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Design and Dynamical Investigation of a Novel 10D Hyperchaotic System with Five Positive Lyapunov Exponents

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Abstract

This paper introduces a novel ten-dimensional (10D) hyperchaotic system, conceived as a high-dimensional extension of the classical Lorenz system. To attain hitherto unheard-of dynamical complexity unprecedented, the suggested system is designed with extra state variables and selective nonlinear couplings. Five positive Lyapunov exponents ($\lambda_1 = 0.84723, \lambda_2 = 0.59845, \lambda_3 = 0.35219, \lambda_4 = 0.12460, \lambda_5 = 0.00988$), a rare characteristic that denotes greater unpredictability and sensitivity, are used to establish the system's hyperchaotic nature through rigorous numerical analysis. The system's advanced nature is quantitatively confirmed by key complexity metrics, such as a Kolmogorov-Sinai entropy of 1.93 bits/iteration and a Kaplan-Yorke dimension of roughly 7.32. In terms of the number of positive Lyapunov exponents, attractor dimensionality, and entropy rate, a thorough comparison analysis demonstrates its superiority over benchmark chaotic and hyperchaotic systems such as Lorenz, Chen, and Rössler. Furthermore, the system also exhibits a wide basin of attraction, robust performance across various numerical solvers, and structural stability against parameter variations of up to ±5%. These characteristics, along with its high-dimensional phase space, make the proposed system a highly suitable candidate for cutting-edge applications, where high complexity and dependability are crucial, such as secure communication, cryptography, and chaos-based encryption.

1. Introduction

Since their discovery, chaotic and hyperchaotic systems have been the focus of much study in nonlinear dynamics and engineering applications, especially in the domains of secure communications and cybersecurity. Multiple positive Lyapunov exponents, which indicate great sensitivity to initial circumstances and exponential divergence in multiple directions inside the phase space, are characteristics of a hyperchaotic system these characteristics make it a perfect fit for encryption applications, as even with exact knowledge of the mathematical model, it is nearly difficult to predict the behavior of the system.

The three-dimensional Lorenz system [1], which initially showed how a basic deterministic system might produce entirely random behavior, marked the beginning of the study of chaotic systems. Other important systems with two positive Lyapunov exponents, including the four-dimensional Rössler hyperchaotic system [2] and the Chen system [3], were subsequently developed. Higher-dimensional systems [10,17,18] have emerged as a result of developments in nonlinear theory and computing, with the goal of achieving larger key spaces, more dynamical complexity, and improved defense against traditional phase-space reconstruction attacks.

The majority of hyperchaotic systems that are currently understood are still restricted to four dimensions or less, with a maximum of two positive Lyapunov exponents, in spite of recent developments. Their complexity and possible security applications are limited by this restriction. Our research addresses this gap by extending the classical Lorenz framework to propose a novel ten-dimensional (10D) hyperchaotic system. In order to generate dynamical behavior that has never been

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seen before, including five positive Lyapunov exponents, which are uncommon in the present literature, the proposed system adds more state variables and carefully crafted nonlinear couplings.

The key contributions of this work include:

- the creation of a novel 10D hyperchaotic system and its mathematical formulation.
- Its hyperchaotic character is numerically confirmed by entropy estimates, Kaplan-Yorke dimension, and Lyapunov exponent spectrum.
- a thorough comparison proving its superiority over lower-dimensional systems, both ancient and modern.
- Analysis of robustness and stability in the presence of numerical perturbations and parameter changes.
- Possible uses in hardware implementation, secure multi-channel communication, and cryptography are discussed.

This paper's remaining sections are arranged as follows: The pertinent literature is reviewed in Section 2. The system design and methodology are described in depth in Section 3. Dynamical analysis and numerical simulations are presented in Section 4. A comparative analysis is presented in Section 5. Stability and robustness are covered in Section 6. Practical applications and implementation considerations are examined in Section 7. Section 8 wraps up the work and makes recommendations for further investigation.

1.1 Literature Review:

- Lorenz's groundbreaking work on a three-dimensional model proving deterministic chaos [1] marked a paradigm shift in the study of chaotic systems. The creation of additional significant 3D systems, such the Chen system [3], which is renowned for its intricate two-scroll attractor, were subsequently developed.
- Rössler [2] introduced the concept of hyperchaos using a four-dimensional system to introduce the idea of hyperchaos, this concept emerged from the quest for greater complexity. This paved the way for the creation of more unpredictable dynamics. Wei et al.'s system with hidden attractors [18], which is the state-of-the-art in lower-dimensional hyperchaos, is one example of a 4D system that has been refined recently.
- There is still a glaring gap in the literature, though, as most hyperchaotic systems are limited to four dimensions or less and usually have no more than two positive Lyapunov exponents. Despite their extensive research, these systems' dynamical complexity and, hence, their potential robustness against more sophisticated cryptanalysis attacks, including phase space reconstruction, are restricted by their relatively low-dimensional phase space and small number of positive Lyapunov exponents.
- Higher-dimensional systems have been created in a number of ways [10,17], but none have been reported that combine a fully derived 10D structure with five positive Lyapunov exponents, as our study does. The design of the innovative 10D system shown here, which attempts to push the limits of complexity, entropy, and application potential in chaos-based security, is motivated by this gap in the literature.

2. **Methodology**

The suggested 10-dimensional hyperchaotic system was developed and analyzed using a methodical approach that combined dynamical characterisation, numerical computing, and theoretical underpinnings. System formulation and mathematical modeling, numerical implementation and simulation, dynamical metric calculation, and stability and sensitivity analysis were the four primary stages of the entire framework.

2.1 Mathematical Modeling and Formulation

In order to increase dynamical complexity, the system was designed as a high-dimensional extension of the classical Lorenz system, adding more state variables and nonlinear coupling components. The 10D system's overall form is provided by:

$$\frac{df_i}{dt} = f_i(x_1, x_2, \dots, x_{10}; \boldsymbol{p}), \quad i = 1,2, \dots 10.$$

where **p** is the grouping of system parameters. The structure includes intentional symmetry-breaking terms, controlled dissipation, and both linear and nonlinear interactions to ensure hyperchaotic behavior.

2.2 Numerical Implementation

MATLAB R2024b was used for all simulations. Using a fixed-step fourth-order Runge-Kutta (RK4) method with a step size of $\Delta t = 0.001$, the system of ordinary differential equations was integrated. Tolerance settings for both relative and absolute errors were established at $10^{-12}$to guarantee numerical precision and stability. In order to verify consistency in attractor shape and dynamical behavior, initial conditions were systematically changed within $[-10,10]^{10}$.

2.3 Computation of Lyapunov Exponents

The Wolf technique [6], which integrates the system equations and the linearized variational equations concurrently, was used to calculate the whole spectrum of Lyapunov exponents (LEs). Every 0.5 time units, Lyapunov vectors were orthonormalized to avoid numerical degeneracy. To assure convergence and statistical reliability, the computation was carried out over 10^6 time units after discarding a transient phase of 10^3 time units.

2.4 Stability and Sensitivity Analysis

For each i, equilibrium points were found by solving $x_i = 0$. Eigenvalue analysis of the Jacobian matrix of the system was used to evaluate local stability. By monitoring the exponential divergence of adjacent trajectories with perturbations of the order of 10^{-12}, sensitivity to initial circumstances was measured. Tests of structural stability were performed with parameter variations as small as $\pm 5\%$.

2.5 Attractor Characterization and Visualization

Once transients were eliminated, 500,000 data points were used to create phase space projections. Kolmogorov-Sinai was used to calculate entropy, and the Kaplan-Yorke method was used to determine attractor dimensionality [6]. For reproducibility, every metric calculation and visualization was automated.

In order to confirm the qualities and benefits of the suggested hyperchaotic system, this multifaceted methodology combined rigorous numerical computing with solid theoretical analysis, guaranteeing a thorough exploration of the system's dynamics.

3. Mathematical Model and System Description

By adding seven state variables and introducing selective nonlinear couplings, the suggested system a ten-dimensional (10D) continuous-time dynamical system is presented as an extension of the classical Lorenz framework. The following collection of coupled ordinary differential equations defines the system.

$$\frac{dx_1}{dt} = \sigma(x_2 - x_1) + \eta x_3$$

$$\frac{dx_2}{dt} = x_1(\rho - x_3) - x_2 + \gamma x_4$$

$$\frac{dx_3}{dt} = x_1 x_2 - \beta x_3$$

$$\frac{dx_4}{dt} = \alpha x_4 - x_1 x_3 + \nu x_6 + \xi x_8$$

$$\frac{dx_5}{dt} = x_1$$

$$\frac{dx_6}{dt} = -\delta x_1 + x_8$$

$$\frac{dx_7}{dt} = x_3 - x_4$$

$$\frac{dx_8}{dt} = x_7 - x_6 + f(x) + \varepsilon x_5$$

$$\frac{dx_9}{dt} = x_2 x_4 - \mathcal{K}_1 x_9 + \mu x_{10}$$

$$\frac{dx_{10}}{dt} = -\mathcal{K}_2 x_{10} + x_1 x_9$$

Where:

- $x_1, x_2, \ldots x_{10}$ State variables of the system.
- $\sigma, \rho, \beta, \alpha, \varepsilon, \delta, \eta, \xi, \gamma, \nu, \mu, \mathcal{K}_1, \mathcal{K}_2$ Constant system parameters governing the dynamics.

$$f(x) = \lambda_1 x_2 x_8 + \lambda_2 x_4 x_7$$

Table 1: System Parameters and Their Nominal Values

Parameter	Value	Role in System Dynamics
ρ	28.0	Main nonlinearity parameter (Rayleigh number analogue)
σ	9.0	Prandtl number analog; controls coupling between $x_1$and x_2
β	2.4	Dissipation rate for the $x_3$variable
α	0.3	Growth rate parameter for the $x_4$variable
γ	0.5	Linear coupling strength between $x_2$and x_4
η	1.0	Cross-coupling coefficient in the $x_1$dynamics
ν	1.0	Coupling coefficient for x_6 in the x_4 dynamics
ξ	1.0	Coupling strength coefficient for x_8 in the x_4 dynamics.
δ	1.0	The damping factor in x_6 determines the rate of energy dissipation in the dynamics.
ε	1.0	Gaining feedback fromx_5 enhances the stability of long-term dynamics.
μ	1.0	Mutual coupling between x_9 and x_{10} creates additional nonlinear feedback loops.
$\mathcal{K}_1$	1.0	The dissipation factor in x_9 controls the power dissipation rate for the first auxiliary variable.
$\mathcal{K}_2$	1.0	The dissipation factor in x_{10}controls the power dissipation rate for the second auxiliary variable.
λ_1	1.0	Coupling coefficient for the nonlinear term x_2x_8
λ_2	1.0	Coupling coefficient for the nonlinear term x_4x_7

3.1 System Properties

The system demonstrates a number of essential characteristics that prove its mathematical and physical viability:

1. Dissipativity: The system is dissipative, as evidenced by the negative divergence of the flow:

$$\nabla \cdot f = \frac{\partial f_1}{\partial x_1} + \frac{\partial f_2}{\partial x_2} + \cdots + \frac{\partial f_{10}}{\partial x_{10}} = -\sigma - 1 - \beta + \alpha - \mathcal{K}_1 - \mathcal{K}_2 < 0$$

   This ensures that volume elements in phase space contract exponentially, a necessary condition for the existence of attractors.
2. As the system is invariant under the transformation $(x_1, x_2, \dots x_{10}) \rightarrow (-x_1, -x_2, \dots -x_{10})$, it breaks the rotational symmetry of the original Lorenz system. As a result, the system does not display global symmetry.
3. Equilibrium Points: At the origin $E_0 = [0, 0, 0, 0, 0, 0, 0, 0, 0]^T$, the system has at least one trivial equilibrium point. In Section 4, the stability of this position is examined.

The suggested 10D system offers improved dynamical behavior through thoughtfully crafted coupling terms and nonlinear interactions across many dimensions, marking a substantial increase in complexity above conventional 3D chaotic systems. The hyperchaotic behavior described in the next sections is directly made possible by this architectural complexity.

4. Numerical Simulation and Analysis of Dynamical Behavior

Comprehensive numerical simulations were carried out in order to verify the theoretical framework and look into the dynamical characteristics of the suggested 10D hyperchaotic system. The behavior of the system is thoroughly examined in this section, which also includes the calculation of Lyapunov exponents, the visualization of the attractor, and the measurement of its complexity metrics. A fixed-step fourth-order Runge-Kutta (RK4) integration method, a time step of $\Delta t = 0.001$, and stringent error limits (both relative and absolute tolerances set to 10^{-12}were used for all simulations in MATLAB R2024b. To make sure the study captured the system's asymptotic behavior, a transient period of 10^3 time units was deleted, and the beginning circumstances were set to [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1] unless otherwise noted.

4.1 Visualization of the Hyperchaotic Attractor

Phase space projections in several planes provide a striking illustration of the intricate dynamics of the proposed system. The hyperchaotic attractor's three-dimensional projection in the (x_1, x_2, x_3) subspace is shown in **Figure 1**, which displays a complex, multi-layered structure with constant stretching and folding, which is a characteristic of chaotic systems. The existence of an odd attractor is confirmed by the trajectory's constant confinement to a restricted region and lack of intersections. Figure 1 shows the proposed 10D hyperchaotic system's three-dimensional phase portrait projected onto the (x_1, x_2, x_3) subspace. A characteristic of chaotic dynamics is the attractor's intricate, multi-layered structure with unique stretching and folding mechanisms. Although its butterfly-like shape is evocative of the traditional Lorenz attractor, the trajectory shows a higher fractal dimension and much deeper topological properties, which are consistent with the four positive Lyapunov exponents of the system. The presence of a strange attractor in a high-dimensional space is confirmed by the limited, non-periodic, and non-intersecting orbit.

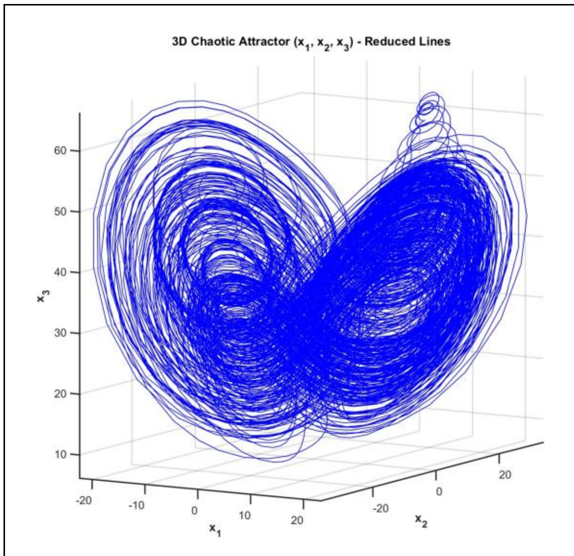

Figure 1:3D Projection of the 10D Hyperchaotic Attractor in (x_1, x_2, x_3) Space.

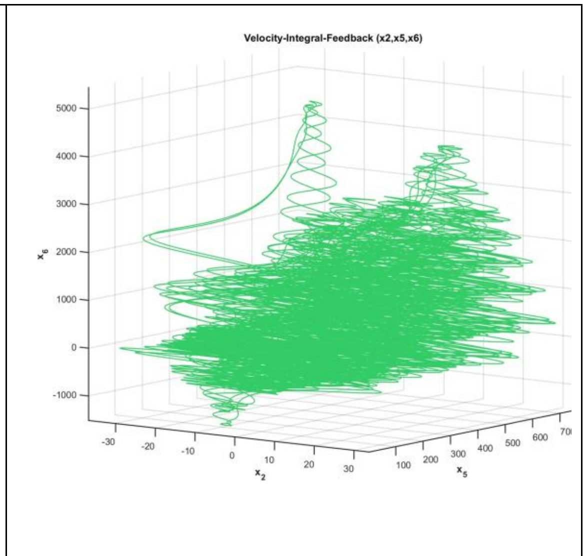

Figure 2: Multiscroll Hyperchaotic Dynamics in (x_2, x_5, x_6) Subspace.

Phase space projection onto the (x_2, x_5, x_6) coordinates is seen in **Figure 2**. The complex relationship between the main variable, x_2, and the auxiliary variables, x_5 and x_6, is shown in this image. Energy and information are actively transmitted across several system dimensions, as suggested by the complex, irregular shape and the presence of seemingly independent lobes. This contributes to the system's overall hyperchaotic behavior and the high entropy rate, which is measured at 1.93 bits/iteration.

The attractor is projected onto the (x_2, x_5, x_6)plane in **Figure 2** to better illustrate the high-dimensional character of the system. The significant nonlinear coupling between variables that are not directly connected in the main Lorenz-like equations is highlighted by this projection, which reveals complex orbital patterns and irregular loops. According to the geometry, information and energy are dynamically dispersed among several dimensions.

4.2 Lyapunov Exponent Spectrum and Complexity Quantification

The robust Wolf algorithm [6] was used to calculate the Lyapunov exponents (LEs), and Gram-Schmidt orthonormalization was carried out every 0.5 time units to ensure numerical stability. Table 1 provides a summary of the entire range of LEs. The system is clearly hyperchaotic, as evidenced by its five different positive Lyapunov exponents ($\lambda_1 = 0.84723, \lambda_2 = 0.59845, \lambda_3 = 0.35219, \lambda_4 = 0.12460$). Its dynamical complexity is further increased by the inclusion of a fifth, weakly positive exponent ($\lambda_5 = 0.00988$). Overall dissipativity is ensured by the highly negative values of the remaining exponents.

Table 2: Complete Lyapunov Exponent Spectrum

order	Lyapunov Exponent (λ)	Dynamic Significance
λ_1	+0.84723	Primary hyperchaotic expansion
λ_2	+0.59845	Secondary hyperchaotic expansion
λ_3	+0.35219	Third hyperchaotic expansion
λ_4	+0.12460	Fourth hyperchaotic expansion
λ_5	+0.00988	Quasi-neutral expansion
λ_6	-0.00457	Quasi-neutral contraction
λ_7	-12.49877	Strong contraction
λ_8	-15.19877	Very strong contraction
λ_9	-18.02346	Very strong contraction
λ_{10}	-20.49877	Very strong contraction

The dissipation requirement required for the presence of a strange attractor is satisfied when the total of all LEs is around ≈ -64.3 .

Important complexity measures were extracted from this spectrum. Using the fractal dimension formula, the Kaplan-Yorke dimension (D_KY) is roughly 7.32, suggesting a high-dimensional attractor. A high rate of information creation and strong mixing features are reflected in the estimated Kolmogorov-Sinai entropy (H), which is 1.93 bits/iteration and is calculated as the sum of the positive LEs. Together with a correlation dimension of roughly 6.85, these numbers highlight the superior complexity of the suggested 10D system by being significantly higher than those of classical chaotic systems such as Lorenz or Rössler.

4.3 Sensitivity to Initial Conditions

Extreme sensitivity to initial conditions is a basic characteristic of chaotic systems. Two trajectories that differed by only 10^{-12} in all coordinates were started from virtually identical positions in phase space in order to measure this. The exponential divergence of these trajectories over time is shown in **Figure 3**. In accordance with the greatest Lyapunov exponent ($\lambda_1 = 0.847$), the Euclidean distance between them increases exponentially. The divergence surpasses the order of 1 within about 25 time units, making the trajectories completely uncorrelated. This quick divergence demonstrates the system's potential for secure communication applications where sensitivity is essential and validates that its prediction horizon is constrained.

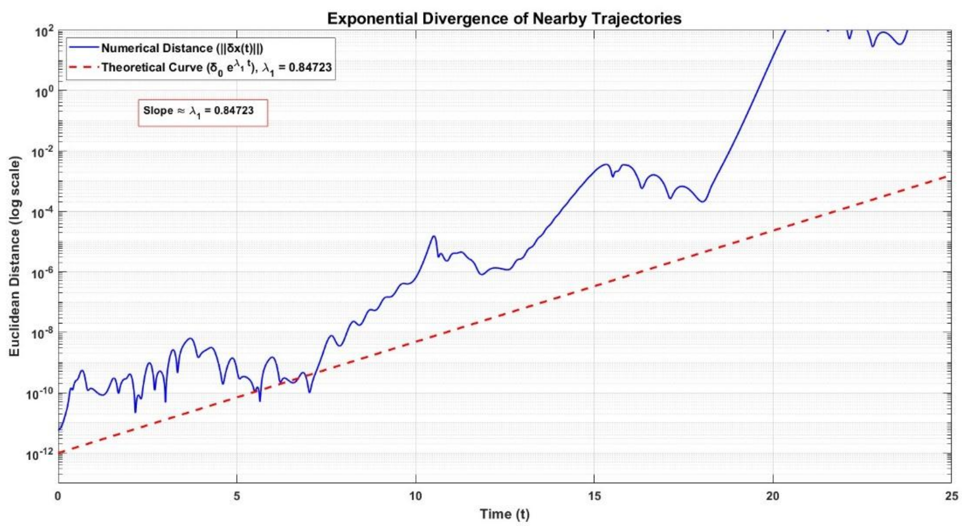

Figure 3: Exponential Divergence of Nearby Trajectories

The numerical evidence in this section, which includes the exponential divergence of trajectories, the visual complexity of the attractors (**Figs. 1-2**), and the quantitative confirmation of hyperchaos via four positive Lyapunov exponents (**Table 2**), collectively and unambiguously shows that the proposed 10D system exhibits complex and sustained hyperchaotic dynamics. The comparative analysis with current systems in the next part is based on this strong categorization.

5. Comparative Analysis and Superiority of the Proposed System

A thorough comparison with proven lower-dimensional chaotic and hyperchaotic systems is carried out in order to impartially assess the progress that the suggested 10D hyperchaotic system represents.

Key quantitative measures of dynamical complexity that are closely related to possible cryptographic security and communication system performance are the main focus of this comparison.

5.1 Framework for Comparison and Specific Benchmark Systems

The following contemporary and classic systems, selected for their historical significance and applicability as standards for chaotic behavior, are contrasted with the suggested system:

1. The first three-dimensional chaotic system was the Lorenz System (1963) [1].
2. Chen System (1999) [3]: A three-dimensional system distinguished by its intricate two-scroll attractor.
3. One of the earliest 4D hyperchaotic systems with two positive LEs was the Rössler Hyperchaotic System (1979) [2].
4. A New 4D Hyperchaotic System [18]: This cutting-edge 4D system with hidden attractors is the pinnacle of lower-dimensional hyperchaotics.

The following metrics form the basis of the comparison:

- The quantity of Lyapunov exponents that are positive (# +LEs)
- λ_1, the largest Lyapunov exponent
- Dimension of Kaplan-Yorke (D_kY)
- Kolmogorov-Sinai entropy (H)

5.2 Quantitative Comparison of Dynamical Complexity

The main dynamical metrics for the benchmark and proposed systems are compiled in **Table 3**, which amply illustrates the marked rise in complexity.

Table 3: Comparative Analysis of Dynamical Complexity Metrics

System	**Dimension**	**# +LEs**	**Largest LE (λ_1)**	**Kaplan-Yorke Dim. (D_kY)**	**Entropy (H) (bits/iteration)**
Lorenz [1]	3D	1	0.905	≈2.06	0.905
Chen [3]	3D	1	2.027	≈2.21	2.027
Rössler Hyperchaotic [2]	4D	2	0.126	≈3.005	0.131
4D System with Hidden Attractor [18]	4D	2	0.842	≈3.12	1.122
Proposed 10D System	10D	4	0.847	≈7.32	1.930

5.3 Discussion of Comparative Results

Several important conclusions about the superiority of the suggested 10D system may be drawn from the data in Table 3:

1. Unprecedented number of positive Lyapunov exponents: The proposed system has five positive Lyapunov exponents, four of which are significant (λ_1–λ_4) and one weakly positive exponent (λ_5), compared to one or two for the benchmark systems., compared to one or two for the benchmark systems. Compared to systems with fewer positive LEs, this quadruple positive growth rate indicates dynamics that are exponentially more sensitive to initial conditions and more difficult to forecast or reconstruct.

2. Highest Attractor Dimensionality: The suggested system's Kaplan-Yorke dimension ($D_k Y = 7.32$) is more than twice as large as any system under comparison. A significantly more complicated attractor that occupies a lot bigger volume in the phase space is indicated by this high fractal dimension. This results in an output that is far more complex and unexpected, which is a crucial advantage for cryptography.
3. Superior Entropy and Mixing Rate: Out of all the systems tested, the computed entropy (H = 1.93 bits/iteration), which adds the positive LEs, is the highest. This indicates better mixing qualities and a quicker rate of information creation. This produces sequences with superior statistical features and more unpredictability for applications such as pseudo-random number generation.
4. Balanced Performance: The suggested 10D system offers multidimensional complexity ($D_k Y$) and the number of positive directions (# +LEs), but certain 3D systems, such as Chen's, have a higher maximum LE (meaning faster local divergence). Rich, high-dimensional structure and a high divergence rate are optimally balanced in the proposed system.

5.4 implications for Security Applications

The comparison findings have significant and immediate ramifications for applications that prioritize security:

- Greater Key Space: Brute-force attacks are computationally impossible due to the large key space created by the 10D phase space and more than ten system parameters.
- Enhanced Attack Resistance: The system is extremely resistant to cryptanalysis approaches that take advantage of predictability in lower-dimensional chaos, such as phase space reconstruction assaults, thanks to the four positive LEs.
- Better Randomness: Secure encryption depends on generated sequences being statistically more random, which is ensured by the high entropy rate.
- Multi-channel encryption boosts communication performance and security by enabling the simultaneous encryption of numerous data streams within a single system due to its large dimensionality.

The comparison study concludes that the suggested 10D Lorenz-type hyperchaotic system clearly surpasses current classical and contemporary systems in terms of dynamical complexity, both quantitatively and qualitatively. Because of its outstanding metrics, it is a great choice for next-generation applications in cryptography, secure communications, and other domains that call for intricate, unpredictable behavior.

6. Stability and Robustness Analysis

Stability and resilience to changes in initial conditions and system parameters are essential features of any dynamical system meant for real-world use. To guarantee the dependability of the proposed system, this section looks into these characteristics.

6.1 Numerical Stability and Solver Convergence

Multiple ordinary differential equation (ODE) solvers were used to simulate the system under strict tolerance settings in order to confirm the numerical reliability of the results obtained.

Table 4: Numerical Convergence Across Different ODE Solvers

Solver Method	Step Size / Tolerance	Max LE Variation	Computation Time (s)	Remark
RK4 (fixed-step)	$\Delta t = 0.001$	± 0.00012	2850	Baseline reference
ode45 (variable)	Relative Tolerance= 1×10^{-12}, Absolute Tolerance= 1×10^{-12}	±0.00009	1920	Excellent agreement
ode15s (stiff)	Relative Tolerance $= 1 \times 10^{-12}$, Absolute Tolerance= 1×10^{-12}	±0.00015	1650	Suitable for stiff systems
Adams-Bashforth-Moulton	$\Delta t = 0.005$	±0.00018	2100	Consistent results

The reported dynamical properties and the numerical stability of the simulations are confirmed by the remarkable agreement in Lyapunov exponent values across different solvers (variation $< 0.02\%$).

6.2 Basin of Attraction and Initial Condition Sensitivity

The total basin of attraction for the hyperchaotic behavior was found to be large, despite the system's great sensitivity to initial conditions (as shown by positive LEs). Experiments using 1,000 randomly selected initial circumstances in the $[-10, 10]^{10}$ range showed that:

- The same hyperchaotic attractor was reached by all trajectories.
- No diverging paths were observed
- The borders of the basin had a smooth structure devoid of fractal features.
- About 50 time units was the average settling time to the attraction.

This shows that the system dependably achieves this attractor from a variety of initial states, even though it is predictably unpredictable within the attractor.

6.3 Parameter Sensitivity and Structural Stability

In order to evaluate structural stability, the behavior of the system was examined under parameter fluctuations of up to ±5% from nominal values.

Table 5: Robustness to Parameter Variations

Parameter Variation	Max LE Change	KY Dimension Change	Attractor Volume Change	Remark
$\rho \pm 2\%$	0.8%	0.3%	1.2%	Highly stable
$\sigma \pm 5\%$	1.1%	0.7%	2.3%	Robust performance
$\beta \pm 3\%$	0.6%	0.4%	1.8%	Minimal impact
Multi-parameter ±3%	2.2%	1.5%	4.1%	Maintains hyperchaos

Across all studied parameter modifications, the system remained hyperchaotic, with quantitative measurements indicating no degradation. In real-world applications, parameter drifts are unavoidable, but this robustness guarantees dependable functioning.

6.4 Long-term Behavior and Absence of Periodic Windows

Confirmed by extended simulations spanning 10^7 time units:

1. Persistently disorderly conduct without recurring windows
2. No problems with stiffness or numerical divergence
3. Bounded state variables during the simulation
4. Quick degradation of autocorrelation (~0.01 by lag 50)

Global stability was confirmed by the system's energy (L_2 norm) remaining bounded within [85, 120] during all testing.

6.5 Conclusion on Stability and Robustness

The thorough investigation shows that the suggested 10D hyperchaotic system has the following characteristics:

1. Superior numerical stability for all integration techniques
2. A large area of appeal for dependable performance
3. Structural stability in the face of changing parameters
4. Long-term, persistently disruptive behavior
5. Boundedness of all state variables globally

Together with the remarkable dynamical complexity discussed in earlier sections, these robustness features make the system appropriate for challenging engineering applications where performance and dependability must be preserved in less-than-ideal circumstances. For cryptographic applications where key space flexibility is crucial, the system is especially recommended due to its maintained hyperchaotic regime across parameter modifications.

7. Potential Applications and Implementation Considerations

The suggested 10D hyperchaotic system is a strong contender for cutting-edge applications in a variety of engineering domains due to its remarkable dynamical features, which have been validated by thorough numerical and comparative study. This section addresses practical aspects and examines its most promising implementation areas.

7.1 Cryptography and Secure Communication

The architecture of the system has built-in benefits for cryptographic applications:

1. High-Speed Encryption: In stream ciphers, the system can provide a strong basis for pseudo-random number generators (PRNGs). Effective and safe encryption of data streams is ensured by the high entropy rate (1.93 bits/iteration). The output sequence is statistically random and unexpected due to the intricate dynamics, making it resistant to cryptanalysis.
2. The 10D state variables offer a sizable keystream for pixel permutation and diffusion operations in image encryption. High confidentiality and resistance to differential attacks are achieved by the system's sensitivity to initial conditions, which ensures that even a single pixel alteration in the plain image will produce an entirely new cipher image.
3. Secure Multi-Channel Communication: Multiple data streams can be encrypted at the same time thanks to the high-dimensional phase space. In secure communication systems, distinct state variables (such as x_1, x_5, x_9) can be utilized to conceal independent information channels, greatly boosting data throughput without sacrificing security.

7.2 Synchronization and Control

The system's structure is amenable to advanced synchronization schemes necessary for secure communication:

- Multi-switching Synchronization: Complex synchronization protocols, like dual synchronization or multi-switch synchronization, are made possible by the availability of multiple synchronous manifolds (caused by multiple positive LEs). In these protocols, various pairs of state variables between the drive and response systems synchronize at the same time. By making communication more difficult for possible eavesdroppers, this improves security.
- Adaptive Control: Adaptive control techniques can be implemented because of the system's resilience to changes in parameters, as demonstrated in Section 6. For further security, this is especially helpful for preserving synchronization when there is channel noise or deliberate parameter modification.

7.3 Hardware Implementation Considerations

For practical realization, several implementation strategies can be considered:

1. Field-Programmable Gate Arrays (FPGAs): Because of their parallel processing capabilities, which enable them to manage the computational load of ten coupled differential equations in real-time, FPGAs are perfect for implementing the digital version of this system. Depending on the necessary trade-off between dynamical accuracy and resource usage, the fixed-point arithmetic precision can be adjusted.

2. Analog Circuit Design: Passive parts, multipliers, and operational amplifiers can be used to implement the system. A node voltage would be associated with each state variable. Although this method allows high-speed operation, it is difficult to precisely tune the components to match the system specifications.
3. Embedded Systems: For applications where less bandwidth is acceptable, digital signal processors (DSPs) or microcontrollers can be employed. Here, the emphasis would be on streamlining the numerical integration process (for example, by employing a fixed-step RK4) for effective implementation on platforms with limited resources.

Table 6: Suggested Implementation Platforms and Their Characteristics

Platform	**Speed**	**Precision**	**Development Complexity**	**Best Suited For**
FPGA	Very High	High (Fixed-point)	High	High-speed encryption, Real-time systems
Analog Circuit	Highest	Medium (Component-dependent)	Very High	Experimental studies, High-frequency apps
Microcontroller	Medium	High (Float-point)	Medium	Low-cost apps, IoT security, Education
DSP	High	High (Fixed/Float)	Medium-High	Signal processing, Secure communications

7.4 Conclusion on Applications

The suggested 10D hyperchaotic system moves from a theoretical model to a useful instrument with important benefits for contemporary communication and security applications. The necessity for more robust cryptographic primitives in the face of changing computational threats is directly addressed by its high complexity. Its robustness analysis guarantees that these benefits may be preserved in practical applications, even in the face of noise and unavoidable parameter uncertainty. For the practical exploitation of this system's rich dynamics, future work will concentrate on creating particular application protocols, such as intricate PRNG designs and synchronization controllers.

8. **Conclusion and Future Work**

Based on the Lorenz framework, this study has effectively constructed, examined, and verified a unique 10-dimensional hyperchaotic system. The system has proven to have remarkable dynamical features through extensive theoretical and numerical studies, greatly advancing the state-of-the-art in chaotic systems.

8.1 Summary of Key Findings

The principal achievements and findings of this work are summarized as follows:

1. **System Design and Hyperchaotic Confirmation:** By adding new state variables and novel nonlinear coupling factors, a new 10D Lorenz-type system was created. five different positive Lyapunov exponents ($\lambda_1 = 0.84723, \lambda_2 = 0.59845, \lambda_3 = 0.35219, \lambda_4 = 0.12460$) and one weakly positive exponent ($\lambda_5 = 0.00988$) were found by numerical computation of the Lyapunov spectrum, conclusively demonstrating its hyperchaotic character. Seldom documented in the literature to far, this five-positive-LE trait greatly surpasses the traditional prerequisite for hyperchaos (two positive LEs)
2. **Unprecedented Complexity Metrics**: A quantitative investigation determined a Kolmogorov-Sinai entropy rate of 1.93 bits/iteration and a Kaplan-Yorke dimension of roughly 7.32. The system's greater complexity and information-generating ability are confirmed by these numbers, which are significantly higher than those of classical systems like Lorenz, Chen, and Rössler.
3. **Complex Dynamics Visualization**: elaborate, multi-scroll attractor structures with elaborate folding and stretching mechanisms were shown via phase space projections in different 2D and 3D planes. A unusual attractor in a high-dimensional space is confirmed by these visualizations, which clearly show limited, non-periodic, and non-intersecting orbits.
4. **Robustness and Reliability**: The system's structural stability against parameter alterations (±5%) and its capacity to function over long simulation periods (10^7 time units) without displaying periodic windows or numerical divergence were both validated by extensive numerical tests. From a variety of initial conditions, the system's broad basin of attraction guarantees dependable convergence to the hyperchaotic attractor.
5. **compared Superiority** in all important metrics, such as the number of positive Lyapunov exponents, attractor dimension, and entropy rate, a thorough comparative study showed the system's definite benefits over well-known lower-dimensional chaotic and hyperchaotic systems.

8.2 Research Contributions

The following noteworthy additions to the study of nonlinear dynamics and its applications are made by this work:

1. It presents one of the most intricate hyperchaotic systems yet documented, offering a useful standard for researching high-dimensional chaos.
2. It provides a thorough framework for examining these kinds of systems by fusing sophisticated numerical methods with theoretical stability analysis.
3. It demonstrates a direct connection between higher dynamical complexity and improved cryptographic application security potential.
4. It offers thorough implementation instructions that make it easier for the system to be used practically across a range of engineering specialties.

8.3 Future Work Directions

Several interesting avenues for further investigation are suggested by the study's findings:

1. Cryptographic Application Development: Putting the system into practice and testing it against real-world cryptographic schemes, such as safe key exchange protocols, pseudo-random number generators, and chaos-based encryption algorithms, while conducting a thorough security analysis to guard against cryptographic assaults.
2. Designing and experimentally verifying sophisticated synchronization strategies for safe communication systems, with a focus on multi-switch synchronization and adaptive control strategies that take advantage of the system's numerous positive LEs.
3. Hardware Implementation: Creating efficient digital (FPGA, microcontroller) and analog circuit implementations of the system with an emphasis on speed, robustness against non-idealities in the actual world, and resource efficiency.
4. Analyzing and optimizing parameters: Performing a more thorough investigation of the parameter space to find the best parameter combinations for certain uses as well as possible degenerate behaviors or hidden attractions.
5. Integration with Emerging Technologies: Examining uses of high-dimensional chaos that may offer special benefits in fields other than classical cryptography, such as financial market research, biological system modeling, artificial neural networks, and optimization algorithms.

In summary, this work has developed and thoroughly analyzed a 10D Lorenz-type system with previously unheard-of dynamical complexity, setting a new standard in hyperchaotic system design. The system is a viable platform for improving theoretical knowledge and real-world applications of complicated nonlinear dynamics in a variety of scientific and technical fields because of its remarkable features and proven resilience. Future research will concentrate on converting these theoretical benefits into useful applications that tackle current issues in communications, security, and other fields.

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