Completely Regular and Weak Completely Regular In Intuitionistic Topological Spaces

Taha, H. Jasim[1] , Samer, R.Yaseen[1] , Luma S. Abdual baqi[2]
[1] Dept .of Mathematics , College of computer Science and mathematics , Tikrit University , Tikrit , Iraq
[2] Dept .of Mathematics , College of education for women , Tikrit University , Tikrit , Iraq

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Abstract

This paper we introduces types of completely regular in intuitionistic topological spaces and study the relation among them. Is also we introduces types of weak completely regular in intuitionistic topological spaces and study the relation among them. Finally we study the relation between completely regular and weak completely regular in intuitionistic topological spaces .

Introduction

The concept of fuzzy set was introduced by Zadeh [15] in his classical paper 1965. After the discovery of the fuzzy sets much attention has been paid to generalize the basic concepts of classical topology in fuzzy setting and thus a subset naturally plays a very significant role in the study of fuzzy topology which introduced by Chang 1968 [6],and later by Malghan and Benchalli in 1981 [10]. In 1983, Atanassov introduced the concept of" Intuitionistic fuzzy set " [1],[2],[3],[4] using a type of generalized fuzzy set , Later, the concept is used to define intuitionistic fuzzy special sets by Coker [7], and intuitionistic fuzzy topological spaces are introduced by Coker [8]. In this direction, the concept of separation axioms in intuitionistic fuzzy topological spaces which introduced by Bayhan, S. and Coker, D [5]. Also concept of intuitionistic topological spaces which introduced by Coker in 2000 [9] .

In this paper we introduce completely regular in intuitionistic topological spaces. Also we introduces a weak completely regular in intuitionistic topological spaces and. Finally we study the relation between completely regular and weak completely regular in intuitionistic topological spaces .

Preliminaries

Definition 1.1 [7]

Let X be a non empty set . An intuitionistic set A is an object having the form

$A = \langle x, A_1, A_2 \rangle$,where A_1 and A_2 are subsets of X satisfying $A_1 \cap A_2 = \emptyset$. The set A_1 is called the set of members of A , while A_2 is called the set of nonmembers of A .

Remark

Any subset A of X can be regarded as intuitionistic set having the for

$\widetilde{A} = \langle x, A, A^c \rangle$.

Definition 1. 2 [7]

Let X be a nonempty set , and let $A = \langle x, A_1, A_2 \rangle$ and $B = \langle x, B_1, B_2 \rangle$ be intuitionistic sets respectively, furthermore, let $\{ A_i \ ; i \in J \}$ be an arbitrary and only if $A_1 \subseteq B_1$ and $B_2 \subseteq A_2$, $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$, The family of intuitionistic sets in X , where $A_i = \langle x, A_i^{(1)}, A_i^{(2)} \rangle$, then $A \subseteq B$ if complement of A is denoted by $\overline{A}$ and defined by $\overline{A} = \langle x, A_2, A_1 \rangle$, $A = \langle x, A_1, A_1^c \rangle$, $SA = \langle x, A_2^c, A_2 \rangle$, $\cup A_i = \langle x, \cup A_i^{(1)}, \cap A_i^{(2)} \rangle$, $\cap A_i = \langle x, \cap A_i^{(1)}, \cup A_i^{(2)} \rangle$, $\widetilde{\Phi} = \langle x, \emptyset, X \rangle$, $\widetilde{X} = \langle x, X, \emptyset \rangle$.

Definition 1.3 [7]

Let X be a nonempty set , $p \in X$ a fixed element in X , and let A $\langle x, A_1, A_2 \rangle$ be an intuitionistic set (IS, for short).The IS $\dot{p}$ defined by $\dot{p} = \langle x, \{p\}, \{p\}^c \rangle$ is called an intuitionistic point (IP for short) in X . The IS $\ddot{p} = \langle x, \emptyset, \{p\}^c \rangle$ is called a vanishing intuitionistic point (VIP, for short) in X .

The IS $\dot{p}$ is said to be contained in A ($\dot{p} \in A$, for short) if and only if $p \in A_1$, and similarly IS $\ddot{p}$ contained in A. ($\ddot{p} \in A$, for short) if and only if $p \notin A_2$. For a given IS A in X , we may write $A = (\cup \{\dot{p} : \dot{p} \in A\}) \cup (\cup \{\ddot{p} : \ddot{p} \in A\})$, and whenever A is not a proper IS (i.e., if A is not of the form $A = \langle x, A_1, A_2 \rangle$ where $A_1 \cup A_2 \neq X$) , then $A = \cup \{ \dot{p} : \dot{p} \in A\}$ hold . In general, any IS A in X can be written in the form $A = \dot{A} \cup \ddot{A}$ where $\dot{A} = \cup \{\dot{p} : \dot{p} \in A\}$ and $\ddot{A} = \cup \{\ddot{p} : \ddot{p} \in A\}$.

Definition 1. 4 [7]

Let X and Y be two nonempty sets and f : $X \longrightarrow Y$ be a function.

a) If $B = \langle y, B_1, B_2 \rangle$ is an IS in Y, then the preimage (inverse image) of B under f is denoted by $f^{-1}(B)$ is an IS in X and defined by $f^{-1}(B) = \langle x, f^{-1}(B_1), f^{-1}(B_2) \rangle$.

b) If $A = \langle x, A_1, A_2 \rangle$ is an IS in X, then the image of A under f denoted by $f(A)$ is an IS in Y defined by $f(A) = \langle y, f(A_1), \underline{f}(A_2) \rangle$, where $\underline{f}(A) = \left(f(A_2^c)\right)^c$.

Definition 1.5 [7],[8]

An intuitionistic topology on a nonempty set X is a family T of an intuitionistic sets in X satisfying the following conditions.

(1) $\widetilde{\Phi}, X \in T$.
(2) T is closed under finite intersections.
(3) T is closed under arbitrary unions.

The pair (X, T) is called an intuitionistic topological space (ITS,for short) .

Any element in T is usually called intuitionistic open set (IOS , for short).

The complement of an IOS in a ITS (X, T) is called intuitionistic closed set (ICS , for short).

Definition 1. 6 [14]

Let (X,T) be an ITS and let $A=\langle x,A_1,A_2\rangle$ be an intuitionistic subset (IS's, for short) in a set X. The interior (IntA, for short) and closure (ClA, for short) of a set A of X are defined : IntA $=\cup\{G : G\subseteq A, G\in T\}$, ClA $=\cap\{F : A\subseteq F, \overline{F}\in T\}$. In other words: The IntA is the largest intuitionistic open set contained in A, and ClA is the smallest intuitionistic closed set contain A i.e., Int A $\subseteq$ A and A $\subseteq$ ClA. In the following definition we give a product of an intuitionistic set and a product of an intuitionistic topological space.

Definition 1. 7 [5] ,[14]

Giving the nonempty set X, we define the diagonal Δ_x as IS in $X\times X$ in the following way: $\Delta_x \langle(x_1,x_2),\{(x_1,x_2): x_1=x_2\},\{(x_1,x_2): x_1\neq x_2\}\rangle$. Now we are ready to give the definition of IP and VIP of the product $X\times Y$.

Definition 1.8. [5]

Let X and Y be two nonempty set's and $(p,q)\in X\times Y$ be a fixed element in $X\times Y$, then the IP $(\dot{p,q})$ is contained in $U\times V$ $((p,q)\,U\times V$, for short$)$ if and only if $(p,q)\in U_1\times V_1$, and IVP $(\ddot{p,q})$ is contained in $U\times V$ $((\ddot{p,q})\in U\times V$, for short$)$ if and only if $(p,q)\notin (U_2^c\times V_2^c)^c$, or equivalently $(p,q)\in (U_2^c\times V_2^c)$.

Definition 1.9.[12]

Let (X,T) be an ITS, then (X,T) is said to be :

a) R(i) if and only if for each $x\in X$ and $F\subseteq X$, F is ICS and $\tilde{x}\notin F$ there exist U,V $\in T$ such that $\tilde{x}\in U$, $F\subseteq V$ and $U\cap V=\tilde{\emptyset}$.

b) R(ii) if and only if for each $x\in X$ and $F\subseteq X$, F is ICS and $\tilde{\tilde{x}}\notin F$ there exist U,V $\in T$ such that $\tilde{\tilde{x}}\in U$, $F\subseteq V$ and $U\cap V=\tilde{\emptyset}$.

c) R(iii) if and only if for each $x\in X$ and $F\subseteq X$, F is ICS and $\tilde{x}\notin F$ there exist U,V $\in T$ such that $\tilde{x}\in U$, $F\subseteq V$ and $U\subseteq\overline{V}$.

d) R(iv) if and only if for each $x\in X$ and $F\subseteq X$, F is ICS and $\tilde{\tilde{x}}\notin F$ there exist U,V $\in T$ such that $\tilde{\tilde{x}}\in U$, $F\subseteq V$ and $U\subseteq\overline{V}$.

Definition 1.10.[12]

Let (X,T) be an ITS, then (X,T) is said to be :

a) wR(i) if and only if for each $x\in X$ and $F\subseteq X$, F is ICS and $\tilde{x}\notin F$ there exist U,V $\in$ WO(X) such that $\tilde{x}\in U$, $F\subseteq V$ and $U\cap V=\tilde{\emptyset}$. Where WO(X) is set of all weak intuitionistic open sets in X.

b) wR(ii) if and only if for each $x\in X$ and $F\subseteq X$, F is ICS and $\tilde{\tilde{x}}\notin F$ there exist U,V $\in$ WO(X) such that $\tilde{\tilde{x}}\in U$, $F\subseteq V$ and $U\cap V=\tilde{\emptyset}$. Where WO(X) is set of all weak intuitionistic open set in X.

Definition 1.11. [15]

A map $f:(X,T)\to(Y,\gamma)$ is called intuitionistic-continuous if the inverse image is intuitionistic - open set of (Y,γ) for every intuitionistic open set of (Y,γ).

Section .2 completely regular and weak completely regular in intuitionistic topological spaces.

In this section we introduce completely regular in intuitionistic topological spaces. Also we introduce a weak completely regular in intuitionistic topological spaces and study the relation among them.

Definition 2-1

Let (X,T) be an ITS, then (X,T) is said to be :

a) CR(i) if and only if for each $x\in X$ and $F\subseteq X$, F is ICS, $\tilde{x}\notin F$ there exists a continuous function $f: X\longrightarrow[0,1]$ such that $f(F)=\{1\}$, $f(\tilde{x})=\{0\}$.

b) CR(ii) if and only if for each $x\in X$ and $F\subseteq X$, F is ICS, $\tilde{\tilde{x}}\notin F$ there exists a continuous function $f: X\longrightarrow[0,1]$ such that $f(F)=\{1\}$, $f(\tilde{\tilde{x}})=\{0\}$.

Proposition 2-3

Let (X,T) be an ITS, then the following implication is valid:

CR(i) implies to CR(ii)

Proof: CR(i) $\longrightarrow$ CR(ii)

Let (X,T) be satisfies CR(i) if for each $x\in X$ and $F\subseteq X$, F is ICS $\tilde{x}\notin F$ there exist a continuous function $f: X\longrightarrow[0,1]$ such that $f(F)=\{1\}$, $f(\tilde{x})=\{0\}$.

Since if $\tilde{x}\notin F$ then $\tilde{\tilde{x}}\notin F$ and since $f(F)=\{1\}$. Thus $f(\tilde{\tilde{x}})=\{0\}$. Therefore (X,T) be satisfies CR(ii).

The reserve implication in Proposition 2-3is not true in general. The following counter example show the cases.

Example 2-4: Let $X=\{a,b,c\}$ and define $T=\{\tilde{\Phi},\tilde{X},A,B,C\}$, where $A=\langle x,\emptyset,\{a,c\}\rangle$, $B=\langle x,\emptyset,\{b\}\rangle$, $C=\langle x,\emptyset,\emptyset\rangle$. Define a function $f: X\to[0,1]$ by $f(a)=f(b)=f(c)=0$. Then (X,T) satisfies CR(ii), because if $a\in X$, then $f(F)=\overline{B}=\{1\}$, $f(\tilde{\tilde{a}})=\{0\}$ and $b\in X$, then $f(F)=\overline{A}=\{1\}$, $f(\tilde{\tilde{b}})=\{0\}$, finally $c\in X$, then $f(F)=\overline{B}=\{1\}$, $f(\tilde{\tilde{c}})=\{0\}$. But (X,T) is not satisfiesCR(i), because there is no continuous function satisfies condition of CR(i).

Definition 2-5

Let (X,T) be an ITS, then (X,T) is said to be :

a) CR(i) if and only if for each $x\in X$ and $F\subseteq X$, F is IWCS, $\tilde{x}\notin F$ there exists a continuous function $f: X\longrightarrow[0,1]$ such that $f(F)=\{1\}$, $f(\tilde{x})=\{0\}$.

b) WCR(ii) if and only if for each $x\in X$ and $F\subseteq X$, F is IWCS, $\tilde{\tilde{x}}\notin F$ there exists a continuous function $f: X\longrightarrow[0,1]$ such that $f(F)=\{1\}$, $f(\tilde{\tilde{x}})=\{0\}$.

Proposition 2-6

Let (X,T) be an ITS, then the following implication is valid:

WCR(i) implies to WCR(ii)

Proof: WCR(i) $\longrightarrow$ WCR(ii)

Let (X,T) be satisfies $WCR(i)$ if for each $x \in X$ and $F \subseteq X$, F is ICS $\tilde{x} \notin F$ there exist a continuous function $f: X \longrightarrow [0,1]$ such that $f(F)=\{1\}$, $f(\tilde{x})=\{0\}$.

Since if $\tilde{x} \notin F$ then $\tilde{\tilde{x}} \notin F$ and since $f(F)=\{1\}$.Thus $f(\tilde{\tilde{x}})=\{0\}$. Therefore (X,T) be satisfies $WCR(ii)$.

The reserve implication in Proposition 2-6 is not true in general. The following counter example show the cases.

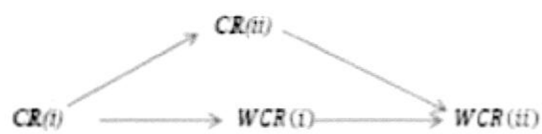
Example 2-7: Let $X=\{a,b,c\}$ and define $T=\{\tilde{\Phi},\tilde{X},A,B\}$, where $A=\langle x,\emptyset,\{a\}\rangle$ $B=\langle x,\emptyset,\emptyset\rangle$ and $WO(X)=T \cup \{C,D,E\}$, where $C=\langle x,\emptyset,\{b,c\}\rangle$, $D=\langle x,\emptyset,\{b\}\rangle$.$E=\langle x,\emptyset,\{a,b\}\rangle$. Define a function f: $X \rightarrow [0,1]$ by $f(a)=f(b)=f(c)=0$. Then (X,T) satisfies $WCR(ii)$, because if $a \in X$, then $f(F)=\bar{C}=\{1\}$, $f(\tilde{\tilde{a}})=\{0\}$ and $b \in X$, then $f(F)=\bar{D}=\{1\}$, $f\left(\tilde{\tilde{b}}\right)=\{0\}$, finally $c \in X$,then $f(F)=\bar{A}=\{1\}$,$f(\tilde{\tilde{c}})=\{0\}$. But (X,T) is not satisfies $WCR(i)$, because there is no exist continuous function satisfies condition of $WCR(i)$.

3- The Relations Between Completely Regular and Weak Completely regular

In this section we introduce the relation between completely regular and weak completely regular in intuitionistic topological spaces.

Proposition 3-1

Let (X,T) be an ITS, then the following implications are valid:



Proof: $CR(i) \longrightarrow WCR(i)$

Let (X,T) be satisfies $CR(i)$ if for each $x \in X$ and $F \subseteq X$, F is ICS, $\tilde{x} \notin F$ there exists for E there exists a continuous function $f: X \longrightarrow [0,1]$ such that $f(F)=\{1\}$, $f(\tilde{x})=\{0\}$. Since every intuitionistic open set is intuitionistic weak open set,

and F is ICS, so that F is IWCS such that $f(F)=\{1\}$, $f(\tilde{x})=\{0\}$.Therefore (X,T) be satisfies $WCR(i)$.

$CR(ii) \longrightarrow WCR(ii)$

Let (X,T) be satisfies $CR(ii)$ if for each $x \in X$ and $F \subseteq X$, F is ICS, $\tilde{\tilde{x}} \notin F$ there exists a continuous function $f: X \longrightarrow [0,1]$ such that $f(F)=\{1\}$, $f(\tilde{\tilde{x}})=\{0\}$. Since every intuitionistic open set is intuitionistic weak open set, and F is ICS, so that is IWCS such that $f(F)=\{1\}$, $f(\tilde{x})=\{0\}$ .Therefore (X,T) be satisfies $CR(ii)$.

Remark 3-2 : by transitive: $CR(i) \longrightarrow WCR(ii)$.

In general the converse of the diagram appears in Proposition 3-1 is not true in general. The following counter examples show the cases.

Example 3-3: Let $X=\{a,b,c\}$ and define $T=\{\tilde{\Phi},\tilde{X},A,B\}$, where $A=\langle x,\emptyset,\{a,b\}\rangle$, $B=\langle x,\emptyset,\emptyset\rangle$ and $WO(X)=T \cup \{C,D,E,F,G,H,I,J,K,L,M\}$ where $C=\langle x,\{a\},\{b,c\}\rangle$, $D=\langle x,\{b\},\{a,c\}\rangle$, $E=\langle x,\emptyset,\{c\}\rangle$, $F=\langle x,\emptyset,\{b\}\rangle$, $G=\langle x,\emptyset,\{a\}\rangle$,$H=\langle x,\emptyset,\{a,c\}\rangle$, $I=\langle x,\emptyset,\{b,c\}\rangle$, $J=\langle x,\{c\},\{a,b\}\rangle$, $K=\langle x,\{b,c\},\emptyset\rangle$, $L=\langle x,\{a,c\},\emptyset\rangle$, $M=\langle x,\{a,b\},\emptyset\rangle$ Define a function f: $X \rightarrow [0,1]$ by $f(a)=f(b)=f(c)=0$. Then (X,T) satisfies $WCR(i)$, because if $a \in X$, then $f(F)=\bar{C}=\{1\}$, $f(\tilde{a})=\{0\}$ and $b \in X$, then $f(F)=\bar{D}=\{1\}$, $f(\tilde{b})=\{0\}$, finally $c \in X$,then $f(F)=\bar{A}=\{1\}$, $f(\tilde{c})=\{0\}$. But (X,T) is not satisfiesCR(i), because there is no exist continuous function satisfies condition of $CR(i)$.

Example 3-4: Let $X=\{a,b\}$ and define $T=\{\tilde{\Phi},\tilde{X},A\}$, where $A=\langle x,\emptyset,\{b,c\}\rangle$,

and $WO(X)=T \cup \{B,C,D,E\}$, where $B=\langle x,\emptyset,\emptyset\rangle$, $C=\langle x,\emptyset,\{a,c\}\rangle$,$D=\langle x,\emptyset,\{a,b\}\rangle$, $E=\langle x,\emptyset,\{a\}\rangle$. Define a function f: $X \rightarrow [0,1]$ by $f(a)=f(b)=f(c)=0$. Then (X,T) satisfies $WCR(ii)$, because if $a \in X$, then $f(F)=\bar{A}=\{1\}$, $f(\tilde{\tilde{a}})=\{0\}$ and $b \in X$, then $f(F)=\bar{C}=\{1\}$, $f\left(\tilde{\tilde{b}}\right)=\{0\}$, finally $c \in X$,then $f(F)=\bar{D}=\{1\}$, $f(\tilde{\tilde{c}})=\{0\}$. But (X,T) is not satisfies $CR(ii)$, because there is no exist continuous function satisfies condition of $CR(ii)$ .Also (X,T) is not satisfies $CR(i)$, because there is no exist continuous function satisfies condition of $CR(ii)$.

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الانتظام الكامل والانتظام الكامل الضعيف في الفضاءات التبولوجية الحدسية

طه حميد جاسم[1] ، سامر رعد ياسين[1] ، لمى سعد عبد الباقي[2]

[1] قسم الرياضيات ، كلية علوم الحاسوب والرياضيات ، جامعة تكريت ، تكريت ، العراق

[2] قسم الرياضيات ، كلية التربية للبنات ، جامعة تكريت ، تكريت ، العراق

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الملخص

هذا البحث قدم انواع الانتظام الكامل في الفضاءات التبولوجية الحدسية ودرس العلاقة بينها، وقدم ايضا انواع الانتظام الكامل الضعيف في الفضاءات التبولوجية الحدسية ودرس العلاقة بينها ، واخيرا دراس العلاقة بين الانتظام الكامل والانتظام الكامل الضعيف في الفضاءات التبولوجية الحدسية .