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A New Class of Univalent Harmonic Mappings Associated with a Binomail-Series

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A b s t r a c t

This study presents a study of a new class of single-valued harmonic mappings on the unit disc using a new operator based on the binomial series. The work defines the class of mappings $\bar{T}_{\lambda,\alpha}^{s,m,k}$ and presents its most important properties, including parameter estimation, closure under convolutions and convex fitting, invariance under an integral operator, and characterization of the extrema and neighborhood of the mappings in this class. The results enhance the development of the theory of single-valued harmonic mappings.

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1. INTRODUCTION

Let $\mathfrak{U}$ belong to the class of arbitrary mappings normalized by $f(0) = f(z)' - 1 = 0 \text{ with } f = \mathrm{h} + \bar{g} \in S_H$ If f is an orientation- preserving harmonic mapping , where h and g in the class $\mathfrak{U}$ is analytic univalent mapping in the unite disk in complex number $N = \{z \in \mathbb{C}: |z| < 1\}$. As the form

$$f(z) = z(1 + \sum_{n=2}^{\infty} a_n z^n) \qquad (1.1)$$

Let $S^*, and\ K\ be$ any subclasses of $\mathfrak{U}$ which are univalent in N and represent the starlike and convex mappings , respectively (Rossdy, Omar, & Soh, 2024).
from the binomial series

$$(1-\alpha)^k = \sum_{m=0}^{k} \binom{k}{m} (-a)^m, (k \in \mathbb{N} = \{1,2,\dots\}, m \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}.$$

For $f \in \mathfrak{U}, \alpha, \lambda \in [0.1], \mathrm{b} \in \mathbb{C}\backslash Z_0^-, s \in \mathbb{C}, k, m \in \mathbb{N}_0$ introduce

$$\mathrm{N}_{\lambda,\alpha}^{s,m,k} f(z) = z + \sum_{n=2}^{\infty} \left(\frac{1+\mathrm{b}}{n+\mathrm{b}}\right)^s [1 + (n\lambda - \lambda)(1-\alpha)^m]^k a_n z^n.$$

The operator $\mathrm{N}_{\lambda,\alpha}^{s,m,k} f(z)$ Generalization of other previous triggers.

1. For $m = 1$ the operator $\mathrm{N}_{\lambda,\alpha}^{s,1,k} = \vartheta_{\alpha,\lambda}^{n,s}$ was presented by . (Yunus et al., 2017)
2. For $k = 0$ the operator $\mathrm{N}_{\lambda,\alpha}^{s,1,0} = J_{s,\mathrm{b}}$ was introduced by (Srivastava & Attiya, 2007).
3. For $\mathrm{k} = 0$, $\alpha = m = 0$ the operator $\mathrm{N}_{\lambda,0}^{0,m,k} = \mathrm{N}_{\lambda,\alpha}^{0,0,k} = \mathrm{N}_{\lambda}^{n} =$ was introduced by (Al-Oboud, 2004).
4. For $\mathrm{k} = \alpha = 0$, $m = 0$ and $\lambda = 1$ the operator $\mathrm{N}_{\lambda,0}^{0,m,k} = \mathrm{N}_{\lambda}^{n}$ was presented by (Srivastava & Attiya, 2007).

Assume that $\mathfrak{U}$ represents the mappings in N as

$$\mathrm{h}(z) = \sum_{n=1}^{\infty} a_n z^n \quad , g(z) = \mathrm{b}_1 z + \sum_{n=2}^{\infty} \mathrm{b}_n z^n \quad |\mathrm{b}_1| < 1. \qquad (1.2)$$

We denote that h is the holomorphic part and g is a anti - holomorphic part, if the class $\mathfrak{U}$ reduces to S for each normalization mapping, whenever the co-analytic part of f vanishes (Srivastava & Attiya, 2007).

The mapping $f = \mathrm{h} + \bar{g}$ where $|(g(z)')| < |(f(z)')|$ in D (where D is simply connected in $\mathbb{C}$).

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Suppose T_s for every subclass mappings of $\mathfrak{U}$ is given by

$$\underline{h}(z) = z + \sum_{n=2}^{\infty} (-|a_n)|z^n \quad , g(z) = |b_1|z + \sum_{n=2}^{\infty} |b_n|z^n \qquad |b_1| < 1. \tag{1.3}$$

Our work employs the operator given in (1.2) for (H.M) $f = \underline{h} + \bar{g}$

$$N_{\lambda,\alpha}^{s,m,k} f(z) = N_{\lambda,\alpha}^{s,m,k} \underline{h}(z) + \overline{N_{\lambda,\alpha}^{s,m,k} g(z)}$$

Where

$$N_{\lambda,\alpha}^{s,m,k} \underline{h}(z) = z + \sum_{n=2}^{\infty} \left(\frac{1+b}{n+b}\right)^s [1 + (n\lambda - \lambda)(1-\alpha)^m]^k a_n z^n \tag{1.4}$$

$$\overline{N_{\lambda,\alpha}^{s,m,k} g(z)} = \sum_{n=2}^{\infty} \left(\frac{1+b}{n+b}\right)^s [1 + (n\lambda - \lambda)(1-\alpha)^m]^k b_n z^n \tag{1.5}$$

(Rossdy et al., 2024). we denote $\bar{T}_{\lambda,\alpha}^{s,m,k}$ subclass of $T_{\lambda,\alpha}^{s,m,k}$ where $T_s \cap T_{\lambda,\alpha}^{s,m,k} = \bar{T}_{\lambda,\alpha}^{s,m,k}$

2- Mains Results

Definition 2.1 For $f = \underline{h} + \bar{g}$ given by (1.2), let $\bar{T}_{\lambda,\alpha}^{s,m,k}$ denote the family of univalent (H.M) that satisfy the following conditions

$$Re\left\{\frac{\left(N_{\lambda,\alpha}^{s,m,k} \underline{h}(z)\right)' + \left(\overline{N_{\lambda,\alpha}^{s,m,k} g(z)}\right)'}{z}\right\} \geq \alpha \quad \forall z \in \mathbb{C} - \{0\} \tag{1,.6}$$

$m \in \mathbb{N}_0, \alpha, \lambda \in [0.1], b \in \mathbb{C}\backslash Z_0^-, s \in \mathbb{C}, k \in \mathbb{N}_0$

Theorem 2.2 Let $f = \underline{h} + \bar{g}$, where $\underline{h}(z)$ *and* $g(z)$ are defined by (1.2). If

$$z + \sum_{n=2}^{\infty} \left(\frac{1+b}{n+b}\right)^s [1 + (n\lambda - \lambda)(1-\alpha)^m]^k |a_n| + \sum_{n=2}^{\infty} \left(\frac{1+b}{n+b}\right)^s [1 + (n\lambda - \lambda)(1-\alpha)^m]^k |b_n|z^n \leq \frac{2(\alpha+1)}{n} \tag{1.7}$$

Proof. Since$\bar{T}_{\lambda,\alpha}^{s,m,k} \subset T_{\lambda,\alpha}^{s,m,k}$, the proof can be carried out in two directions. For brevity, we restrict out attention to form (1.3), in view of that condition (1.6) follows from it

$$Re\left\{\frac{\left(N_{\lambda,\alpha}^{s,m,k} \underline{h}(z)\right)' + \left(\overline{N_{\lambda,\alpha}^{s,m,k} g(z)}\right)'}{z} - \alpha\right\} \geq 0$$

$$Re\left\{\frac{\left(\begin{array}{c}1 - \sum_{n=2}^{\infty} n\left(\frac{1+b}{n+b}\right)^s [1 + (n\lambda - \lambda)(1-\alpha)^m]^k |a_n||z|^{n-1} - \\ \sum_{n=1}^{\infty} n\left(\frac{1+b}{n+b}\right)^s [1 + (n\lambda - \lambda)(1-\alpha)^m]^k |b_n||\bar{z}|^{n-1} - \alpha z\end{array}\right)}{z}\right\} \geq 0 \tag{1.13}$$

let $z \to 1^-$ we became

$$\frac{\left(\begin{array}{c}2(\alpha+1) - \sum_{n=2}^{\infty} n\left(\frac{1+b}{n+b}\right)^s [1 + (n\lambda - \lambda)(1-\alpha)^m]^k |a_n z^{n-1} - \\ \sum_{n=1}^{\infty} n\left(\frac{1+b}{n+b}\right)^s [1 + (n\lambda - \lambda)(1-\alpha)^m]^k |b_n||\bar{z}|^{n-1}\end{array}\right)}{z} \geq 0$$

The constraint in equation (1.13) for all z$\in \mathbb{R}^+$, such that $|z| \in (0,1)$ we must have

$$2(1+\alpha) - \sum_{n=2}^{\infty} n\left(\frac{1+b}{n+b}\right)^s [1 + (n\lambda - 1\lambda)(1-\alpha)^m]^k a_n - \sum_{n=1}^{\infty} n\left(\frac{1+b}{n+b}\right)^s [1 + (n\lambda - \lambda)(1-\alpha)^m]^k b_n \geq 0$$

Definition 2.3:

If $f(z) = z - \sum_{n=2}^{\infty} |a_n|z^n + \sum_{n=1}^{\infty} |b_n|\bar{z}^n$ and $F(z) = z - \sum_{n=2}^{\infty} |A_n|z^n + \sum_{n=1}^{\infty} |B_n|z^n$

The Hadamard product of $f(z)$ and $F(z)$ hold

$$(f * F)(z) = f(z) * F(z) = z - \sum_{n=2}^{\infty} |a_n A_n|z^n + \sum_{n=1}^{\infty} |b_n B_n|z^n \tag{1.8}$$

Theorem 2.4 for $0 \leq \acute{\Gamma} \leq \alpha < 1$ let $f(z) \in \bar{T}_{\lambda,\alpha}^{s,m,k}$ and $F(z) \in \bar{T}_{\lambda,\Gamma}^{s,m,k}$.then $f * F \in \bar{T}_{\lambda,\alpha}^{s,m,k} \subset \bar{T}_{\lambda,\Gamma}^{s,m,k}$

Proof. To establish the coefficient $f * F$ meets the condition in theorem (1.2) for $F(z) \in \bar{T}_{\lambda,\alpha}^{s,m,k}$. Let $|A_n| \leq 1$ and $|B_n| \leq 1$ now, then $f * F$ is given

$$\sum_{n=2}^{\infty} \frac{\left(\frac{1+b}{n+b}\right)^s [1 + (n\lambda - \lambda)(1-\alpha)^m]^k |a_n|\,|A_n|}{\frac{2(\alpha+1)}{n}} + \frac{\sum_{n=1}^{\infty} \left(\frac{1+b}{n+b}\right)^s [1 + (n\lambda - \lambda)(1-\alpha)^m]^k |b_n||B_n|}{\frac{2(\alpha+1)}{n}}$$

$$\leq \sum_{n=2}^{\infty} \frac{\left(\frac{1+ḃ}{n+ḃ}\right)^s [1+(n\lambda-\lambda)(1-\alpha)^m]^k |a_n|}{\frac{2(\alpha+1)}{n}} + \frac{\sum_{n=1}^{\infty}\left(\frac{1+ḃ}{n+ḃ}\right)^s [1+(n\lambda-\lambda)(1-\alpha)^m]^k |ḃ_n|}{\frac{2(\alpha+1)}{n}} \leq 1$$

Since $0 \leq Ѓ \leq \alpha < 1$ and $f(z) \in \bar{T}^{m}_{\mu,\lambda,\sigma}(\alpha,\beta)$ therefore $f * F \in \bar{T}^{s,m,k}_{\lambda,\alpha} \subset \bar{T}^{s,m,k}_{\lambda,\Gamma}$

Definition 2.5 the mappings $f_{k_i}(z)$ we can define it for *i =1,2,….,m* by

$$f_{k_i}(z) = z - \sum_{n=2}^{\infty} |a_{n,i}| z^n + \sum_{n=1}^{\infty} |ḃ_n| z^n \quad , for\ i = 1,2,\ldots,m$$ **(1.9)**

Theorem 2.6 consider the mapping $f_{k_i}(z)$, according to (1.9) in $\bar{T}^{s,m,k}_{\lambda,\alpha}$, where i *=1, 2… m*. Then the mapping $t_i(z)$ was transformed by

$$t_i(z) = \sum_{i=1}^{\infty} C_i\, f_{k_i}(z), 0 \leq C_i \leq 1$$

Such that the class $\bar{T}^{s,m,k}_{\lambda,\alpha}$, where $\sum_{i=1}^{\infty} C_i = 1$

Proof. Under the condition of $t_i(z)$, is written as

$$t_i(z) = z - \sum_{n=2}^{\infty}\left(\sum_{i=1}^{\infty} C_i |a_{n,i}|\right) z^n + \sum_{n=1}^{\infty}\left(\sum_{i=1}^{\infty} C_i |ḃ_{n,i}|\right) \bar{z}^n$$

moreover, since $f_{k_i}(z)$ is in $\bar{T}^{s,m,k}_{\lambda,\alpha}$ for every i=1, 2… m, then

$$\sum_{n=2}^{\infty}\left(\frac{1+ḃ}{n+ḃ}\right)^s [1+(n\lambda-\lambda)(1-\alpha)^m]^k \left(\sum_{i=1}^{\infty} C_i |a_{n,i}|\right) + \sum_{n=1}^{\infty}\left(\frac{1+ḃ}{n+ḃ}\right)^s [1+(n\lambda-\lambda)(1-\alpha)^m]^k \sum_{i=1}^{\infty} C_i |ḃ_{n,i}|$$

$$= \sum_{i=1}^{\infty} C_i \left[\sum_{n=2}^{\infty}\left(\frac{1+ḃ}{n+ḃ}\right)^s [1+(n\lambda-\lambda)(1-\alpha)^m]^k C_i |a_{n,i}| + \sum_{n=1}^{\infty}\left(\frac{1+ḃ}{n+ḃ}\right)^s [1+(n\lambda-\lambda)(1-\alpha)^m]^k C_i |ḃ_{n,i}|\right]$$

$$\leq \sum_{i=1}^{\infty} C_i \frac{2(\alpha+1)}{n} \leq \frac{2(\alpha+1)}{n}.$$

Definition 2.7 by circular Bernardi- Libera-Livingston integral $T_u(f)$ (Saheb & Buti, 2024), (Libera, 1965),. That is expressed as :

$$T_u(f) = \frac{u+1}{z^u}\int_0^2 t^{u-1} f(t)dt, u > -1 \quad (1.10)$$

We must prove quality the class $\bar{H}^{s,m,k}_{\lambda,\alpha}$

Theorem 2.8 Suppose that $f \in \bar{T}^{s,m,k}_{\lambda,\alpha}$ therefore $\bar{H}^{s,m,k}_{\lambda,\alpha} \in \bar{T}^{s,m,k}_{\lambda,\alpha}$

Proof. since $T_u(f(z)) = \frac{u+1}{z^u}\int_0^2 t^{u-1}\,(t - \sum_{n=2}^{\infty}|a_n|z^n + \sum_{n=1}^{\infty}|ḃ_n|\bar{z}^n)dt,$

$$= z - \sum_{n=2}^{\infty} \frac{u+1}{u+n}|a_n| z^n + \sum_{n=1}^{\infty} \frac{u+1}{u+n}|ḃ_n|\bar{z}^n$$

$$z - \sum_{n=2}^{\infty} Ŋ_n z^n + \sum_{n=1}^{\infty} L_n \bar{z}^n$$

Wherever

$$Ŋ_n = \frac{u+1}{u+n}|a_n| and\ L_n = \frac{u+1}{u+n}|ḃ_n|$$

Therefore

$$\sum_{n=2}^{\infty} \frac{\left(\frac{1+ḃ}{n+ḃ}\right)^s [1+(n\lambda-\lambda)(1-\alpha)^m]^k \left[\frac{u+1}{u+n}\right] |a_n|}{\frac{2(\alpha+1)}{n}} + \frac{\sum_{n=1}^{\infty}\left(\frac{1+ḃ}{n+ḃ}\right)^s [1+(n\lambda-\lambda)(1-\alpha)^m]^k \left[\frac{u+1}{u+n}\right] |ḃ_n|}{\frac{2(\alpha+1)}{n}}$$

$$\leq \sum_{n=2}^{\infty} \frac{\left(\frac{1+ḃ}{n+ḃ}\right)^s [1+(n\lambda-\lambda)(1-\alpha)^m]^k |a_n|}{\frac{2(\alpha+1)}{n}} + \frac{\sum_{n=1}^{\infty}\left(\frac{1+ḃ}{n+ḃ}\right)^s [1+(n\lambda-\lambda)(1-\alpha)^m]^k |ḃ_n|}{\frac{2(\alpha+1)}{n}} \leq 1$$

From theorem 1.4

Hence, we have $\bar{H}^{s,m,k}_{\lambda,\alpha} \in \bar{T}^{s,m,k}_{\lambda,\alpha}$

Theorem 2,9 Let $f(z)$ given by (1.3) then $f(z) \in \bar{T}^{s,m,k}_{\lambda,\alpha}$ if and only if

$$f(z) = \sum_{n=1}^{\infty}(X_n ḫ_n + Y_n ℊ_n(z))$$

$$\mathfrak{h}_1(z) = z, \mathfrak{h}_n(z) = z - \left(\frac{\frac{2(\alpha+1)}{n}}{\left(\frac{1+b}{n+b}\right)^s [1+(n\lambda-\lambda)(1-\alpha)^m]^k} \right) z^n, \quad n = 2,3,\dots$$

$$g_n(z) = z + \left(\frac{\frac{2(\alpha+1)}{n}}{\left(\frac{1+b}{n+b}\right)^s [1+(n\lambda-\lambda)(1-\alpha)^m]^k} \right) \bar{z}^n, n = 1,2,3,\dots \text{ and } \sum_{n=1}^{\infty}(X_n+Y_n) = 1, X_n \ge 0, Y_n \ge 0$$

Proof.

$$f(z) = \sum_{n=1}^{\infty}\left(X_n \mathfrak{h}_n + Y_n g_n(z)\right)$$

$$= \sum_{n=2}^{\infty} X_n \left\{ z - \left(\frac{\frac{2(\alpha+1)}{n}}{\left(\frac{1+b}{n+b}\right)^s [1+(n\lambda-\lambda)(1-\alpha)^m]^k} \right) z^n \right\} + \sum_{n=1}^{\infty} Y_n \left\{ z + \left(\frac{\frac{2(\alpha+1)}{n}}{\left(\frac{1+b}{n+b}\right)^s [1+(n\lambda-\lambda)(1-\alpha)^m]^k} \right) \bar{z}^n \right\}$$

$$= z - \sum_{n=2}^{\infty} \left(\frac{\frac{2(\alpha+1)}{n}}{\left(\frac{1+b}{n+b}\right)^s [1+(n\lambda-\lambda)(1-\alpha)^m]^k} \right) X_n z^n + \sum_{n=1}^{\infty} \left(\frac{\frac{2(\alpha+1)}{n}}{\left(\frac{1+b}{n+b}\right)^s [1+(n\lambda-\lambda)(1-\alpha)^m]^k} \right) Y_n \bar{z}^n$$

Therefore

$$\sum_{n=2}^{\infty} \frac{\left[\frac{(\mu+\lambda+(n-1)(\lambda-\alpha)(\beta-\sigma))}{\mu+\lambda}\right]^m |a_n|}{\frac{2(\alpha+1)}{n}} + \frac{\sum_{n=1}^{\infty}\left[\frac{(\mu+\lambda+(n-1)(\lambda-\alpha)(\beta-\sigma))}{\mu+\lambda}\right]^m |b_n|}{\frac{2(\alpha+1)}{n}} = \sum_{n=2}^{\infty} X_{\boldsymbol{n}} + \sum_{\boldsymbol{n=1}}^{\infty} Y_n$$

$= \sum_{n=2}^{\infty} X_{\boldsymbol{n}} + \sum_{\boldsymbol{n=1}}^{\infty} Y_n - X_1 = 1 - X_1 \le 1$ *and also* $f(z) \in \bar{T}^m_{\mu,\lambda,\sigma}(\alpha,\beta)$

Conversely, suppose that $f(z) \in \bar{T}^{s,m,k}_{\lambda,\alpha}$, Setting

$$X_n = \frac{\left(\frac{1+b}{n+b}\right)^s [1+(n\lambda-\lambda)(1-\alpha)^m]^k |a_n|}{\frac{2(\alpha+1)}{n}}, 0 \le X_n \le 1, \quad n = 2,3,\dots.$$

$$Y_n = \frac{\left(\frac{1+b}{n+b}\right)^s [1+(n\lambda-\lambda)(1-\alpha)^m]^k |b_n|}{\frac{2(\alpha+1)}{n}}, 0 \le X_n \le 1, \quad n = 1,2,3,\dots.$$

And

$$X_1 = 1 - \left(\sum_{n=2}^{\infty} X_n + \sum_{n=1}^{\infty} Y_n\right).$$

Therefore $f(z)$ can be written as

$$f(z) = z - \sum_{n=2}^{\infty}|a_n| z^n + \sum_{n=1}^{\infty}|b_n|\bar{z}^n$$

$$= \boldsymbol{z} - \sum_{n=2}^{\infty} \left(\frac{\frac{2(\alpha+1)}{n}}{\left(\frac{1+b}{n+b}\right)^s [1+(n\lambda-\lambda)(1-\alpha)^m]^k} \right) X_n z^n + \sum_{n=1}^{\infty} \left(\frac{\frac{2(\alpha+1)}{n}}{\left(\frac{1+b}{n+b}\right)^s [1+(n\lambda-\lambda)(1-\alpha)^m]^k} \right) Y_n \bar{z}^n$$

$$= h_1(z)X_1 + \sum_{n=2}^{\infty} h_n(z)X_n + \sum_{n=1}^{\infty} g_n(z)Y_n$$

$$= \sum_{\boldsymbol{n=1}}^{\infty}(X_n h_n(z) + Y_n g_n(z)).$$

3- Neighborhood.

The main results obtained in this section concern the Neighborhood for the class $\bar{T}^{s,m,k}_{\lambda,\alpha}$.(Libera, 1965), (Avcı & Złotkiewicz, 1990) and (Ruscheweyh, 1981) , the δ-Neighborhood of the mappings (1.2) takes the form

$$N_\delta(f) = \left\{ F = z + \sum_{n=2}^{\infty} A_n z^n + \sum_{n=1}^{\infty} \overline{B_n z^n} : \sum_{n=2}^{\infty} n(|a_n - A_n| + |b_n - B_n|) + |b_1 - B_1| \le \delta \right\}. \quad (1.11)$$

We must prove $m-\delta$-Neighborhood of the mapping f

$$N_\delta^m(f) = \left\{ F = z + \sum_{n=2}^{\infty} A_n z^n + \sum_{n=1}^{\infty} \overline{B_n z^n} : \sum_{n=2}^{\infty} (|a_n - A_n| + |\dot{b}_n - B_n|) + |\dot{b}_1 - B_1| \leq \delta \right\} \quad (1.12)$$

In this case we define $m-\delta$-Neighborhood of f $N_\delta^m(f) = \left\{ F: \sum_{n=2}^{\infty} \left(\frac{1+\dot{b}}{n+b}\right)^s [1+(n-1)\lambda(1-\alpha)^m]^k |a_n - A_n| + \left(\frac{1+\dot{b}}{n+\dot{b}}\right)^s [1+(n\lambda-\lambda)(1-\alpha)^m]^k |\dot{b}_n - B_n| + (\alpha+1)|\dot{b}_1 - B_1| \leq (\alpha+1)\delta \right\}$ (1.13)

Theorem 3.1 Suppose f is defined by (1.2) satisfies the conditions

$$\sum_{n=2}^{\infty} \left(\frac{1+\dot{b}}{n+\dot{b}}\right)^s [1+(n\lambda-\lambda)(1-\alpha)^m]^k + \sum_{n=1}^{\infty} \left(\frac{1+\dot{b}}{n+\dot{b}}\right)^s [1+(n\lambda-\lambda)(1-\alpha)^m]^k \leq (\alpha+1) \quad (1.14)$$

where $0 \leq \alpha < 1, \delta \leq \frac{2(\alpha+1)}{n}$

Then $N_\delta^m(f) \subset \bar{T}_{\mu,\lambda,\sigma}^m(\alpha, \beta)$

Proof. Consider f such that condition (7.4) is fulfilled and let

$$\boldsymbol{F(z) = z + \overline{B_1 z} + \sum_{n=2}^{\infty} (A_n z^n + B_n z^n)} \quad \mathbf{(1.15)}$$

Belong to $N_\delta^m(f)$ we have

$$(\alpha+1)|B_1| + \sum_{n=2}^{\infty} \left(\frac{1+\dot{b}}{n+\dot{b}}\right)^s [1+(n\lambda-\lambda)(1-\alpha)^m]^k |A_n| + \left(\frac{1+\dot{b}}{n+\dot{b}}\right)^s [1+(n\lambda-\lambda)(1-\alpha)^m]^k |B_n|$$
$$\leq (\alpha+1)|\dot{b}_1 - B_1| + (\alpha+1)|\dot{b}_1| + \sum_{n=2}^{\infty} \left(\frac{1+\dot{b}}{n+\dot{b}}\right)^s [1+(n\lambda-\lambda)(1-\alpha)^m]^k |A_n - a_n| + \left(\frac{1+\dot{b}}{n+\dot{b}}\right)^s [1+(n\lambda-\lambda)(1-\alpha)^m]^k |B_n - \dot{b}_n| + \sum_{n=2}^{\infty} \left(\frac{1+\dot{b}}{n+\dot{b}}\right)^s [1+(n\lambda-\lambda)(1-\alpha)^m]^k |A_n| + \left(\frac{1+\dot{b}}{n+\dot{b}}\right)^s [1+(n\lambda-\lambda)(1-\alpha)^m]^k |B_n|$$
$$\leq (\alpha+1)\delta + (\alpha+1)|\dot{b}_1| + \sum_{n=2}^{\infty} \left(\frac{1+\dot{b}}{n+\dot{b}}\right)^s [1+(n\lambda-\lambda)(1-\alpha)^m]^k |a_n| + \left(\frac{1+\dot{b}}{n+\dot{b}}\right)^s [1+(n\lambda-\lambda)(1-\alpha)^m]^k |b_n| \leq (\alpha+1)$$

Hence

$$\delta \leq \frac{2(\alpha+1)}{n}$$
$$N_\delta^m(f) \subset \bar{T}_{\lambda,\alpha}^{s,m,k}$$

Conclusion :

This study successfully defined and investigated a new class of univalent harmonic mappings ($\bar{T}_{\lambda,\alpha}^{s,m,k}$) on the unit disk, utilizing a novel differential operator derived from the binomial series. The most important properties of this new class are determined, which include finding coefficient estimates for the harmonic functions belonging to it, and proving the property of closure under both the convolution operation (Hadamard product) and convex combinations. Furthermore, a sufficient condition for a harmonic function to belong to the mentioned class is presented, and the extreme points and neighborhoods of these mappings are characterized. These results represent a valuable contribution to the development of the theory of harmonic functions by providing a new analytical framework for studying their geometric behavior.

REFERENCES

[1]. Rossdy, M., Omar, R., & Soh, S. C. (2024, January). New differential operator for analytic univalent functions associated with binomial series. In *AIP Conference Proceedings* (Vol. 2905, No. 1, p. 070001). AIP Publishing LLC.

[2]. Yunus, Y., Akbarally, A., & Halim, S. A. (2017, August). Strongly starlike functions associated with a new operator. In *AIP Conference Proceedings* (Vol. 1870, No. 1, p. 040001). AIP Publishing LLC.

[3]. Srivastava, H. M., & Attiya, A. A. (2007). An integral operator associated with the Hurwitz–Lerch Zeta function and differential subordination. *Integral Transforms and special functions*, *18*(3), 207-216.

[4]. Al-Oboud, F.M. (2004) , International Journal of Mathematics and Mathematical Sciences, 1429.

[5]. Sălăgean, G. S. (1983). Subclasses of Univalent Functions. In C. A. Cazacu, N. Boboc, M. Jurchescu, & I. Suciu (Eds.), Complex Analysis — Fifth Romanian-Finnish Seminar (pp. 362–372). Springer Berlin Heidelberg

[6]. Rossdy, M., Omar, R., & Soh, S. C. (2024, January). New differential operator for analytic univalent functions associated with binomial series. In *AIP Conference Proceedings* (Vol. 2905, No. 1, p. 070001). AIP Publishing LLC..

[7] Saheb, A. H., & Buti, R. H. (2024). A Specific Category of Harmonic Functions Characterized By A Generalized Komatu Operator in Conjunction With The (RK) Integral Operator and Applications to Neutrosophic Complex Field. *International Journal of Neutrosophic Science (IJNS)*, *23*(3**).**

[8].. Libera, R. J. (1965). Some Classes of Regular Univalent Functions. Proceedings of the American Mathematical Society, 16(4), 755–758.

[9]. Avcı, Y., & Złotkiewicz, E. (1990). On harmonic univalent mappings. Annales Universitatis Mariae Curie-Skłodowska A, 44, 1–7..

[10]. Ruscheweyh, S. (1981). Neighborhoods of Univalent Mappings. Proceedings of the American Mathematical Society, 81(4), 521–527.