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Fuzzy Normal Operator in Fuzzy n-Hilbert Space and Its Consequent Results

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ABSTRACT

In this paper, we systematically study the concept of a fuzzy normal operator defined on a fuzzy n-Hilbert space (briefly denoted as $F - n - H$ space). The work begins with a rigorous presentation of the fundamental definitions related to fuzzy n-inner product spaces and fuzzy n-Hilbert spaces, establishing the necessary theoretical framework for our study.

We then formally define fuzzy normal operators within this setting and investigate their structural and algebraic properties. Several key theorems are presented and proved to characterize the behavior of fuzzy normal operators in $F - n - H$ spaces. In particular, we explore conditions for normality in the fuzzy context, relationships between fuzzy adjoint operators and fuzzy normal operators, as well as important operator-theoretic properties that arise from the interaction between fuzziness and the n-inner product structure.

1. Introduction

Zadeh L. was first introduced fuzzy set theory in 1965[9]. Katsaras is the first to study the concept of a fuzzy norm on a linear space in 1984[1]. In 1992 Felbin was introduced finite dimensional fuzzy normed linear space [2]. Cheng and Mordeson were study fuzzy linear operators and fuzzy normed linear spaces in 1994 [3]. Biswas was introduced fuzzy inner product spaces and fuzzy norm functions in 1991[4]. On F IPS & F Co- IP S was introduced by Kohil and Kumar in 1993[5]. Majundarand and Samanta were study on fuzzy inner product spaces in 2008[6].

Goudarzi and Vaezpour was on the definition of Fuzzy Hilbert spaces in 2009[7].

Radharamani, Brindha and Maheswari were introduced fuzzy normal operator in fuzzy Hilbert space & it is qualities in 2018[8]. In the present work they defined fuzzy normal operator in F−n− H S & divided in two sections as follow:

In section one: basic definitions and preliminary results are provided.

In section two: we defined fuzzy−n− Hilbert space & fuzzy normal operator on fuzzy−n− Hilbert space.

2. Preliminaries

The section, we introduce definitions & preliminary results that shall be to utilize in the portion.

Definition 2.1. ([2]) Assum X is a linear space over a field $\mathscr{F}$. A F subset $\text{Ʉ} : X^{n+1} \times R$ was said a F−n−inner product on X Provided that:

(1) $\forall \text{ŧ} \in R$ & $\text{ŧ} \leq 0, \text{Ʉ}(\text{ꝑ}, \text{ꝑ} | \text{ꝑ}_2, \ldots, \text{ꝑ}_n, \text{ŧ}) = 0$,

(2) $\forall \text{ŧ} \in R$ &$\text{ŧ} > 0, \text{Ʉ}(\text{ꝑ}, \text{ꝑ} | \text{ꝑ}_2, \ldots, \text{ꝑ}_n, \text{ŧ}) = 1$ if and only if $\text{ꝑ}, \text{ꝑ}_2, \ldots, \text{ꝑ}_n$ are linear dependent,

(3) $\forall \text{ŧ} > 0, \text{Ʉ}(\text{ꝑ}, \text{ꞡ} | \text{ꝑ}_2, \ldots, \text{ꝑ}_n, \text{ŧ}) = \text{Ʉ}(\text{ꞡ}, x | \text{ꝑ}_2, \ldots, \text{ꝑ}_n, \text{ŧ})$,

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(4) $Ų(ꝑ, ᶃ|ꝑ_2, \dots, ꝑ_n, ŧ)$ is remains the same under any permutation of $ꝑ_2, \dots, ꝑ_n,$
(5) ∀ ŧ > 0, $Ų(ꝑ, ꝑ|ꝑ_2, \dots, ꝑ_n, ŧ) = Ų(ꝑ_2, ꝑ_2|ꝑ, ꝑ_3, \dots, ꝑ_n, ŧ),$
(6) ∀ ŧ > 0, $Ų(aꝑ, bꝑ|ꝑ_2, \dots, ꝑ_n, ŧ) = Ų(ꝑ, ꝑ|ꝑ_2, \dots, ꝑ_n, ŧ)_{,} \frac{ŧ}{|ab|}), a, b \in R$ (real)
(7) $\forall\, ɧ, ŧ \in R$:
$Ų(ꝑ + ᶃ, z|ꝑ_2, \dots, ꝑ_n, ŧ + ɧ) \geq Ų(ꝑ, z|ꝑ_2, \dots, ꝑ_n, ŧ) * Ų(y, z|ꝑ_2, \dots, ꝑ_n, ɧ)$
(8) ∀ ɧ, ŧ ∈ R with $ɧ, ŧ > 0$:
$Ų\left(ꝑ, ᶃ\middle|ꝑ_2, \dots, ꝑ_n, \sqrt{ŧɧ}\right) \geq Ų(ꝑ, ꝑ|ꝑ_2, \dots, ꝑ_n, ŧ) * Ų(ᶃ, ᶃ|ꝑ_2, \dots, ꝑ_n, ɧ).$
(9) $Ų(ꝑ, ᶃ|ꝑ_2, \dots, ꝑ_n, ŧ)$is a monotonically increasing function of ŧ ∈ R and:
$\lim_{ŧ\to\infty} Ų(ꝑ, ᶃ|ꝑ_2, \dots, ꝑ_n, ŧ)$=1. Then the triple $(X, Ų,*)$ is F $-n$ $-$IPS.

Definition 2.2. ([1]) The 3- tuple $(X, ₦,*)$ be called F $-n$ $-$ normed space, where X a linear space over the filed F,$*$ is a continuous t – norm and Ų be a F set on X^{n} × (0,∞) (i.e, ₦: X × (0, ∞) → [0,1]) check the following six condition:
∀ $ȩ_1, ȩ_2, \dots, ȩ_n, t, ᶆ \in X$ and $t, r \in R$:
1) ₦ $(ȩ_1, ȩ_2, \dots, ȩ_n, t)$ = zero, $\forall t \in R$ and $t \leq zero,$
2) ₦ $(ȩ_1, ȩ_2, \dots, ȩ_n, t))$ =1 if and only if $ȩ_1, ȩ_2, \dots, ȩ_n$ are linearly dependent,
3) ₦ $(ȩ_1, ȩ_2, \dots, ȩ_n, t))$ is remains the same under any permutation of $ȩ_1, ȩ_2, \dots, ȩ_n$
4) ₦ $(αȩ_1, αȩ_2, \dots, αȩ_n, t)$ = μ $(ȩ_1, ȩ_2, \dots, ȩ_n, t\backslash|α|)$, if $\alpha \neq 0, \alpha \in F,$
5) ₦ $(ȩ_1, ȩ_2, \dots, ȩ_n + ᶆ, t + r) \geq N (ȩ_1, ȩ_2, \dots, ȩ_n, t)) * ₦ (ȩ_1, ȩ_2, \dots, ȩ_n, ᶆ, r),$
6) ₦ $(ȩ_1, ȩ_2, \dots, ȩ_n, t)$ was a weakly increasing function of t in R & $\lim_{t\to\infty}$ ₦ $(ȩ_1, ȩ_2, \dots, ȩ_n, t)$ = 1.

Theorem 2.3. ([7]) Assum $(X, (.,.|.,\dots,.)_f)$ is a F$-n$ $-$IPS for n ≥1.
explain: $\|ȩ_1, ȩ_2, \dots, ȩ_n\|_f = (ȩ_1, ȩ_1 \mid ȩ_2, \dots, ȩ_n)_f^{1/2}$, then $(X, \|.,.| \dots,.\|_f)$ is fuzzy $-n-$ normed space on X induced by fuzzy$-n-$ inner product space.

Definition 2.4. ([5]) A sequence $\{x_k\}$in a linear n-normed space $H$is said to converge to an element $x \in H$if $\lim_{k\to\infty} \| x_k - x, e_2, \dots, e_n \| = 0,$ for every $e_2, \dots, e_n \in H$.
The sequence is called a Cauchy sequence if $\lim_{k,l\to\infty} \| x_k - x_l, e_2, \dots, e_n \| = 0$
for every $e_2, \dots, e_n \in H$. The space $H$is said to be Cauchy-complete if every Cauchy sequence in Hconverges in H. Finally, an n-inner product space $H$is called a complete $n - HS$ if it is complete with respect to the norm induced by its $n - IPS$.

Definition 2.5. ([6]) Assum $(X, J,*)$ be a FHS using IP: $< h, k > = sup\,\{x \in \mathbb{R} : Ų(a, b, t) < 1\}, \forall a, b \in X$ and let Ś ∈$ƑB(E)$, then S is self –adjoint F operator. If Ś = Ś* at which Ś*is adjoint F operator of Ś.
Remark 2.6: $ƑB(X)$ the set of every F linear operators on X.

3. Main Results and Properties of Fuzzy Normal Operator in Fuzzy n-Hilbert Space

Theorem 3.1: Assum $(X, Ų,*)$ is a F$-n$ $-$IPS, using $*$ is a strong t $-$norm as well as all
$ß, ß_2, \dots, ß_n$ ∈ X, sup$\{t$ in R: $Ų(ß, c|ß_2, \dots, ß_n\ t) < one\} < \infty$. where $(.,.|.,\dots,.)$: $X \times X$ ×

$\times X \to R$by means of$(ß, ß\ |ß_2, \dots, ß_n) = \sup\{t \in R{:}\ Ų(ß, c|ß_2, \dots, ß_n\ t) < 1\}$. thus $(X, (.,.|., \dots, .))$ was $n - IP$ space.

Definition 3.2: Assum $(X, Ų)$ is a fuzzy $-n-$inner product space, where X be a vector space over filed R as well as $Ų$ is a F set in X and $n - IP{:}\ (ƛ, ℏ|ƛ_2, \dots, ƛ_n) = \sup\{t \in \mathrm{R}{:}\ Ų(ƛ, ℏ|ƛ_2, \dots, ƛ_n, t) < 1\}, \forall ƛ_1, ƛ_2, \dots, ƛ_n \in X$. If X was complete space concerning fuzzy $-n-$norm induced by fuzzy $n-$inner product space $\left||ƛ_1, ƛ_2, \dots, ƛ_n, t|\right|_f = \sqrt{(ƛ_1, ƛ_2, \dots, ƛ_n, t)}_f$ then is called fuzzy$-n-$Hilbert space.

Definition 3.3: Assum $(X, Ų,*)$ is fuzzy$-n-$Hilbert space and $n - inner{:}\ (ɉ, ℏ|ɉ_2, \dots, ɉ_n) = \sup\{t \in \mathrm{R}{:}\ Ų(ɉ, ℏ|ɉ_2, \dots, ɉ_n, t) < 1\}$, $\forall\ ɉ_1, ɉ_2, \dots, ɉ_n \in \mathrm{X}$ as well as, let $Ŧ \in ƑB(X)$, hence $Ŧ$ a fuzzy normal operator (in short $F - N$ operator) if it commutes with it is (fuzzy) adjoint, i.e $\mathbb{T}\mathbb{T}^* = \mathbb{T}^*\mathbb{T}$.

Theorem 3.4: If $Ɠ_1, Ɠ_2$ are fuzzy normal operators on $(X, J,*)$ be a fuzzy$-n-$Hilbert space with property that either commutes with fuzzy adjoint of other, then $Ɠ_1 + Ɠ_2$and $Ɠ_1 . Ɠ_2$are fuzzy normal operators.

Proof: It is luminous by taking fuzzy adjoint, that is $Ɠ_1Ɠ_2{}^* = Ɠ_2{}^*Ɠ_1 \Leftrightarrow Ɠ_2Ɠ_1{}^* = Ɠ_1{}^*Ɠ_2$

Thus, the assumption entails in which every operator commute and F adjoint of the other.

1) $Ŋ((Ɠ_1 + Ɠ_2)\ (Ɠ_1 + Ɠ_2)^*x, t) = Ŋ((Ɠ_1 + Ɠ_2)\left(Ɠ_1{}^* + Ɠ_2{}^*\right)ь, t)$

$= Ŋ((Ɠ_1Ɠ_1{}^* + Ɠ_1Ɠ_2{}^* + Ɠ_2Ɠ_1{}^* + Ɠ_2Ɠ_2{}^*)ь, t)$... (1)

$Ŋ((Ɠ_1 + Ɠ_2)^*\ (Ɠ_1 + Ɠ_2)x, t) = Ŋ\left(\left(Ɠ_1{}^* + Ɠ_2{}^*\right)(Ɠ_1 + Ɠ_2\right)ь, t)$

$= Ŋ(Ɠ_1{}^*Ɠ_1 + Ɠ_2{}^*Ɠ_1 + Ɠ_1{}^*Ɠ_2 + Ɠ_2{}^*Ɠ_2)ь, t)\dots$ (2)

From (1) and (2) we get: $Ŋ((Ɠ_1 + Ɠ_2)^*\ (Ɠ_1 + Ɠ_2)ь, t) = Ŋ((Ɠ_1 + Ɠ_2)\ (Ɠ + Ɠ_2)^*ь, t)$

Then $Ɠ_1 + Ɠ_2$ is fuzzy normal operator on $Ƒ - n - H$ space.

2) $Ŋ((Ɠ_1 . Ɠ_2)(Ɠ_1 . Ɠ_2)^*ь, t) = Ŋ((Ɠ_1 . Ɠ_2Ɠ_2{}^*Ɠ_1{}^*)ь, t)$

$= Ŋ((Ɠ_1\ Ɠ_2{}^*Ɠ_2Ɠ_1{}^*)ь, t) \quad = Ŋ((Ɠ_2{}^*\ Ɠ_1Ɠ_1{}^*\ Ɠ_2)ь, t)$

$= Ŋ((Ɠ_2{}^*\ Ɠ_1{}^*\ Ɠ_1Ɠ_2)ь, t) = Ŋ((Ɠ_1 . Ɠ_2)(Ɠ_1 . Ɠ_2)^*ь, t)$

Then, $(Ɠ_1 . Ɠ_2)(Ɠ_1 . Ɠ_2)^* = (Ɠ_1 . Ɠ_2)(Ɠ_1 . Ɠ_2)^*$ .Thus $Ɠ_1 . Ɠ_2$ is F normal operator on F$-n-$HS.

Theorem 3.5: Assum $(X, Ų,*)$ is fuzzy$-n-$Hilbert space & $(Ŧᶒ, Ŧɖ\ |ᶒ_2, \dots, ᶒ_n) = \sup\{$ $\in R{:}\ Ų(ᶒ, ɖ|ᶒ_2, \dots, ᶒ_n, t) < 1\}$ and let $Ŧ \in ƑB(X)$ is F normal opertor on $F - n - H$ space $\Leftrightarrow$ $||Ŧ^*ᶒ|| = ||Ŧᶒ||\ \forall ᶒ \in X$.

Proof: Let $||Ŧ^*ᶒ|| = ||Ŧᶒ||$

$\Leftrightarrow ||Ŧ^*x||^2 = ||Ŧx||^2$

$\Leftrightarrow (Ŧ^*ᶒ, Ŧ^*ᶒ\ |ᶒ_2, \dots, ᶒ_n) = (Ŧᶒ, Ŧᶒ\ |ᶒ_2, \dots, ᶒ_n)$

$\Leftrightarrow \sup\{t$ belongs to $R{:}\ Ų(Ŧ^*ᶒ, Ŧ^*ᶒ, t) < one\} = \sup\{t$ belongs to $R{:}\ Ų(Ŧᶒ, Ŧᶒ, t) < one\}$

$\Leftrightarrow \sup\{t$ belongs to $R{:}\ Ų(ŦŦ^*ᶒ, ᶒ, t) < one\} = \sup\{t$ belongs to $R{:}\ Ų(Ŧ^*Ŧᶒ, ᶒ, t) < one\}$

$\Leftrightarrow (ŦŦ^*ᶒ, ᶒ|ᶒ_2, \dots, ᶒ_n) = (Ŧ^*Ŧᶒ, x|ᶒ_2, \dots, ᶒ_n)$

$\Leftrightarrow (ŦŦ^*ᶒ - Ŧ^*Tᶒ, ᶒ|ᶒ_2, \dots, ᶒ_n) = 0$

$\Leftrightarrow ((ŦŦ^* - Ŧ^*T)ᶒ, ᶒ|ᶒ_2, \dots, ᶒ_n) = 0$

$\Leftrightarrow \left((ŦŦ^* - Ŧ^*T)ᶒ\middle|ᶒ_2, \dots, ᶒ_n\right) = 0$, $is\ equivalent\ to\ ŦŦ^* - Ŧ^*Ŧ = zero$

$Is\ equivalent\ to\ ŦŦ^* = Ŧ^*Ŧ$. Then $ŦŦ^* = Ŧ^*Ŧ$.

Result 3.6: Assum $(X, Ų,*)$ be a fuzzy$-n-$Hilbert space with $(ȩ, ɖ\,|ȩ_2, \dots, ȩ_n) = \sup\{t \in R: J(ȩ,$ $ɖ|ȩ_2, \dots, ȩ_n\ t) < 1\}$ as well as let Ŧ belongs to $ƒB(X)$ is fuzzy normal operator hence $||Ŧ^2|| = ||Ŧ||^2$.
Proof : Assum $||Ŧ^2ȩ|| = ||ŦŦȩ||^2 = (ŦŦȩ, ŦŦȩ|ȩ_2, \dots, ȩ_n) = \sup\{t\ is\ in\ R: Ų(ŦŦȩ, ŦŦȩ, t) < one\}$
$= \sup\{t$ is in R: $Ų(Ŧ^*Ŧȩ, Ŧ^*Ŧȩ, t) < 1\}\ equl\ ||Ŧ^*Ŧȩ||^2$
I.e $||Ŧ^{*2}x||^2 = ||Ŧ^*Ŧȩ||^2$, then $||Ŧ^2x||equl\ ||Ŧ^*Ŧȩ||$
$||Ŧ^{*2}||equl\ ||Ŧ^*Ŧ||, \forall\ ȩ_2, \dots, ȩ_n \in X$.By know result $||Ŧ^*Ŧ||^2 equl\ ||Ŧ^2||$
Therefore, we get: $||Ŧ^2||equl\ ||Ŧ||^2$.
Remark 3.7: A complex number $z$can be uniquely written as $z = a + ib$, where $a = \Re(z)$is the real part and $b = \Im(z)$is the imaginary part, with $\bar{z}$denoting the complex conjugate.
By analogy, for any operator $S \in \mathcal{B}(X)$, we define $T_1 = \frac{S+S^*}{2}, T_2 = \frac{S-S^*}{2i}$ where $T_1$and $T_2$are fuzzy self-adjoint, then: $S = T_1 + iT_2, S^* = T_1 - iT_2$
This shows that any fuzzy operator can be decomposed into its self-adjoint components, analogous to a complex number.

Result 3.8: Assum $(X, Ų,*)$ is a fuzzy$-n-$Hilbert space and $(ȩ, ɖ\,|ȩ_2, \dots, ȩ_n)$ $= \sup\{t \in R: Ų(ȩ, ɖ|ȩ_2, \dots, ȩ_n\ t) < 1\}$ as well as let Ŧ in $ƒB(X)$ be fuzzy normal operator thenŦ fuzzy normal operator if and only if its real also imaginary parts of Ŧ is commutative with.
Proof: Let $Ś_1, Ś_2$ are real and imaginary components of Ŧ, in such a way thatŊ $(Ŧx, t) = Ś_1 + iŚ_2$ and $Ŋ(Ŧ^*x, t) = Ś_1 - iS_2$
$Ŋ(ŦŦ^*x, t) = (Ś_1 + iŚ_2)\,(Ś_1 - iS_2) = Ś_1{}^2 - iŚ_1Ś_2 + iŚ_2Ś_1 - i^2Ś_2{}^2$
$= Ś_1{}^2 - iŚ_1Ś_2 + iŚ_2Ś_1 + Ś_2{}^2 = Ś_1{}^2 + Ś_2{}^2 + i(Ś_2Ś_1 - Ś_1Ś_2)$... (1)
$Ŋ(Ŧ^*Ŧx, t) = (Ś_1 - iŚ_2)(Ś_1 + iŚ_2) = Ś_1{}^2 + iŚ_1Ś_2 - iŚ_2Ś_1 - i^2Ś_2{}^2$
$= Ś_1{}^2 + iŚ_1Ś_2 - iŚ_2Ś_1 + S_2{}^2 = Ś_1{}^2 + Ś_2{}^2 + i(Ś_1S_2 - Ś_2S_1)\dots$ (2)
It is clear that if $Ś_1 . Ś_2 = Ś_2 . Ś_1$, originating in (1) and (2) they get $ŦŦ^*$ equl $Ŧ^*Ŧ$
Suppose that $Ŋ(ŦŦ^*x, t) = Ŋ(Ŧ^*Ŧx, t)$. Implies $Ś_1 . Ś_2 - Ś_2 . Ś_1 = Ś_2 . Ś_1 - Ś_1 . Ś_2$
$2\,Ś_1 . Ś_2 = 2Ś_2 . Ś_1$. Hence $Ś_1 . Ś_2 = Ś_2 . Ś_1$
Theorem 3.9: Let $(X, Ų,*)$ be a fuzzy$-n-$Hilbert space with $(ȩ, ɖ\,|ȩ_2, \dots, ȩ_n) = \sup\{t \in$ $R: Ų(ȩ, ɖ|ȩ_2, \dots, ȩ_n\ t) < 1\}$ also let $Ŧ \in ƒB(X)$ is arbitrary but fixed fuzzy operator also if α, β for which $|\alpha| = |\ \beta|$ prove $Ŋ((\alpha Ŧ + \beta\ Ŧ^*)x, t)$ is fuzzy normal operator.
Proof: From theorem (3.5) we must prove $||Ŋ((\alpha Ŧ + \beta\ Ŧ^*)^*ȩ, t)|| = ||\mathcal{N}((\alpha Ŧ + \beta\ Ŧ^*)ȩ, t)||$
Let $||Ŋ((\alpha Ŧ + \beta\ Ŧ^*)^*ȩ, t)||^2 = ((\alpha Ŧ + \beta\ Ŧ^*)ȩ, (\alpha Ŧ + \beta\ Ŧ^*)^*ȩ|ȩ_2, \dots, ȩ_n)$
$= (\alpha Ŧ^* + \beta\ (Ŧ^*)^*)ȩ, (\alpha Ŧ^* + \beta\ (Ŧ^*)^*)ȩ|ȩ_2, \dots, ȩ_n)$
$= ((\alpha Ŧ^* + \beta\ Ŧ)ȩ, (\alpha Ŧ^* + \beta\ Ŧ)ȩ|ȩ_2, \dots, ȩ_n)$
$= \sup\{t \in R: Ų((\alpha Ŧ^* + \beta\ Ŧ)ȩ, (\alpha Ŧ^* + \beta\ Ŧ)ȩ, t) < 1\}$
$= \sup\{t \in R: Ų((\alpha Ŧ^*ȩ, \alpha Ŧ^*ȩ, t\ ) < 1\} + \sup\{t$ is in $R: Ų((\beta Ŧȩ, \beta Ŧȩ, t\ ) < one\}$
$= \sup\{t$is in $R: Ų((\beta Ŧȩ, \beta Ŧȩ, t\ ) < 1\} + \sup\{t\ is$ in $R: Ų((\alpha Ŧ^*ȩ, \alpha Ŧ^*ȩ, t\ ) < 1\}$
Since $Ŋ(Ŧ^*x,)$ is equal to $Ŋ(Ŧx, t)$
$= \sup\{t$ in $R: Ų((\alpha Ŧ + \beta\ Ŧ^*)ȩ, (\alpha Ŧ + \beta\ Ŧ^*)ȩ, t) < 1\}$
$= ((\alpha Ŧ + \beta\ Ŧ^*)ȩ, (\alpha Ŧ + \beta\ Ŧ^*)ȩ|ȩ_2, \dots, ȩ_n)$

$= ||\text{Ŋ}((\alpha\text{Ŧ} + \beta\ \text{Ŧ}^*)\text{e}, t)||^2$

i.e. $||\text{Ŋ}((\alpha\text{Ŧ} + \beta\ \text{Ŧ}^*)^*\text{e}, t)||^2 = ||\text{Ŋ}((\alpha\text{Ŧ} + \beta\ \text{Ŧ}^*)\text{e}, t)||^2$

$\text{Ŋ}((\alpha\text{Ŧ} + \beta\ \text{Ŧ}^*)^*\text{e}, t) = \text{Ŋ}((\alpha\text{Ŧ} + \beta\ \text{Ŧ}^*)\text{e}, t)$. Therefore $\alpha\text{Ŧ} + \beta\ \text{Ŧ}^*$is fuzzy normal operator.

References

[1] K. Katsaras, Fuzzy topological vector space-II, Fuzzy Sets and Systems,12,1984,143-154.

[2] C. Felbin, Finite dimensional Fuzzy normed linear space, Fuzzy Sets and Systems, 48, 1992,239-248.

[3] E. Kreyszig, Introductory Functional Analysis with Applications. New York: Wiley & Sons, (1978).

[4]. J. K.Kohil and R.Kumar, Linear mappings, Fuzzy linear spaces, Fuzzy inner product spaces and Fuzzy Co-inner product spaces, Bull Calcutta Math. Soc., 87,1995, 237-246.

[5]. J. K.Kohil and R. Kumar, On Fuzzy Inner Product Spaces and Fuzzy Co- Inner Product Spaces, Fuzzy Setsand system", Bull Calcutta Math.Soc.,53,1993,227-232.

[6]. M. Goudarzi and S. M.Vaezpour, On the definition of Fuzzy Hilbert spaces andits Applications, J. NonlinearSci.Appl,2(1), 2009,46-59.

[7]. R. Biswas, Fuzzy inner product spaces &Fuzzy norm functions, Information Sciences ,53,1991,185-190.