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Differential System in algebra SBH

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ABSTRACT

In this research , we will be study some types of an alge.- SBH ,we prove some properties and solve some examples to relationship by the definitions.

1. Introduction

The notion In this paper.. of alge -BCK. introduced in 1966 by K. Isiki [6]., introduced In 1983 by Q.P.Hu with X. Lie the notion of in alge.- BCH [5], introduced In 1998 byY. Jon, and E. Rof and H. Kiem the notion of alge. some type of BH [5}.

2. Some Basic About alge.-BH (BH-algebra)

We will take some definitions In this section for alge.-BCI, alge.-BCK, BCH in alge.and alge.- BH, with some of basics of, theorems and some examples.

Definition (2.1)[1]: A BCI in alge. If (C,*,CONS 0) of a type (2,0) , when C is a non - empty set with CONS 0 is a constant , hold, for all z, y, a ∈ C and for all"*" is a binary operation

i. ((a*z)*(a*y))*(y*z) = CONS 0, ,
ii. (y*(y*a))*a = CONS 0,
iii. y * y = CONS 0, ,
iv. y * a =CONS 0 and a * y = CONS 0 → y= a,.

Definition(2.2)[3]:A BCK in alge.is a BCI in alge. hold the one axiom: CONS 0 * y = CONS 0 , for all y ∈C.

Definition(2.3)[5]: A BCH in alge.is an alge.(C,*,CONS 0), when C is set with CONS 0 , holding the following, ∀z,y,a ∈C and, for all"*" is a binary operation

i. y*y=CONS 0, ∀y ∈C.
ii. y*a=CONS 0 and a*y=CONS 0 imply y=a,.
iii. (z*y)*a=(z*a)*y,.

Definition(2.4)[7]: An alge.- BH is a set C , CONS 0 with "*" an operation and hold a binary, ∀y, a ∈ C. holding the following:

i. y*y=CONS 0,
ii. y*CONS 0 =y,
iii. . y*a=CONS 0 and a*y =CONS 0 imply y = a,

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THE MAIN RESULTS

In this paper study, the types of alge. and a Differential System In SBH-Alge. and. So that we prove theorems and some examples about alge. the differential System for First order and defined the function y = y(x) “ is written as
$Y^{(1)}$ = FUNTI (x, y)

When FUNTI (x, y) be a function and x be independent variable with y be dependent variable

3. **Solve Alge.-SBH (SBH)**

we will introduce in this section, the some of types concepts Alge.-SBH, Also we state some examples and prove theorems

Definition(3.1): An alge.- SBH is a set C , non - empty , a constant CONS 0 with "*" a binary operation, holding the following:

i. y*CONS 0 =y, ∀ y ∈C.
ii. . y*a=CONS 0 and a*y =CONS 0 iff y = a, ∀y, a ∈ C.
iii. ∀ a≠CONS 0 , y≠CONS 0 and a≠ y then [a*y)]≠CONS 0,
Then it is a. D. S. in SBH

Example(3.2):- let C={CONS 0,c,v,p} and operation * is in the following table.

*	CONS 0	c	v	p
CONS 0	CONS 0	c	c	v
c	c	CONS 0	c	c
v	v	c	CONS 0	c
p	p	p	p	CONS 0

Then it is a. D. S. in SBH

Example(3.3):- let C={CONS 0,c,v,p} and operation * is in the following table.

*	CONS 0	c	v	p
CONS 0	CONS 0	c	c	v
c	c	CONS 0	c	0
v	v	c	CONS 0	c
p	p	p	p	CONS 0

∀c≠CONS 0 , p≠CONS 0 and c≠p, but c*p=CONS 0. Then it is not a D. S. in SBH

4. Solve Differential System by Alge.-SBH (D. S. in SBH)

we will explain In this section , the concepts a Differential System by Alge.-SBH, Also we state some examples and prove some theorems about the Differential System In SBH-Alge.or in alge.-SBH.

Definition (4.1):- a Differential System In SBH-Alge.or in alge.-SBH ,FUNTI (X,Y) Be A Function And (*) Be Any binary operation And Let (CONS 0) Denoted By (D. S. in SBH)

Let, y=1 → FUNTI (x,1) = FUNTI (x) ……….(1)

Let, x=1 → FUNTI (1,y) = FUNTI (y) ……….(2)

and satisfies

i. [FUNTI (x)*CONS 0]=FUNTI (x)
ii. [FUNTI (x)*FUNTI (y)] = [FUNTI (y)*FUNTI (x)]=CONS 0 iff [FUNTI (x) = FUNTI (y)]
iii. ∀ FUNTI (x)≠CONS 0 , FUNTI (y)≠CONS 0 and FUNTI (x)≠ FUNTI (y) then [FUNTI (x)*FUNTI (y)] ≠CONS 0,

Example (4.2) : let (R,-,0) be a field in a differential System and a function and called by FUNTI (x,y) =x.y is it (D. S. in SBH), FUNTI (y) = y and FUNTI (x) = x, then

i. [FUNTI (x)*CONS 0]

→ [x-0]=x by def.

→ FUNTI (x)

ii. [FUNTI (x)*FUNTI (y)]

→ [x-y] = 0 =[y-x] by def.

→ [FUNTI (y)*FUNTI (x)]

then [x=y]

Conversely
[x=y] then [FUNTI (x)*FUNTI (y)]
→ [x-x] = 0 =[x-x] by def.
→ [FUNTI (y)*FUNTI (x)]

iii. ∀ FUNTI (x)≠CONS 0 , FUNTI (y)≠CONS 0 and FUNTI (x)≠ FUNTI (y) then [FUNTI (x)*FUNTI (y)] ≠CONS 0,
s.t (R,-,0) is (D. S. in SBH)

Example (4.3):- let (Z/1,-,0) be a field in a differential System and the function defined be FUNTI (x,y) = xy+y Is it (D. S. in SBH)

Solve :- let FUNTI (x) = x+1, FUNTI (y)=2y then

i. [FUNTI (x)*CONS 0]

→ [x+1-0]=[x+1] by def.

→ FUNTI (x)

ii. [FUNTI (x)*FUNTI (y)] = [FUNTI (y)*FUNTI (x)]=0 by def.

→ [x+1-2y] =[2y-x-1]= 0 by def.

Conversely
[FUNTI (x)*FUNTI (x)]
→ [x+1-x-1]=0 by def.

iii. . ∀ FUNTI (x)≠CONS 0 , FUNTI (y)≠CONS 0 and FUNTI (x)≠ FUNTI (y) then [FUNTI (x)*FUNTI (y)] ≠CONS 0,

s.t (Z/1,-,0) is (D. S. in SBH)

since, it is hold if y=x=.

Theorem (4.4) : Every an Alge.- SBH of X iff a D. S. in SBH of X

Proof.

Let C be a set and D. S. in SBH of X

T. p. is an Alge.- SBH

∀x, y ∈C , FUNTI (x) = x and FUNTI (y) = y

i. x*CONS 0 =[FUNTI (x)*CONS 0=FUNTI (x)] =x
ii. x*y =[FUNTI (x)*FUNTI (y)]=CONS 0

and y*x = [FUNTI (y)*FUNTI (x)]=CONS 0

imply [FUNTI (x)=FUNTI (y)] by def.

→ x=y

and x*x = [FUNTI (x)*FUNTI (x)]=CONS 0

iii. ∀ FUNTI (x)≠CONS 0 , FUNTI (y)≠CONS 0 and FUNTI (x)≠ FUNTI (y) then [FUNTI (x)*FUNTI (y)] ≠CONS 0,

Then, it is an Alge.- SBH

Conversely

Let, C be an Alge.- SBH of X,

To prove is a D. S. in SBH

Since, FUNTI (x)=x and FUNTI (y)=y

i. [FUNTI (x)*CONS 0]

=x*CONS 0

=x by def.

= FUNTI (x)

ii. [FUNTI (x)*FUNTI (y)]

= x*y by def.

= CONS 0

and [FUNTI (y)*FUNTI (x)]

=y*x by def.

= CONS 0

imply x=y by def.

→ [FUNTI (x)=FUNTI (y)]

And [FUNTI (x)*FUNTI (x)]

=x*x by def.

= CONS 0

iii. ∀ FUNTI (x)≠CONS 0 , FUNTI (y)≠CONS 0 and FUNTI (x)≠ FUNTI (y) then [FUNTI (x)*FUNTI (y)] ≠CONS 0,

Then, it is a D. S. in SBH

***Proposition* (4.5)**: *Let C be a set* , SBH-Alge. *and { FUNTI* $_i$*(x), x∈C, i∈λ} be a* D. S. SBH. *Then* $\bigcap_{i\in\lambda}$ *FUNTI* $_i$*(x) is a* D. S. in SBH.

Proof: *To prove* $\bigcap_{i\in\lambda}$ *FUNTI* $_i$*(x) is a* D. S. in SBH.

Since FUNTI $_i$(x)=x and FUNTI $_i$(y)=y

i. $\bigcap_{i\in\lambda} FUNTI_i(x) * CONS\ 0 = \bigcap_{i\in\lambda} (FUNTI_i(x) * CONS\ 0) = \bigcap_{i\in\lambda} FUNTI_i(x)$

ii. $\bigcap_{i\in\lambda} FUNTI_i(x) * \bigcap_{i\in\lambda} FUNTI_i(y) = \bigcap_{i\in\lambda} (FUNTI_i(x) * FUNTI_i(y)) = CONS\ 0$

And FUNTI $\bigcap_{i\in\lambda} FUNTI_i(y) * \bigcap_{i\in\lambda} FUNTI_i(x) = \bigcap_{i\in\lambda} (FUNTI_i(y) * FUNTI_i(x)) = CONS\ 0$

→ $\bigcap_{i\in\lambda}$ FUNTI $_i$(x)= $\bigcap_{i\in\lambda}$ FUNTI $_i$(y) imply x=y

And $\bigcap_{i\in\lambda} FUNTI_i(x) * \bigcap_{i\in\lambda} FUNTI_i(x) = \bigcap_{i\in\lambda} (FUNTI_i(x) * FUNTI_i(x)) = CONS\ 0$

iii. ∀ FUNTI (x)≠CONS 0 , FUNTI (y)≠CONS 0 and FUNTI (x)≠ FUNTI (y) then [FUNTI (x)*FUNTI (y)] ≠CONS 0, it is hold

Then it is hold a D. S. in SBH

Example *(4.6)*:. Let *C* =R a real numbers , with operation * and defined by:

$$\text{FUNTI (x)*FUNTI (y)=} \begin{cases} FUNTI(x), when, x \neq y \\ 0, when, x = y \end{cases}$$

For all, x, y be R

Then, *C is a D. S.* in *SBH of C*

Proposition (4.7) : Let { FUNTI $_i$(x) i∈λ} it is a chain of a D. S. SBH. Then $\bigcup_{i\in\lambda}$ FUNTI $_i$(x) is a D. S. in SBH of C.

Proof: *to* prove $\bigcup_{i\in\lambda}$ *FUNTI* $_i$*(x) is a* D. S. in SBH.

Since FUNTI $_i$(x)=x and FUNTI $_i$(y)=y

i. $\bigcup_{i\in\lambda} FUNTI_i(x) * CONS\ 0 = \bigcup_{i\in\lambda} (FUNTI_i(x) * CONS\ 0) = \bigcup_{i\in\lambda} FUNTI_i(x)$

ii. FUNTI $\bigcup_{i\in\lambda} FUNTI_i(x) * \bigcup_{i\in\lambda} {}_i\, FUNTI_i(y) = \bigcup_{i\in\lambda} (FUNTI_i(x) * FUNTI_i(y)) = CONS\ 0$

And FUNTI $\bigcup_{i\in\lambda}$ $FUNTI_i(y) * \bigcup_{i\in\lambda} FUNTI_i(x) = \bigcup_{i\in\lambda} (FUNTI_i(y) * FUNTI_i(x))$ =CONS 0

→ $\bigcup_{i\in\lambda}$ FUNTI $_i$(x)= $\bigcup_{i\in\lambda}$ FUNTI $_i$(y) imply x=y

And $\bigcup_{i\in\lambda} FUNTI_i(x) * \bigcup_{i\in\lambda} FUNTI_i(x) = \bigcup_{i\in\lambda} (FUNTI_i(x) * FUNTI_i(x))$ = CONS 0

iii. ∀ FUNTI (x)≠CONS 0 , FUNTI (y)≠CONS 0 and FUNTI (x)≠ FUNTI (y) then [FUNTI (x)*FUNTI (y)] ≠CONS 0, it is hold

Then, it is a D. S. in SBH

Definition (4.8): Let C be a SBH-alge. and G(X)= { x∈C \ CONS 0*x=x} is denoted by a .G-part of C.

Proposition (4.9): Let X=G(X) . Then every D. S. SBH is a commutation D. S. SBH

<u>Proof</u> :Let X be a D.S. in SBH

i. FUNTI (x)*CONS 0 by def.

=x*CONS 0 (by a comm).

CONS 0*x by def. G(x)

=x= FUNTI (x)

ii FUNTI (x)*FUNTI (y) = x*y (by a comm). y * x (by def. SBH)

= CONS 0

and FUNTI (y)*FUNTI (x) = y*x (by a comm). x * y (by def. SBH)

= CONS 0

then x=y → FUNTI (x)=FUNTI (y) and FUNTI (x)*FUNTI (x)

=x*x (by a comm.)

= x*x =CONS 0

iii.∀ FUNTI (x)≠CONS 0 , FUNTI (y)≠CONS 0 and FUNTI (x)≠ FUNTI (y) then [FUNTI (x)*FUNTI (y)]= [FUNTI (y)*FUNTI (y)]≠CONS 0, it is hold

Then it is a comm. D. S. in SBH

Proposition (4.10): Let C=G(X) . Then every D. S. in SBH is hold:-

i. FUNTI (x)*(FUNTI (x)*FUNTI (x)) = (FUNTI (x)*FUNTI (x))*FUNTI (x)=FUNTI (x)

ii. FUNTI (x)*(FUNTI (y)*FUNTI (x))*FUNTI (x)=FUNTI (x)*FUNTI (y)

iii. (FUNTI (x)*FUNTI (y)) * (FUNTI (y)*FUNTI (x))= CONS 0

iv. (FUNTI (x)*FUNTI (y) *FUNTI (z)) * (FUNTI (z) *FUNTI (y)*FUNTI (x))= CONS 0

v. (FUNTI (x)*FUNTI (y) *FUNTI (x)) * (FUNTI (x) *FUNTI (y)*FUNTI (x))= CONS 0

Proof:-

i. FUNTI (x)*(FUNTI (x)*FUNTI (x)) (by def. D.S in SBH)

= FUNTI (x)*CONS 0=FUNTI (x)

and (FUNTI (x)*FUNTI (x))*FUNTI (x) (by def. D.S in SBH)

= CONS 0*FUNTI (x) = FUNTI (x) * CONS 0 by def. G(x)

= FUNTI (x)

ii. FUNTI (x)*(FUNTI (y)*FUNTI (x))*FUNTI (x)= FUNTI (x) *FUNTI (x)*(FUNTI (y)*FUNTI (x)) by def. G(x)

= CONS 0*(FUNTI (y)*FUNTI (x)) (by def. D.S in SBH)

= CONS 0*(FUNTI (y)*FUNTI (x))= FUNTI (y)*FUNTI (x) by def. G(x)

=FUNTI (x)*FUNTI (y) by a comm.

iii. (FUNTI (x)*FUNTI (y)) * (FUNTI (y)*FUNTI (x))= (FUNTI (x)*FUNTI (y)) * (FUNTI (x)*FUNTI (y)) (by a comm).

= CONS 0 by def. D.S in SBH

iv. (FUNTI (x)*FUNTI (y) *FUNTI (z)) * (FUNTI (z) *FUNTI (y)*FUNTI (x))

= (FUNTI (x)*FUNTI (y) *FUNTI (z)) * (FUNTI (y) *FUNTI (z)*FUNTI (x))

= (FUNTI (x)*FUNTI (y) *FUNTI (z)) * (FUNTI (y) *FUNTI (x)*FUNTI (z))

= (FUNTI (x)*FUNTI (y) *FUNTI (z)) * (FUNTI (x) *FUNTI (y)*FUNTI (x))=CONS 0

v. (FUNTI (y)*FUNTI (x) *FUNTI (x)) * (FUNTI (y) *FUNTI (x)*FUNTI (x))= CONS 0

\(FUNTI (y)*CONS 0) * (FUNTI (y) *CONS 0)

FUNTI (y)* FUNTI (y) = CONS 0

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