ON bi- INTUITIONSTIC TOPOLOGICAL SPACE

Taha H. Jassim[1] , Zenah T. Abdulqader[2] , Hiba O. Mousa[2]

[1] *Department of mathematics , College of Sinece , Tikrit University , Tikrit , Iraq*

[2] *Department of mathematics , College of education for women , Tikrit University , Tikrit , Iraq*

(Received: 26 / 2 / 2013 ---- Accepted: 22 / 4 / 2013)

Abstract:

In this paper we introduce a new definition is called bi- intuitionistic topological space and from this concept we present some kinds of closed set (semi –closed , pre-closed ,β-closed ,α-closed)setes in bi- intuitionistic topological space, define generalized closed set (sg-closed ,gs-closed ,gp-closed , gα-closed , αg- closed , gβ-closed) sets in bi - intuitionistic topological space and give relationship among them , we introduce the definition of T_{gs}-space in bi- intuitionistic topological space and from this consept we get new results .

1- Introduction and terminologies:

A triple (X ,T_i , T_j) (i ≠j) where X≠Ø and T_i , T_j are topologies on X is called a bi – topological space in 1963 ,Kelly[3] , The concept of fuzzy set introduced for first time in1965 by Zadeh , 1996 Coker [4] introduced the concept of intuitionistic set and intuitionistic topology as a special case of intuitionistic fuzzy topological spaces. Now we define the bi- intuitionistic topological space if X ≠ Ø , T_i , T_j are tow intuitionistic topologies on X then (X ,T_i , T_j) are bi- intuitionistic topological space(bi-ITS for short).

Now we recall the definition of an intuitionistic set and intuitionistic topology and some basic properties which are needed .

(1-1) definition:[5],[6]

Let X be a non-empty set. An intuitionistic set A (IS, for short) is an object having the form $A = \langle X, A_1, A_2\rangle$ where A_1 and A_2 are subsets of X satisfying $A_1 \cap A_2 = \emptyset$. The set A_1 is called set of members of A, while A_2 is called set of nonmember of A.

(1-2) definition:[6],[7]

Let A and B be two IS having the form $A = \langle x, A_1, A_2\rangle$ and $B = \langle x, B_1, B_2\rangle$ respectively, then,

a) $A \subseteq B \Leftrightarrow A_1 \subseteq B_1 \ \&\ A_2 \supseteq B_2$

b) $A = B \Leftrightarrow A \subseteq B \ \&\ B \subseteq A$

c) $\overline{A} = \langle x, A_2, A_1\rangle$

d) $A \cap B = \langle x, A_1 \cap B_1, A_2 \cup B_2\rangle$

e) $A \cup B = \langle x, A_1 \cup B_1, A_2 \cap B_2\rangle$

f) $\widetilde{X} = \langle x, X, \emptyset\rangle$

g) $\widetilde{\emptyset} = \langle x, \emptyset, X\rangle$.

(1-3) definition[4]:

Let X and Y be two non-empty sets and $f: X \to Y$ be a function.

a) If $B = \langle y, B_1, B_2\rangle$ is an IS in Y , then the preimage (inverse image) of B under f is denoted by $f^{-1}(B)$ is an IS in X and defined by $f^{-1}(B) = \langle x, f^{-1}(B_1), f^{-1}(B_2)\rangle$.

b) If $A = \langle x, A_1, A_2\rangle$ is an IS in X, then the image of A under f is denoted by f(A)is IS in Y defined by $f(A) = \langle y, f(A_1), \underline{f}(A_2)\rangle$ where $\underline{f}(A2) = (f(A^c2)^c)$, A any sub set of X.

(1-4) definition:[6]

An intuitionistic topology (IT for short) on a nonempty set X is a family T of IS's in X containing $\widetilde{\emptyset}, \widetilde{X}$, and closed under finite intersection and arbitrary union. In this case the pair (X, IT)is called an intuitionistic topological spaces ,(ITS for short), and any IS in T is known as an intuitionistic open set (IOS, for short) in X, the complement of IOS is called intuitionistic closed set (ICS, for short) in X .

(1-5) definition [2],[7]:

Let (X, IT) be ITS , and let $A = \langle x, A_1, A_2\rangle$ be IS in X. then the intuitionistic interior of ISA (int A ,for short) and intuitionistic closure of ISA (cl A , for short) are defined by

$\text{int}\, A = \cup \{G \in T : G \subseteq A\}$

$\text{cl}\, A = \cap \{F : A \subseteq F,\ \overline{F} \in T\}$

(1-6) definition: [1]

a) Let $P_{\sim}$ be an IP in X and $A = \langle x, A_1, A_2\rangle$ be an IS in X. $P_{\sim}$ is said to be contained in A (for short $P_{\sim} \in A$, if $p \in A_1$).

b) Let $P_{\approx}$ be VIP in X and $A = \langle x, A_1, A_2\rangle$ be an IS in X. $P_{\approx}$ is said to be contained in A, ($P_{\approx} \in A$, for short if, $p \notin A_2$).

Now we introduce a new definitions which is needed in our work .

(2-1) definition:

We say that (X, IT_i ,IT_j) bi- intuitionistic topological space if for each of (X,IT_i) and (X,IT_j)is intuitionistic topological space on X .

(2-2) definition:

Let (X, IT_i ,IT_j) bi-ITS and G be a sub set of X then G is said to be (i,j)- intuitionistic open set((i, j)IOS for short) if $G = A \cup B$ where $A \in IT_i$ and $B \in IT_j$ the complement of (i, j)-open set is (i, j)-intuitionistic closed set ((i, j)ICSfor short).

(2-3)Example:

Let $X = \{1,2,3\}$ and $IT_i = \{\widetilde{\emptyset}, \widetilde{X}, A, B, C\}$ where $A = \langle X, \{3\}, \{1,2\}\rangle, B = \langle X, \{1\}, \{3\}\rangle, C = \langle X, \{1,3\}, \emptyset\rangle$.and

$IT_j = \{\widetilde{\emptyset}, \widetilde{X}, D, E\}$ where $D = \langle X, \{1\}, \{2\}\rangle, E = \langle X, \{1\}, \{2,3\}\rangle$

(i, j)- open set $= \{\widetilde{\emptyset}, \widetilde{X}, A, B, C, D, E, F, G\}$ where $F = \langle X, \{1,3\}, \{2\}\rangle$,$G = \langle X, \{1\}, \emptyset\rangle$.

(i, j)- closed set$= \{\widetilde{\emptyset}, \widetilde{X}, \overline{A}, \overline{B}, \overline{C}, \overline{D}, \overline{E}, \overline{F}, \overline{G}\}$

(2-4) definition

Let (X, IT_i ,IT_j) bi-ITS and $A = \langle x, A_1, A_2\rangle$ is IS in X. then the intuitionistic interior and intuitionistic closure of A are denoted by (i, j)int(A) and(i, j) cl(A) respectively and defined as a union of all (i, j)- IOS of X that contained in A and the intersection of all (i, j) -ICS in X that contain A respectively .

(2-5)Remark

Let (X, IT_i ,IT_j) bi-ITS, and $A = \langle x, A_1, A_2 \rangle$ be IS in X. Then$(i,j)cl(\overline{A}) = (i,j)\overline{int(A)}$ & $(i,j)\ int(\overline{A}) = (i,j)\ \overline{cl(A)}$.

Now we give the definition of semi, pre ,semi pre (β) , pre semi (α) in bi ITS,s .

(2-6) definitions

Let be(X, IT_i ,IT_j) bi- ITS, and $A = \langle x, A_1, A_2 \rangle$ be IS in X. Then A is called:

1) (i, j) intuitionistic semi-open set ((i, j)ISOS, for short) if $A \subseteq IT_j\ cl(IT_i int(A))$

2(i, j)intuitionisticα-ope set ((i, j)(IαOS, for short) if $A \subseteq IT_i int\left(IT_j cl(IT_i int(A))\right)$.

3)(i, j) intuitionistic pre-open set ((i, j)(IPOS, for short)if $A \subseteq IT_i int\left(IT_j cl(A)\right)$

4) (i,j)intuitionistic β-open set (i,j)(IβOS, for short)if $A \subseteq IT_j cl\left(IT_i int(IT_j cl(A))\right)$.

The complement of (i,j)ISOS (resp. (i,j) IαOS, (i,j)IPOS, and(i,j) IβOS) is called(i,j) intuitionistic semi- closed set (resp. (i,j) intuitionistic α-closed, (i,j)intuitionistic pre-closed, , and (i,j) intuitionistic β-closed) set in X. (((i,j) ISCS, (i,j) IαCS, (i,j) IPCS, and (i,j) IβCS, for short).

(2-7)Theorem :

Let (X, IT_i ,IT_j) bi-ITS, and $A = \langle x, A_1, A_2 \rangle$ IS in X. then

i.A is (i, j)ICS then A is(i, j) IαCS , (i, j)ISCS, (i, j) IPCS AND(i, j) IβCS .

ii.A is (i, j) IαOS then A is (i, j)ISOS,((i, j) IPOS, (i, j) IβOS.)

iii.A is (i, j) ISOS then A is(i, j) IβOS.

iv.A is (i, j) IPOS then A is (i, j)IβOS.

Proof: [clear from definition]

(2-8)Example:

Let $X = \{1,2,3\}$ and $IT_i = \{\tilde{\emptyset}, \tilde{X}, A, B, C\}$ where $A = \langle X, \{3\}, \{1,2\}\rangle, B = \langle X, \{1\}, \{3\}\rangle, C = \langle X, \{1,3\}, \emptyset\rangle$.and

$IT_j = \{\tilde{\emptyset}, \tilde{X}, D, E\}$ where $D = \langle X, \{1\}, \{2\}\rangle, E = \langle X, \{1\}, \{2,3\}\rangle$,

ISCX=$\{\tilde{\emptyset}, \tilde{X}$,A,B,E,$K_1$,$K_2$,$K_3$,$K_4$,$K_5$ }.

IPCX=$\{\tilde{\emptyset}, \tilde{X}$,A, E,$K_1$,$K_3$,$K_4$,$K_5$,$K_6$,$K_7$,$K_8$,$K_9$,$K_{10}$,$K_{11}$,$K_{12}$,$K_{13}$,$K_{14}$,$K_{15}$, K_{16},K_{17}}

IαCX=$\{\tilde{\emptyset}, \tilde{X}$,A,E ,$K_1$,$K_3$,$K_4$}

IβCX=$\{\tilde{\emptyset}, \tilde{X}$,A,B,C,E,$K_1$,$K_2$,K_3,K_4,K_5,K_6,K_7,K_8,K_9,K_{10},K_{11},K_{12},K_{13},K_{13},K_{14},K_{15} ,K_{16},K_{17}}

Where $K_1=\langle X,\{3\},\{1\}\rangle$, $K_2=\langle X,\{1,2\},\{3\}\rangle$, $K_3=\langle X,\emptyset,\{1\}\rangle$, $K_4=\langle X,\emptyset,\{1,2\}\rangle$, $K_5=\langle X,\emptyset,\{1,3\}\rangle$, $K_6=\langle X,\{2\},\{1\}\rangle$, $K_7=\langle X,\{2\},\{3\}\rangle$, $K_8=\langle X,\{2\},\{1,3\}\rangle$,

$K_9=\langle X,\{2\},\emptyset\rangle$, $K_{10}=\langle X,\{3\},\{2\}\rangle$, $K_{11}=\langle X,\{3\},\emptyset\rangle$, $K_{12}=\langle X,\{2,3\},\{1\}\rangle$$K_{13}=\langle X,\{2,3\},\emptyset\rangle$, $K_{14}=\langle X,\emptyset,\{2\}\rangle$, $K_{15}=\langle X,\emptyset,\{2\}\rangle$, $K_{16}=\langle X,\emptyset,\{2,3\}\rangle$

$K_{17}=\langle X,\emptyset,\emptyset\rangle$

(2-9)Remark

Let (X, IT_i ,IT_j) bi-ITS, and $A = \langle x, A_1, A_2 \rangle$ be IS in X .then

(i,j)ISOS and (i,j)IPOS is independed) from example (2-3) $K_2=\langle X,\{1,2\},\{3\}\rangle$ is (i,j)ISOS but not (i,j)IPOS and $K_{15}=\langle X,\emptyset,\{2\}\rangle$ is (i,j)IPOS but not (i,j)ISOS .

(2-10) definition

Let (X, IT_i ,IT_j) bi-ITS, and $A = \langle x, A_1, A_2 \rangle$ be IS in X. Then the intersection of all (i,j) ISCS (resp (i,j)IαCS, (i,j)IPCS and(i,j) IβCS) in X that containing A is called the semi-closure (resp. (i,j)α-closure , (i,j) pre-closure, and (i,j) β-closure) of A and denoted by $(i,j)\ scl(A)\ (resp.\ (i,j)\ \alpha cl(A), (i,j)\ pcl(A\ ), and (i,j)\ \beta cl(A))$.

Note It is well-known that:

$$(i,j)scl(A) = A \cup IT_j int(IT_i cl(A)), (\ resp.\ (i,j)\alpha cl(A) = A \cup IT_i cl\ IT_j int(IT_i cl(A)),$$
$$(i,j)pcl(A) = A \cup IT_i cl\ IT_j int(A), (i,j)\beta cl(A) = A \cup IT_j int\left(IT_i cl\ IT_j int\ (A)\right)$$

(2-11) definition

Let (X, IT_i ,IT_j) be bi- ITS, and $A = \langle x, A_1, A_2 \rangle$ be IS in X. Then the union of all(i,j) ISOS (resp. (i,j)IαOS, (i,j) IPOS and(i,j) IβOS) in X that contained A is called the(i,j) semi-interior (resp. (i,j) α-interior, (i,j)pre-interior and(i,j) β-interior) of A and denoted by $(i,j)sint(A)\ (resp.\ (i,j)\ \alpha int(A\ ), (i,j)pint(A)\ and (i,j)\ \beta int(A))$.

Note It is well-known that

$$(i,j)sint(A) = A \cap IT_j cl\ IT_i int(A), (\ resp.\ (i,j)\alpha cl(A) = A \cap IT_i int\ IT_j cl\ IT_i int(A),$$
$$(i,j)pint(A) = A \cap IT_i int\ IT_j cl(A), (i,j)sint(A) = A \cap IT_j cl\ IT_i int IT_j cl(A)\ .$$

(2-13)Proposition

Let (X, IT_i ,IT_j) be bi- ITS , and $A = \langle x, A_1, A_2 \rangle$ be IS in X. Then A is(i,j) IαOS in X if and only if it is both (i,j)ISOS and(i,j) IPOS in X.

Proof: [clear from definition].

3-Generalized closed set in bi- intuitionistic topological spaces

(3-1) definitions

Let (X, IT_i ,IT_j) be bi- ITS, an IS $\tilde{A}$ in X is called:

1) (i,j) Generalized closed (briefly, (i,j) g-closed), if $IT_j cl(A) \subseteq U$, whenever $A \subseteq U$ and U is IT_i- ISOS.

2) (i,j) Semi-generalized closed (briefly, (i,j) sg-closed), if IT_j- $scl(A) \subseteq U$, whenever $A \subseteq U$ and U is IT_i- ISOS,

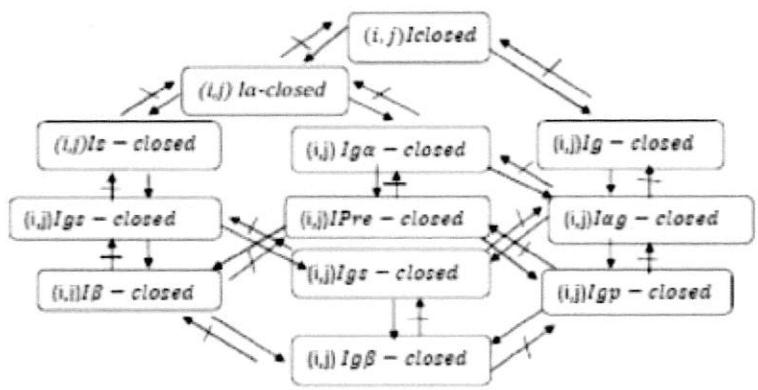
3) (i,j)Generalized semi-closed (briefly, (i,j) gs-closed), if IT_j- $Scl(A\ ) \subseteq U$, whenever $A \subseteq U$ and U is IT_i- IOS,

4) (i,j)Generalized α-closed (briefly, (i,j) gα-closed), if $IT_j-\alpha cl(A) \subseteq U$, whenever $A \subseteq U$ and U is IT_i- IαOS,
5) (i,j)α-generalized closed (briefly, (i,j) αg-closed), if $IT_j-\alpha cl(A) \subseteq U$, whenever $A \subseteq U$ and U is IT_i IOS,
6) (i,j)Generalized β-closed (briefly, (i,j) gβ-closed), if $IT_j-\beta cl(A) \subseteq U$, whenever $A \subseteq U$ and U is IT_i- IOS,
7) (i,j)Generalized pre-closed (briefly, (i,j) gp-closed), if IT_j- $pcl(A) \subseteq U$, whenever $A \subseteq U$ and U is IT_i -IOS.
An IS A in X is(i,j) g-open (resp. (i,j) sg-open, (i,j) gs-open, (i,j) gα-open, (i,j) αg-open, (i,j) gβ-open, and (i,j)gp-open), if the $\overline{A}$is(i,j) g-closed (resp. (i,j) sg-closed, (i,j) gs-closed, (i,j) gα-closed, (i,j)αg-closed, (i,j) gβ-closed and(i,j) gp-closed).

(3-2)Theorem:
Let (X, IT_i ,IT_j) bi- ITS. An intuitionistic subset A of X is (i,j)g-open if and only if, for each (i,j)ICS F in X such that $F \subseteq (i,j)int(A)$ whenever $F \subseteq A$

Proof
⇒ Suppose that A is(i,j) g-open set in X, and let F be any closed set such that $F \subseteq A$, so by definition$\overline{A}$ is (i,j)g-closed set in X. Therefore, for each(i,j) IOS U say $U = \overline{F}$ in X ,$\overline{A} \subseteq \overline{F}$, then(i,j) $cl(\overline{A}) \subseteq \overline{F}$, so $\overline{\overline{F}} = F \subseteq \overline{(i,j)cl(\overline{A})} = (i,j)int(A)$ by Remark (2-5).
⇐ suppose that for each(i,j) ICS F $\subseteq A$ then $F \subseteq (i,j)int(A)$, we have to prove that A is (i,j)g-open, i.e. we have to prove that $\overline{A}$ is(i,j) g-closed, let U be any IOS in X such that $\overline{A} \subseteq U$, we have to prove that(i,j) $cl(\overline{A}) \subseteq U$. For if, since U is$(i,j)$ IOS, then $\overline{U}$ is (i,j)ICS and $\overline{U}\, A$, so by hypothesis $\overline{U} \subseteq (i,j)int(\overline{A})$. Therefore$(i,j)$, $\overline{Int(A)} = (i,j)cl(\overline{A}) \subseteq \overline{\overline{U}} = U$. By Remark (2-5) we get that $\overline{A}$ is(i,j) g-closed.

(3-3)Theorem
Let (X,IT_i, IT_{ji})bi- ITS. Then the following implications in the diagram are true but not reversible.



Proof
The method of prove this theorem is to take one implication and prove truth one and give a counter example for the other at the end of the proof.
1)(i,j)Closed⇒)(i,j)g-closed, but the converse is not true.
We have to prove that, if A is(i,j)- closed set then A is(i,j) g-closed. For if, since A is(i,j)- closed, then(i,j) $cl(A) = A$. Now, for each (i,j)- IOS U , $A \subseteq U$. We have(i,j) $cl(A) = A \subseteq U$.
2) (i,j) IClosed⇒ $(i,j)I\alpha$-closed, but the converse is not true.
We have to prove that, if A is (i,j)- closed, then A is $(i,j)I\alpha$-closed. For if, since A is (i,j)closed, then $IT_i cl(A) = A$, so $IT_j\ int\ IT_i cl(A) \subseteq IT_i cl\ (A)$, therefore $IT_i cl\ IT_j int IT_i\ cl(A) \subseteq IT_i clA = A$. But $\alpha cl(A) = A \cup IT_i\ cl(IT_j\ int(IT_i\ cl(A)\,) \subseteq A \cup IT_i cl(A) = IT_i cl(A) = A$, Therefore(i,j) $\alpha cl(A) \subseteq A$, and we have from definition of $A \subseteq (i,j)\alpha cl(A)$, so we get that$(i,j)$ $\alpha cl(A) = A$. i.e. A is$(i,j)I$ α-closed..

3) (i,j)Ig-closed ⇒ (i,j)Iαg-closed and the converse is not true.
We have to prove that, if a is(i,j)Ig-closed, then A is (i,j) Iαg-closed. For if, since A is Ig-closed, so for each U∈ $IT_i, A \subseteq U$, then$IT_i cl(A) \subseteq U$. SinceIT_i $cl(A) \subseteq U$, then $IT_j int IT_i cl(A) \subseteq A \ \subseteq U$, so$IT_j int IT_i cl(A) \subseteq IT_i cl IT_j int IT_i cl(A) \subseteq IT_i cl(A) \subseteq U$, so $(i,j)\alpha cl(A) \subseteq IT_i cl(A) \subseteq U$. That is,(i,j)$\alpha cl(A) \subseteq U$. Therefore A Iαg-closed.
4) (i,j) Iαg-closed⇒ $(i,j)I$gp-closed and the converse is not true
We have to prove that, if A is(i,j)αg-closed, then A is (i,j) gp-closed. For if, since A is Iαg-closed, so for each U∈ IT_i , $A \subseteq U$, then $(i,j)\alpha cl(A) \subseteq U$.
$(i,j)\alpha cl(A) =$
$A \cup IT_i cl IT_j int IT_i cl(A), (i,j)pcl(A) = A \cup IT_i cl IT_j int(A) \subseteq A \cup IT_i cl IT_j int IT_i cl(A) = (i,j)\alpha cl(A)$ Since $IT_i cl(A) \subseteq U$, then $IT_j int IT_i cl(A) \subseteq A \ \subseteq U$, so$IT_j\ int IT_i cl(A) \subseteq IT_i cl IT_j int IT_i cl(A) \subseteq IT_i cl(A) \subseteq U$, so (i,j)$pcl(A) \subseteq (i,j)\alpha cl(A) \subseteq U$. That is, $(i,j)pcl(A) \subseteq U$. Therefore A (i,j)Igp-closed.

6) $(i,j)I$gs-closed⇒ $(i,j)I$gβ-closed and the converse is not true.

We have to prove that, if A is $(i,j)I$ gs-closed, then A is(i,j)Igβ-closed. For if, since A is(i,j)Igs-closed, so for each U∈ IT_i , A $\subseteq U$, then $(i,j)scl(A) \subseteq U$.

Since $(i,j)scl(A) = A \cup IT_j intIT_i cl(A) \subseteq U$

, and since $IT_j intIT_i cl(A) \subseteq IT_j intIT_i cl\ IT_j int(A)$ $\subseteq U$ $(i,j)\beta cl(A) \subseteq U$.

Therefore A is (i,j) Igβ-closed.

7)- (i,j) Iβ-closed⇒ $(i,j)I$gβ-closed and the converse is not true.

We have to prove that, if A is(i,j)Iβ-closed, then A is(i,j) Igβ-closed. For if,

since A is (i,j)Iβ-closed, $IT_j intIT_i clIT_j int(A) \subseteq A$. Let U be any IOS in IT_i such that $A \subseteq U$ Since $(i,j)\beta cl(A) = A \cup IT_j intIT_i clIT_j int(A) = A \subseteq U$.

Therefore A(i,j)Igβ-closed.

8) (i,j)Iαg-closed⇒ (i,j)Igs-closed and the converse is not true in general.

We have to prove that, if A is (i,j) Iαg-closed, then A is(i,j)I gs-closed. For if,

since A is(i,j) Iαg-closed, so for each U ∈ IT_i ,$A \subseteq U$, then(i,j) $\alpha cl(A) \subseteq U$.

$(i,j)scl(A) = A \cup IT_j intIT_i cl(A) \subseteq A \cup$
$IT_i\ clIT_j intIT_i cl(A) = (i,j)\alpha cl(A) \subseteq U$

Since (i,j) $scl(A) \subseteq (i,j)\alpha cl(A) \subseteq U$, That is, $(i,j)scl(A) \subseteq U$.

Therefore, A is (i,j) Igs-closed.

9) (i,j)Igp-closed⇒ $(i,j)I$gβ-closed and the converse is not true in general.

We have to prove that, if A is $(i,j)I$gp-closed, then A is $(i,j)I$gβ-closed. For if,

since A is⇒ $(i,j)I$gp-closed, so for each U ∈ IT_i ,$A \subseteq U$, then(i,j) $pcl(A) \subseteq U$

sine U ∈ IT_i, IT_j $int\ A \subseteq A$, then $\beta cl(A) = A \cup$ $IT_j intIT_i clIT_j int(A) \subseteq A \cup IT_i clIT_j int(A) \subseteq A \cup$ $IT_i clIT_j int(A) = A \subseteq U$,so(i,j)I$\beta cl(A) \subseteq U$.

That is, A is (i,j)Igβ-closed.

10)(i,j)Iα-Closed⇒ $(i,j)I$s-closed, but the converse is not true

Since IT_j $intIT_i cl(A) \subseteq IT_i clIT_j intIT_i cl(A) \subseteq A$, so the result follows

11(i,j)I p-Closed⇒ $(i,j)I$β-closed, but the converse is not true

Since $IT_i clIT_j int(A) \subseteq IT_j intIT_i clIT_j intc(A) \subseteq A$, so the result follows.

12)(i,j)Iα-closed⇒ (i,j)Igα-closed and the converse is not true in general.

We have to prove that, if A is (i,j)Iα-closed, then A is (i,j)Igα-closed. For if,

since A is (i,j)Iα-closed, thenIT_i $clIT_j intIT_i clA \subseteq A$. Let $A \subseteq U$, where U is any IT_i α-open , Since $(i,j)\alpha cl(A) = A \cup IT_i clIT_j intIT_i cl(A) \subseteq A \subseteq U$.

Therefore A is(i,j)Igα-closed.

13) (i,j)Is-closed⇒ $(i,j)I$sg-closed and the converse is not true in general.

We have to prove that, if A is (i,j)Is-closed, then A is (i,j)Isg-closed. For if,

since A is (i,j)Is-closed , then $IT_j intIT_i clA \subseteq A$. Let $A \subseteq U$, where U is any IT_is-open , Since $(i,j)scl(A) = A \cup IT_j intIT_i cl(A) \subseteq A \subseteq U$.

Therefore A is (i,j)Isg-closed.

14)(i,j)Igα-closed⇒ (i,j)Ipre-closed and the converse is not true.

We have to prove that, if A is (i,j)Igα-closed, then A is(i,j)Ipre-closed. For if,

since A is (i,j)Igα-closed , then , if for each U is IT_iαOS , $A \subseteq U$ then $(i,j)I\alpha clA \subseteq U$. $(i,j)\alpha cl(A) = A \cup IT_i cIT_j lintIT_i cl(A) \subseteq U$ and since $IT_i clIT_j int(A) \subseteq IT_i clIT_j intIT_i cl(A) \subseteq U$. IT_i $clIT_j int\ (A) \subseteq A$.

Therefore A is (i,j)Ipre-closed

15) (i,j)Isg-closed⇒(i,j)I β-closed, and the converse is not true in general

We have to prove that, if A is(i,j)Isg-closed, then A is (i,j)Iβ-closed. For if, since A is (i,j)Isg-closed then , if for each U∈ IT_iSOS , $A \subseteq U$ then $(i,j)sclA \subseteq U$. $(i,j)scl(A) = A \cup IT_j intIT_i cl(A) \subseteq U$ and since $IT_j intIT_i clIT_j int(A) \subseteq IT_j intIT_i cl(A) \subseteq U$

$(IT_j intA \subseteq A)$ then $IT_i intIT_i clIT_j int\ (A) \subseteq A$.

Therefore A (i,j)Iβ-closed.

16) (i,j)Igα-closed⇒ (i,j)Iαg-closed, and the converse is not true in general.

We have to prove that, if A is(i,j)I gα-closed, then A is(i,j)I αg-closed. For if,

since A is (i,j)Igα-closed , then , if for each U∈ IT_iαOS , $A \subseteq U$ then $(i,j)\alpha clA \subseteq U$. Since $(i,j)\alpha cl(A) \subseteq IT_i clA \subseteq U$ for each IOS G , A⊆ G(G is(i,j)IαOS)

Therefore, (i,j)$\alpha cl(A) \subseteq IT_i clA \subseteq G$, A(i,j)I αg-closed

17)(i,j)I sg-closed⇒ (i,j)Igs-closed, an the converse is not true in general.

We have to prove that, if A is (i,j)Isg-closed, then A is (i,j)Igs-closed. For if,

since A is (i,j)Isg-closed , then , if for each U∈ ISOS , $A \subseteq U$ then(i,j)$sclA \subseteq U$. Since $(i,j)scl(A) \subseteq IT_i clA \subseteq U$ for each(i,j)IOS G , $A \subseteq G$(G is (i,j)ISOS)

Therefore, $(i,j)scl(A) \subseteq IT_i clA \subseteq G$, A is (i ,j)Isg-closed.

The following example shows that;

1) (i,j) I αg-closed $\nrightarrow$ (i,j) I gα-closed

2) $(i,j)I$gp-closed ,$(i,j)I$p-closed , $(i,j)I$gs-closed ,$(i,j)I$gβ-closed

and $(i,j)I$ β-closed $\nrightarrow$ (i,j) gα-closed

3) $(i,j)I$ gs-closed , (i,j) I β-closed and (i,j) Igβ-closed $\nrightarrow$,$(i,j)I$sg-closed .

4) ,(i,j) Ig-closed $\nrightarrow$ (i,j)I closed

Let $X = \{a,b,c\}$ and IT_i $= \{\tilde{\emptyset}, \tilde{X}, A, B, C\}$ where $A = \langle X, \{a\}, \{b,c\}\rangle, B = \langle X, \{c\}, \{a,b\}\rangle, C = \langle X, \{a,c\}, \{b\}\rangle$ and

$IT_j=\{\tilde{\emptyset},\tilde{X},D,E\}$ where $D=\langle X,\{a\},\{b\}\rangle$, $E=\langle X,\{a,c\},\emptyset\rangle$

$I\alpha O(X)=\{\tilde{\emptyset},\tilde{X},A,B,C,D,E,G_1,G_2,G_3,G_4,G_5,G_6,G_7,G_8,G_9,G_{10},G_{11},G_{12},G_{13}\}$ $=ISO(X)$. where $G_1=\langle X,\{a\},\{C\}\rangle$, $G_2=\langle X,\{a\},\emptyset\rangle$, $G_3=\langle X,\{b\},\{a\}\rangle$, $G_4=\langle X,\{b\},\{c\}\rangle$, $G_5=\langle X,\{b\},\{a,c\}\rangle$, $G_6=\langle X,\{b\},\emptyset\rangle$, $G_7=\langle X,\{c\},\{a\}\rangle$, $G_8=\langle X,\{c\},\{b\}\rangle$, $G_9=\langle X,\{c\},\emptyset\rangle$, $G_{10}=\langle X,\{a,b\},\{c\}\rangle$, $G_{11}=\langle X,\{a,b\},\emptyset\rangle$, $G_{12}=\langle X,\{b,c\},\{a\}\rangle$, $G_{13}=\langle X,\{b,c\},\emptyset\rangle$.

1) Let $L=\langle X,\{b\},\emptyset\rangle\subseteq U=X$. then L is $(i,j)I\alpha g$-closed set because

$(i,j)\ \alpha cl(L)=A\cup ITi-cl(ITj-int(ITi-cl(L))$

$=(i,j)\ \alpha cl(L)=A\cup ITi-cl(ITj-int(X))$

$=L\cup X=X\subseteq U$.

But is not $(i,j)I\alpha$-closed set because the only(i,j) $I\alpha OX$ in X containing L is G_{11} ,G_{13} But $(i,j)Ig\alpha$-cl $=X\not\subseteq G_{11}$,G_{13} .

2) L is $(i,j)Igp$-closed ($(i,j)Ip$-closed, $(i,j)Igs$-closed, $(i,j)Ig\beta$-closed and $(i,j)I\beta$-closed) because :

$(i,j)pcl(L)=L\subseteq U$, $(i,j)Ip$-closed $=IT_i\ cl(IT_j\text{-}int(L))\subseteq L=IT_i\text{-}cl(\emptyset)=\emptyset\subseteq L$.

, (i,j) gscl(L) =X $\subseteq$ U and ,$(i,j)g\beta cl(L)=L\subseteq U$. but are not $(i,j)g\alpha$-closed set because by (1).

3)L is (i,j) gs-closed set since (i,j) gscl(L)=L $\cup$ $ITj\ int(ITi\ cl(L))$

(i,j) gscl(L)=L $\cup$ X = X $\subseteq$ U .and L is $(i,j)I$ β-closed , ,$(i,j)Ig\beta$-closed

Because $(i,j)I$ β-closed : $ITj\ int(ITi\ clITj\ int(L))\subseteq L$ so : $ITj\ int(ITi\ cl\ \emptyset)=\emptyset\subseteq L$, $(i,j)Ig\beta cl(L)=L\subseteq U$ .But not (i,j) Isg – closed because the only ISOX in X containing L is G_{11} ,G_{13} But (i,j) gscl(L) = X $\not\subseteq G_{11}$,G_{13}

4) since $L=\langle X,\{b\},\emptyset\rangle\subseteq U=X$ then cl(L)=X $\subseteq$ U therefore L is $(i,j)Ig$-closed set but not closed set $L\notin IT_i$-closed set.

The following examples show that;

1) $(i,j)Ig\beta$-closed $\not\longrightarrow$ (i,j) Isg –closed, (i,j) I gs –closed ,and (i,j) Igα –closed .

2) $(i,j)I\beta$-closed $\not\longrightarrow$ (i,j) Isg –closed

3) (i,j) Ip -closed $\not\longrightarrow$ (i,j) Igα –closed

4) (i,j) Igp -closed $\not\longrightarrow$ (i,j) Igα -closed

(3-5)Example:

Let $X=\{a,b,c\}$ and $IT_i=\{\tilde{\emptyset},\tilde{X},A,B\}$ where $A=\langle X,\{a,b\},\emptyset\rangle$, $B\langle X,\{a\},\{c\}\rangle$ and $IT_j=\{\tilde{\emptyset},\tilde{X},C,D\}$ where $C=\langle X,\emptyset,\{b\}\rangle$, $D=\langle X,\{a\},\{b\}\rangle$.

Let $H=\langle X,\emptyset,\{c\}\rangle\subseteq U=\langle X,\{a\},\{c\}\rangle$

1) H is $(i,j)Ig\beta$-closed set because $(i,j)Ig\beta$-cl(H)= H $\subseteq$ U.But H is not (i,j)Isg –closed, (i,j) I gs – closed because (i,j) I gscl(H)= X $\not\subseteq$ U , and not (i,j)Igα –closed because (i,j) I gαcl(H)= X $\not\subseteq$ U

2) H is $(i,j)I\beta$-closed: $IT_j int(IT_i cl\ IT_j int(H))\subseteq H$ then

$IT_j int(\ IT_i\ cl(\emptyset))=\emptyset\subseteq H$ since (i,j) I gαcl(H)= X $\not\subseteq$ U then H is not (i,j) Isg –closed.

3) since (i,j) Ip -closed set $\left(IT_i\ cl\ IT_j int(H)\right)\subseteq H$ then$IT_i c\ l\emptyset=\emptyset\subseteq H$ so H is (i,j) Ip -closed set but H is not (i,j) Igα –closed because (i,j) I gαcl(H)= X $\not\subseteq$ U

4) H is (i,j) Igp –closed because (i,j) Ipcl(H)=H$\cup$ $IT_i\text{cl}\left(\text{IT}_j\ \text{int}\ (\text{H})\right)$

=H$\cup$ IT_icl$\emptyset$ =H $\subseteq$ U but is not (i,j) Igα –closed because (i, j) I αcl(H)= X $\not\subseteq$ U

(3-6)Example:

From example (2-3) let $M=\langle X,\{1\},\{2\}\rangle\subseteq U=X$ then

1) M is (i, j) Igp –closed set because (i, j) I pcl(M)= M $\cup$ $IT_i\text{cl}\left(\text{IT}_j\text{int}\ (\text{M})\right)$

=M $\cup$ IT_icl(D) = M $\cup$ X= X $\subseteq$ U, But M $\notin$ IPCX .

And (i, j) I βcl(M)= M $\cup$ $IT_j\text{int}\left(IT_i\text{cl}\ (IT_j\text{int}\ (M)\right)$ = M $\cup$ X = X $\subseteq$ U so M is (i, j) Igβ –closed set but M $\notin$ IβCX..

2) Let $F=\langle X,\{1\},\emptyset\rangle\subseteq U=X$ then (i, j) I Scl(F)= F$\cup$ $IT_j\text{int}(IT_i\text{cl}\ (F))$ = F $\cup$ X = X $\subseteq$ U so F is (i, j) Igs –closed set but F $\notin$ ISCX IT_icl(ITj And (i, j) I αcl(F)= F $\cup$ $IT_j\text{int}(IT_i\text{cl}\ (F))$ = F $\cup$ X =X so F is (i, j) Igα –closed But F $\notin$ IαCX..

Now we introduce the definition of T_{gs}-space in bi-ITS.

(3-7)definition:

(X, IT_i, IT_j) bi-ITS is said to be Tgs-space, if every (i,j)Igs-closed set in X is (i,j)Isg-closed set in X.

(3-8)proposition:

A subset A of (X, IT_i, IT_j) bi-ITS is (i,j)Igα- closed if and only if $X_1\cap(i,j)\alpha cl(A)\subseteq A$, where

$X_1=\{P_\sim=\langle x,\{p\},\{p\}^c\rangle\,\tilde{\in}\,\tilde{X}: P_\sim \text{ is no where dense in } \tilde{X}\}$

Proof: The same method of proof in ITS see [1]

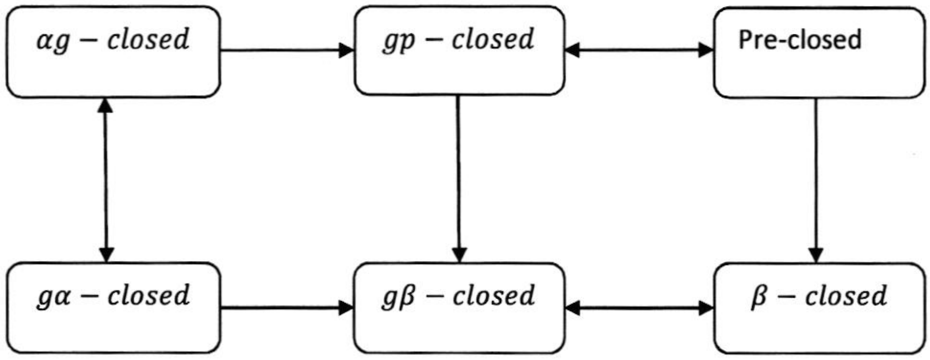
(3-9)theorem:

For (X, IT_i, IT_j) bi-ITS , the following statements are equivalent.

1. (X, IT_i, IT_j) is Tgs-space,
2. $P_\sim$ is either (i,j)Ipre-open or (i,j)I closed for each $P_\sim\in\tilde{X}$.
3. Every(i,j)I αg-closed in X is (i,j)Igα-closed.
4. Every gp-closed set in X is pre-closed.
5. Every gβ-closed set in X is β-closed in X.
6. Every gp-closed in X is β-closed.

Proof: by similar way on the Tgs space in ITS see [1].

Now we get tow equavilant relation and one new implication in theorem (3-9) .



References

[1]. Al-hawez, Z. T. (2008), generalized slightly continuous function between ITS, M.Sc. thesis college of Education Tikrit university.

[2]. Andrijeric, D. (1996) "On b-open sets" M. Bechnk. 48 pp. 59-64.

[3]. J. C. Kelly, Bitopological spaces, Proc, LondonMath. Soc., 13 (1963), 71–89.

[4]. Coker, D. (1996) "a note on intuitionistic set and intuitionistic points" Turkish J. of Math. Vol.20 , pp.343-351.

[5]. Ozcelik, A. Z. and Narli, S. (2007) "On submaximality in intuitionistic topological space" International J. of Math. And Math. Sci. Vol. 1. No.1. pp. 139-141

[9].

[6]. Özcač , S. and Coker, D. (2000) " A note on connectedness in intuitionistic fuzzy special topological spaces" International J. of Math. and Math. Sci. Vol. 23. No.1. pp. 45-54.

[7]. Jeon, J. K., Baejun Y. and Park, J. H. (2005) "Intuitionistic fuzzy alpha-continuity and intuitionistic fuzzy precontinuity" Inter. J. of Math. Sci. 3041-3101.

[8]. .Noiri, T. and Popa, V. (2006) "Slightly m-continuous multifunction" Bull. Of the institute of mathematic Academia Sinica Vol. 1, No. 4, pp. 484-505.

حول الفضاء ثنائي التبولوجي الحدسي

طه حميد جاسم[1] ، زينة طه عبد القادر[2] ، هبة عمر موسى[2]

[1]*قسم الرياضيات ، كلية العلوم ، جامعة تكريت ، تكريت ، العراق*

[2]*قسم الرياضيات ، كلية التربية للبنات ، جامعة تكريت ، تكريت ، العراق*

(تاريخ الاستلام: 26 / 2 / 2013 ---- تاريخ القبول: 22 / 4 /2013)

الملخص

في هذا البحث نقدم تعريفا جديدا يسمى الفضاء ثنائي التبولوجي الحدسي وعن هذا المفهوم نقدم بعض أنواع المجموعات المغلقة (المجموعة شبه مغلقة، المجموعة قبل المغلقة، المجموعة β المغلقة، المجموعة مغلقة α) في الفضاء ثنائي التبولوجي الحدسي. وتعريف المجموعات المغلقة المعممة (sg–مغلقة، gs المغلقة، gp المغلقة، $g\alpha$ المغلقة، αg المغلقة، $g\beta$ مغلقة) في الفضاء ثنائي التبولوجي الحدسي وا عطاء العلاقة فيما بينها وكذلك قدمنا تعريف الفضاء T_{gs} ومن خلال هذا المفهوم توصلنا الى علاقات جديدة .