



**Tikrit Journal of Administrative  
and Economics Sciences**

مجلة تكريت للعلوم الإدارية والاقتصادية

EISSN: 3006-9149

PISSN: 1813-1719



**New Huber weighted function based on wavelet process in robust  
estimation: A Simulation Study**

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**Keywords:**

Wavelet thresholding, outliers, Multiple regression, robust estimation, simulation

**ARTICLE INFO**

**Article history:**

Received	19 May. 2025
Received in revised form	08 Jul. 2025
Accepted	08 Jul. 2025
Available online	31 Mar. 2026

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**Abstract:** One of the primary problems affecting the accuracy of linear regression parameter estimates is contamination, including noise and outliers. To reduce the impact of data contamination, a novel Huber-weighted function based on wavelet processing is evaluated in this study and compared with the classical Huber-weighted function. The efficiency of the suggested approach is demonstrated through simulation, which also compares it with the traditional Huber method using the root-mean-square errors of the calculated parameters. A simulation program was implemented using MATLAB. The study's findings demonstrated that the suggested approach effectively handled contamination (noise and outliers) and improved the precision of the multiple linear regression's parameter estimates when outliers were present.

## وزن هوبر جديدة بناءً على عملية الموجات في التقدير الحصين: دراسة محاكاة دالة

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### المستخلص

تعد التلوثات (مثل الضوضاء والقيم المتطرفة) من أبرز المشكلات التي تؤثر على دقة المعلمات المقدرة للانحدار الخطي. وللحد من تأثير هذه التلوثات في البيانات، تم في هذه الدراسة تقييم دالة وزنية جديدة من نوع هوبر، مبنية على عملية الموجات، ومقارنتها بالدالة الوزنية التقليدية لهوبر. وقد أظهرت نتائج المحاكاة كفاءة النهج المقترح، حيث تم اجراء مقارنة بينه وبين الطريقة التقليدية لهوبر باستخدام متوسط مربع الخطأ للمعلمات المقدرة. وقد تم تنفيذ برنامج محاكاة باستخدام بيئة MATLA. أظهرت نتائج فاعلية الطريقة المقترحة مع التلوثات (الضوضاء والقيم المتطرفة)، كما ساهمت في تحسين دقة تقدير معلمات الانحدار الخطي المتعدد في ظل وجود القيم المتطرفة. **الكلمات المفتاحية:** العتبة باستخدام الموجات، القيم الشاذة، الانحدار المتعدد، التقدير الحصين، المحاكات.

### 1. Introduction

A statistical method for examining and quantifying the link between one or more independent variables, also denoted as "predictors" or "explanatory variables," and a dependent variable, often referred to as the "outcome" or "response" variable, is the regression model. A regression model seeks a mathematical equation that best fits the observed data to predict the dependent variable's value from the independent variables' values. If all conditions are satisfied, ordinary least squares is the most effective approach for traditional multiple regression. However, if the data do not meet some of these assumptions, sample estimates and results may be misleading. Outliers specifically defy the assumption of least-squares regression that residuals are normally distributed. (Susanti et al., 2014). Since the weight function's shape is directly derived from the optimal control in this study, integrating efficiency into the weight function is difficult, as it must be expressed as a function in the equation. To increase robustness's role more generally, research focuses on how to express the weight function's rise and fall using robust function attributes. Because outliers or influential data can significantly affect regression findings, these regression domains are often studied (Choi, 2009).

However, when controlled homoscedasticity or potentially crossover heteroscedasticity was applied to the data, no prior discussion was started or

developed. In contrast to least square estimators, where all observations are assigned weight one, robust M estimators lower the weighted functions at the tails.

Wavelet analysis is one of the main areas of interest for regression analysis. The goal of wavelet regression is to minimise noise in a sampled function that has been contaminated by noise in general, and to deal with the problem of outlier values (Gürünlü Alma, 2011).

In this paper, the New Huber Weighted Function Based on Wavelet Process in Robust Estimation is a hybrid method that combines the Huber weight function (used in robust statistics to reduce the influence of outliers) with wavelet-based denoising (used in signal processing to separate noise from useful signals).

**2. Multiple Linear Regression Model:** Regression analysis is a statistical method used to explain the relationship between response and predictor variables. The models may also be used to determine the relative importance of the dependent variable compared to the independent variable. The following notation is used to denote a multiple regression model, which frequently includes numerous independent variables:

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i3} + \dots + \beta_k X_{ik} + \varepsilon_i \quad (1)$$

where,

$Y_i$  is the dependent variable.

$X_{i1}, X_{i2}, \dots, X_{ik}$  the independent variable's value on the  $i$ -th observation.

Regression parameters are  $\beta_0, \beta_1, \dots, \beta_k$ .

$\varepsilon_i$  is a normally distributed random variable.

A multiple linear regression model with  $n$  observations and  $k$  explanatory variables, for instance, might be described as follows:

$$Y = X\beta + \varepsilon \quad (2)$$

Where  $Y_{n \times 1}$  is a response vector,  $X_{n \times (k+1)}$  is a non-stochastic input matrix,  $\beta_{(k+1) \times 1}$  is an unknown coefficient vector and  $\varepsilon_{n \times 1}$  is an error vector distributed normally with  $E(\varepsilon) = 0$  and  $V(\varepsilon) = \sigma^2 I_n$  (Hastie et al., n.d, 2015)

One of the most popular approaches for estimating the model parameters in equation (3) is the Ordinary Least Squares (OLS) method. This approach minimises the Sum of Squares of Residuals (SSR), which is defined as follows: The least-squares method provides the best unbiased linear estimate.

$$SSR = (Y - X\beta)^T(Y - X\beta) \quad (3)$$

By minimising the sum of squares of the errors in equation (4), the OLS estimator is produced, which has the following form:

$$\hat{\beta} = (X^T X)^{-1} X^T Y \quad (4)$$

In many practical investigations, a linear regression model often includes a large number of explanatory variables. Despite its widespread use, the OLS approach has limitations in OLS estimations. (T. H. Ali & D. M., Salah, 2022)

**3. Outliers in Regression Analysis:** In regression analysis, outliers can have a significant effect on the dependent variable, distorting the findings and leading to misunderstandings. There are several approaches, each with advantages and disadvantages, for identifying and controlling these outliers. "Outliers increase residual variability and induce heteroscedasticity, which distorts parameter estimates. Outliers in the dependent variable can substantially affect regression analysis. By reducing the influence of outliers on the regression model, the Weighted Least Squares approach effectively resolves this problem and produces more accurate estimates. (Prasetya, R.P., 2022.)

This large residual is taken into account and significantly affects the average squared residual, which is minimised using the least-squares approach.

Usually, two distribution types are employed to produce contaminated data. The second type, known as the contaminating distribution  $f_{c(.)}$  and  $g$  is the ratio of contamination by distribution  $f_{c(.)}$ , followed by the distribution of an arbitrary observation, the first form, known as the primary distribution  $f_{dc(.)}$ . It produces clear data. (Fox, J. and Weisberg, S., 2002)

$$f_{mix}(.) = (1 - g) \times f_{dc(.)} + g \times f_{c(.)} \quad (5)$$

**4. Robust estimation in regression:** When outliers affect the model or the residual distribution is not normal, robust regression is employed. This approach is crucial for assessing data affected by outliers and creating models that can withstand them. Researchers examined the widely accepted notion that regression models fail to satisfy the regression assumptions and found that the transformation did not remove or reduce the impact of outliers, ultimately leading to skewed forecasts. The ideal approach in these situations is robust regression, which is impervious to the impact of outliers. To

identify outliers and produce results that are resilient to them, robust regression is used. When data is contaminated, there are several ways to improve the effectiveness of a linear regression model. These methods frequently rely on the concept of a robust estimator, which is resistant to the influence of anomalous data points. There is a wealth of literature in this field that offers several strategies for strengthening estimates in the face of severe observations. (Susanti et al., 2014).

**4-1. M – Estimation:** One trustworthy method for estimating Regression is M- estimation; the letter M in M -estimation stands for the estimate of the maximum likelihood type. M-estimation, first presented by Huber in 1964, is the most often used generic technique for robust regression and is almost as effective as OLS. When the error distribution is nonnormal, especially when errors are heavy-tailed, linear least-squares estimates may exhibit poor performance. Eliminating significant observations from the least-squares fit is one solution. Another strategy, known as robust regression, utilises a fitting criterion that is less susceptible to anomalous data than least squares. "M-estimation" refers to the most popular universal robust regression technique. This family of estimators may be thought of as a generalisation of maximum-likelihood estimation. (Gürünlü Alma, 2011)

Consider the linear model:

$$y_i = \alpha + \beta_1 x_{i1} + \beta_2 x_{i2} + \dots + \beta_k x_{ik} + \varepsilon_i \\ = \mathbf{x}_i \boldsymbol{\beta} + \varepsilon_i \quad (6)$$

for the  $i$ -th of  $n$  observations. Given an estimator  $\mathbf{b}$  for  $\boldsymbol{\beta}$ , the fitted model is

$$\hat{y}_i = a + b_1 x_{i1} + b_2 x_{i2} + \dots + b_k x_{ik} = \mathbf{x}_i \mathbf{b} \quad (7)$$

$y_i$ : is the actual observed value.

$\hat{y}_i$ : is the fitted value, obtained by substituting the predictor values into the estimated model coefficients  $b_1, b_2, \dots, b_k$ .

$\mathbf{x}_i$ : is the vector of predictor variables for observation  $i$ .

$\mathbf{b}$ : is the vector of estimated coefficients.

and the residuals are given by

$$e_i = y_i - \hat{y}_i \quad (8)$$

With M-estimation, the estimates  $\mathbf{b}$  is determined by minimising a particular objective function over all  $\mathbf{b}$ ,

$$\sum_{i=1}^n \rho(e_i) = \sum_{i=1}^n \rho(y_i - \mathbf{x}_i \mathbf{b}) \quad (9)$$

Each residual's contribution to the goal function is given by the function  $\rho$ .

The following characteristics should be present in a decent  $\rho$ .

- ❖ Always nonnegative,  $\rho(e) \geq 0$
- ❖ Equal to zero when its argument is zero,  $\rho(0) = 0$
- ❖ Symmetric,  $\rho(e) = \rho(-e)$
- ❖ Monotone in  $\rho(e_i) \geq \rho(e'_i)$  for  $|e_i| > |e'_i|$  (Fox, J. and Weisberg, S., 2002)

#### Objective Function in M-Estimation

The regression coefficients  $\beta$  are estimated by solving:

$$\min_{\beta} \sum_{i=1}^n \rho(e_i) \quad (10)$$

Where  $\rho(e_i)$  is a loss function, and  $e_i$  is the residual for observation  $i$ :

$$e_i = y_i - \hat{X}_i \beta \quad (11)$$

The derivative of  $\rho$  with respect to residuals, denoted as  $\psi(e_i)$ , determines the influence of each observation:

$$\psi(e_i) = \frac{\partial \rho(e_i)}{\partial (e_i)} \quad (12)$$

$\psi(e_i)$  is also known as the influence function, which controls how much an observation contributes to the solution. (Satman, 2013)

**5. Weighted function:** Weighted functions are employed in regression models to give data points different degrees of significance, which affects how well the model fits the data. When certain observations are more trustworthy or pertinent than others, this is very helpful. The typical weighted function types used in regression are listed below. A weighted function is, in general, a mathematical or computational function in which certain inputs or components are assigned a higher weight than others. These weights alter each element's influence on the result. Weighted functions are widely used in statistics, machine learning, economics, and other areas of optimisation. (Čížek, P., 2011.)

This modification enables more precise parameter estimation, especially in datasets where outliers can distort results. (Saputri, S., 2023)

**5-1. Huber weighting function:** A reliable substitute for standard square error loss, the Huber function limits the influence of outliers on regression estimates by lowering their contributions to the squared error loss. In robust regression, the Huber function is a loss function that effectively handles outliers. The Huber function combines the squared loss for small residuals with the absolute loss for large residuals, unlike the quadratic loss function, which is susceptible to extreme values (outliers).

The Huber weight function provides a compromise between the L1 and L2 norms, combining the robustness of least absolute deviations with the efficiency of least squares.

Huber's weight function combines the least absolute deviation for big residuals with the least squares for small residuals (Feng, Y. and Wu, Q., 2022)

$$W_{(u_i)} = \begin{cases} 1 & \text{for } |u_i| \leq c \\ \frac{c}{|u_i|} & \text{for } |u_i| > c \end{cases} \quad (13)$$

where

$u$ : is the residual.

$C$ : is a tuning constant.

**6. Wavelet process:** Wavelet transformations are used in wavelet processes to analyse signals, time series, and regression data. A signal can be decomposed into several frequency components and time scales using mathematical tools known as wavelet transforms. Programming languages or tools that facilitate wavelet analysis are commonly used when implementing wavelet processes. (Gençay, Selçuk & Whitherer, 2002)(Yu & Yao, 2017)

#### **6-1. Wavelet Types:**

1. Beylkin (18) wavelet.
2. Coiflet
3. Daubechies wavelet (2, 4, 6, 8, 10, 12, 14, 16, 18, 20).
4. Binomial- QMF.
5. Haar wavelet.
6. Symlet wavelet.

**6-2. Coiflet Wavelet:** The Coiflet wavelet is a type of wavelet function used in signal processing and image compression. It is designed to have a compact support in both time and frequency domains, making it useful for analysing signals with localised features. The Coiflet wavelet was proposed by Ingrid Daubechies, a Belgian mathematician, in the late 1980s. It is commonly used in applications such as image compression and signal processing. (T. H. Ali & D. M., Salah, 2022).

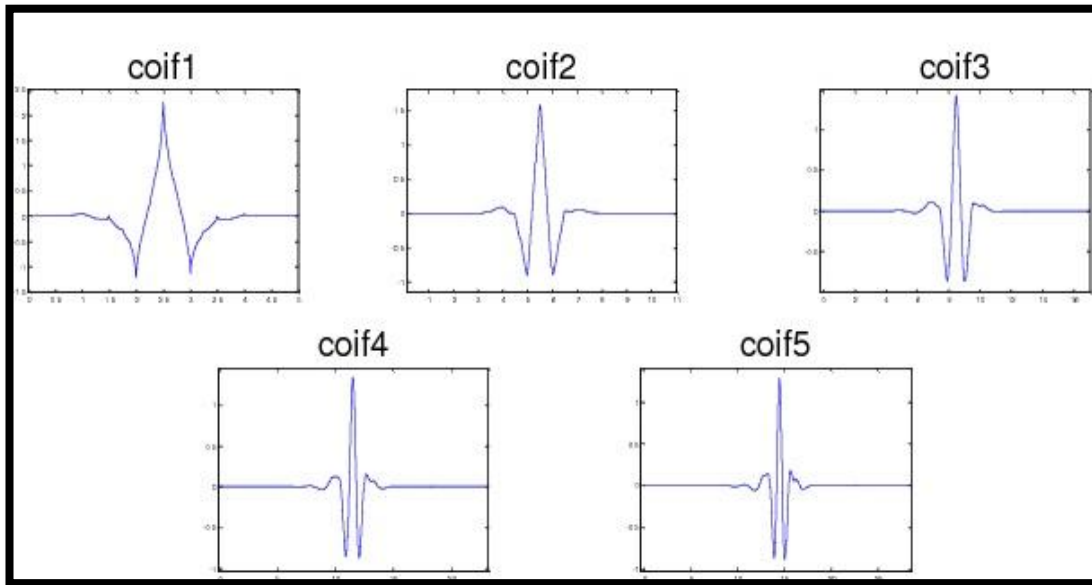


Figure 1: Coiflet Wavelet.

**6-3. Wavelet Transform:** A mathematical technique for signal processing and image compression is the wavelet transform. It breaks down a signal into distinct frequency components, enabling the simultaneous analysis of time and frequency data. When managing non-stationary signals with fluctuating frequency content, it is especially helpful. (Jianhui & Li, 2019)(Salh et al., 2021).

**6-4. Discrete Wavelet Transformation (DWT):** A popular observation-processing tool, the discrete wavelet transform (DWT) is used in various fields, including computer science, mathematics, engineering, and science. DWT provides a multiresolution representation of an observation by decomposing it into scaled and shifted copies of a compactly supported basis function (the mother wavelet). The DWT of  $y$ , as determined by the formula (14), is given as a vector of observations  $y$ , where  $k$  is an integer.

$$D = dy \tag{14}$$

$D$  is a vector with  $(n * 1)$  dimensions that includes both scaling and wavelet coefficients, where  $w$  is a wavelet matrix with  $(n * n)$  dimensions. It is possible to structure the wavelet coefficient vector into  $(k+1)$  vectors.

$D = [[d_1, d_2, \dots, d_k, V_{k0}]^T$ . The details are appended with the details of the most recent decomposition at each DWT after the approximation coefficients are separated into bands using the same wavelet as previously, as shown in the following formula:

$$Y = D d^T = \sum_{k=1}^{k_0} D_k^T D_k + V_{k_0}^T V_{k_0} \tag{15}$$

The inverse DWT can recreate the observations from the denoised data (lowering the contamination) at each level (k). (Qadir, J.R., 2022)

**6-5. Wavelet Shrinkage:** Wavelet shrinkage is a technique used in signal and image processing, particularly with the Discrete Wavelet Transform (DWT). It involves thresholding the wavelet coefficients to reduce or eliminate noise while preserving important signal features. (Sedeeq B. S., 2024).

**6-6. Thresholding method:** In signal and image processing, thresholding is a technique that sets values above or below a threshold to particular levels to simplify or segment data. For instance, thresholding successfully eliminates noise and preserves pertinent information in wavelet shrinkage by setting coefficients below a predetermined threshold to zero. (T. H. Ali & D. M., Salah, 2021).

**6-7. Thresholding rules:** In information and image processing, particularly in wavelet shrinkage, several thresholding methods are frequently employed. Two common types are (Daubechies, 1992):

1. Hard Thresholding:
2. Soft Thresholding:

The properties of the data and the intended balance between noise reduction and the preservation of significant features determine which thresholding rule is best.

**6-7-1. Soft Thresholding:** Soft thresholding is a method used in signal and image processing, especially for wavelet shrinkage. It entails reducing the magnitude of wavelet coefficients by a specified amount without setting them exactly to zero. The definition of the soft thresholding function is as follows (Kumar & Kusagur, 2017; Shahriari & Ahmadi, 2017).

$$D_n^{(s)} = \text{sign} \{D_n\} (|D_n| - \vartheta)$$

(16)

- 7. New Huber weighted function:** Researchers proposed a hybrid wavelet analysis with a Huber-weighted function (M-robust) to address contamination problems (outliers and noise) in the multiple linear regression model, which uses wavelets after thresholding. And then, using the outputs to find the inverse of the Discrete Wavelet Transformation (DWT), get filtered data, and later use this data, modified to estimate parameters by the M-Robust method of multiple linear regression models. Furthermore, RMSE can be calculated and compared with the aforementioned classical methods.

To remove outliers or noise from the values of observations of the response variable, one of the types of thresholds, such as (hard or soft), is usually used by shrinking the detail coefficients, which can be obtained by recovering the original observations and splitting them into two components using wavelets. The first represents the sum of the coefficient details, while the second represents the smoothing coefficients based on Multiple Resolution Analysis (MRA) (Sulaiman, N.G. and Rahim, A.G., 2022)(Amin, 2022)

$$Y = d D = \sum_{j=1}^{j_0} d_j \hat{D}_j + v_{j_0} \hat{V}_{j_0} \quad (17)$$

One of the well-known methods estimates the threshold level, including the fixed threshold method at level  $j = 1$  only ( $D_1$ ). The DWT coefficients of the modified wavelet, typically represented by ( $\tilde{D}$ ), can then be obtained by applying the soft threshold to the DWT coefficients and returning the remaining coefficients to the vector elements ( $D$ ). This allows recovery of the observations for the treating response variable.

$$Y_{\text{wavelet}} = d' \tilde{D} \quad (18)$$

Depending on the wavelet matrix, such as (coif2), the values of (observations of the processed dependent variable) can be obtained, and a weighted function based on wavelet analysis:

### 7-1. Huber weighted function based on wavelet process"

$$W_{(u_{\text{wavelet}(i)})} = \begin{cases} 1 & |u_{\text{wavelet}(i)}| \leq c \\ \frac{c}{|u_{\text{wavelet}(i)}|} & |u_{\text{wavelet}(i)}| < c \end{cases} \quad (19)$$

which are used with the independent variables in estimating the parameters of the multiple linear regression model, depending on the methods, i.e.:

$$\hat{\beta}_{\text{wavelet M-robust}} = (X'WX)^{-1}X'WY_{\text{wavelet}} \quad (20)$$

The following diagram represents a summary of the resultant steps of the wavelet downsizing procedure (Amin, 2022)

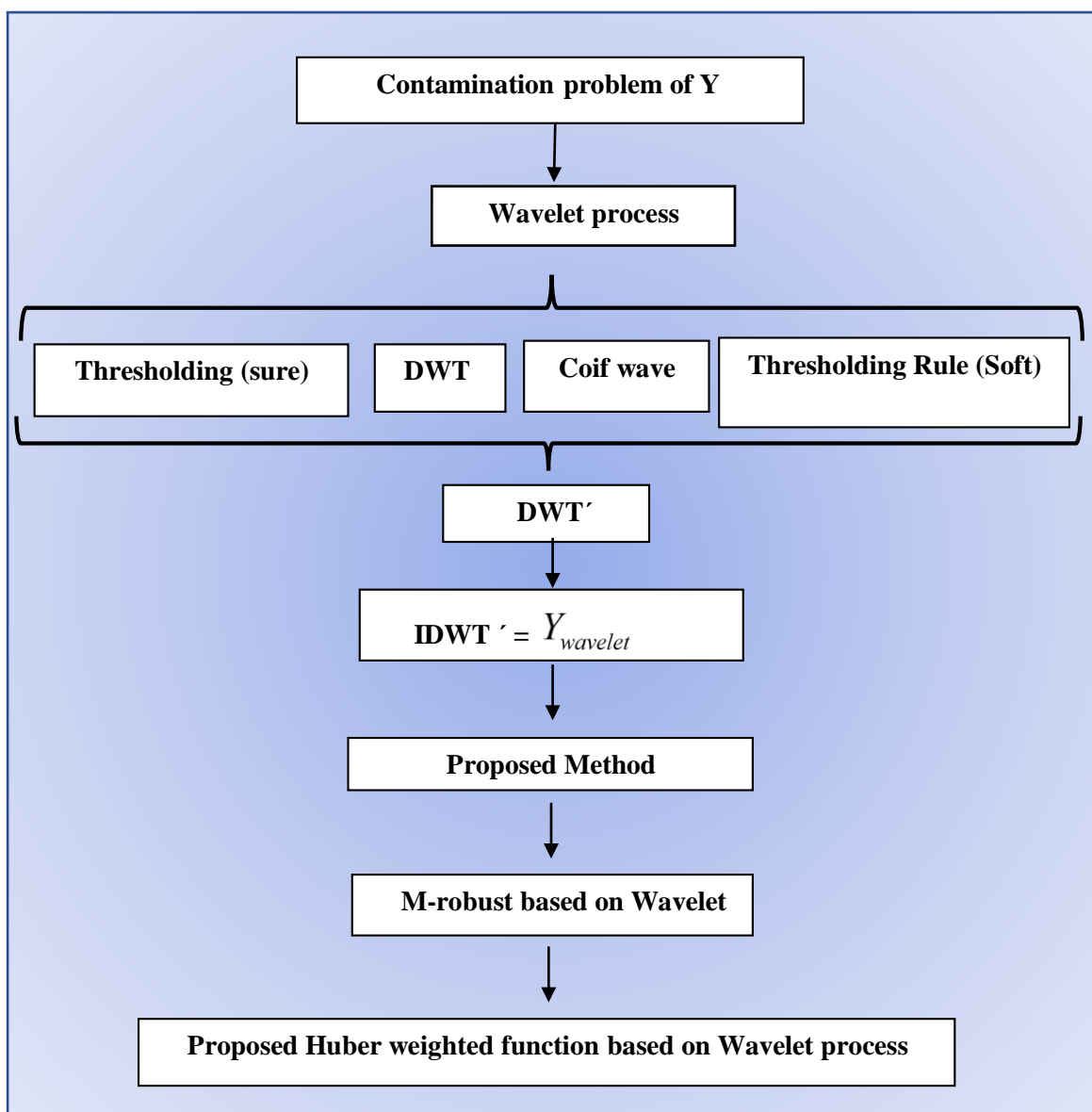


Diagram (1): proposed an algorithm

**8. Root Mean Squared Error (RMSE) criterion:** Model evaluation is one of the most crucial phases in developing a system model. When the model's objective is prediction, the mean squared error is a useful metric for evaluating its predictive quality. Regression models trained on different data samples can be compared using their mean squared errors (MSEs). A lower MSE indicates a higher quality regression model. The following is the name given to the MSE:

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{n}} \quad (21)$$

$y_i$ : is the actual value for the  $i$ -th observation.

$\hat{y}_i$ : is the predicted value for the  $i$ -th observation.

$n$ : is the total number of observations. (Sulaiman, N.G. and Rahim, A.G., 2022.)

**9. Application part:** In this chapter, we compare the Classical weighted function and the Improved weighted function based on wavelets. This analysis uses one of the robust methods, specifically Robust M-estimation, as robust regression relies heavily on the choice of weight function. Then we combined two functions to reduce contamination in our data. The weight function plays a critical role in Robust M-estimation for estimating parameters in multiple linear regression models, particularly in the presence of contamination.

The study involves an experimental phase using simulations to determine which weight function is more efficient for parameter estimation in robust regression. To assess the efficiency and accuracy of the estimated models, we use the comparative measure ARMSE (Average Root Mean Square Error). This is especially useful in simulation studies or when evaluating model robustness. The results are generated and analysed using statistical software tools, including MATLAB R2021b, which is employed for simulation.

**9-1. Simulation for Generating Random Data:** The combination under discussion is repeated (1000) times for various scenarios using simulation of the trials as follows:

1. Three sample sizes (100,200,400).
2. Standard deviation of the noise (1.5, 3).
3. The number of explanatory variables, denoted by k, can be 4 or 7.
4. The contamination at these rates (10% and 20%). Using the following formula:

$$f_{\text{mix}}(\cdot) = (1 - g) \times f_{\text{dc}}(\cdot) + g \times f_{\text{c}}(\cdot) \quad (22)$$

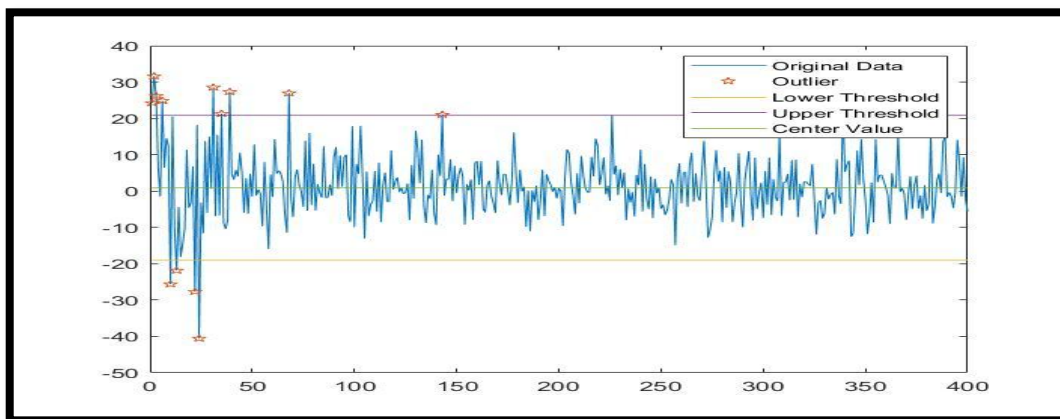


Figure (2) shows the outliers where (10%) contaminate,  $k=7$ , and  $\sigma=1.5$  for  $(n=400)$

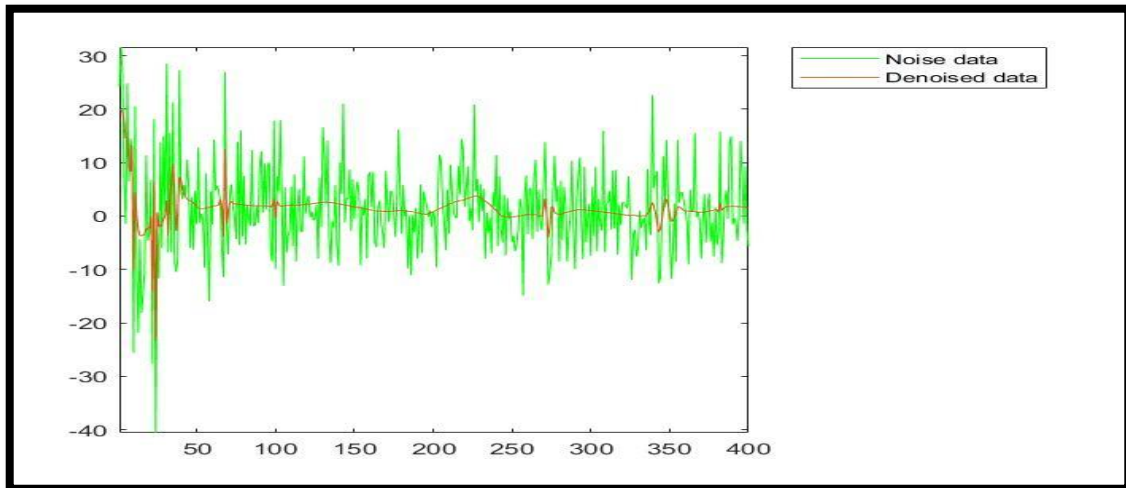


Figure (3) shows the Wavelet analysis where (10%) contamination,  $k=7$ , and  $\sigma=1.5$  for ( $n=400$ ). When we use this method, Robust M-Estimation, the replication is equal to (1000)

Table 1: ARMSE values for Robust M-Estimation for Improved Weighted Functions depend on Wavelet and Classical Weighted Functions when ( $\sigma = 1.5, k=4$ ),

	<b>Robust M-Estimation</b>		<b>ARMSE %10 contamination</b>	<b>ARMSE %20 contamination</b>
n=100	Classical Weighted Functions	Huber	2.8634	3.4289
	Improve Weighted Functions that depend on wavelets	Huber - Wavelet (coif 2)	2.5824	3.2029
n=200	Classical Weighted Functions	Huber	2.4818	2.8414
	Improve Weighted Functions that depend on wavelets	Huber - Wavelet (coif 2)	2.0928	2.5283
n=400	Classical Weighted Functions	Huber	2.1891	2.3728
	Improve Weighted Functions that depend on wavelets	Huber - Wavelet (coif 2)	1.6503	1.9589

This table shows the (ARMSE) value in different (Robust M-estimation) methods, when ( $\sigma = 1.5, k=4$ ) under the different sample sizes ( $n=100,200,400$ ) with contamination levels for simulation data (10%,

20%)—trusting in wavelet, while the proposed and classical methods are compared to choose the most convenient method. The smaller ARMSE is a more suitable measure when we compare models to estimate parameters.

At the 10% and 20% levels of contamination, across all sample sizes, it is observed that (Huber\_wavelet coif2) has the lowest (ARMSE) value, indicating that it is the most suitable method compared with the classical method.

Table 2: ARMSE values for Robust M-Estimation for Improved Weighted Functions depend on Wavelet and Classical Weighted Functions when ( $\sigma = 3, k=4$ )

	<b>Robust M- Estimation</b>		<b>ARMSE %10 contamination</b>	<b>ARMSE %20 contamination</b>
n=100	Classical Weighted Functions	Huber	5.7955	6.9568
	Improve Weighted Functions that depend on wavelets	Huber - Wavelet (coif 2)	4.6749	6.0721
n=200	Classical Weighted Functions	Huber	4.9712	5.6586
	Improve Weighted Functions that depend on wavelets	Huber - Wavelet (coif 2)	3.6241	4.5701
n=400	Classical Weighted Functions	Huber	4.3746	4.7357
	Improve Weighted Functions that depend on wavelets	Huber - Wavelet (coif 2)	2.7438	3.4684

In Table 2, when ( $\sigma = 3, k=4$ ) ARMSE values for two different forms of weighted functions, Classical Weighted Functions and Improved Weighted Functions based on wavelet (coif 2), are compared in the table "ARMSE values for Robust M-Estimation for Improved Weighted Functions depend on Wavelet and Classical Weighted Functions when contamination our data by %10, %20. The results show that the Improved Weighted Functions based on wavelet (coif 2) are better than the classical method, because the ARMSE in the Improved Weighted Functions based on wavelet

(coif 2) is smaller than the ARMSE in the Classical Weighted Functions in all cases for sample sizes (100, 200, 400).

Table 3: ARMSE values for Robust M-Estimation for Improved Weighted Functions depend on Wavelet and Classical Weighted Functions when ( $\sigma = 1.5, k=7$ )

	Robust M-Estimation		ARMSE %10 contamination	ARMSE %20 contamination
n=100	Classical Weighted Functions	Huber	3.6146	4.5300
	Improve Weighted Functions that depend on Wavelet	Huber - Wavelet (coif 2)	3.4087	4.2492
n=200	Classical Weighted Functions	Huber	3.0794	3.7849
	Improve Weighted Functions that depend on wavelets	Huber - Wavelet (coif 2)	2.7811	3.4621
n=400	Classical Weighted Functions	Huber	2.6176	3.0680
	Improve Weighted Functions that depend on wavelets	Huber - Wavelet (coif 2)	2.2055	2.7034

In this table When ( $\sigma = 1.5, k=7$ ), and for all sample sizes (100, 200, 400) and contaminated at 10% and 20% levels, the Average Root Mean Square Error (ARMSE) of the two robust estimation methods under different conditions (Classical Weighted Function (Huber)) and (Improved Weighted Functions (Huber-Wavelet, coif 2)) is smaller than the ARMSE values in Classical Weighted Functions, indicating better robustness against outliers. The data compares the Asymptotic Root Mean Squared Error (ARMSE) between Classical Huber and Huber-Wavelet (coif 2) methods across different sample sizes and contamination levels (10% and 20%). The Huber-Wavelet method consistently outperforms Classical Huber, with greater improvements observed at larger sample sizes.

Table 4: ARMSE values for Robust M-Estimation for Improved Weighted Functions depend on Wavelet and Classical Weighted Functions when ( $\sigma = 3, k=7$ )

	Robust M-Estimation		ARMSE %10 contamination	ARMSE %20 contamination
n=100	Classical Weighted Functions	Huber	7.3006	9.0979
	Improve Weighted Functions that depend on wavelets	Huber - Wavelet (coif 2)	6.0278	7.9964
n=200	Classical Weighted Functions	Huber	6.1759	7.6213
	Improve Weighted Functions that depend on wavelets	Huber - Wavelet (coif 2)	4.7844	6.5806
n=400	Classical Weighted Functions	Huber	5.2545	6.1030
	Improve Weighted Functions that depend on wavelets	Huber - Wavelet (coif 2)	3.7664	4.9597

Table 4 compares ARMSE values under conditions ( $\sigma = 3, k = 7$ ). Results show that the wavelet-based Huber function consistently outperforms the classical method in reducing error, especially at higher contamination levels. The analysis relies on wavelet-based Robust M-estimation techniques to determine the best parameters for both 10% and 20% contamination levels. The goal is to assess how effectively these methods handle contamination and achieve the smallest ARMSE values. The results indicate that the Improved Weighted Functions yield smaller ARMSE values than the Classical Weighted Function, making it the superior choice. Additionally, within the Improved Weighted Functions, the (Huber-Wavelet (coif 2)) approach stands out as the best method, as it produces the smallest ARMSE values for both 10% and 20% contamination levels, outperforming the Classical Weighted Function (Huber). Consequently, this method is the most suitable for handling contaminated data under these conditions.

**Compare Classical Huber and Wavelet-Huber in Robust Regression:**

Classical Huber	Wavelet-Huber
General-purpose robust estimation	Multiscale structured data
Moderate outlier contamination	Localised contamination
Low-dimensional parametric models	High-dimensional/nonparametric models

**10. Conclusion:** During the conduct of the simulated data and according to the results from the practical part, the following conclusions have been drawn:

- A. It is determined that the Huber weighted function based on the wavelet process is the most appropriate approach for all the examples utilised in this study, after comparing it with the Huber classical weighted function.
- B. The proposed method addresses contamination.
- C. However, the ARMSE value changes as the sample size increases.
4. ARMSE values decrease at a 10% contamination level, but increase slightly at 20%, although they remain lower than the classical method."

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