

Certain Spaces and Functions Through $c\mathcal{O}l_\omega$ -Open Set

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Abstract

In this work, we present a generalization of open and $c\mathcal{O}$ -Lindelof open (simply $c\mathcal{O}l$ -open) sets termed $c\mathcal{O}$ -Lindelof- ω -open (simply $c\mathcal{O}l_\omega$ -open or $c\mathcal{O}l_\omega-\mathbb{O}$) sets. Moreover, we investigate relationships between open, $c\mathcal{O}l$ -open, and $c\mathcal{O}l_\omega$ -open sets. Furthermore, we give new forms of $\mathcal{M}c$ -function, namely $\mathcal{M}c\mathcal{O}l_\omega c$, $c\mathcal{O}l_\omega \mathcal{M}c$ and $c\mathcal{O}l_\omega \mathcal{M}c\mathcal{O}l_\omega c$ -functions. Additionally, we present a new form of $\mathcal{M}c$ -function termed by $\mathcal{L}c$ -function. Furthermore, we introduce new forms of $\mathcal{L}c$ -space termed $\mathcal{L}c\mathcal{O}l_\omega c$ -space, $c\mathcal{O}l_\omega \mathcal{L}c$ -space, Locally $\mathcal{L}c\mathcal{O}l_\omega c$ -space, $c\mathcal{O}l_\omega \mathcal{L}c\mathcal{O}l_\omega c$ -space and Locally $c\mathcal{O}l_\omega \mathcal{L}c\mathcal{O}l_\omega c$ -space. Finally, several facts, theorems, and examples are given to illustrate the results.



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1. INTRODUCTION

The notion of a co-Lindelof open (briefly $co\mathcal{O}l$ -open) set was introduced by authors: (Abushaheen, Alrimawi, Kawariq & Jerash, 2024) in that manner: A set \mathfrak{B} is termed a $c\mathcal{O}$ -Lindelof open (simply $c\mathcal{O}l$ -open) set which is a subsection from a space $(\mathbb{X}, \mathfrak{F})$ if, for each $j \in \mathfrak{B}$, there is a Lindelof subsection $\mathbb{H} \in L(\mathbb{X}, \mathfrak{F})$ and $\mathbb{K} \in \mathfrak{F}$ in which $j \in \mathbb{K} \setminus \mathbb{H} \subseteq \mathfrak{B}$. A $c\mathcal{O}l$ -closed set is the complementary of a $c\mathcal{O}l$ -open set. The complication of all $c\mathcal{O}l$ -open subsections of $(\mathbb{X}, \mathfrak{F})$ is signified by \mathfrak{F}_L .

In 1982, the idea of an ω -open and an ω -closed sets was presented by (Hdeib, 1982) as follows: An ω -open set \mathfrak{D} , which is a subsection from a space, $(\mathbb{X}, \mathfrak{F})$ is formulated as follows: for all $n \in \mathfrak{D}$, it follows that there is an open set, say $\mathcal{U}_n \subseteq \mathbb{X}$, consisting of n , in order that $\mathcal{U}_n \setminus \mathfrak{D}$ is countable. An ω -closed is the complementary of ω -open set. The complication of all ω -open subsections of $(\mathbb{X}, \mathfrak{F})$ is symbolized by \mathfrak{F}_ω . The ω -closure and ω -interior, that can be defined in a

manner to $\mathcal{C}l(\mathfrak{D})$ and $Int(\mathfrak{D})$, respec., will be referred to as $\mathcal{C}l_\omega(\mathfrak{D})$ and $Int_\omega(\mathfrak{D})$, respec. (Ganster, Kanibir, & Reilly, 1998) introduced the class of locally $\mathcal{L}c$ -spaces.

The concept of generalized open sets has received significant attention from many researchers; for instance, (Mohammed & Al-Jumaili, 2024), (Ali & Al-Jumaili, 2025) studied the new class of ω -open sets, (Suliman & Esmaeel, 2023), (Sadek & Esmaeel, 2024) studied the compactness. Subsequent developments have been presented in several studies, for instance (Yousif, 2021), (Ashaea & Yousif, 2020), (Mohammed, Noorani & Ghareeb, 2016), (Mutawek & Al-Abdulla, 2023). In this work, we present a generalization of $co\mathcal{O}l$ -open set by using ω -open set name, it $c\mathcal{O}$ -Lindelof- ω -open (simply $c\mathcal{O}l_\omega$ -open or $c\mathcal{O}l_\omega-\mathbb{O}$) set, and we introduce several related concepts and results.

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2. PRELIMINARIES:

In this section, we present some definitions, propositions, and notations that will be used throughout the paper.

Definition(2.1) (Hdeib, 1982, Jubair, & Ali, 2022): An ω –open set \mathfrak{D} which is a subsection from a space $(\mathbb{X}, \mathfrak{S})$ is formulated as if, for all $n \in \mathfrak{D}$, it follows that there is an open set ,say $\mathcal{U}_n \subseteq \mathbb{X}$,consisting of n , in order that $\mathcal{U}_n \setminus \mathfrak{D}$ is countable. An ω –closed is the complementary of ω –open set. The complication of all ω –open subsections of $(\mathbb{X}, \mathfrak{S})$ is symbolized by \mathfrak{S}_ω .

Definition(2.2) (Al Ghour & Samarah, 2012): A subdivision \mathfrak{B} of a space $(\mathbb{X}, \mathfrak{S})$ and for all $h \in \mathfrak{B}$, there is $\mathcal{U} \in \mathfrak{S}$ and a compact subdivision $\mathfrak{M} \in C(\mathbb{X}, \mathfrak{S})$ ($C(\mathbb{X}, \mathfrak{S})$ is the set of all compact subsets of a space $(\mathbb{X}, \mathfrak{S})$) for which $h \in \mathcal{U} - \mathfrak{M} \subseteq \mathfrak{B}$, so, a subsection \mathfrak{B} of a space $(\mathbb{X}, \mathfrak{S})$, is termed $c\sigma c$ -open set. The complementary of $c\sigma c$ -open set is $c\sigma c$ -closed. The complication of all $c\sigma c$ -open subdivisions of $(\mathbb{X}, \mathfrak{S})$ is regarded by \mathfrak{S}_K , and the complication of all $\{\mathcal{U} - \mathfrak{M} : \mathcal{U} \in \tau \text{ and } \mathfrak{M} \in C(\mathbb{X}, \mathfrak{S})\}$ is deemed by $\mathcal{B}^K(\tau)$.

Definition(2.3)(Abushaheen, Alrimawi, Kawariq & Jerash, 2024): A set \mathfrak{B} is termed a $c\sigma$ -Lindelof open (simply $c\sigma l$ -open) set which is a subsection from a space $(\mathbb{X}, \mathfrak{S})$ if, for each $j \in \mathfrak{B}$, there is a Lindelof subsection $\mathfrak{H}_j \in L(\mathbb{X}, \mathfrak{S})$ ($L(\mathbb{X}, \mathfrak{S})$ is the set of all Lindelof subsets of a space $(\mathbb{X}, \mathfrak{S})$) and $\mathfrak{K} \in \mathfrak{S}$ in which $j \in \mathfrak{K} \setminus \mathfrak{H}_j \subseteq \mathfrak{B}$. A $c\sigma l$ -closed is the complementary of $c\sigma l$ -open set. The complication of all $c\sigma l$ -open subsections of $(\mathbb{X}, \mathfrak{S})$ is signified by \mathfrak{S}_L .

Theorem(2.1)(Abushaheen, Alrimawi, Kawariq & Jerash, 2024): For a topological space $(\mathbb{X}, \mathfrak{S})$, we have $\mathfrak{S} \subset \mathfrak{S}_K \subset \mathfrak{S}_L$.

Remark(2.1): The opposite of theorem 2.1 is not necessarily true ,as in the following examples:

Example(2.1): In a space $(\mathbb{Z}, \mathfrak{S}_E)$ with $\mathfrak{S}_E = \{\mathcal{U} \subseteq \mathbb{Z} : 0 \notin \mathcal{U}\} \cup \{\mathbb{Z}\}$. Now for any subset \mathfrak{K} of \mathbb{Z} with $0 \in \mathfrak{K}$, then \mathfrak{K} is a $c\sigma l$ -open set but not $c\sigma c$ -open.

Example(2.2): In a space $(\mathbb{R}, \mathfrak{S}_{ind})$, Q is a $c\sigma c$ -open set but not open.

Proposition(2.1): Each ω –open set is $c\sigma l$ -open; however, the reverse implication is not necessarily true.

Proof: Let \mathfrak{M} be an ω –open set ,so for all $t \in \mathfrak{M}$, we can find an open set \mathcal{U}_t with $t \in \mathcal{U}_t$ and $\mathcal{U}_t \setminus \mathfrak{M} = \mathcal{G}$ is countable ,which is Lindelof , then $t \in \mathcal{U}_t \setminus \mathcal{G} = \mathcal{U}_t \setminus (\mathcal{U}_t \setminus \mathfrak{M}) = \mathcal{U}_t \setminus (\mathcal{U}_t \cap \mathfrak{M}^c) = \mathcal{U}_t \cap (\mathcal{U}_t \cap \mathfrak{M}^c)^c = \mathcal{U}_t \cap (\mathcal{U}_t^c \cup \mathfrak{M}) = (\mathcal{U}_t \cap \mathcal{U}_t^c) \cup (\mathcal{U}_t \cap \mathfrak{M}) = \emptyset \cup (\mathcal{U}_t \cap \mathfrak{M}) \subseteq \mathfrak{M}$. So, \mathfrak{M} is a $c\sigma l$ -open set.

Example(2.3): In the cofinite space $(\mathbb{R}, \mathfrak{S}_{cof})$, Q is not an ω –open set because for all $n \in Q$, there will find just open set \mathcal{R} with $n \in \mathcal{R}$, and $\mathcal{R} \setminus Q = Q^c$ which is uncountable set, but Q is $c\sigma l$ -open, since for all $n \in Q$, there is an open set \mathcal{R} with $n \in \mathcal{R}$ and Lindelof set Q^c for which $\mathcal{R} \setminus Q^c = Q$ which is subdivision for itself. Thereafter Q is a $c\sigma l$ -open set.

Definition(2.4) (Abdulrazak & Ali, 2025) : $f: (\mathbb{X}, \mathfrak{S}) \rightarrow (\mathbb{Y}, \xi)$ is regarded as an $\mathcal{M}c$ -function if $f^{-1}(\mathcal{K})$ is a closed in \mathbb{X} , for each \mathcal{K} is compact in \mathbb{Y} .

Example(2.4): Let $\mathbb{X} = \mathbb{Y} = \mathbb{R}$. A function $f: (\mathbb{X}, \mathfrak{S}_{dis}) \rightarrow (\mathbb{Y}, \mathfrak{S}_u)$ is defined by $f(x) = x$, and let \mathfrak{B} be any compact subset of \mathbb{Y} , since each subset of a discrete space is closed, then $f^{-1}(\mathfrak{B})$ is a closed in \mathbb{X} . Therefore, f is an $\mathcal{M}c$ -function.

Definition(2.5) (Nadhim, Ali & Majeed, 2020): A space $(\mathbb{X}, \mathfrak{S})$ is regarded as a $\mathcal{K}c$ –space if each compact subdivision of \mathbb{X} is closed.

Example(2.5): In a space $(\mathbb{R}, \mathfrak{S}_u)$, let $\mathfrak{H} \subseteq \mathbb{R}$ be a compact set, because \mathbb{R} is a Housdorff space, then each compact subset of \mathbb{R} is closed. Thus, \mathfrak{H} is a closed in \mathbb{R} . Thereafter, each compact subset of \mathbb{R} is closed, and so $(\mathbb{R}, \mathfrak{S}_u)$ is a $\mathcal{K}c$ –space.

Definition(2.6) (Ali, 2020): A space $(\mathbb{X}, \mathfrak{S})$ is regarded as a $\mathcal{L}c$ –space if each Lindelof subdivision of \mathbb{X} is closed.

Example(2.6): Let \mathbb{X} be any finite discrete space, because each subset of \mathbb{X} is both Lindelof and closed , then each Lindelof subset of \mathbb{X} is closed. Thereafter, \mathbb{X} is a $\mathcal{L}c$ –space.

Remark(2.2) (Ali & Saleh, 2014): Each $\mathcal{L}c$ –space is a $\mathcal{K}c$ –space, but the reverse is not true.

Example(2.7): A space $(\mathbb{R}, \mathfrak{S}_u)$ is a $\mathcal{K}c$ –space but not $\mathcal{L}c$ –space.

Definition(2.7): $f: (\mathbb{X}, \mathfrak{S}) \rightarrow (\mathbb{Y}, \xi)$ is referred to as the Lindelof function if $f^{-1}(\mathcal{H})$ is Lindelof in \mathbb{X} , whenever \mathcal{H} is a Lindelof subsection in \mathbb{Y} .

Example(2.8): In topological spaces, any function $f: (X, \mathfrak{T}) \rightarrow (Y, \xi)$ is a Lindelof function, whenever X is a countable space.

Definition(2.8) (Ganster, Kanibir, & Reilly, 1998): A space (X, \mathfrak{T}) is regarded as a Locally $\mathfrak{L}c$ –space if each point has a neighborhood which is a $\mathfrak{L}c$ –subspace.

3. $c\sigma\ell_\omega$ -OPEN SET

In this section, we give a generalization of a $c\sigma\ell$ -open set by using an ω –open set name it $c\sigma$ -Lindelof- ω -open set.

Definition(3.1): A subsection \mathcal{M} of a space (X, \mathfrak{T}) is referred to as a $c\sigma$ -Lindelof- ω -open (simply $c\sigma\ell_\omega$ -open or $c\sigma\ell_\omega$ - \mathfrak{O}) set if, for all $\mathcal{G} \in \mathcal{M}$, there is $H \in \mathfrak{T}_\omega$ and a Lindelof set $\mathfrak{B} \in L(X, \mathfrak{T})$ in which $\mathcal{G} \in H \setminus \mathfrak{B} \subseteq \mathcal{M}$. An $c\sigma\ell_\omega$ -closed (simply $c\sigma\ell_\omega$ - \mathfrak{C}) is the complementary of $c\sigma\ell_\omega$ -open set. The complication of all $c\sigma\ell_\omega$ -open subsections of (X, \mathfrak{T}) is signified by $\mathfrak{T}_{\omega L}$.

Lemma(3.1): Each open set is a $c\sigma\ell_\omega$ -open; however, the reverse implication is not necessarily true.

Proof: Assume that \mathfrak{D} is an open set and $u \in \mathfrak{D}$, so \mathfrak{D} is an ω –open set also, one can find a Lindelof set \mathcal{G} consists a finite subdivision of \mathfrak{D} not including u , then $u \in \mathfrak{D} \setminus \mathcal{G} \subseteq \mathfrak{D}$. Thereafter, \mathfrak{D} is a $c\sigma\ell_\omega$ -open set.

Example(3.1): In $(\mathbb{R}, \mathfrak{T}_{ind})$, \mathbb{Q}^c is a $c\sigma\ell_\omega$ -open, because for each $r \in \mathbb{Q}^c$, there will be find \mathbb{R} is an ω –open set also \mathbb{Q} is a Lindelof set in which, $r \in \mathbb{R} \setminus \mathbb{Q} \subseteq \mathbb{Q}^c$. Subsequently, \mathbb{Q}^c is $c\sigma\ell_\omega$ -open and ω –open ,but it is not open.

Lemma(3.2): Each ω –open set is a $c\sigma\ell_\omega$ -open; however, the reverse implication is not necessarily true.

Proof: Assume that H is an ω –open set and $h \in H$, then there is an open set \mathcal{U} with $h \in \mathcal{U}$ s.t. $\mathcal{U} \setminus H = \mathfrak{P}$ is countable. Thus, \mathfrak{P} is a $c\sigma\ell$ -open set, so $h \in H \setminus \mathfrak{P} \subseteq H$. Hence, H is a $c\sigma\ell_\omega$ -open set.

Example(3.2): In $(\mathbb{R}, \mathfrak{T}_{ind})$, \mathbb{Q} is a $c\sigma\ell_\omega$ -open set (because for each $q \in \mathbb{Q}$, there will be find an ω –open set \mathbb{R} including q also, \mathbb{Q}^c is a Lindelof set in which, $q \in \mathbb{R} \setminus \mathbb{Q}^c \subseteq \mathbb{Q}$). Consequently, \mathbb{Q} is a $c\sigma\ell_\omega$ -open but, it is not ω –open (since for all $v \in \mathbb{Q}$, $\exists!$ an open set which is \mathbb{R} including v , that is $\mathbb{R} \setminus \mathbb{Q} = \mathbb{Q}^c$, which is not countable .

Thus \mathbb{Q} is not an ω –open set.

Lemma(3.3): Each $c\sigma\ell$ -open set is a $c\sigma\ell_\omega$ -open.

Proof: If K is a $c\sigma\ell$ -open set and $g \in K$, then there is an open set \mathcal{U} with $g \in \mathcal{U}$, also a Lindelof set \mathfrak{B} , in which $g \in \mathcal{U} \setminus \mathfrak{B} \subseteq K$. But, \mathcal{U} is also ω –open, thereafter, K is a $c\sigma\ell_\omega$ -open set.

Example(3.3): In example 3.2 \mathbb{Q} is $c\sigma\ell_\omega$ -open set and $c\sigma\ell$ -open.

Proposition(3.1): Consider a space (X, \mathfrak{T}) , then $\mathfrak{T}_{\omega L}$ is a topology on X .

Proof:

1. φ, X are open sets, then φ, X are ω –open sets. Clearly $\varphi \in \mathfrak{T}_{\omega L}$. Now, let $x \in X$ and X is an ω –open set, then there exists a Lindelof set φ s.t. $x \in X \setminus \varphi = X \subseteq X$. Hence $X \in \mathfrak{T}_{\omega L}$.

2. Let \mathfrak{B} and \mathfrak{W} be any $c\sigma\ell_\omega$ -open sets to deduce that $\mathfrak{B} \cap \mathfrak{W}$ is a $c\sigma\ell_\omega$ -open set. Claim that if we have $n \in \mathfrak{B} \cap \mathfrak{W}$, then $n \in \mathfrak{B}$ also $n \in \mathfrak{W}$. It is clear that there are two ω –open subsections V_n, K_n of X and two Lindelof subsections \mathcal{U}_n, L_n of X in which, $n \in V_n \setminus \mathcal{U}_n \subseteq \mathfrak{B}$ (by $V_n \setminus \mathcal{U}_n = V_n \cap X \setminus \mathcal{U}_n$) we get $n \in V_n \cap X \setminus \mathcal{U}_n \subseteq \mathfrak{B}$ also $n \in K_n \setminus L_n \subseteq \mathfrak{W}$, we get $n \in K_n \cap X \setminus L_n \subseteq \mathfrak{W}$. Consequently, $n \in V_n \cap (X \setminus \mathcal{U}_n) \cap K_n \cap (X \setminus L_n) \subseteq \mathfrak{B} \cap \mathfrak{W}$, so we achieve that $n \in (V_n \cap K_n) \cap (X \setminus \mathcal{U}_n \cap X \setminus L_n) \subseteq \mathfrak{B} \cap \mathfrak{W}$, then $n \in (V_n \cap K_n) \cap X \setminus (\mathcal{U}_n \cup L_n) \subseteq \mathfrak{B} \cap \mathfrak{W}$, also achieve that $n \in (V_n \cap K_n) \setminus (\mathcal{U}_n \cup L_n) \subseteq \mathfrak{B} \cap \mathfrak{W}$. Because, $V_n \cap K_n$ is an ω –open set (since finite intersection of ω –open sets is also ω –open (Al-Zoubi & Al-Nashef, 2003)) also $\mathcal{U}_n \cup L_n$ is a Lindelof set in X (since countable union of Lindelof sets is also Lindelof). Subsequently, $\mathfrak{B} \cap \mathfrak{W}$ is a $c\sigma\ell_\omega$ -open set.

3. Let $\{\mathfrak{B}_\delta: \delta \in \gamma\}$ be a family of $c\sigma\ell_\omega$ -open sets. To deduce that $\bigcup_{\delta \in \gamma} \mathfrak{B}_\delta$ is a $c\sigma\ell_\omega$ -open set. Let $r \in \bigcup_{\delta \in \gamma} \mathfrak{B}_\delta$, then $r \in \mathfrak{B}_{\delta_0}$ for some $\delta_0 \in \gamma$. Because \mathfrak{B}_{δ_0} is a $c\sigma\ell_\omega$ -open set, then one can find an ω –open subsections V_n of X , also a Lindelof subsections \mathcal{U}_n of X in which, $r \in V_n \setminus \mathcal{U}_n \subseteq \mathfrak{B}_{\delta_0}$ for some $\delta_0 \in \gamma$, since $\mathfrak{B}_{\delta_0} \subseteq \bigcup_{\delta \in \gamma} \mathfrak{B}_\delta$, thereafter, $\bigcup_{\delta \in \gamma} \mathfrak{B}_\delta$ is a $c\sigma\ell_\omega$ -open set.

Definition(3.2): A subsection \mathfrak{H} of a space (X, \mathfrak{T}) is deemed by $c\sigma\ell_\omega$ -compact set if for all $c\sigma\ell_\omega$ -open cover to \mathfrak{H} has a finite sub cover.

Remark(3.1): Each $c\sigma\ell_\omega$ -compact set is compact; however, the reverse implication is not necessarily true.

Example(3.4): The space $(\mathbb{R}, \mathfrak{S}_E)$ with $\mathfrak{S}_E = \{\mathbb{U} \subseteq \mathbb{R} : x_0 \in \mathbb{U} \text{ for some } x_0 \in \mathbb{R}\} \cup \{\mathbb{R}\}$ is a $c\mathcal{O}l_\omega$ -compact and a compact space.

Example(3.5): Example 3.2 is a compact space , but neither $c\mathcal{O}l$ -compact nor $c\mathcal{O}l_\omega$ -compact space.

4. TYPES OF $\mathcal{M}c$ –FUNCTIONS

In this section, we give new forms of $\mathcal{M}c$ -function ,namely $\mathcal{M}c\mathcal{O}l_\omega c$, $c\mathcal{O}l_\omega \mathcal{M}c$ and $c\mathcal{O}l_\omega \mathcal{M}c\mathcal{O}l_\omega c$ –functions as subsequent definitions:

Definition(4.1): $f: (\mathbb{X}, \mathfrak{S}) \rightarrow (\mathbb{Y}, \xi)$ is regarded as an $\mathcal{M}c\mathcal{O}l_\omega c$ –function if $f^{-1}(\mathcal{H})$ is a $c\mathcal{O}l_\omega$ -closed in \mathbb{X} , for each \mathcal{H} is compact subsection in \mathbb{Y} .

Definition(4.2): $f: (\mathbb{X}, \mathfrak{S}) \rightarrow (\mathbb{Y}, \xi)$ is referred to as a $c\mathcal{O}l_\omega \mathcal{M}c$ –function if $f^{-1}(\mathcal{H})$ is closed in \mathbb{X} , whenever \mathcal{H} is $c\mathcal{O}l_\omega$ -compact subsection in \mathbb{Y} .

Definition(4.3): $f: (\mathbb{X}, \mathfrak{S}) \rightarrow (\mathbb{Y}, \xi)$ is deemed to be a $c\mathcal{O}l_\omega \mathcal{M}c\mathcal{O}l_\omega c$ –function if $f^{-1}(\mathcal{H})$ is $c\mathcal{O}l_\omega$ -closed in \mathbb{X} , whenever \mathcal{H} is $c\mathcal{O}l_\omega$ -closed subsection in \mathbb{Y} .

Example(4.1): Any function has discrete domain satisfies all kinds of $\mathcal{M}c$ -function which illustrate us in our work.

Remark(4.1): Every $\mathcal{M}c\mathcal{O}l_\omega c$ –function is a $c\mathcal{O}l_\omega \mathcal{M}c\mathcal{O}l_\omega c$,and every $c\mathcal{O}l_\omega \mathcal{M}c$ – function is a $c\mathcal{O}l_\omega \mathcal{M}c\mathcal{O}l_\omega c$.

Example(4.2): In $f: (\mathbb{R}, \mathfrak{S}_{ind}) \rightarrow (\mathbb{R}, \xi)$, consider V is $c\mathcal{O}l_\omega$ -compact in \mathbb{Y} , then V is compact in \mathbb{Y} , but f is compact, then $f^{-1}(V)$ is compact, which implies it will be Lindelof in \mathbb{X} . Let $a_1 \in (f^{-1}(V))^c$, then there will be $\mathbb{R} \setminus \{a_2\}$ is an ω –open including a_1 , then $a_1 \in (\mathbb{R} \setminus \{a_2\}) \setminus f^{-1}(V) \subseteq (f^{-1}(V))^c$, so $(f^{-1}(V))^c$ is $c\mathcal{O}l_\omega$ -open. Thereafter, $f^{-1}(V)$ is $c\mathcal{O}l_\omega$ -closed.

Proposition(4.1): If \mathbb{Y} is a \mathcal{T}_2 –space, then a continuous function $f: (\mathbb{X}, \mathfrak{S}) \rightarrow (\mathbb{Y}, \xi)$ is $\mathcal{M}c\mathcal{O}l_\omega c$.

Proof: Assume that J be a compact set in \mathcal{T}_2 –space \mathbb{Y} , so it will be closed set also, we own f is continuous. Accordingly, $f^{-1}(J)$ is closed set in \mathbb{X} , which led us to $f^{-1}(J)$ is a $c\mathcal{O}l_\omega c$ set.

Lemma(4.1): If \mathbb{Y} is a \mathcal{T}_2 –space, then a continuous function $f: (\mathbb{X}, \mathfrak{S}) \rightarrow (\mathbb{Y}, \xi)$ is a $c\mathcal{O}l_\omega \mathcal{M}c$

$(c\mathcal{O}l_\omega \mathcal{M}c\mathcal{O}l_\omega c)$.

Proof: We claim that \mathbb{K} be a $c\mathcal{O}l_\omega$ -compact set in \mathcal{T}_2 –space \mathbb{Y} , so it will be compact set, but \mathbb{Y} is \mathcal{T}_2 –space, then \mathbb{K} is closed. Also, we own f is continuous. Consequently, $f^{-1}(J)$ is a closed ($c\mathcal{O}l_\omega$ -closed) set in \mathbb{X} .

Proposition(4.2): If \mathbb{Y} is $\mathcal{K}c$ –space, then a continuous function $f: (\mathbb{X}, \mathfrak{S}) \rightarrow (\mathbb{Y}, \xi)$ is $\mathcal{M}c\mathcal{O}l_\omega c$.

Proof: Assume that \mathcal{D} is compact subsection of $\mathcal{K}c$ –space \mathbb{Y} , hence it will be closed by continuity of f we get, $f^{-1}(\mathcal{D})$ is a closed set in \mathbb{X} . Thus, it is a $c\mathcal{O}l_\omega$ -closed set.

Corollary(4.1): Each continuous function $f: (\mathbb{X}, \mathfrak{S}) \rightarrow (\mathbb{Y}, \xi)$ is a $c\mathcal{O}l_\omega \mathcal{M}c$ –function, whenever \mathbb{Y} is a $\mathcal{K}c$ –space.

Corollary(4.2): Each continuous function $f: (\mathbb{X}, \mathfrak{S}) \rightarrow (\mathbb{Y}, \xi)$ is a $c\mathcal{O}l_\omega \mathcal{M}c\mathcal{O}l_\omega c$ –function, whenever \mathbb{Y} is a $\mathcal{K}c$ –space.

Corollary(4.3): Each compact function $f: (\mathbb{X}, \mathfrak{S}) \rightarrow (\mathbb{Y}, \xi)$ is $\mathcal{M}c\mathcal{O}l_\omega c$, $c\mathcal{O}l_\omega \mathcal{M}c$ and $c\mathcal{O}l_\omega \mathcal{M}c\mathcal{O}l_\omega c$ –function, whenever \mathbb{X} is a \mathcal{T}_2 or ($\mathcal{K}c$) –space.

5. $\mathcal{L}c$ –FUNCTIONS

In this section, we give a new form of $\mathcal{M}c$ -function termed by $\mathcal{L}c$ –function and we present new types of this form.

Definition(5.1): $f: (\mathbb{X}, \mathfrak{S}) \rightarrow (\mathbb{Y}, \xi)$ is referred to as an $\mathcal{L}c$ –function if $f^{-1}(\mathcal{G})$ is closed in \mathbb{X} , whenever \mathcal{G} is a Lindelof subsection in \mathbb{Y} .

Definition(5.2): $f: (\mathbb{X}, \mathfrak{S}) \rightarrow (\mathbb{Y}, \xi)$ is described as an $\mathcal{L}c\mathcal{O}l_\omega c$ –function if $f^{-1}(\mathfrak{B})$ is a $c\mathcal{O}l_\omega$ -closed in \mathbb{X} , whenever \mathfrak{B} is a Lindelof subsection in \mathbb{Y} .

Definition(5.3): A space \mathbb{X} is referred to as a $c\mathcal{O}l_\omega$ -Lindelof space, if for all $c\mathcal{O}l_\omega$ -open cover of \mathbb{X} has a countable sub cover.

Definition(5.4): $f: (\mathbb{X}, \mathfrak{S}) \rightarrow (\mathbb{Y}, \xi)$ is termed by $c\mathcal{O}l_\omega \mathcal{L}c$ –function if $f^{-1}(\mathfrak{M})$ is closed in \mathbb{X} , whenever \mathfrak{M} is a $c\mathcal{O}l_\omega$ -Lindelof subsection in \mathbb{Y} .

Definition(5.5): $f: (\mathbb{X}, \mathfrak{S}) \rightarrow (\mathbb{Y}, \xi)$ is referred to as $c\mathcal{O}l_\omega \mathcal{L}c\mathcal{O}l_\omega c$ –function if $f^{-1}(\mathcal{U})$ is $c\mathcal{O}l_\omega$ -closed in \mathbb{X} , whenever \mathcal{U} is a $c\mathcal{O}l_\omega$ -Lindelof subsection in \mathbb{Y} .

Lemma(5.1): Let $f: (X, \mathfrak{S}) \rightarrow (Y, \xi)$ be a Lindelof function and if X is an $\mathcal{L}c$ -space, then f is an $\mathcal{L}c$ -function.

Proof: Let \mathfrak{B} be a Lindelof set in Y , then by f is a Lindelof function we get, $f^{-1}(\mathfrak{B})$ is a Lindelof in X , but X is an $\mathcal{L}c$ -space, then $f^{-1}(\mathfrak{B})$ is a closed set which led us to f is an $\mathcal{L}c$ -function.

Definition(5.6): A space X is described as an $\mathcal{L}c\mathcal{O}\omega c$ -space if each Lindelof subdivision of X is $c\mathcal{O}\omega c$ -closed.

Definition(5.7): A space X is described as a $c\mathcal{O}\omega\mathcal{L}c$ -space if each $c\mathcal{O}\omega$ -Lindelof subdivision of X is closed.

Corollary(5.1): $f: (X, \mathfrak{S}) \rightarrow (Y, \xi)$ is a Lindelof function and X is an $\mathcal{L}c\mathcal{O}\omega c$ or $\mathcal{L}c$ -space, then f is a $c\mathcal{O}\omega\mathcal{L}c\mathcal{O}\omega c$ -function.

Definition(5.8): A space X is regarded as a Locally $\mathcal{L}c\mathcal{O}\omega c$ -space if for all points has a proper neighborhood which is an $\mathcal{L}c\mathcal{O}\omega c$ -subspace.

Remark(5.1): Each $\mathcal{L}c$ -space is an $\mathcal{L}c\mathcal{O}\omega c$ -space. But, the reverse is not necessarily true.

Example(5.1): The indiscrete space $(\mathbb{Z}, \mathfrak{S}_{ind})$ is an $\mathcal{L}c\mathcal{O}\omega c$ -space but not $\mathcal{L}c$ -space.

Remark(5.2): Each Locally $\mathcal{L}c$ -space is a Locally $\mathcal{L}c\mathcal{O}\omega c$ -space. But, the reverse is not necessarily true.

Example(5.2): The indiscrete space $(\mathbb{Z}, \mathfrak{S}_{ind})$ is a Locally $\mathcal{L}c\mathcal{O}\omega c$ -space but not Locally $\mathcal{L}c$ -space.

Lemma(5.2): $f: (X, \mathfrak{S}) \rightarrow (Y, \xi)$ is a bijective closed Lindelof function. If X is an $\mathcal{L}c$ -space then Y is an $\mathcal{L}c$ -space.

Proof: Assume that \mathcal{H} is Lindelof in Y . Then $f^{-1}(\mathcal{H})$ is a Lindelof in X (since f is Lindelof function). But, X is an $\mathcal{L}c$ -space., then $f^{-1}(\mathcal{H})$ is a closed set in X , also f is a bijective closed function , then $\mathcal{H} = f(f^{-1}(\mathcal{H}))$ is closed in Y . Accordingly, Y is an $\mathcal{L}c$ -space.

Proposition(5.1): In a bijective closed Lindelof function $f: (X, \mathfrak{S}) \rightarrow (Y, \xi)$, Y is an Locally $\mathcal{L}c\mathcal{O}\omega c$ -space, whenever X is a Locally $\mathcal{L}c$ -space.

Proof: Claim that X is a Locally $\mathcal{L}c$ -space, thus for all $r \in X$, there will be find a neighborhood K of r in which K is an $\mathcal{L}c$ -subspace, but K is a neighborhood of r , then there is an open set U in which $r \in U \subseteq K$, thus $v = f(r) \in f(U) \subseteq f(K) \subseteq Y$. So we get $f(K)$ is a neighborhood which is an $\mathcal{L}c$ -subspace, because $f(U)$ is an open set (since each bijective closed function is open). Thereafter, Y is a Locally $\mathcal{L}c\mathcal{O}\omega c$ -space, because for all $v \in Y$, one can find a neighborhood $f(K)$ which is an $\mathcal{L}c$ -space (via Lemma 5.2). Also, (via Remark 5.1) we get $f(K)$ is an $\mathcal{L}c\mathcal{O}\omega c$ -subspace of Y , but v is arbitrary. Hence Y is a Locally $\mathcal{L}c\mathcal{O}\omega c$ -space.

Remark(5.3): A subspace M of a space X in which, $\mathcal{H} \subseteq M$ and \mathcal{H} is a $c\mathcal{O}\omega$ -closed (respe., $c\mathcal{O}\omega$ -open) subset in X . Then \mathcal{H} is a $c\mathcal{O}\omega$ -closed (respe., $c\mathcal{O}\omega$ -open) set in M .

Proof: We proof only $c\mathcal{O}\omega$ -closed case. Claim that $g \in X \setminus \mathcal{H}$, then there is $G \subseteq X$ is an ω -open set and $B \subseteq X$ is a Lindelof set s.t. $g \in G \setminus B \subseteq X \setminus \mathcal{H} \dots (1)$, but $\mathcal{H} \subseteq M$, then $M \setminus \mathcal{H} = M \cap X \setminus \mathcal{H}$. Now, we intersect with M in (1) we get $g \in (G \cap M) \setminus (B \cap M) \subseteq M \cap X \setminus \mathcal{H} = M \setminus \mathcal{H}$, where $G \cap M$ ω -open in M and $B \cap M$ is Lindelof in M , so $M \setminus \mathcal{H}$ is $c\mathcal{O}\omega$ -open set in M . Thereafter, $M \setminus (M \setminus \mathcal{H}) = \mathcal{H}$ is a $c\mathcal{O}\omega$ -closed in X .

Proposition(5.2): Each subspace of $\mathcal{L}c\mathcal{O}\omega c$ -space is an $\mathcal{L}c\mathcal{O}\omega c$ -space.

Proof: Let U be a Lindelof subset of \mathcal{B} , and \mathcal{B} be a subspace of X , it follows that $U \subseteq X$. Then U is Lindelof in X . Now, we claim that X is an $\mathcal{L}c\mathcal{O}\omega c$ -space, then each Lindelof subset of X is $c\mathcal{O}\omega$ -closed in X . Thus U is $c\mathcal{O}\omega$ -closed in X , (via remark 5.3), we get U is $c\mathcal{O}\omega$ -closed in \mathcal{B} . Hence each Lindelof subset of \mathcal{B} is $c\mathcal{O}\omega$ -closed in \mathcal{B} . Accordingly, \mathcal{B} is a $\mathcal{L}c\mathcal{O}\omega c$ -space.

Theorem(5.1): Each subspace of Locally $\mathcal{L}c\mathcal{O}\omega c$ -space is a Locally $\mathcal{L}c\mathcal{O}\omega c$ -space.

Proof: Assume that K be a subspace of X , and $s \in K$ then $s \in X$, and let X be a Locally $\mathcal{L}c\mathcal{O}\omega c$ -space. Then there is a neighborhood \mathcal{D} of s in X s.t. \mathcal{D} is a $\mathcal{L}c\mathcal{O}\omega c$ -subspace in X . Now, let $V = K \cap \mathcal{D}$, then $V \subseteq \mathcal{D}$, thus V is an $\mathcal{L}c\mathcal{O}\omega c$ -subspace (via proposition 5.2). Because, $s \in \mathcal{D}$ also $s \in K$ then $s \in K \cap \mathcal{D}$, hence $s \in V = K \cap \mathcal{D}$, which mean that V is a neighborhood of s in K . Subsequently, K is a Locally $\mathcal{L}c\mathcal{O}\omega c$ -subspace of X .

Definition(5.9): A space X is referred to as a $c\mathcal{O}\omega\mathcal{L}c\mathcal{O}\omega c$ -space if for all $c\mathcal{O}\omega$ -Lindelof set in X is $c\mathcal{O}\omega$ -closed.

Example(5.3): Let $\mathbb{X} = \{w, t\}$ with the discrete topology. Because each subset of \mathbb{X} is both open and closed, then each $c\sigma\ell_\omega$ -Lindelof subset of \mathbb{X} is $c\sigma\ell_\omega$ -closed. Hence, \mathbb{X} a $c\sigma\ell_\omega\text{-}\mathbb{L}c\sigma\ell_\omega c$ -space.

Definition(5.10): A space \mathbb{X} is regarded as a Locally $c\sigma\ell_\omega\text{-}\mathbb{L}c\sigma\ell_\omega c$ -space if for all point has a neighborhood which is a $c\sigma\ell_\omega\text{-}\mathbb{L}c\sigma\ell_\omega c$ -subspace.

Example(5.4): Let $\mathbb{X} = \{w, t, \ell\}$ with the discrete topology. Because each singleton $\{w\}$ is an open neighborhood of w and each subset of \mathbb{X} is both open and closed, then each neighborhood of a point in \mathbb{X} is a $c\sigma\ell_\omega\text{-}\mathbb{L}c\sigma\ell_\omega c$ -subspace. Thus, for each point $w \in \mathbb{X}$ there is a neighborhood $\mathbb{U} = \{w\}$ s.t. \mathbb{U} is a $c\sigma\ell_\omega\text{-}\mathbb{L}c\sigma\ell_\omega c$ -subspace. Thereafter, \mathbb{X} is a Locally $c\sigma\ell_\omega\text{-}\mathbb{L}c\sigma\ell_\omega c$ -space.

Remark(5.4): Each $c\sigma\ell_\omega\text{-}\mathbb{L}c\sigma\ell_\omega c$ -space is a Locally $c\sigma\ell_\omega\text{-}\mathbb{L}c\sigma\ell_\omega c$ -space.

Proposition(5.3): In a bijective $c\sigma\ell_\omega$ -open function $f: (\mathbb{X}, \mathfrak{S}) \rightarrow (\mathbb{Y}, \xi)$, if \mathcal{G} is a $c\sigma\ell_\omega$ -Lindelof subdivision of \mathbb{Y} , then $f^{-1}(\mathcal{G})$ is a Lindelof subdivision of \mathbb{X} .

Proof: Assume that \mathcal{G} is a $c\sigma\ell_\omega$ -Lindelof subdivision of \mathbb{Y} , let $\{\mathbb{G}_\delta: \delta \in \eta\}$ be an open cover of $f^{-1}(\mathcal{G})$ in \mathbb{X} , that is $f^{-1}(\mathcal{G}) \subseteq \bigcup_{\delta \in \eta} \mathbb{G}_\delta$, because f is bijective, we get $\mathcal{G} = f(f^{-1}(\mathcal{G})) \subseteq f(\bigcup_{\delta \in \eta} \mathbb{G}_\delta) = \bigcup_{\delta \in \eta} f(\mathbb{G}_\delta)$, but f is $c\sigma\ell_\omega$ -open, then each $f(\mathbb{G}_\delta)$ is a $c\sigma\ell_\omega$ -open subdivision of \mathbb{Y} for each $\delta \in \eta$. Thereafter, $\{f(\mathbb{G}_\delta): \delta \in \eta\}$ is $c\sigma\ell_\omega$ -open cover of \mathcal{G} . Because \mathcal{G} is a $c\sigma\ell_\omega$ -Lindelof subdivision of \mathbb{Y} , thus $\mathcal{G} \subseteq \bigcup_{\delta \in \eta'} f(\mathbb{G}_\delta)$, η' is a countable subdivision of η . Also $f^{-1}(\mathcal{G}) \subseteq f^{-1}(\bigcup_{\delta \in \eta'} f(\mathbb{G}_\delta)) = \bigcup_{\delta \in \eta'} f^{-1}(f(\mathbb{G}_\delta)) = \bigcup_{\delta \in \eta'} \mathbb{G}_\delta$. Thus $\{\mathbb{G}_\delta: \delta \in \eta'\}$ is a countable subcover of $f^{-1}(\mathcal{G})$. Thereafter, $f^{-1}(\mathcal{G})$ is a Lindelof subdivision of \mathbb{X} .

Theorem(5.2): In a bijective $c\sigma\ell_\omega$ -open function $f: (\mathbb{X}, \mathfrak{S}) \rightarrow (\mathbb{Y}, \xi)$, \mathbb{Y} is a $c\sigma\ell_\omega\text{-}\mathbb{L}c\sigma\ell_\omega c$ -space, whenever \mathbb{X} is an $\mathbb{L}c$ -space.

Proof: Claim that \mathfrak{B} be a $c\sigma\ell_\omega$ -Lindelof subdivision of \mathbb{Y} , then $f^{-1}(\mathfrak{B})$ is a Lindelof set in \mathbb{X} (via proposition 5.3), but \mathbb{X} is an $\mathbb{L}c$ -space, then $f^{-1}(\mathfrak{B})$ is a closed set in \mathbb{X} , so $(f^{-1}(\mathfrak{B}))^c$ is an open set in \mathbb{X} , also f is a $c\sigma\ell_\omega$ -open function, thus $f((f^{-1}(\mathfrak{B}))^c) = f(\mathbb{X}/f^{-1}(\mathfrak{B})) = f(\mathbb{X})/f(f^{-1}(\mathfrak{B})) = \mathbb{Y}/\mathfrak{B} = \mathfrak{B}^c$ is $c\sigma\ell_\omega$ -open in \mathbb{Y} , so \mathfrak{B} is $c\sigma\ell_\omega$ -closed in \mathbb{Y} . Consequently, \mathbb{Y} is a

$c\sigma\ell_\omega\text{-}\mathbb{L}c\sigma\ell_\omega c$ -space.

Corollary(5.2): In a bijective $c\sigma\ell_\omega$ -open function $f: (\mathbb{X}, \mathfrak{S}) \rightarrow (\mathbb{Y}, \xi)$, \mathbb{Y} is a Locally $c\sigma\ell_\omega\text{-}\mathbb{L}c\sigma\ell_\omega c$ -space, whenever \mathbb{X} is an $\mathbb{L}c$ -space.

Proof: Via theorem 5.2 and remark 5.4, we get \mathbb{Y} is a Locally $c\sigma\ell_\omega\text{-}\mathbb{L}c\sigma\ell_\omega c$ -space.

CONCLUSION

In this study, we introduced $c\sigma$ -Lindelof- ω -open (simply $c\sigma\ell_\omega$ -open or $c\sigma\ell_\omega\text{-}\mathbb{O}$) set. Through this new concept, we deduced new forms of Mc-function, new forms of $\mathbb{L}c$ -function. In addition, we present new types of $\mathbb{L}c$ -space and new types of locally $\mathbb{L}c$ -space. Theorems and examples illustrate the properties and distinctions of these concepts. These results provide a foundation for further investigations into related topological structures and functions.

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