# On Some New Classes of Separation Axioms in Topological spaces

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## Abstract

In this paper we introduce a new class of separation axioms through the concept $e^*$ - open set which was studied ,we study the relation among them and some hereditary property. At last we study and investigate the characterization of $e^*$- completely regular space and $e^*$- regular space.

**Keywords**: $e^*$ - open set, $e^*$- completely regular, $e^*$- regular space, $e^*$- Normal spaces and $e^*$- completely Normal .

## Introduction

Stone (1937) introduce regular open (resp . regular closed) if A=Int (cl(A)) (resp . A=cl (Int(A))) [5] . Velicko (1968) introduce $\delta-$ open if for each $x \in A$, there exist regular open set V such that $x \in V \subseteq A$ [6] .The $\delta$ –Interior of a subset A of X is the union of all regular open set of X contained in A and it is denoted by$\delta-\mathrm{Int}(A)$.

The complement ofa $\delta-$ open set is called$\delta$ –closed set .Then $\delta$ –closure of a set A in a space $(X,\tau)$ is defined by$\{x \in X : A \cap \mathrm{Int}(cl(B)) \neq \emptyset, B \in \tau \text{and } x \in B\}$ and it is denoted $by\ \delta - cl(A)\ [cl_\delta(A)]$ . Ekici (2008) introduce e-open set if $A \subseteq cl(\delta\text{-}Int(A) \cup (Int(\delta\text{-}cl(A))$ .The complement of an e-open set is called e-closed [1] .

Aagian (2009) Ekici introduce$e^*$-open set [ 3].In this paper we introduce new type of $e^*$ -separation axioms,we study the relations among these types of separation axioms and standard separation axioms .

## Preliminaries

Throughout this paper $(X,\tau_x)$, $(Y,\tau_y)$and $(Z,\tau_z)$ mean topological spaces. For a subset A of X, the interior and closure of A are denoted by Int (A) and cl(A) respectively .If Y be a subset of X then classes $\tau_y$ of all interseations of Ywith $\tau_y$ –open subset of X belong to $\tau_x$ is a topology on Y it is called relative topology[4].Let p is any property in X, then we called is hereditary property if every relative topological space has property p [4] .

**Definition 2.1**

A subset A of a space $(X,\tau)$ is called an $e^*$-open if $A \subset cl\,(Int\,(\delta - cl(A)$ , the complement of an $e^*$-open set is called $e^*$-closed . The family of all $e^*$-open (resp $e^*$-closed) in X denoted by$E^*(X,\tau)$ (resp $E^*C\,(X,\tau)$) .

**Definition 2.2**

Let A be a subset of a topology space$(X,\tau)$,Then

1.The intersection of all $e^*$- closed sets containing A is called the $e^*$- closure of A, denoted by of $cl_{e^*}(A)$.

2.The union of all $e^*$- open sets of X contained in A is called the $e^*$-interior denoted by $Int_{e^*}(A)$ .

**Example 2.3**

Let X={a,b,c}, $\tau_x=\{\emptyset, X, \{a\},\{b\},\{a,b\}\}$ $\tau_x^C=\{X,\emptyset,\{b,c\},\{a,c\},\{c\}\}$,since regular open $=\{\emptyset, X,\{a\},\{b\}\}$.

Hence - open=$\{\emptyset, X,\{a\},\{b\},\{a,b\},\{b,c\},\{a,c\}\}$ then $E^*(X,\tau)=\{\emptyset, X,\{a\},\{b\},\{a,b\},\{b,c\},\{a,c\}\}$, and $E^*C(X,\tau)=\{X,\emptyset,\{b,c\},\{a,c\},\{c\},\{b\},\{a\}\}$ .

**Definition 2.4**

A function $f:(X,\tau_x) \rightarrow (Y,\tau_y)$ is to be $e^*$- continuous if $f^{-1}(V)$ is $e^*$- open in X for every open set V of Y.

**Definition 2.5**

A function $h:(X,\tau_x) \rightarrow (Y,\tau_y)$ is to be $e^*$- irresolute if $h^{-1}(V)$ is $e^*$- open in X for every $e^*$- open set V in Y.

**Remark 2.6**

Every $e^*$ - continuous is $e^*$ - irresolute .but the converse is not true.

**Example 2.7**

Suppose that X={a , b ,c}, $\tau_x=\{X, \emptyset, \{a\}, \{c\},\{a,c\}\}$ Since $E^*(X,\tau_x)=\{X,\emptyset,\{a\},\{c\},\{a,c\},\{b,c\},\{a,b\}\}$, and Y={3,5,6} , $\tau_y=\{\emptyset,Y,\{5\},\{6\},\{3,6\},\{6\}\}$ , since $E^*(Y,\tau_y)=\{\emptyset,Y,\{5,6\},\{3,6\},\{6\}\}$.Let $f:(X,\tau_x) \rightarrow (Y,\tau_y)$ defined f (a)=6 ,f (b) =3 , f (c) =5 .

Hence f is $e^*$- irresolute but not $e^*$- continuous .

**Theorem 2.8**

For function $f:(X,\tau_x) \rightarrow (Y,\tau_y)$ and $h:(Y,\tau_y) \rightarrow (Z,\tau_z)$ the following hold :

1- If f is $e^*$- continuous and h is $e^*$- irresolute then $h \circ f: X \rightarrow Z$is $e^*$- irresolute.

2-If f is $e^*$- irresolute and h is $e^*$- continuous then $h \circ f: X \rightarrow Z$ is $e^*$- continuous .

3- If f and h are $e^*$- irresolute then $h \circ f: X \rightarrow Z$ is $e^*$- irresolute .

4- If f and h are $e^*$- continuous then $h \circ f: X \rightarrow Z$ is $e^*$- continuous .

**Proof**: It is obvious, it is follows from their respective definitions.

## 3- e*-separation axioms

As an application of $e^*$- open sets ,we introduce new types of spaces namely $e^*$- $\tau_0$- space, $e^*$- $\tau_1$- space, $e^*$- $\tau_2$- space .

**Definition 3.1**

A topological space $(X,\tau)$ is called an $e^*$- $\tau_0$- space and denoted ($e^*$- $\tau_0$) if for any two distinct points x , y of X there is an $e^*$- open set of X containing one of them but not the other .

**Example 3.2**

Let X = {a,b,c},$\tau_x=\{\emptyset, X, \{a\},\{b\},\{a,b\}\}$ Since $E^*(X,\tau)=\{X, \emptyset, \{a\},\{b\},\{a,b\}\{a,c\},\{b,c\}\}$.it clearly $(X,\tau)$ is $e^*$-$\tau_0$- space. since every open (closed) set is an $e^*$- open ( $e^*$- closed) set ,then we have the following theorem

**Theorem 3-3**

Every $\tau_0$- space is an $e^*$-$\tau_0$- space.

**Proof**: It is obvious .

But the converse above Theorem (3.3) may not be true in general . Consider the following example

**Example 3.4**

let X={a,b,c},$\tau_x$ = { X , ∅ ,{a,c}}. Since E*(X, $\tau$) ={∅ , X ,{a,c}}.Then(X, $\tau$) is $e^*$-$\tau_0$- space but not $\tau_0$-space.

**Theorem 3.5**

A topological space (X, $\tau$) is $e^*$-$\tau_0$- space if and only if for each pair of distinct points x , y of X . $cl_{e^*}\{x\} \neq cl_{e^*}\{y\}$

**Proof**: For each x,y∈X, x ≠y, $cl_{e^*}\{x\} \neq cl_{e^*}\{y\}$.let G∈X such that G∈ $cl_{e^*}${x} but G∉ $cl_{e^*}${y} .assume that x ∉ $cl_{e^*}\{y\}n$ .if x ∈ $cl_{e^*}${y} then $cl_{e^*}\{x\} \subseteq cl_{e^*}\{y\}$This contradict that fact that G∉ $cl_{e^*}${y}

Consequently x ∈ $cl_{e^*}\{y\})^C$ to which y does not belong .

Necessity let (X, $\tau$) be $e^*$-$\tau_0$- space and x ,y∈X,x≠ y,∃$e^*$-open set U such that x∈u or y∈u then $U^C$ is an $e^*$-open set which x∈u and y∈ $U^C$ since$cl_{e^*}$ {y} is the smalleste*- closed set containing y $cl_{e^*}\{y\} \subseteq U^C$ and there for x∈ $cl_{e^*}$ {y}. Hence$cl_{e^*}\{y\} \neq cl_{e^*}\{x\}$.

**Theorem 3.6**

Let (X,$\tau_x$)be any $e^*$-$\tau_0$- space then every relative topological space is $e^*$-$\tau_0$

**Proof**: Let (Y,$\tau_y$) be relative topological space .To show is (Y,$\tau_y$) is $e^*$-$\tau_0$- let $y_1, y_2 \in$ Y and $y_1 \neq y_2$ then $y_1, y_2$ ∈X.Because(X,$\tau_x$) is $e^*$-$\tau_0$.∃$e^*$-open set U⊆X Such that U containing one of $y_1, y_2$ but not both .Now , $y_1 \in$ U then$y_1 \in$ Y∩U $=U^*$ if $y_2 \in$ U then $y_2 \in$ Y ∩U=$U^*$ therefore (Y,$\tau_y$) is $e^*$-$\tau_0$- space .

**Definition 3.7**

A topological space (X, τ) is called an $e^*$-$\tau_1$- space and denoted by ( $e^*$-$\tau_1$) if for any two distinct points x and y of X there exists two $e^*$-open sets U,V in τ such that x∈ U, x∉V and y ∈V,y ∉ U.

**Remark 3.8**

Every $e^*$-$\tau_1$- space is $e^*$-$\tau_0$- space but the converse is not true .

**Example 3.9**

Let X ={a , b, c}

1-$\tau_x$={X, ∅,{a},{b},{a,b}}$\tau_x^c$ ={X, ∅,{b,c},{a,c},{ c}}.since $E^*$(X, τ)={X, ∅,{a},{b},{a,b},{a,c},{b,c}}. Then (X,$\tau_x$) are $e^*$-$\tau_1$and $e^*$-$\tau_0$- spaces .

2- $\tau_x$={∅,X, {a ,b},{b}} , $\tau_x^c$ ={X, ∅ ,{c},{a,c}}. Since $E^*$(X, τ)={X, ∅,{b},{a,b},{b,c}}.Then (X,$\tau_x$) are $e^*$-$\tau_0$but not $e^*$-$\tau_1$- spaces.

**Theorem 3.10**

Every $\tau_1$- space is an $e^*$-$\tau_1$- space .

**Proof**: It is obvious

from example (3.9) every $\tau_1$- space is $e^*$-$\tau_1$- space.

**Remark 3.11**

The converse of theorem (3.10) may not be true in general consider the following example

Let X={a,b,c}, $\tau_x$={X, ∅, {a,b},{a}} since $E^*$(X, τ) ={X, ∅ , {a},{b},{c},{a,b},{a,c},{b,c}} . Hence (X,$\tau_x$) is $e^*$-$\tau_1$- space but not $\tau_1$- space .

From example (3.9) then (X,$\tau_x$) is $e^*$-$\tau_1$- space but not $\tau_1$- space .

**Theorem 3.12**

A topological space (X, τ) is an $e^*$-$\tau_1$- space if every singleten is $e^*$-closed .

**Proof**: Suppose that every singleton is $e^*$-closed in (X, τ) .

To prove (X, τ) is an $e^*$-$\tau_1$- space, let x and y be any two distinct points of (X, τ). Assume that U=X /{x} and V=X/{y} since {x} and {y} are $e^*$-closed set in (X, τ) .Then U and V are $e^*$- open sets in (X, τ) ,since y∈U ,x∉U and x∈V,y∉V thus (X, τ) is an $e^*$-$\tau_1$.

**Theorem 3.13**

Let (X, τ) be any $e^*$-$\tau_1$ then every a topological space (Y ,$\tau_y$) is $e^*$-$\tau_1$.

**Proof**: A topology space (X, τ) be $e^*$-$\tau_1$,let y ⊆X and let $y_1, y_2$ ∈X such that $y_1 \neq y_2$. ∃two $e^*$-open sets U,V such that $y_1$ ∈U and $y_1$ ∈Y then $y_1$ ∈Y ∩U $=U^*$ and $y_2$ ∈V and $y_2$ ∈Y then $y_2$ ∈Y∩V= $V^*$.Hence (Y ,$\tau_y$) is $e^*$-$\tau_1$.

**Definition 3.14**

A topological space(X, τ) is called an $e^*$-$\tau_2$- space if for any two distinct points x and y of X there exists two disjoint $e^*$-open sets U,V in τ such that x∈U and y∈V.

**Theorem 3.15**

Every $\tau_2$- space is an $e^*$-$\tau_2$- space

**Proof**: It is obvious .

The converse of Theorem (3.15) may not be true in general .

**Example 3.16**

Let X ={a , b, c} ,$\tau_x$={X, ∅, {a},{c}, {a,c}} since $E^*$(X, τ)={X, ∅ , {a},{c}, {a,c},{a,b},{b,c}}.Then (X,$\tau_x$) is $e^*$-$\tau_2$- space but not $\tau_2$- space .

**Theorem 3.17**

Every $e^*$-$\tau_2$- space is an $e^*$-$\tau_1$.

**Proof**: Let (X, τ) be an $e^*$-$\tau_2$- space, to prove (X, τ) be an $e^*$-$\tau_1$.

Let x and y be two distinct points of X since (X, τ) is an $e^*$-$\tau_2$ then ∃ two $e^*$-open sets U, V in X assume x ∈ U , y ∈ V and

V ∩U= ∅ since y∉U and x∉ V then (X, τ) is an $e^*$-$\tau_1$ . The converse of Theorem (3.17) may not be true in general . Consider the following example

**Example 3.18**

Suppose that X ={1,2,3} and$\tau_x$={X, ∅, {3} ,{1,3},{2, 3},{1,2}}. Since $E^*$(X, τ)={X, ∅,{3} ,{1,3} ,{2, 3}, {1,2} then (X,$\tau_x$) is $e^*$-$\tau_1$- space but not $e^*$-$\tau_2$-space .

**Theorem 3.19**

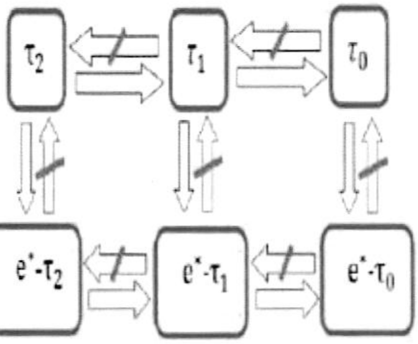
Let (X,$\tau_x$) be any $e^*$-$\tau_2$- space then every relative a topological space (Y ,$\tau_y$) is $e^*$-$\tau_2$- space .

**Proof:** Let (Y,$\tau_y$) be relative a topological space and let $y_1, y_2$ ∈Y such that $y_1 \neq y_2$.Since y ⊆X so $y_1$, $y_2$ ∈X .But (Y,$\tau_y$)is $e^*$-$\tau_2$- space ,thus there exist two disjoint $e^*$-open sets U,V in X such that $y_1$ ∈U and

$y_2 \in V$ and $V \cap U = \emptyset$ thus $y_1 \in Y \cap U = U^*$ and $y_2 \in Y \cap V = V^*$ and $U^* \cap V^* = \emptyset$ .Hence $(Y,\tau_y)$ is $e^*$-$\tau_2$.

**Remark 3.20**

$\tau_i$-space (i=0,1) and $e^*$-$\tau_2$ -space are independent the space of example (3.18) is $\tau_i$-space (i=0,1) but not an $e^*$ - $\tau_2$ –space . From above theorems and definitions we have the following diagram .




## 4- $e^*$- regular and $e^*$- Normal spaces

In this section we introduce new concepts in topological spaces, namely $e^*$-regular,$e^*$-Normal, $e^*$-Complety regular and $e^*$-Complety Normal.

**Definition 4.1**

A topological space $(X,\tau)$ is called an $e^*$- regular space denoted by $[e^*R]$ if for any closed subset F of X and any point x of X which is not in F, there are two $e^*$-open sets U and V such that $x \in U, F \subseteq V$ and $U \cap V = \emptyset$.

**Remark 4.2**

Every regular space is $e^*$- regular space but the converse is not true in general . We observe that in the space (Example 3.16) is an $[e^*R]$ but not [R] .

**Definitions 4.3**

An $e^*$- regular space and $e^*$-$\tau_1$ –space is called an $e^*$-$\tau_3$–space . i.e $e^*$-$\tau_3$= $e^*$-$\tau_1$+$[e^*R]$ .

Hence every $e^*$-$\tau_3$ is $[e^*R]$ but the converse is not true Consider the following example .

**Example 4.4**

Let X={1 , 2 , 3}

1-$\tau_x$={X , $\emptyset$, {1},{1,2}}and $E^*(X,\tau)$={X ,$\emptyset$,{1}, {2}, {3}, {1,2} , {1,3},{2,3}}.Then $(X,\tau)$ is $e^*$-$\tau_3$.

2- $\tau_x$={X , $\emptyset$, {1},{2},{1,2},{1,3}} and $E^*(X,\tau)$ ={X , $\emptyset$, {1},{2},{1,2},{1,3}} .Then $(X,\tau)$ is $[e^*R]$ but not $e^*$-$\tau_3$.

**Proposition 4.5**

Every $\tau_3$- space is an $e^*$-$\tau_3$- space.

**Proof**: It is obvious.

**Theorem 4.6**

Every $e^*$-$\tau_3$- space is an $e^*$-$\tau_2$-space.

**Proof**: Let $(X,\tau)$ be an $e^*$-$\tau_3$- space .To prove that $(X,\tau)$ is an $e^*$-$\tau_2$- space.Assume that x and y be two distinct points of $(X,\tau)$ .Since $(X,\tau)$ is a $e^*$-$\tau_1$-space ,then by theorem(3.12) {y}is closed in X and $x \notin \{y\}$ . Since $(X,\tau)$ is $e^*$-regular ,then there exists two $e^*$-open sets U and V of $(X,\tau)$ such that $x \in U$ ,$\{y\} \subseteq V$ and $U \cap V = \emptyset$. Hence $x \in U$ and $y \in V$.Thus $(X,\tau)$ is an $e^*$-$\tau_2$- space.

From (Example 3.16) hence $(X,\tau_x)$ is $e^*$-$\tau_3$ and $e^*$-$\tau_2$.

The converse of theorem (4.6) may not be true in general .

**Example 4.7**

Let X={a,b,c,d} ,$\tau_x$={X, $\emptyset$ ,{a},{b},{c}, {a,b} ,{a,c} ,{b,c} ,{b,d} ,{b,c,d} , {a,b,c} , {a,b,d}} and $\tau_x^C$={$\emptyset$, X,{b,c,d},{a,c,d},{a,b,d},{c,d},{b,d},{a,d},{a,c},{a},{d},{c}}.Since $E^*(X,\tau)$={X, $\emptyset$ , {a},{b},{c},{a,c}, {a,b},{b,c},{b,d},{c,d},{a,b,d},{b,c,d},{a,b,c}}.Then $(X,\tau_x)$ is $e^*$-$\tau_2$-space but not $e^*$-$\tau_3$.

**Theorem 4.8**

$e^*$-$\tau_3$ property is hereditary property .

**Proof:** Let $(Y,\tau_y)$ be subspace of an $e^*$-$\tau_3$-space $(X,\tau)$.To prove that $(Y,\tau_y)$ is an $e^*$-$\tau_3$-space.Since $(X,\tau)$ is an $e^*$-$\tau_3$-space then $(X,\tau)$ is an $e^*$- regular and $e^*$-$\tau_1$-space by theorem (3.13) $(Y,\tau_y)$ is $e^*$-$\tau_1$-space . Therefore to prove $(Y,\tau_y)$ is $e^*$- regular .Let G be a closed subset of Y and $y \in Y$ such that $y \notin G$ then $G = G \cap Y$,where $G_1$ is closed in X and $y \notin G_1$.Since $(X,\tau)$ is $e^*$- regular then $\exists$ two $e^*$-open sets U and V of $(X,\tau)$ such that $y \in U, G_1 \subseteq V$and $U \cap V = \emptyset$.Since $U \cap Y$and $V \cap Y$ are $e^*$-open sets in$(Y,\tau_y)$ such that $y \in U \cap Y$ ,$G = G_1 \cap Y \subseteq V \cap Y$ and $(U \cap Y) \cap (V \cap Y) = (U \cap V) \cap Y = \emptyset \cap Y = \emptyset$ .Hence $(Y,\tau_y)$ is an $e^*$- regular-space thus $(Y,\tau_y)$ is an $e^*$-$\tau_3$-space.

**Definition 4.9**

A topological space $(X,\tau)$ is $e^*$- normal space denoted by $[e^*N]$if for any $G_1$ and $G_2$ are disjoint closed subset of X ,there are two $e^*$- open sets U and V such that $G_1 \subseteq U$ and $G_2 \subseteq V$ .

**Remark 4.10**

Every Normal space [N] is $e^*$- normal $[e^*N]$ but the converse is not true in general .

**Example 4.11**

Let X={1,2,3,4}, and$\tau_x$={X, $\emptyset$ ,{1,2,3},{1,2,4},{1,2} ,{1},{2},{1,3}},and $E^*(X,\tau)$={ $\emptyset$,X,{1},{2},{2,4},{1, 3,4},{1,4},1,2,3 },{1,2,4},{1,2},{1,3}}

hence $(X,\tau)$ is $[e^*N]$ but not [N] .

**Definition 4.12**

An $e^*$- normal $e^*$-$\tau_1$-space is called an $e^*$-$\tau_4$-space. (i.e) $e^*$-$\tau_4$= $e^*$-$\tau_1$ +$[e^*N]$.

Hence every $e^*$-$\tau_4$ is $[e^*N]$ but the converse is not true ,we observe that the space of (Example 4.11) is an $[e^*N]$ but not $e^*$-$\tau_4$.

**Proposition 4.13**

Every $\tau_4$-space is an $e^*$-$\tau_4$-space .

**Proof:** It is obvious .

**Theorem 4.14**

Every $e^*$-$\tau_4$-space is an $e^*$-$\tau_3$-space.

**Proof**: Let$(X,\tau_x)$ be an $e^*$-$\tau_4$-space. To prove $(X,\tau_x)$ is an $e^*$-$\tau_3$-space. Assume that G is $e^*$- closed subset of X and $x \in X$ such that $x \notin G$ .Since $(X,\tau_x)$ is $e^*$-$\tau_1$-space by Theorem( 3.12) let F={x} is closed such that $F \cap G = \emptyset$. since $(X,\tau_x)$ $e^*$-$\tau_4$-space then there exist two distinct $e^*$- open sets $H_1, H_2$ such that $H_1 \cap$

$H_2=\emptyset$and $\{x\}\subseteq H_1, G\subseteq H_2$ then $x\in H_1$, $G\subseteq H_2$.Hence $(X,\tau_x)$ is $e^*$-$\tau_3$-space.

**Remark 4.15**

The converse of theorem (4.14) may not be true in general .

We observe that the space of (Example 3.16) is an $e^*$-$\tau_3$-space but not an $e^*$-$\tau_4$-space .

**Theorem 4.16**

Let $(X,\tau_x)$ be a topological space ,Then the following conditions are equivalent:

i-$(X,\tau_x)$ is $e^*$-normal .

ii-If H is an $e^*$-open subset of a $e^*$- closed set F then there exists an $e^*$-open set G such that $F\subset G\subset cl\ G \subset H$ .

**Proof**: (i)→(ii)

Let $F\subset H$,with F is $e^*$- closed and H $e^*$- open then $H^C$is $e^*$- closed , and $F\cap H^C=\emptyset$ ,but X is $e^*$- normal . Hence $\exists$ $e^*$- open sets G ,$G^*$ such that $F\subset G$, $H^C\subset G^*$and $G\cap G^*=\emptyset$ but $G\cap G^*=\emptyset$ then $G\subseteq (G^*)^C$ and $H^C\subset G^*$ then $(G^*)^C\subset H$. Hence $(G^*)^C$ is $e^*$- closed then $F\subset G\subseteq cl\ G(G^*)^C\subseteq H$ .

(ii)→(i)

Let $F_1$ and $F_2$ be disjoint $e^*$- closed sets then $F_1\subseteq {F_2}^C$,and ${F_2}^C$ is $e^*$- open by (ii) $\exists$an $e^*$- open set G such that $F_1\subset G\subseteq cl\ G\subseteq {F_2}^C$ but $cl\ G\subset {F_2}^C\rightarrow F_2\subseteq G^C$ and $G\subset cl\ G\rightarrow G\subseteq cl(G^C)=\emptyset$ then $cl(G^C)$ is $e^*$-open thus $F_1\subset G$ and $F_2\subseteq cl(G^C)$ with G, $cl(G^C)$ disjoint $e^*$- open set. Hence X is $e^*$- normal.

**Theorem 4.17**

$e^*$-$\tau_4$-space property is hereditary property.

**Proof**: :Let $(Y,\tau_y)$ be subspace of $(X,\tau)$.To prove that $(Y,\tau_y)$ is an $e^*$-$\tau_4$- space. Since $(X,\tau_x)$ is an $e^*$-$\tau_4$- space then $(X,\tau_x)$ is an $e^*$- normal and $e^*$-$\tau_1$-space hence by theorem( 3.13) $(Y,\tau_y)$ is an $e^*$-$\tau_1$-space to prove that $(Y,\tau_y)$ is $e^*$- normal .

Let $F_1$ and $F_2$ be disjoint closed sets of Y then $F_1=G_1\cap Y$and $F_2=G_2\cap Y$ where $G_1$and $G_2$ are closed in X and $G_1\cap G_2=\emptyset$ .Since$(X,\tau_x)$ is an $e^*$- normal then $\exists$ two $e^*$- open sets U and V such that $G_1\subseteq U$ and $G_2\subseteq V$ and $U\cap V=\emptyset$ therefore $U\cap Y$and $V\cap Y$ are $e^*$- open sets in $(Y,\tau_y)$ such that $F_1=G_1\cap Y\subseteq U\cap Y$,$F_2=G_2\cap Y\subseteq V\cap Y$ and $(U\cap Y)\cap(V\cap Y)=\emptyset$ Hence $(Y,\tau_y)$ is $e^*$- normal space thus $(Y,\tau_y)$ is an $e^*$-$\tau_4$-space .

**Definition 4.18**

A topological space $(X,\tau)$ is called an $e^*$- completely regular space denoted by [ $e^*c\mathcal{R}$] if for any closed subset F of X and any point x of X which is not in F , there is an $e^*$-continuous function f: $(X,\tau_x)\rightarrow([0,1],\sigma)$ such that f(x)=0 and f(F)={1}.

Hence every [CR] completely regular space is $e^*$-completely regular space but the converse is not true in general .

**Proposition 4.19**

$e^*$-completely regular space is $e^*$- regular space.

**Proof**: Let F be a closed subset of X and suppose $p\in X$ and $p\notin F$ by hypothesis X is $e^*$- completely regular space .Hence $\exists$ $e^*$-continuous function f: X→ [0,1] such that f(p)=0 ,f(F)={1}. But $\mathcal{R}$ and its subspace [0,1] are $e^*$-$\tau_2$-space .Then $\exists$ two disjoint $e^*$-open sets G and H containing 0 and 1 respectively .Accordingly their inverse $f^{-1}[G]$ and $f^{-1}[H]$ are disjoint $e^*$-open and contain P and F then X is [$e^*\mathcal{R}$].

**Theorem 4.20**

$e^*$-completely regular space property is hereditary property.

**Proof:** Let $(Y,\tau_y)$ be subspace of $(X,\tau)$ .To prove that $(Y,\tau_y)$ is an $e^*$- completely regular space .Let $G_1$ be closed subset of Y and $x\in Y$ such that $x\notin G_1$since $(Y,\tau_y)$ be subset of $(X,\tau_x)$ then there exist G closed subset of X such that $G_1=G\cap Y$ ,now $G\in X$ and $x\notin G$ .Since $(X,\tau_x)$ is $e^*$- completely regular space there is an $e^*$-continuous function f: X→[0,1] such that f(x)=0 ,f( $G_1$ )={1} therefore by Bases $f(x)=f^*(x)\forall x\in Y$.Hence $f^*:Y\rightarrow[0,1]$ statefly $f^*(x)=0$ because $x\in Y$ and $f^*(G_1)=\{1\}$ because $G_1=G\cap Y$ then $(Y,\tau_y)$ is $e^*$- completely regular space.

**Definition 4.21**

A topological space $(X,\tau)$ is called $e^*$- completely Normal space denoted by [ $e^*$CN] if for any A and B separation sets there exist H ,G two disjoint $e^*$- open sets such that $A\subseteq H$ and $B\subseteq G$.

**Remark 4.22**

Every completely normal space [CN] is [$e^*$CN] but the converse is not true in general .

**Example 4.23**

Let X={a ,b , c}

1- $\tau_x=\{X,\emptyset,\{a\},\{c\},\{a,c\},\{b,c\}\}$

$\tau_x^C=\{\emptyset,X,\{a\},\{b\},\{a,b\},\{b,c\}\}$ and $E^*(X,\tau)=\{\emptyset,X,\{a\},\{c\},\{a,c\},\{b,c\}\}$. Then $(X,\tau)$ are [CN] and [ $e^*$CN].

2- $\tau_x=\{X,\emptyset,\{a\},\{c\},\{a,c\}\}$and $E^*(X,\tau)=\{\emptyset,X,\{a\},\{c\},\{a,c\},\{b,c\},\{a,b\}\}$then $(X,\tau_x)$ is [ $e^*$CN] but not [CN].

**Proposition 4.24**

Every $e^*$ -completely normal space is $e^*$ - normal space.

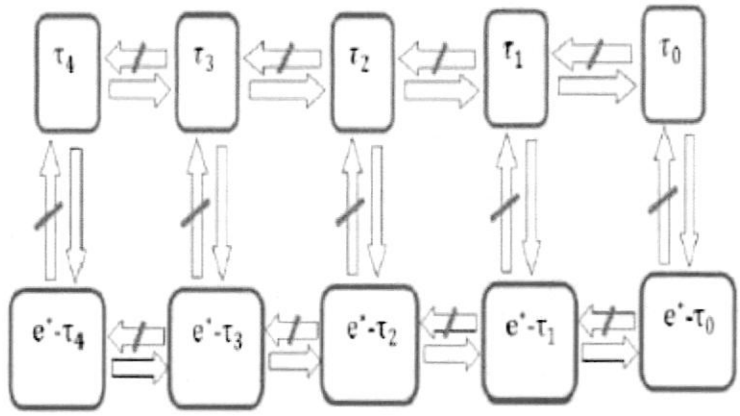
**Proof**: Suppose that $(X,\tau)$ is [ $e^*$CN] by definition let A and B are disjoint closed subset over x , therefore $A\cap cl(B)=cl(A)\cap B=A\cap B=\emptyset$ Then A and B are separation ,Since $(X,\tau)$ is[ $e^*$CN] there exist H,G are two $e^*$- open sets ,$H\cap G=\emptyset$ such that $A\subseteq G$ ,$B\subseteq H$.Then $(X,\tau)$ is [ $e^*$N] .

The converse of proposition( 4.24) may not be true in general . Consider the following example

Let X={a,b} , $\tau=\{\emptyset,X\}$ then $(X,\tau)$ is[ $e^*$N] but not [ $e^*$CN] .

**Remark 4.25**

From theorems and definitions above we have the following diagram .

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# بعض الأصناف الجديدة من بديهيات الفصل في الفضاءات التبولوجية

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**الملخص**

في هذا البحث قدمنا صف جديد من بديهيات الفصل من خلال مفهوم المجموعة المفتوحة -$e^*$ المقدمة سابقا ودرسنا العلاقات وبعض الصفات الوراثية لهذه البديهيات وأخيرا درسنا وتحققنا من بعض الخواص للفضاءات المنتظمة و كاملة الانتظام$e^*$و السوية و الفضاءات الكاملة السوية$e^*$.