The Minimal Blocking Set Of Size ($[\frac{3}{2}(q+1)+1]$) In *PG* (2 , 19)

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Abstract

A non-Redei type minimal blocking set of size $\frac{3}{2}(q+1)$ was known in $PG(2,7)$. Blokhuis , Brouwer and Wibrink [1] in (2003) studied the minimal blocking sets of size $\frac{3}{2}(q+1)$ in $PG(2,q)$ for $q<41$, q is prime , and we found a second minimal blocking set of this size that is non-Re'dei type in $PG(2,13)$, also show that no other non-Re'dei sets of size $\frac{3}{2}(q+1)$ occur in $PG(2,q)$, $q<41$.

In this paper we prove that the projective plane $PG(2,q)$, $q=19$ does not contain minimal blocking set of size $[\frac{3}{2}(q+1)+1]$ which have non-Re'dei type .

Introduction

Let GF (q) denote the Galois field of q elements. A projective plane PG(2,q) over GF (q) is a two-dimensional projective space contains q^2+q+1 points and q^2+q+1 lines. Every line contains q+1 points and every point is on q+1 lins .

A blocking set B in $PG(2,q)$ is a set of points meet any line in at least one point. It is called non-trivial when it does not contain a line. A blocking set B in PG (2,q) is minimal if B\{p} not blocking set for every point p in B .

Let M be aset of points in any projective plane. An i-secant is a line meeting M in exactly i points, if i=0,1,2,…,n we called respectively external line, 1-secant , 2-secant , …, n-secant .

A blocking set B is called of Re'dei type when there is a line L in PG(2,q) contains K points from B such that | B | =q+K, that is $|B\setminus L|=q$.

A projective triangle B of side n in $PG(2,q)$ is a set of $3(n-1)$ points , n of which lie on each side of a triangle $P_0P_1P_2$ such that the vertices are in B , and if Q_0 on P_1P_2 and Q_1 on P_2P_0 are in B, then $Q_2=Q_0Q_1\cap P_0P_1$ is in B , which gives an example of a blocking set of size $\frac{3}{2}(q+1)$ in projective plane of odd order .

The projective plane *PG* (2,19)

The plane $PG(2,19)$ contains 381 points and 381 lines , 20 points on every line and 20 lines through every point . Given acyclic projectivity $T=\begin{bmatrix}0&1&0\\0&0&1\\1&1&4\end{bmatrix}$ on $PG(2,19)$, let P_1 be the point (1,0,0) then $P_i=P_1T^i, i=1,\ldots,381$ are 381 points of $PG(2,19)$.

Let L_1 be the line which contains the poin P_1 , P_2 , P_{24} , P_{37} , P_{52} , $P_{56}, P_{82}, P_{93}, P_{126}, P_{157}, P_{234}, P_{244}, P_{252}, P_{261}, P_{268}, P_{273}, P_{320}, P_{334}$, P_{340} and P_{380} then $L_i=L_1T^i, i=1,\ldots,381$ are the lines of $PG(2,19)$.

Theorem

Let B be a minimal blocking set of size 31 in $PG(2,19)$, and let L be an n-secant of B then $1\le n\le 12$.

Proof

Let L be 13-secant line of B and let $P=(x,y,z)$ be a point in $PG(2,19), P\in L\setminus B$.

Since there are 19 lines passes through the point P and intersect B at least with one point, so by allocating the points of B on these 19 lines plus the (13) points of L we get the size of B=32, contradiction .

Theorem

Let B be a minimal blocking set of size 31 in PG(2,19) , then, there are at most 19 (1 – secant) passes through a point not belong to *B*.

Proof

Suppose that there are 20 (1 – secant) passes through a point p not belong to *B*.

But we know that in PG (2,19) there are (20) lines passes through any point p implies p∈B, contradiction. Before we introduce the main result, we must recall the following theorem:

Ca'cs Theorem

[4] In $PG(2,q)$, Let B be a minimal blocking set of size $q+K$.

If a line L intersect B in exactly $K-1$ points, then there is a point $Q\notin B$ such that any line joining the point Q with any point of $L\setminus B$ contains exactly two points of B, and $K\ge(q+3)/2$.

Theorem

There is no minimal blocking set B of size 31 in $PG(2,19)$ having 11–secant or 12– secant.

Proof

Let $P=(x,y,z)$ be apoint in $PG(2,19)$ and L_1 be 11–secant, suppose that L_1 be an infinite line$(L_1: z=0)$,and $L_1\setminus B=\{A_1,A_2,A_3,A_4,\ldots,A_9\}$.

By Ca'cs Theorem there is a point $Q \notin B$ such that $QA_i, i = 1,...,9$ be a 2-seconts and these lines contains 18 points of B , let U_1 be the nineteenth point, since A_i lies on one 2-secant and eighteen 1-secant, then the lines $U_iA_i, i = 1,...,9$ are 1-secants and U_1Q is a line passing through one point of $B \cap L_1$.

By using the following Linear transformations

$T_1 : (x, y, z) \to (y + z, x + 18z, 0)$

$T_2 : (x, y, z) \to (18y + z, x + y, 0)$

Then the effect of the Linear group $S_3 = (T_1, T_2)$ about the infinite line gives five orbits .

First orbit :{(1,0,0),(0,1,0),(1,18,0)} .

Second orbit:
{(1,13,0),(1,3,0),(1,15,0),(1,4,0),(1,5,0),(1,14,0)} .

Third orbit :
{(1,12,0),(1,8,0),(1,10,0),(1,16,0),(1,6,0),(1,2,0)}.

Fourth orbit :{(1,17,0),(1,9,0),(1,1,0)} .

Fifth orbit :{(1,7,0),(1,11,0)}.

Let

$A_1 = (1,0,0), A_2 = (0,1,0), A_3 = (1,18,0), A_4 = (1,13,0), A_5 = (1,12,0), A_6 = (1,17,0), A_7 = (1,7,0), A_8 = (1,5,0)$ $and A_9 = (1,16,0)$,

the third source point Q = (0,0,1) and the fourth source point $U_1 = (1,1,1)$.

We know that $AG(2,19) = PG(2,19)/L_1$ is the affine plane of the projective plane $PG(2,19)$.

Let $B_1 = B \setminus (L_1 \cup \{U_1, U_2\})$ which contains the affine points $P_i = (x, y), i = 0,...,18$.

The following lines represent the intersection of the lines passes through the points $A_1,..., A_9$ with the two points Q and U_1.

Table (1)

Lines	Points
L3	(10, 0), (6 , 0), (3 , 0), (5 , 0), (15, 0), (18, 0), (8 , 0), (4 , 0), (7 , 0), (13, 0)
L2	(0 , 9), (0 ,11), (0, 5), (0 ,17), (0 , 6), (0 ,10), (0 ,14), (0 ,12), (0 ,18), (0 ,16)
L228	(17, 2), (13, 6), (4 ,15), (10, 9), (8 ,11), (11, 8), (6 ,13), (12, 7), (16, 3), (5 ,14)
L51	(7,15), (2 , 7), (14,11), (17,12), (15, 5), (13,17), (6 , 2), (12, 4), (10,16),(16 ,18)
L348	(5 , 3), (3 ,17), (16, 2) , (15, 9) , (2 , 5), (10, 6), (17,14), (4 ,10), (9 ,13), (11,18)
L133	(15, 8),(2 ,15), (18, 2), (12,14), (11,16), (8 , 3), (3 ,13), (14,10), (10,18), (5 , 9)
L151	(14, 3), (3 , 2), (15,10), (12, 8), (7 ,11), (10,13), (4 , 9), (18,12) ,(9 , 6), (16,17)
L5	(12, 3), (9 , 7), (17, 9) , (15,18), (5 , 6), (8 , 2), (6 ,11), (18,14), (2 ,10), (16, 4)
L292	(13,18), (7,17), (9 ,11), (5 , 4), (14,15), (11, 5), (3 ,10), (8 ,14), (4 , 7), (17, 6)

By choosing any two points of line L_3 we get (45) cases as follows :

Case-1

Choose the two points (10,0) , (6,0) of the line L_3 then the point (10,0) delete all the following points on the other lines (table-1) which are :
{(14,15),(0,10),(11,18),(9,6),(12,7),(15,8),(2,10),(16,2),(13,17),(8,14),(14,10),(9,7),(14,11),(15,9),(16,4),(5,3),(12,14),(0,6),(4,15),(11,5),(18,2),(3,2),(11,16),(18,14),(2,5), (0,11),(17,12),(8,2),(7,17)}.

And the point (6,0) delete all the following points on the other lines (table-1) which are
{(15,10),(0,6),(14,11),(11,8),(2,5),(0,17),(5,6),(3,2),(16,18),(12,7),(14,3),(0,12),(18,14),(9,13),(7,17),(3,17),(8,14),(11,16),(10,9),(12,4),(2,10),(4,9),(2,5),(13,17),(18,2),(0,18), (5,3)}
except the point (4,10) which is a contradiction .

Case-2

Choose the two points (10,0), (3,0) of the line L_3 then the point (10,0) delete all the following points on the other lines (table-1) which are
{(14,15),(0,10),(11,18),(9,6),(12,7),(15,8),(2,10),(16,2),(13,17),(8,14),(14,10),(9,7),(14,11),(15,9),(16,4),(5,3),(12,14),(0,6),(4,15),(11,5),(18,2),(3,2),(11,16),(18,14),(2,5), (0,11),(17,12),(8,2),(7,17)} .

And the point (3,0) delete all the following points on the other lines (table-1) which are
{(8,14),(9,13),(7,15),(16,17),(0,18),(12,3),(14,10),(16,4),(8,3),(2,7),(13,6),(9,7),(6,13),(7,11),(0,6),(13,18),(6,2),(5,14),(4,7),(11,18),(0,17),(15,8),(12,7),(10,16),(9,11),(0,9), (18,12),(16,18)} .
except the point (4,10) which is a contradiction .

Case-3

Choose the two points (10,0) , (5,0) of the line L_3 then the points on the other lines in (table-1) deleted except the point (17,6) which is a contradiction .

Case-4

Choose the two points (10,0) , (15,0) of the line L_3 then the points on the other lines in (table-1) deleted except the point (6,11) which is a contradiction .

Case-5

Choose the two points (10,0) , (18,0) of the line L_3 then all the points on the other lines in (table-1) deleted except the point (5,9) which is a contradiction

Case-6

Choose the two points (10,0) , (8,0) of the line L_3 then the points on the other lines in (table-1) deleted which is a contradiction.

Case-7

Choose the two points (10,0) , (4,0) of the line L_3 then the points on the other lines in (table-1) deleted except the point (8,3) which is a contradiction .

Case-8

Choose the two points (10,0) , (7,0) of the line L_3 then the points on the other lines in (table-1) deleted except the point (6,11) which is a contradiction .

Case-9

Choose the two points (10,0) , (13,0) of the line L_3 then the points on the other lines in (table-1) deleted except the point (5,9) which is a contradiction .

Case-10

Choose the two points (6,0) , (3,0) of the line L_3 then the points on the other lines in (table-1) deleted except the point (15,5) which is a contradiction .

Case-11

Choose the two points (6,0) , (5,0) of the line L_3 then the points on the other lines in (table-1) deleted except the point (12,3) which is a contradiction .

Case-12

Choose the two points (6,0) , (15,0) of the line L_3 then the points on the other lines in (table-1) deleted except the point (7,15) which is a contradiction .

Case-13

Choose the two points (6,0) , (18,0) of the line L_3 then the points on the other lines in (table-1) deleted except the point (16,17) which is a contradiction .

Case-14

Choose the two points (6,0) , (8,0) of the line L_3 then the points on the other lines in (table-1) deleted except the point (14,10) which is a contradiction .

Case-15

Choose the two points (6,0) , (4,0) of the line L_3 then the points on the other lines in (table-1) deleted except the point (17,2) which is a contradiction .

Case-16

Choose the two points (6,0) , (7,0) of the line L_3 then the points on the other lines in (table-1) deleted except the point (15,5) which is a contradiction .

Case-17

Choose the two points (6,0) , (13,0) of the line L_3 then the points on the other lines in (table-1) deleted except the point (10,16) which is a contradiction .

Case-18

Choose the two points (3,0) , (5,0) of the line L_3 then all the points on the other lines in (table-1) deleted which is a contradiction .

Case-19

Choose the two points (3,0) , (15,0) of the line L_3 then the points on the other lines in (table-1) deleted except the point (12,8) which is a contradiction .

Case-20

Choose the two points (3,0) , (18,0) of the line L_3 then all the points on the other lines in (table-1) deleted which is a contradiction .

Case-21

Choose the two points (3,0) , (8,0) of the line L_3 then all the points on the other lines in (table-1) deleted which is a contradiction .

Case-22

Choose the two points (3,0) , (4,0) of the line L_3 then the points on the other lines in (table-1) deleted except the point (12,4) which is a contradiction .

Case-23

Choose the two points (3,0) , (7,0) of the line L_3 then the points on the other lines in (table-1) deleted except the point (11,15) which is a contradiction .

Case-24

Choose the two points (3,0) , (13,0) of the line L_3 then the points on the other lines in (table-1) deleted except the point (5,4) which is a contradiction .

Case-25

Choose the two points (5,0) , (15,0) of the line L_3 then the points on the other lines in (table-1) deleted except the point (7,11) which is a contradiction .

Case-26

Choose the two points (5,0) , (18,0) of the line L_3 then the points on the other lines in (table-1) deleted except the point (10,13) which is a contradiction .

Case-27

Choose the two points (5,0) , (8,0) of the line L_3 then the points on the other lines in (table-1) deleted except the point (11,15) which is a contradiction .

Case-28

Choose the two points (5,0) , (4,0) of the line L_3 then the points on the other lines in (table-1) deleted except the point (7,11) which is a contradiction .

Case-29

Choose the two points (5,0) , (7,0) of the line L_3 then the points on the other lines in (table-1) deleted except the point (9,6) which is a contradiction .

Case-30

Choose the two points (5,0) , (13,0) of the line L_3 then the points on the other lines in (table-1) deleted except the point (18,2) which is a contradiction .

Case-31

Choose the two points (15,0) , (18,0) of the line L_3 then the points on the other lines in (table-1) deleted except the point (0,6) which is a contradiction .

Case-32

Choose the two points (15,0) , (8,0) of the line L_3 then the points on the other lines in (table-1) deleted except the point (14,10) which is a contradiction .

Case-33

Choose the two points (15,0) , (4,0) of the line L_3 then the points on the other lines in (table-1) deleted except the point (9,13) which is a contradiction .

Case-34

Choose the two points (15,0) , (7,0) of the line L_3 then the points on the other lines in (table-1) deleted except the point (13,17) which is a contradiction .

Case-35

Choose the two points (15,0) , (13,0) of the line L_3 then the points on the other lines in (table-1) deleted except the point (14,11) which is a contradiction .

Case-36

Choose the two points (18,0) , (8,0) of the line L_3 then the points on the other lines in (table-1) deleted except the point (15,8) which is a contradiction .

Case-37

Choose the two points (18,0) , (4,0) of the line L_3 then all the points on the other lines in (table-1) deleted which is a contradiction .

Case-38

Choose the two points (18,0) , (7,0) of the line L_3 then the points on the other lines in (table-1) deleted except the point (8,0) which is a contradiction .

Case-39

Choose the two points (18,0) , (13,0) of the line L_3 then the points on the other lines in (table-1) deleted except the point (10,13) which is a contradiction .

Case-40

Choose the two points (8,0) , (4,0) of the line L_3 then the point (8,0) delete the points {(3,5),(6,2) ,(11,16) ,(12,14),(0,10),(9,13),(15,8),(0,18),(4,9),(14,15),(0,16),(15,5),(16,3),(3,10),(5,6),(4,10),(17,6),(9,7),(16,18), (2,15),(0,17),(13,6),(5,4),(16,2),(3,13),(18,12), (14,11),(10,13),(0,5),(12,7).(5,9)} .

And the point (4,0) delete the points {(17,6),(10,13),(15,8),(16,4),(0,5),(8,14),(15,10),(0,9) ,(15,18),(7,17),(11,8),(11,5),(8,11),(12,3),(17,12),(3,2),(13,6),(0,10),(16,3),(11,16),(0,18),(7,15),(9,6),(0,12) ,(16,2),(15,5),(12,4)}

We obtained the following probabilities, each set contains (18) point plus the points A_1 ,..., A_9 , Q and U_1 we get (29) point .And by adding any two points from the remaining points of the plane PG (2,19) ,we obtain a set of (31) point but it is not a blocking set in each case , because it contains 0-secants which is a contradiction to the definition of a blocking set So there is no a blocking set of size 31 in PG (2,19) .

And there is no line L such that $|B \setminus L| = q$. So there is no a blocking set of Re`dei type of size 31 in PG (2,19)

The probabilities are :

1){(8,0),(4,0),(0,11),(0,6),(17,2),(5,14),(12,4),(10,16), (3,17),(15,9),(18,2),(14,10),(7,11),(16,17),(6,11),(2,1 0) , (13,18),(9,11)} .

2){(8,0),(4,0),(0,11),(0,6),(17,2),(5,14),(2,7),(12,4),,(15,9),(11,18),(14,10),(10,18),(7,11),(16,17),(6,11),(18 ,14),(13,18),(9,11)} .

3){(8,0),(4,0),(0,11),(0,6),(17,2),(5,14),(2,7),(12,4),(3 ,17),(15,9),(14,10),(10,18),(7,11),(16,17),(6,11),(18,1 4),(13,18),(9,11)} .

4){(8,0),(4,0),(0,11),(0,6),(17,2),(5,14),(12,4),(10,16), (3,17),(11,18),(18,2),(14,10),(7,11),(16,17),(6,11),(2, 10),(13,18).(9,11)} .

5){(8,0),(4,0),(0,11),(0,6),(17,2),(5,14),(12,4),(10,16), (15,9),(11,18),(18,2),(14,10),(7,11),(16,7),(6,11),(2,1 0),(13,18).(9,11)} .

6){(8,0),(4,0),(0,11),(0,6),(10,9),(5,14),(2,7),(12,4),(3 ,17),(15,9),(18,2),(14,10),(7,11),(16,7),(17,9),(6,11),(13,18).(9,11)}.

7){(8,0),(4,0),(0,11),(0,6),(10,9),(5,14),(2,7),(12,4),(3 ,17),(11,18),(18,2),(14,10),(7,11),(16,17),(17,9),(6,11),(13,18).(9,11)}.

8){(8,0),(4,0),(0,11),(0,6),(10,9),(5,14),(2,7),(12,4),(1 5,9),(11,18),(18,2),(14,10),(7,11),(16,17),(17,9),(6,11),(13,18).(9,11)} .

Case-41

Choose the two points (8,0) , (7,0) of the line L_3 then the points on the other lines in (table-1) deleted except the point (9,11) which is a contradiction .

Case-42

Choose the two points (8,0) , (13,0) of the line L_3 then the point (8,0) delete the points{(14,15),(4,9) ,(0,18), (15,8),(0,10),(9,13),(12,14),(11,16),(6,2),(5,3) ,(2,15),(16,18),(0,17),(13,6),(5,4),(16,2),(3,13),(18,12),(14,11),(10,13),(0,5),(12,7),(5,9)} .

And the point (13,0) delete the points{(4,9),(3,10) ,(18,14),(10,18),(17,14),(8,11),(7,17),(16,17),(15,5),(6,11),(3,13),(12,7),(10,6),(11,5),(8,3),(16,2),(7,15),(1 7,9),(15,10),(0,11),(12,14),(6,2),(10,9)}.

To get the following (24) probabilities .

1){(8,0),(13,0),(0,9),(0,6),(17,2),(6,13),(2,7),(10,16),(3,17),(11,18),(18,2),(14,10),(12,8),(7,11),(15,18),(16, 4),(9,11) ,(4,7)}.

2){(8,0),(13,0),(0,9),(0,6),(17,2),(5,14),(2,7),(10,16),(3,17),(11,18),(18,2),(14,10),(12,8),(7,11),(15,18),(16, 4),(9,11),(4,7)}.

3){(8,0),(13,0),(0,9),(0,6),(17,2),(5,14),(12,4),(10,16), (15,9),(11,18),(18,2),(14,10),(3,2),(7,11),(2,10),(16,4) ,(9,11), (4,7)}.

4){(8,0),(13,0),(0,9),(0,6),(17,2),(5,14),(12,4),(10,16), (2,5),(11,18),(18,2),(14,10),(3,2),(7,11),(15,18),(16,4) ,(9,11),(4,7)}.

5){(8,0),(13,0),(0,9),(0,6),(11,8),(6,13),(17,12),(10,16),(3,17),(15,9),(18,2),(14,10),(12,8),(7,11),(2,10),(16, 4),(9,11), (4,7)}.

6){(8,0),(13,0),(0,9),(0,6),(11,8),(6,13),(17,12),(10,16),(3,17),(2,5),(18,2),(14,10),(12,8),(7,11),(15,18),(16, 4),(9,11),(4,7)}.

7){(8,0),(13,0),(0,9),(0,6),(11,8),(5,14),(17,12),(10,16),(3,17),(15,9),(18,2),(14,10),(12,8),(7,11),(2,10),(16, 4),(9,11),(4,7)}.

8){(8,0),(13,0),(0,9),(0,6),(6,13),(5,14),(2,7),(17,12),(3,17),(11,18),(18,2),(14,10),(12,8),(7,11),(15,18),(16, 4),(9,11),(4,7)}.

9){(8,0),(13,0),(0,9),(0,6),(6,13),(5,14),(2,7),(10,16),(3,17),(11,18),(18,2),(14,10),(12,8),(7,11),(15,18),(16, 4),(9,11),(4,7)}.

10){(8,0),(13,0),(0,9),(0,6),(6,13),(5,14),(17,12),(12,4),(15,9),(11,18),(18,2),(14,10),(3,2),(7,11),(2,10),(16, 4),(9,11),(4,7)}.

11){(8,0),(13,0),(0,9),(0,6),(6,13),(5,14),(17,12),(12,4),(15,9),(11,18),(18,2),(14,10),(3,2),(7,11),(2,10),(16, 4),(9,11),(4,7)}.

12){(8,0),(13,0),(0,9),(0,6),(6,13),(5,14),(17,12),(10,1 6),(3,17),(15,9),(18,2),(14,10),(12,8),(7,11),(2,10),(16 ,4),(9,11),(4,7)}.

13){(8,0),(13,0),(0,9),(0,6),(6,13),(5,14),(17,12),(10,1 6),(3,17),(2,5),(18,2),(14,10),(12,8),(7,11),(15,18),(16 ,4),(9,11),(4,7)}.

14){(8,0),(13,0),(0,9),(0,6),(6,13),(5,14),(17,12),(10,1 6),(3,17),(11,18),(18,2),(14,10),(12,8),(7,11),(15,18),(2,10),(9,11),(4,7)}.

15){(8,0),(13,0),(0,9),(0,6),(6,13),(5,14),(17,12),(10,1 6),(3,17),(11,18),(18,2),(14,10),(12,8),(7,11),(15,18),(16,4),(9,11),(4,7)}.

16){(8,0),(13,0),(0,9),(0,6),(6,13),(5,14),(17,12),(10,16),(3,17),(11,18),(18,2),(14,10),(12,8),(7,11),(2,10),(16,4),(9,11),(4,7)}.
17){(8,0),(13,0),(0,9),(0,6),(6,13),(5,14),(17,12),(10,16),(15,9),(11,18),(18,2),(14,10),(3,2),(12,8),(2,10),(16,4),(9,11),(4,7)}.
18){(8,0),(13,0),(0,9),(0,6),(6,13),(5,14),(17,12),(10,16),(15,9),(11,18),(18,2),(14,10),(3,2),(7,11),(2,10),(16,4),(9,11),(4,7)}.
19){(8,0),(13,0),(0,9),(0,6),(6,13),(5,14),(17,12),(10,16),(15,9),(11,18),(18,2),(14,10),(12,8),(7,11),(2,10),(16,4),(9,11),(4,7)}.
20){(8,0),(13,0),(0,9),(0,6),(6,13),(5,14),(17,12),(10,16),(2,5),(11,18),(18,2),(14,10),(3,2),(12,8),(15,18),(16,4),(9,11),(4,7)}.
21){(8,0),(13,0),(0,9),(0,6),(6,13),(5,14),(17,12),(10,16),(2,5),(11,18),(18,2),(14,10),(3,2),(7,11),(15,18),(16,4),(9,11),(4,7)}.
22){(8,0),(13,0),(0,9),(0,6),(6,13),(5,14),(17,12),(10,16),(2,5),(11,18),(18,2),(14,10),(12,8),(7,11),(15,18),(16,4),(9,11),(4,7)}.
23){(8,0),(13,0),(0,9),(0,6),(6,13),(5,14),(12,4),(10,16),(15,9),(11,18),(18,2),(14,10),(3,2),(7,11),(2,10),(16,4),(9,11),(4,7)}.
24){(8,0),(13,0),(0,9),(0,6),(6,13),(5,14),(12,4),(10,16),(2,5),(11,18),(18,2),(14,10),(3,2),(7,11),(15,18),(16,4),(9,11),(4,7)} .

By the same way as in case-40 ,we show that all the probabilities are not a blocking sets of Re`dei type.

Case-43

Choose the two points (4,0) , (7,0) of the line L_3 then the point (4,0) delete the points {(17,6),(10,13),(15,8),(16,4),(0,5),(8,14),(15,10),(0,9),(15,18),(7,17),(11,8),(11,5),(8,11),(12,3),(17,12),(3,2),(2,5),(13,6),(0,10),(16,3),(11,16),(0,18),(7,15),(9,6),(0,12),(16,2),(6,13),(15,5),(12,14)} .

And the point (7,0) delete the points {(10,16),(2,5),(17,9),(12,14),(16,3),(12,8),(15,9),(9,7),(14,15),(12,3),(0,11),(5,14),(17,6),(2,10),(5,4),(6,2),(0,14),(10,13),(14,11),(15,18),(3,10),(5,9),(17,12),(9,13),(4,9),(2,15),(5,6),(12,4)}.

To get the following probability which is not a blocking set of Re`dei type.As in case-40 {(4,0),(7,0),(0,17),(0,6),(17,2),(12,7),(2,7),(16,18),(5,3),(11,18),(3,13),(10,18),(14,3),(18,12),(8,2),(6,11),(13,18),(9,11)}.

Case-44

Choose the two points (4,0), (13,0) of the line L_3 then the points on the other lines in (table-1) deleted except the point (9,11) which is a contradiction .

Case-45

Choose the two points (7,0), (13,0) of the line L_3 then the points on the other lines in (table-1) deleted except the point (9,11) which is a contradiction .

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المجموعة القالبية الأصغرية ذات الحجم ([$\frac{3}{2}(q+1)+1$]) في $PG(2,19)$

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الملخص

المجموعة القالبية الاصغرية ذات الحجم $\frac{3}{2}(q+1)$ وليست من النوع – ريدي المعروفة في المستوي الاسقاطي $PG(2,7)$. في عام (2003) درس Blokhuis,Brouwer and Wibrink [1] المجاميع القالبية الاصغرية ذات حجم $\frac{3}{2}(q+1)$ في المستوي الاسقاطي $PG(2,q)$ لقيم $q<41$ ، q عدد أولي ، ووجدوا مجموعة قالبية اصغرية ثانية من نفس الحجم وليست من النوع – ريدي في المستوي الاسقاطي $PG(2,13)$ كما اثبتوا عدم وجود أي مجموعة قالبية اصغرية اخر ى ذات الحجم $\frac{3}{2}(q+1)$ وليست من النوع – ريدي في المستوي $PG(2,q)$ $q<41$ ، q عدد أولي . في هذا البحث تم إثبات عدم وجود مجموعة قالبية اصغرية ذات حجم [$\frac{3}{2}(q+1)+1$] في المستوي الاسقاطي $PG(2,q)$ $q=19$, وليست من النوع – ريدي .