1}{r}+\frac{(1-\vartheta)(1-\Upsilon)}{\Upsilon-\vartheta(1-\Upsilon)+3}r. \tag{3.4}$$

Similarly ,

$$|I(m,\lambda)f(z)|\geq\frac{1}{|z|}-\sum_{k=1}^{\infty}\left(\frac{k+\lambda}{\lambda+1}\right)^m a_k|z|^k\geq\frac{1}{|z|}-|z|\sum_{k=1}^{\infty}a_k=\frac{1}{r}-r\sum_{k=1}^{\infty}a_k$$

$$\geq\frac{1}{r}-\frac{(1-\vartheta)(1-\Upsilon)}{\Upsilon-\vartheta(1-\Upsilon)+3}r. \tag{3.4}$$

From (3.4) and (3.5) , we have (3.1) and now the proof is completing**.**

Theorem 3.2. If $f\in L(\vartheta,\Upsilon,\lambda,m)$, then

$$\frac{1}{r^2}-\frac{(1-\vartheta)(1-\Upsilon)}{\Upsilon-\vartheta(1-\Upsilon)+3}\leq\left|\left(I(m,\lambda)f(z)\right)'\right|\leq\frac{1}{r^2}+\frac{(1-\vartheta)(1-\Upsilon)}{\Upsilon-\vartheta(1-\Upsilon)+3}\quad(|z|=r<1)\,.$$

The result is sharp and given by (3.2) .

Proof . The proof which obtained like to that of (3.1) .

4. Neighborhood for the class $L(\vartheta, \Upsilon, \lambda, m)$

Earlier works on neighborhoods of analytic functions was studied by ref. [3] and Ruscheweyh [5] , we begin by introducing the δ −neighborhood of a function $f \in T$ of the form (1.1) by the definition below :

$$N_\delta(f) = \left\{ g \in T : g(z) = z^{-1} + \sum_{k=1}^{\infty} b_k z^k \; and \; \sum_{k=1}^{\infty} k|a_k - b_k| \le \delta , 0 \le \delta < 1 \right\}. \qquad (4.1)$$

Particularly for the identity function $e(z) = z^{-1}$, we have

$$N_\delta(e) = \left\{ g \in T : g(z) = z^{-1} + \sum_{k=1}^{\infty} b_k z^k \; and \; \sum_{k=1}^{\infty} k|b_k| \le \delta \right\}. \qquad (4.2)$$

Definition 4.1. A function $f \in T$ is said to be in the class $L_y(\vartheta, \Upsilon, \lambda, m)$ if there exists a function $g \in L(\vartheta, \Upsilon, \lambda, m)$, such that

$$\left| \frac{f(z)}{g(z)} - 1 \right| < 1 - y \quad (z \in U, 0 \le y < 1) .$$

Theorem 4.1. If $g \in L(\vartheta, \Upsilon, \lambda, m)$ and

$$y = 1 - \frac{\delta[\Upsilon - \vartheta(1-\Upsilon) + 3]}{[\Upsilon - \vartheta(1-\Upsilon) + 3] - (1-\vartheta)(1-\Upsilon)} , \qquad (4.3)$$

then $N_\delta(g) \subset L_y(\vartheta, \Upsilon, \lambda, m)$.

proof . Let $f \in N_\delta(g)$. Then from (4.1) obtained that

$$\sum_{k=1}^{\infty} k|a_k - b_k| \le \delta,$$

the coefficient inequality implied by

$$\sum_{k=1}^{\infty} |a_k - b_k| \le \delta , \qquad (k \in \mathbb{N}).$$

Since $g \in L(\vartheta, \Upsilon, \lambda, m)$, then by using Theorem 2.1, we get

$$\sum_{k=1}^{\infty} b_k \le \frac{(1-\vartheta)(1-\Upsilon)}{\Upsilon - \vartheta(1-\Upsilon) + 3} ,$$

so that

$$\left|\frac{f(z)}{g(z)}-1\right| < \frac{\sum\limits_{k=1}^{\infty} |a_k - b_k|}{1-\sum\limits_{k=1}^{\infty} b_k} \leq \frac{\delta[\Upsilon-\vartheta(1-\Upsilon)+3]}{[\Upsilon-\vartheta(1-\Upsilon)+3]-(1-\vartheta)(1-\Upsilon)} = 1-y\,.$$

Hence , by Definition 4.1 , equivalently to $f \in L_y(\alpha, \Upsilon, \lambda, m)$ for y given by (4.3) .
This completes the proof.

Theorem 4.2. If

$$\delta = \frac{(1-\vartheta)(1-\Upsilon)}{\Upsilon-\vartheta(1-\Upsilon)+3},$$

then $L(\vartheta, \Upsilon, \lambda, m) \subset N_\delta(e)$.
proof . Let $f \in L(\vartheta, \Upsilon, \lambda, m)$. Then by using Theorem 2.1 , we have

$$[\Upsilon-\vartheta(1-\Upsilon)+3]\sum_{k=1}^{\infty} a_k \leq (1-\vartheta)(1-\Upsilon)\,. \tag{5.4}$$

On the other hand , from(2.1) and (5.4) that

$$\begin{aligned}\sum_{k=1}^{\infty} k a_k &\leq (1-\vartheta)(1-\Upsilon)-[\Upsilon-\vartheta(1-\Upsilon)+2]\sum_{k=1}^{\infty} a_k \\ &\leq (1-\vartheta)(1-\Upsilon)-[\Upsilon-\vartheta(1-\Upsilon)+2]\frac{(1-\vartheta)(1-\Upsilon)}{\Upsilon-\vartheta(1-\Upsilon)+3} \\ &= \frac{(1-\vartheta)(1-\Upsilon)}{\Upsilon-\vartheta(1-\Upsilon)+3}.\end{aligned}$$

That is

$$\sum_{k=1}^{\infty} k a_k \leq \frac{(1-\vartheta)(1-\Upsilon)}{\Upsilon-\vartheta(1-\Upsilon)+3} = \delta\,.$$

The definition given by (4.2) , and $f \in N_\delta(e)$.

5. Convexity and Radii of Starlike

Theorem 5.1. If $f \in L(\vartheta, \Upsilon, \lambda, m)$, then f is univalent meromorphic starlike of order φ $(0 \leq \varphi < 1)$ in the disk $|z| < R_1$, where

$$R_1 = \inf_k \left\{ \frac{k(1-\varphi)(k+\lambda)^m[k(1+\gamma)-\vartheta(1-\gamma)+2]}{(k-\varphi+2)(1-\vartheta)(1-\gamma)(\lambda+1)^m} \right\}^{\frac{1}{k+1}}$$

The result is sharp for the function f given by (2.2) .

proof . It is sufficient to show that

$$\left| \frac{zf'(z)}{f(z)} + 1 \right| \leq 1-\varphi \quad \text{for } |z| < R_1 . \tag{5.1}$$

But

$$\left| \frac{zf'(z)}{f(z)} + 1 \right| = \left| \frac{zf'(z)+f(z)}{f(z)} \right| \leq \frac{\sum_{k=1}^{\infty} (k+1)a_k|z|^{k+1}}{1-\sum_{k=1}^{\infty} a_k|z|^{k+1}} .$$

Thus (5.1) will be satisfied if

$$\frac{\sum_{k=1}^{\infty} (k+1)a_k|z|^{k+1}}{1-\sum_{k=1}^{\infty} a_k|z|^{k+1}} \leq 1-\varphi ,$$

or if

$$\sum_{k=1}^{\infty} \frac{(k-\varphi+2)}{1-\varphi} a_k|z|^{k+1} \leq 1 . \tag{5.2}$$

Since $f \in L(\vartheta,\gamma,\lambda,m)$, we have

$$\sum_{k=1}^{\infty} \frac{k(k+\lambda)^m[k(1+\gamma)-\vartheta(1-\gamma)+2]}{(1-\vartheta)(1-\gamma)\,(\lambda+1)^m} a_k \leq 1 .$$

Hence (5.2)will be true if

$$\frac{(k-\varphi+2)}{1-\varphi}|z|^{k+1} \leq \frac{k(k+\lambda)^m[k(1+\gamma)-\vartheta(1-\gamma)+2]}{(1-\vartheta)(1-\gamma)\,(\lambda+1)^m} ,$$

Since $f \in L(\vartheta,\gamma,\lambda,m)$, we have

$$\sum_{k=1}^{\infty} \frac{k(k+\lambda)^m[k(1+\gamma)-\vartheta(1-\gamma)+2]}{(1-\vartheta)(1-\gamma)\,(\lambda+1)^m} a_k \leq 1 .$$

Hence (5.2)will be true if

$$\frac{(k-\varphi+2)}{1-\varphi}|z|^{k+1} \leq \frac{k(k+\lambda)^m[k(1+\gamma)-\vartheta(1-\gamma)+2]}{(1-\vartheta)(1-\gamma)\,(\lambda+1)^m} ,$$

or equivalently

$$\|z| \leq \left\{ \frac{k(1-\varphi)(k+\lambda)^m[k(1+\gamma)-\vartheta(1-\gamma)+2]}{(k-\varphi+2)(1-\vartheta)(1-\gamma)(\lambda+1)^m} \right\}^{\frac{1}{k+1}} \quad (k \geq 1) ,$$

which follows the result .

Theorem 5.2. If $f \in L(\vartheta, ɤ, \lambda, m)$, then f is univalent meromorphic convex of order φ $(0 \leq \varphi < 1)$ in the disk $|z| < R_2$, where

$$R_2 = \inf_k \left\{ \frac{(1-\varphi)(k+\lambda)^m[k(1+ɤ) - \vartheta(1-ɤ) + 2]}{(k-\varphi+1)(1-\vartheta)(1-ɤ)(\lambda+1)^m} \right\}^{\frac{1}{k+1}}$$

The result is sharp for the function f given by (2.2) .

proof . It is sufficient to show that

$$\left| \frac{zf''(z)}{f'(z)} + 2 \right| \leq 1 - \varphi \quad \text{for } |z| < R_2 . \tag{5.3}$$

But

$$\left| \frac{zf''(z)}{f'(z)} + 2 \right| = \left| \frac{zf''(z) + 2f'(z)}{f'(z)} \right| \leq \frac{\sum\limits_{k=1}^{\infty} k^2 a_k |z|^{k+1}}{1 - \sum\limits_{k=1}^{\infty} k a_k |z|^{k+1}} .$$

Thus (5.3) will be satisfied if

$$\frac{\sum\limits_{k=1}^{\infty} k^2 a_k |z|^{k+1}}{1 - \sum\limits_{k=1}^{\infty} k a_k |z|^{k+1}} \leq 1 - \varphi ,$$

or if

$$\sum_{k=1}^{\infty} \frac{k(k-\varphi+1)}{1-\varphi} a_k |z|^{k+1} \leq 1 . \tag{5.4}$$

Since $f \in L(\vartheta, ɤ, \lambda, m)$, we have

$$\sum_{k=1}^{\infty} \frac{k(k+\lambda)^m[k(1+ɤ) - \vartheta(1-ɤ) + 2]}{(1-\vartheta)(1-ɤ)\,(\lambda+1)^m} a_k \leq 1 .$$

Hence (5.4)will be true if

$$\frac{k(k-\varphi+1)}{1-\varphi} |z|^{k+1} \leq \frac{k(k+\lambda)^m[k(1+ɤ) - \vartheta(1-ɤ) + 2]}{(1-\vartheta)(1-ɤ)(\lambda+1)^m} ,$$

or equivalently

$$|z| \leq \left\{ \frac{(1-\varphi)(k+\lambda)^m[k(1+ɤ) - \vartheta(1-ɤ) + 2]}{(k-\varphi+1)(1-\vartheta)(1-ɤ)(\lambda+1)^m} \right\}^{\frac{1}{k+1}} \quad (k \geq 1) ,$$

which follows the result .

6. Recommendations , Results , Conclusion:

6.1. Recommendations:

Based on the given document, it is recommended to further study and explore the subclass of meromorphic univalent functions defined in the punctured disk associated with the linear operator. This subclass shows potential for coefficient estimates, distortion and growth theorems, as well as results related to neighborhoods and radii of starlikeness and convexity. Further research can focus on investigating the properties and applications of this subclass in the field of complex analysis.

6.2. Results:

The document presents several results related to the subclass of meromorphic univalent functions. These results include coefficient estimates, distortion and growth theorems, as well as properties related to neighborhoods and radii of starlikeness and convexity. Theorems and proofs are provided to support these results, demonstrating the validity and significance of the findings.

6.3. Conclusion:

In conclusion, the document introduces and studies the subclass of meromorphic univalent functions defined in the punctured unit disk associated with the linear operator. The obtained results, including coefficient estimates, distortion and growth theorems, and properties related to neighborhoods and radii of starlikeness and convexity, contribute to the understanding and analysis of this subclass. Further research and exploration of this subclass can lead to additional insights and applications in the field of complex analysis

References

[1] M. K. Aouf, Subclass of Meromorphic Univalent Function Defined by Ruscheweyh Derivative with Fixed Second Coefficients, Southeast Asian Bulletin of Mathematics, 41(2017), 473–480.

[2] M. K. Aouf and A. O. Mostafa , Properties of some families of meromorphic multivalent functions associated with generalized hypergeometric functions , MATEMATIQKI VESNIK , 2011 , 17 pages .

[3] A. W. Goodman , Univalent functions and non-analytic curves , Proc. Amer. Math. Soc. , 8 (1975), 598-601.

[4] T. R. Reddy, R. B. Sharma and K. Saroja, A new subclass of meromorphic functions with positive coefficients, Indian Journal of Pure and Applied Mathematics, 44(1)(2013), 29-46.

[5] S. Ruscheweyh , Neighborhoods of univalent functions, Proc. Amer. Math. Soc. , 81(1981), 521-527.

[6] H. M. Srivastava , K. Suchithra , B. A. Stephen and S. Sivasubramanian , Inclusion and neighborhood properties of certain subclass of analytic and multivalent functions of complex order , J. Ineq. Pure Appl. Math. , 7(2006) , Article 191 , 8 pages .

[7] A. Tehranchi and S. R. Kulkarni , An application of differential subordination for the class of meromorphic multivalent function with complex order, J. Southeast Asian Bulletin of Mathematics ,32(2008), 379-392.

[8] A. K. Wanas and B. A. Frasin, Applications of fractional calculus for a certain subclass of multivalent analytic functions on complex Hilbert space, General Mathematics, 26(1-2)(2018), 11–23.