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A New Proposed Method for Solving Multi-Objective Optimization Problems (MOP)

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Abstract: Multi-objective Optimization Problems (MOP) (or Pareto optimization) are deeply rooted as an alternative concept of actual optimization where multi-objective identifies the conception of the optimization, as is common in the fields of Operations Research (OR) and decision science. These are often constrained problems where the search for an optimal solution requires some form of analyzing objective achievement trade-offs. Because there are no trade-offs along a Linear Programming (LP), optimization is an essential concept of multiple objectives. In other words, a Decision-Making (DM) process for LP optimization is a traditional method that depends on finding the optimal solution for a single objective function. However, in real life, this logic is considerably classical, where researchers often encounter situations that contain more than one objective function, and these functions usually conflict with each other. It means that when one objective value is either increased or decreased, it comes at the expense of the other in the opposite way. Problems of this type are called MOP, which seeks to find a set of nondominated solutions (or Pareto optimal solution) to satisfy the decision maker. This paper expands and develops the multi-objective optimization concept by using a proposed new method for solving all objective functions together without converting to an individual objective problem, as is common in solving MOP. The paper also introduces a tabular and graphical comparison with a simple numerical problem showing the difference between the proposed method and some essential multiple optimality methods (such as compromise programming, relative weighting, and constraint programming). The suggested method provides a practical initiatory framework for future MOP research and applications.

Keywords:
Nondominated Solutions, Linear Programming, Relative Weighting Method, Constraint Programming.

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طريقة جديدة مقترحة لحل مشاكل الأمثلية متعددة الأهداف

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المستخلص

تُعدّ مسائل الأمثلية متعددة الأهداف (أو أمثلية باريتو) مفهومًا بديلًا راسخًا للتحسين الفعلي، حيث يُشير مصطلح "متعدد الأهداف" إلى مفهوم الأمثلية، كما هو شائع في مجالات بحوث العمليات وعلوم القرار. غالبًا ما تكون هذه المسائل مقيدة، ويتطلب البحث عن الحل الأمثل فيها تحليلًا للمفاضلات بين تحقيق الأهداف. ولأن البرمجة الخطية لا تتضمن مفاضلات، فإن الأمثلية تُعدّ مفهومًا أساسيًا للأهداف المتعددة. بعبارة أخرى، تُعتبر عملية اتخاذ القرار في الأمثلية الخطية طريقة تقليدية تعتمد على إيجاد الحل الأمثل لدالة هدف واحدة. مع ذلك، في الواقع العملي، يُعدّ هذا المنطق تقليديًا إلى حد كبير، حيث يواجه الباحثون غالبًا مشاكل تتضمن أكثر من دالة هدف، وعادةً ما تتضارب هذه الدوال مع بعضها البعض. وهذا يعني أنه عند تعظيم قيمة أحد الأهداف يؤدي الى تخفيض الأهداف الأخرى، أي بمعنى آخر ذلك يأتي على حساب الهدف الآخر في الاتجاه المعاكس. تُسمى هذه المسائل بمسائل الأمثلية متعددة الأهداف (MOP)، والتي تسعى لإيجاد مجموعة من الحلول المهيمنة (أو حلول باريتو المثلى) لإرضاء صانع القرار. تُوسّع هذه الورقة البحثية مفهوم الأمثلية متعددة الأهداف وتُطوّره باستخدام طريقة جديدة مُقترحة لحل جميع دوال الهدف معًا دون تحويلها إلى مسألة هدف فردي (برمجة خطية)، كما هو شائع في حل مسائل التحسين متعدد الأهداف. نُقدّم مقارنة جدولية ورسومية مع مسألة عددية بسيطة تُبيّن الفرق بين الطريقة المُقترحة وبعض طرق الأمثلية متعددة الأهداف الأساسية (مثل البرمجة التوافقية، والأوزان النسبية، وبرمجة القيد). كما وتُوفّر الطريقة الجديدة المُقترحة إطارًا عمليًا تمهيديًا لأبحاث وتطبيقات التحسين متعدد الأهداف المستقبلية.

الكلمات المفتاحية: الحلول غير المهيمنة، البرمجة الخطية، طريقة الأوزان النسبية، برمجة القيد.

1. INTRODUCTION

In general, human life is characterized by cognition and is realistically associated with the DM process, as some decisions are relevant only to the decision maker. In contrast, many other decisions significantly affect the interest of the whole organization. That means the higher the level of DM is, the more serious the consequences and the greater the responsibility of decision makers. The optimal (or best) DM in the case of multiple objectives is an essential topic in OR and decision science; it represents an alternative concept of optimality. Scientists and researchers have realized that most DM problems in real life consider the existence of multi-objectives that often conflict with each other. Because DM in the modern era is no longer based on trial and error but on a quantitative scientific basis to reach more accurate DM to avoid risk as much as possible. All this resulted in the development

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of theories and computation methods for MOP. The MOP is determined by a linear function that achieves the maximization according to a set of linear constraints. This problem arises when two or more (k) scalar-valued objective functions are to be maximized over a set of a feasible solutions, and the formal statement of the general optimization problem according to Zeleny, M., [30] can be written as follows:

$$\text{Optimal}\ \{f_k(\underline{x}) | \underline{x} \in X\}. \quad (1)$$

Whereas: $f_k(\underline{x}) = [f_1(\underline{x}), f_2(\underline{x}), \ldots, f_k(\underline{x})]$, and $X = \mathbb{R}^{\mathcal{N}}$.

A feasible solution x^{**} is a set of nondominated solutions concerning X and $f(\underline{x})$ if this condition is proper: $\{f(\underline{x}^*) \neq f(\underline{x}^{**})\}$ and there exists no $\underline{x}^* \in X$ such that $\{f(\underline{x}^*) \geq f(\underline{x}^{**})\}$.

Multiple Objective Decision Making (MODM) is one of the most important and fundamental topics in optimality problems. This subject began in 1881 when Edgeworth, F., [5] presented the first proposal on the main idea of MOP. Consequently, this proposition changed the traditional concepts of optimization. Because rather than obtaining optimization for an objective, one can optimize several objectives by finding the best trade-offs between those objectives. Kuhn, H.W. and Tucker, A.W., [13] 1951 published the first suggestion on the problems of attaining MOP using the Vector Maximum Problem (VMP) concept. In 1955, Gass, S. and Saaty, T.L., [10] proved how to generate a set of nondominated solutions for Bi-Objective Problems (BOP) by changing the relative weights of the objective function coefficients. Zadeh, L.A., [26], in 1963 was the first to show that the set of nondominated solutions can be found by solving the improvement of the optimization problem in which the objective function is a set of the relative weights of the vector components of the original value function. Additionally, the early time has proposed method of constraint programming (or ϵ- constraint method) by Marglin, S., [16] in 1969 as a new approach to solving MOP. This method works by optimizing one primary objective, and all other objective functions are bounded by taking some small values (ϵ). Further, Cohon, J., and Marks, D., [4] 1973 applied constraint programming to the water resource problem.

In 1974, Zeleny, M., [27] published a book containing fifty papers on Multi-Criteria Decision Making (MCDM). It included multiple objective problems, goal programming, the theory of achieving optimum displacement, Etc. It moreover contained applications for MCDM problems in planning, managing water resources, administering, and so on. Further, Zeleny, M., [28] 1982 published a paper proving that multi-criteria contain multi-objective and attributes, where this paper contains two fundamental theories. The first theory is about multi-objective linear programming problems, and the second is about multi-attribute utility problems. These two theories have become an essential basis for many theoretical differences. In 1986, Steuer, R.E., [22] proposed the relative weighting method (or parametric method) of the goal linear programming problem; this method gives a relative weight to each objective function according to its importance to the decision maker. Furthermore, the sum of these weights is equal to 1. In 1997, Zeleny, M., [29] expanded and developed the notion of optimization as a balance among multi-criteria and introduced eight optimality concepts, E.g., de novo programming, optimal valuation, cognitive equilibrium, and so on. Enrique, B., and Carlos R., [8] 1998 applied the multi-objective compromise programming (or compromise solution method), which showed its effectiveness in solving economic problems, especially the equilibrium problem of a monopolist and the "Leisure-Work" dilemma. In 2000 Ehrgott, M., [6] focused on a different and noticeable mathematical way of presenting the concepts and methods of MCDM, such as the weighting method, constraint programming, and compromise programming. Caramia, M. and Dell'Olmo, P., [2] 2008 introduced modern concepts of multi-objective management in freight logistics services, focusing primarily on multi-objective models (E.g., relative weights, and constraint programming). In 2013, Tzeng, G.H. and Huang, J.J., [23] presented a compromise programming method using mathematical modeling as one of the advantageous methods for solving the MOP. Tzeng, G.H. and Shen, K.Y., [24] 2017 submitted significant research related to solving the problems of MCDM in real life scientifically and efficiently, depending on the development of theories, methods of calculation, and applications. Lee, P.; Kang, S., [14] 2018 proposed a multi-objective interactive optimization approach to the problem of resource selection and demand allocation while including the concept of desire in the optimization model to take into account the principles of diminishing marginal utility. The results were compared with the solutions obtained from the relative weights methods. Rojas, José David., [20] 2021 applied a multi-objective approach to performance comparison of the scalarization methods and analysis of the results from the control theory perspective to obtain the Pareto front (or nondominated solutions). The computational performance of the multi-objective optimization methods has also been considered using the MATLAB program. Rocktäschel, S., [19] 2020 introduced a study of Multi-Objective Mixed-Integer Convex Optimization Problems (MOMICP). He was able to add additional objective functions, which must be minimized, one could derive an additional objective function for this case by introducing a parameter for each facility that measures the negative impact on the environment that occurs, if the respective facility is built. For solving (MOMICP) he was interested in finding the set of efficient points (or nondominated solutions). Abhinav Goel, at et al [1] 2022, introduced a multi-objective approach using a Multi-Objective Genetic Algorithm (MOGA) to do clustering of sensor nodes deployed randomly in any industrial scenario. To measure the quality of clustering, two objectives, namely compactness and separation, are considered as the fitness functions. Eiselt, H. A., [7] 2022, discussed two of the main approaches to Multi-Objective Linear Programming (MOLP): the vector optimization problem and Goal Programming Problems (GPP). Their main distinction is rooted in the decision maker’s input. In vector optimization problems, the decision maker does not provide any input regarding the trade-off between objectives. This means, of course, that many nondominated solutions will be generated, which then must be compared manually by the decision maker, a process in which he will have to use some trade-offs. García Márquez, Pedro, F., [9] 2023, A Multi-Objective Sustainability Control (MOSC) was proposed for a sustainable road traffic network. A multi-objective stochastic program was presented to minimize total delay at links downstream and to reduce maximum risk over links while accounting for travelers' route choices.

In multi-objective optimization, achieving a solution that simultaneously maximizes all objective functions is generally infeasible due to inherent conflicts among objectives, in which improving one objective may lead to deterioration in another. To address this challenge, this study introduces an exact method for generating a rational set of nondominated solutions that better reflects decision makers' preferences. The methodology comprises five stages: (1) introducing slack variables into the model constraints, (2) enumerating feasible candidate solutions using combinatorial principles while enforcing non-negativity, (3) applying the (Gauss–Gordan) matrix method to identify optimal solutions for all feasible combinations, (4) selecting the solution with the highest aggregate objective performance as a reference point, and (5) equating the optimal objective values with the original objective functions to derive a final set of satisfactory nondominated solutions. This approach offers a systematic framework for solving linear multi-objective programming problems, and may be an acceptable alternative for facilitating decision-making under conflicting criteria. Recognizing the significance of this topic, this paper advances the field of multi-objective optimization by introducing a novel method for identifying a set of nondominated solutions. This approach aims to achieve an optimal outcome that maximizes all objectives simultaneously while remaining practical and acceptable to the decision-maker. We can also say that the primary motivation of this paper is to address the challenge of simultaneously optimizing all objective functions by employing an innovative mathematical approach, as previously described. This method aims to achieve optimization of all objective functions for any multi-objective optimization problem across various scientific research domains, as will be demonstrated in subsequent sections. This paper also contributes to the field of scientific knowledge by highlighting the solution to the problem of conflicting objectives in multi-objective optimization problems in an innovative way through the proposed method to address this conflict between objectives by generating a set of nondominated solutions that give an optimal solution for all objective functions by maximizing them together, as explained in paragraph (5.4) of this paper. It also opens the door for future researchers to overcome the challenges posed by the conflict between multiple objectives in multi-objective linear programming.

In this paper, we introduce a proposed method to find a set of non-dominated solutions and some standard multi-objective methods: (a) compromise programming; (b) relative weighting; and (c) constraint programming. The paper is organized as follows: in Section 2, we provide the paper problem. Section 3 presents the paper's objective. Section 4 presents the prevalent methods of MOP. In Section 5, we present the numerical problem. In the last Section (6), we conclude the paper.

2. Paper Problem: Researchers often encounter situations with more than one objective function, which usually conflict with each other; the models of this type are called multi-objective linear programming optimization problems. It looks for the nondominated set (or Pareto set) to provide solutions that satisfy the decision maker. Therefore, this paper examines finding a set of non-dominated solutions without using conventional multi-objective optimization methods and without transforming multiple objective functions into a single objective function. Thus, efforts were devoted to finding a final optimal (or maximal) solution that would satisfy the decision-maker and maximize all objective functions simultaneously.

3. **Paper Objective**: Given the significance of this topic, this paper seeks to advance the concept of multi-objective optimization by introducing a novel method for identifying a set of non-dominated solutions. This approach aims to achieve an efficient, optimal solution that maximizes all objective functions simultaneously while meeting the requirements of decision-makers.
4. **Prevalent methods of MOP:** Most common methods for solving MOP transform the problem into a single objective optimization problem (i.e., traditional LP). In the following, some of these methods are reviewed.

4-1. Compromise programming method: When the parameters of the objective function are determined, the decision maker has one of two things, either accepting the result from the model solution or ignoring it. The decision maker cannot obtain any other information through the model. Contrary to the aforementioned conventional logic, the main goal of MOLP is to find the possible set of nondominated solutions or to find a satisfactory compromise. Consequently, the decision maker can focus effectively on trade-offs (swaps) when there are several conflicts among objectives [28]. This problem can be represented mathematically in general form as follows:

$$\left.\begin{aligned} &\text{Max}\ \mathcal{F} = \left[\mathrm{f}_1\left(\underline{\mathrm{x}}\right), \mathrm{f}_2\left(\underline{\mathrm{x}}\right), \dots, \mathrm{f}_\mathrm{k}\left(\underline{\mathrm{x}}\right)\right] \\ &\ \text{s.t.}\ \underline{\mathrm{x}} \in \mathrm{X} = \left\{\mathcal{A}\underline{\mathrm{x}} \le \mathscr{b}\right\}, \underline{\mathrm{x}} \ge 0. \end{aligned}\right\} \quad (2)$$

Where f_k represent the vector of objective functions with dimension $\mathrm{k} \times 1$, and $\underline{\mathrm{x}} \in \mathrm{X}$ be a set of feasible solutions, $\mathcal{A}$ represents matrix of coefficients with dimension $m \times n$, $\mathscr{b}$ is a vector of resources with dimension $m \times 1$, and x represents a vector of decision variables with dimension $n \times 1$. An optimal solution can be defined as a sign of an optimal value f_i^* defined in the domain of feasible solution $\underline{\mathrm{x}} \in \mathrm{X}$ for each single objective $\left\{\mathrm{f}_i\left(\underline{\mathrm{x}}\right), i = \overline{1, \mathrm{k}}\right\}$ according to the concept of compromise solution,

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should be specified by a point representing the shortest distance d_{p} to the optimum solution from a set of nondominated solutions that can be formulated as follows [23]:

$$\left.\begin{array}{l}\text{Min } f = \{d_{p}\} \\ \text{s.t.}\, \underline{x} \in X = \{\mathcal{A}\underline{x} \leq b\}, \underline{x} \geq 0.\end{array}\right\} \quad (3)$$

So, $d_{p} = \left(\sum_{i=1}^{k}\left(\left(f_i^{max} - f_i(\underline{x})\right) \times \left(f_i^{max} - f_i^{min}\right)^{-1}\right)^{\mathcal{P}}\right)^{1/\mathcal{P}}, 1 \leq \mathcal{P} < \infty;$ represent the scale of distance and the range of $\mathcal{P}$ between one to infinity, when $\mathcal{P} = 1$ and $\mathcal{P} = \infty$, the above mathematical expression becomes [25]: $\{d_{p=1}\} = \sum_{i=1}^{k}\left(f_i^{max} - f_i(\underline{x})\right)/\left(f_i^{max} - f_i^{min}\right), \{d_{p=\infty}\} = \max_{i}\left[\left(f_i^{max} - f_i(\underline{x})\right)/\left(f_i^{max} - f_i^{min}\right) | i = 1,2,\ldots,k\right]$. In the above equation, when taking different values of $\mathcal{P}$, the distance scale d_{p} is different, where it has a variety of meanings. It can be said that it is necessary to consider the closest optimal positive solutions as well as to consider the farthest optimal negative solutions so that the maximum profit can be obtained while avoiding the largest expected amount of risks during the DM process [12]. The general model by which a balanced solution can be found (i.e., finding the general measure of natural distance) for multi-objectives depending on the value of p, can be described as [15]:

$$\left.\begin{array}{l}\text{Min } f = \{d\} \\ \text{s.t.}\, \dfrac{f_i^{max} - f_i(\underline{x})}{f_i^{max} - f_i^{min}} \leq d;\; i = \overline{1,k}, 0 \leq d \leq 1;\; \underline{x} \in X, \underline{x} \geq 0.\end{array}\right\} \quad (4)$$

As the following represents: $\{f_i^{max}; f_i^{min}\}$ maximum and minimum limits of all objective functions i; and $\{d\}$: scale of a distance of multi-objective for the decision maker; $\underline{x} \in X$: set of feasible solutions $\{\mathcal{A}\underline{x} \leq b\}$, where the feasible regain solutions of the above model are: $\{\underline{x} | f_i^{min} \leq f_i(\underline{x}) \leq f_i^{max}, \forall i\}$.

4-1-1. Limitations of compromise programming method in multi-objective linear programming: The assignment of weights is inherently subjective, as the method relies on an ideal point, which may be unrealistic in practice. The normalization process can significantly influence the results, and the obtained solution is sensitive to the choice of norm. And compromise programming may yield solutions that are not Pareto optimal. The approach

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is sensitive to the presence of outlier objectives, such that the computational burden increases substantially for large-scale problems [3].

4-2. Relative weighting method: The relative weighting method is one of the oldest techniques used to solve MOP for obtaining the set of nondominated solutions. Assume that the relative significance of the objectives is known and predetermined. So, the preferred better solution that is a nondominated solutions of the VMP is obtained by solving [3]:

$$\left.\begin{array}{l} \text{Max}\left\{\sum_{i=1}^{k} w_i f_i(\underline{x}) = \mathcal{W}\right\} \\ \text{s.t.}\ \underline{x} \in X = \{\mathcal{A}\underline{x} \le \mathcal{b}\};\ \sum_{i=1}^{k} w_i = 1;\ w_i \ge 0, \underline{x} \ge 0. \end{array}\right\} \quad (5)$$

Where the weighting parameters w_i representing the significance importance of the objectives. Observe that, this method can be used to create a set of nondominated solutions by using different values of w_i. In other words, weight coefficients w_i do not reflect the relative significance of objective functions in the relative sense. However, the difference is found in transactions to determine the points of the nondominated solutions, and if the feasible solution region is convex, it produces a complete set of nondominated solutions.

4-2-1. Limitations of relative weighting method: In conclusion, although the weighting method is straightforward and easy to implement, it presents significant disadvantages, including subjectivity, failure to capture the complete set of efficient solutions, difficulties with scaling and normalization, and constraints in expressing and aggregating preferences in complex real-world scenarios [3].

4-3. Constraint programming method: Another method of multi-objective compromise was proposed by Chankong and Haimes in (1984) [18]. In this method, the decision maker chooses a minimizing objective of one of the n-objectives, and the remaining objectives are in the form of constraints less than or equal to known values. In the convention mathematical, if we have $f_2(\underline{x})$ represents the chosen objective function, then the problem of $\beta(\epsilon_2)$ as in [17]:

$$\left.\begin{array}{l} \text{Min } f_2(\underline{x}) \\ \text{s.t. } f_i(\underline{x}) \le \epsilon_i, \forall i \in \{\overline{1,n}\}\backslash\{2\}, \underline{x} \in \mathcal{T}, \underline{x} \ge 0. \end{array}\right\} \quad (6)$$

Doi: www.doi.org/10.25130/tjaes.22.74.1.42

More generally, if the objective j and vector $\{\epsilon = \epsilon_1, \dots, \epsilon_{j-1}, \epsilon_{j+1}, \dots, \epsilon_n \in \mathbb{R}^{n-1}\}$ exist, where $\underline{x}^*$ represent the optimal solution of the problem $\beta(\epsilon_2)$, the mathematical model becomes:

$$\left.\begin{array}{l} \text{Min } f_j(\underline{x}) \\ \text{s.t. } f_i(\underline{x}) \le \epsilon_1, \forall i \in \{\overline{1,n}\} \backslash \{j\}, \underline{x} \in \mathcal{T}, \underline{x} \ge 0. \end{array}\right\} (7)$$

4-3-1. Limitations of constraint programming method: In summary, the epsilon constraint method effectively generates Pareto optimal solutions; however, it is computationally intensive, necessitates precise selection of constraint levels, may yield weakly efficient solutions, and becomes impractical as the number of objectives increases [3].

4-4. New proposed method: The best solution is the solution that maximizes all the objective functions simultaneously. However, there is a problem that arises in the conflict that occurs between the set of objective functions. That leads to difficulty in obtaining such a solution in many cases, since it is one of the objective functions in the maximal state, the other objective functions in the minimizing case [21]. Therefore, in this paper, a new method is proposed to find the set of nondominated solutions rationally that satisfy the decision makers. The proposed method can be summarized in the following five steps:

Step 1: Adding the slack variables vector $\underline{y}$ to the constraints of the multiobjective problem.

Step 2: Extracting all possible attempts to find a set of nondominated solutions using the principle of combination (nC_r), with neglecting solutions in which the decision variables are negative (considering the solution is infeasible). Because the proposed method is linear, it is necessary to achieve the condition of non-negativity,

Step 3: The matrices of the **(**Gauss-Gordan**)** [11] method is used to find all the optimal solutions to the number of attempts that were found using the formula of the combination mentioned in step 2.

Step 4: Select the maximal solution of the results obtained from the previous step as the hypothesis of the method that the decision maker searches for the maximization of all objective functions simultaneously.

Step 5: After selecting the optimal solution of the objective functions from step 4, we equalize the optimal values with the original objective functions $\{f_1(\underline{x}), f_2(\underline{x}), \dots, f_k(\underline{x}) = f_k^*\}$ and find the final set of nondominated solutions

Doi: www.doi.org/10.25130/tjaes.22.74.1.42

that are satisfactory as closely as possible to decision-makers; i.e., arrive at an optimal solution using the matrices method shown in step 2.

5. **Numerical Problem:** In this section, the traditional methods of MOP shown previously will be applied with the application of the proposed method to find the set of nondominated solutions using the numerical problem mentioned in [30] and then compare the obtained results. Consider the following MOLP Problem with three objective functions, three variables, and five resource constraints, as shown below:

$$\left.\begin{array}{l} Max\{50x_1 + 100x_2 + 17.5x_3 = f_1\}, \\ Max\{92x_1 + 75x_2 + 50x_3 = f_2\}, \\ Max\{25x_1 + 100x_2 + 75x_3 = f_3\} \\ s.t.\, \underline{x} \in X = \begin{cases} 12x_1 + 17x_2 + 0x_3 \leq 1400; \\ 3x_1 + 9x_2 + 8x_3 \leq 1000; \\ 10x_1 + 13x_2 + 15x_3 \leq 1750; \\ 6x_1 + 0x_2 + 16x_3 \leq 1325; \\ 0x_1 + 12x_2 + 7x_3 \leq 900; \\ 9.5x_1 + 9.5x_2 + 4x_3 \leq 1075; \\ \text{all } \underline{x} \geq 0. \end{cases} \end{array}\right\} \quad (8)$$

The first objective represents profit, the second represents quality, and the third represents the time of achievement. Therefore, the optimal solutions are found, and it can be seen how achieving the maximal solution of all objective functions together is possible, as shown below.

5-1. Using the compromise programming method

The model described in paragraph (1.1) can now be applied to the numerical problem according to the following steps:

A. Solve all objective functions $\{f_1, f_2 \; and \; f_3\}$ separately to get the optimal value for each one. Therefore, the optimal solution with respect to f_1 is $x_1^* = 44.94$, $x_2^* = 50.63 \; and \; x_3^* = 41.77$, $f_1(44.94, 50.63 and\; 41.77) = 8041.140$; optimal solution with respect to f_2 is $x_1^* = 92.97$, $x_2^* = 00.00 \; and \; x_3^* = 47.95$, $f_2(92.97, 00.00 \; and \; 47.95) = 10950.590$; optimal solution with respect to f_3 is $x_1^* = 45.22$, $x_2^* = 49.61 \; and \; x_3^* = 43.52$, $f_3(45.22, 49.61 \; and \; 43.52) = 9355.900$.

B. According to the optimal solutions extracted above, it is now possible to find a set of nondominated solutions as shown in the table (1).

Table (1): A set of nondominated solutions

x_1	x_2	x_3	f_1	f_2	f_3
44.94	50.63	41.77	8041.140	10020.230	9319.250
92.97	0.00	47.95	5487.625	10950.590	5920.500
45.22	49.61	43.52	7983.600	10056.990	9355.900
$f_i^{\{max\}}(\underline{x}) - f_i^{\{min\}}(\underline{x})$			2553.515	930.360	3435.400

It is noted from the above table that there is a conflict between the objective functions $\{f_1, f_2, and\ f_3\}$; e.g., when the quality is maximized, at the same time the profit and achievement time, are minimized simultaneously, and if the achievement time is maximum, the profit is minimum at the same time.

C. Therefore, to obtain a balanced solution, compromise programming can be applied to the problem (8) as follows:

$$\left.\begin{array}{l} Min\{f = d\}, \\ s.t.50x_1 + 100x_2 + 17.5x_3 + 2553.515(d) \geq 8041.140; \\ 92x_1 + 75x_2 + 50x_3 + 930.360(d) \geq 10950.590; \\ 25x_1 + 100x_2 + 75x_3 + 3435.400(d) \geq 9355.900; \\ \underline{x} \in X; d \geq 0. \end{array}\right\} \quad (9)$$

The results of the optimal solution areas: $f_1^*(x^*) = 6834.73; f_2^*(x^*) = 10511.04; f_3^*(x^*) = 8069.61;\ x_1^* = 65.12, x_2^* =$

D. $27.07, x_3^* = 49.79, d^* = 0.47$. So, It is now possible, through above optimal results, to find the difference between the objective functions and how far each objective is from the other, as shown below:

$\{f_2^*(x^*) = 10511.04 - f_1^*(x^*) = 6834.73\} = 3676.31;$

$\{f_2^*(x^*) = 10511.04 - f_3^*(x^*) = 8069.61\} = 2441.43;$

E. $\{f_3^*(x^*) = 8069.61 - f_1^*(x^*) = 6834.73\} = 1234.88.$

F. The decision maker notices through Model (9) results that there is a

G. significant divergence between the objective functions $\{f_1, f_2\ and\ f_3\}$.

H. Where the value of scale distance $\{d^*\}$ shows how far the objectives are from each other in terms of the most preferred performance. Although the performance here is almost balanced in maximizing the objective functions in terms of profit, quality, and achievement time, it may need to be more satisfactory to the decision-maker.

Doi: www.doi.org/10.25130/tjaes.22.74.1.42

5-2. Using the relative weighting method : The numerical problem will now be solved by applying the method of relative weights shown in paragraph (1.2) in the following manner:

A. Choosing the vector of relative weights equally for each objective, meaning $\underline{w} = (w_1 = w_2 = w_3 = 1/3)$, and the sum of these weights is equal to one (i.e., $\sum_{i=1}^{3} w_i = 1$).

B. These relative weights are substituted into the problem No. (8) to convert it into a LP model with single objective function, as follows:

$$\left.\begin{aligned} &Max\{w_1 f_1(\underline{x}) + w_2 f_2(\underline{x}) + w_3 f_3(\underline{x})\} = 55.67x_1 + 91.67x_2 + 47.50x_3 = \mathcal{W}, \\ &s.t.\, \underline{x} \in X; \sum_{i=1}^{3} w_i = 1;\ \underline{w} \geq 0, \underline{x} \geq 0. \end{aligned}\right\} \quad (10)$$

Solving the above model will yield the optimal values, as shown below:

$$f_1^*(x^*) = 7983.60; f_2^*(x^*) = 10056.99; f_3^*(x^*) = 9355.50; x_1^* = 45.22, x_2^* = 49.61, x_3^* = 43.52.$$

In the method of relative weighting, a set of nondominated solutions is obtained without the decision maker needing any information of any kind. And then, this set is handed over to the decision maker to choose his preferred solution based on some of the information available. The set of nondominated solutions may be extensive, which is one of the drawbacks of the weighting method, as the decision maker cannot choose his preferred solution since the method depends mainly on relative weighting. i.e., when a change occurs in these weights, a set of nondominated solutions will change with it).

5-3. Using the constraint programming method

The model described in paragraph (1.3) can be applied to the numerical problem according to the following steps:

A. Converting Model No. (8) into a LP problem.

B. Choose a small relative value of the vector $\underline{\epsilon_i}$ to find a set of nondominated solutions, for example $[\epsilon_1 = 0.0010, \epsilon_2 = 0.0015]$, and it can also be seen that the vector $\underline{\epsilon_i}$ is the lowest possible level for the values of the objective functions $\{f_1, f_2\ and\ f_3\}$. A very small $\underline{\epsilon_i}$ in the epsilon constraint method is used to force the solution to achieve a near-optimal value for a particular objective, to explore the boundaries of feasible solutions, and to satisfy strict operational or regulatory requirements. Conversely, choosing very large $\underline{\epsilon_i}$ values in the epsilon constraint method makes the corresponding constraints ineffective, reduces the problem to single-objective optimization, and prevents you from finding meaningful trade-offs or constructing the full Pareto frontier [3].

C. Finding the optimal solutions for the objective functions f_1, f_2 and f_3, and then perform the process of substituting the values of the set of nondominated solutions related to the objective function with the largest result in other objective functions with a relatively lower result, in order to obtain a final optimal solution for all functions together as shown in the following:

- When the objective function f_1 is the main function,

$$\left.\begin{array}{l} Max\{50x_1 + 100x_2 + 17.5x_3 = f_1\}, \\ s.t.92x_1 + 75x_2 + 50x_3 \leq \epsilon_1 = 0.0010; \\ 25x_1 + 100x_2 + 75x_3 \leq \epsilon_2 = 0.0015; \\ \underline{x} \in X; \underline{\epsilon_i} > 0, \underline{x} \geq 0. \end{array}\right\} (11)$$

Then a set of nondominated solutions for the above model is, $f_1^*(x^*) = 8041.14; x_1^* = 44.94, x_2^* = 50.63, x_3^* = 41.77.$

- Whenever the objective function f_2 is the major function,

$$\left.\begin{array}{l} Max\{92x_1 + 75x_2 + 50x_3 = f_2\}, \\ s.t.50x_1 + 100x_2 + 17.5x_3 \leq \epsilon_1 = 0.0010; \\ 25x_1 + 100x_2 + 75x_3 \leq \epsilon_2 = 0.0015; \\ \underline{x} \in X; \underline{\epsilon_i} > 0, \underline{x} \geq 0. \end{array}\right\} (12)$$

Thus, a set of nondominated solutions to the above model is, $f_2^*(x^*) = 10950.59; x_1^* = 92.97, x_2^* = 00.00, x_3^* = 47.95.$

- If **the** objective function f_3 is the primary function,

$$\left.\begin{array}{l} Max\{25x_1 + 100x_2 + 75x_3 = f_3\}, \\ s.t.50x_1 + 100x_2 + 17.5x_3 \leq \epsilon_1 = 0.0010; \\ 92x_1 + 75x_2 + 50x_3 \leq \epsilon_2 = 0.0015; \\ \underline{x} \in X; \underline{\epsilon_i} > 0, \underline{x} \geq 0. \end{array}\right\} (13)$$

Then the set of nondominated solutions to the above model is, $f_3^*(x^*) = 9355.90; x_1^* = 45.22, x_2^* = 49.61, x_3^* = 43.52.$

- Through the above optimal results, it was found that the second objective function achieved the highest possible maximization, so the set of nondominated solutions of the objective function f_2 will be compensated in the objective functions f_1 and f_3 to obtain a final optimal solution for all functions together, as shown below, $f_1^*(x^*) = 5487.625$; $f_2^*(x^*) = 10950.590$; $f_3^*(x^*) = 5920.500$; $x_1^* = 92.97, x_2^* = 00.00,$ $x_3^* = 47.95.$

Doi: www.doi.org/10.25130/tjaes.22.74.1.42

Observe that, in the constraint programming method, after extracting the set of nondominated solutions, it will be given to the decision maker to choose his preferred solution. Nevertheless, if the set of nondominated solutions is enormous, then this leads to the inability of the decision-maker to choose the optimal solution for himself. Because constraint programming depends mainly on the values of the vector ($\underline{\epsilon_i}$), when these values change, the set of nondominated solutions changes with it, which is one of the problems of the method.

5-4. Using a new proposed method: Finally, the proposed method will now be applied to find a set of nondominated solutions to the numerical problem using the steps mentioned in Paragraph (1.4) and as shown below:

Step 1:

$$\left.\begin{array}{l} Max \begin{Bmatrix} 50x_1 + 100x_2 + 17.5x_3 + 0y_1 + 0y_2 + 0y_3 + 0y_4 + 0y_5 + 0y_6 = f_1 \\ 92x_1 + 75x_2 + 50x_3 + 0y_1 + 0y_2 + 0y_3 + 0y_4 + 0y_5 + 0y_6 = f_2 \\ 25x_1 + 100x_2 + 75x_3 + 0y_1 + 0y_2 + 0y_3 + 0y_4 + 0y_5 + 0y_6 = f_3 \end{Bmatrix}, \\ s.t.\, 12x_1 + 17x_2 + 0x_3 + y_1 = 1400;\; 3x_1 + 9x_2 + 8x_3 + y_2 = 1000; \\ \quad 10x_1 + 13x_2 + 15x_3 + y_3 = 1750;\; 6x_1 + 0x_2 + 16x_3 + y_4 = 1325; \\ 0x_1 + 12x_2 + 7x_3 + y_5 = 900;\; 9.5x_1 + 9.5x_2 + 4x_3 + y_6 = 1075;\; \text{all } \underline{x} \geq 0. \end{array}\right\} \quad (14)$$

Step 2: $(nC_r = 9C_6 = 84)$; Where n represents the total number of variables in the three objective functions f_1, f_2 and f_3, and their sum is (9); as for r, it represents the number of slack variables in the objective functions, and its sum is 6; thus, the final result 84 represents the number of possible attempts to find a set of nondominated solutions. The proposed method successfully identified the optimal solution for the research problem, which involved 84 possible combinations for generating the set of nondominated solutions. However, for larger-scale problems with a greater number of decision variables, the computational effort may increase substantially. Accordingly, future research may focus on integrating computer-based optimization techniques, approximation algorithms, or artificial intelligence approaches to efficiently derive nondominated solution sets. Such directions offer significant potential for advancing solution methodologies in multi-objective optimization.

Step 3: After using the (Gauss-Gordan) method to find the set of nondominated solutions for all attempts, then exclude the negative results because there is a non-negativity condition in the numerical problem, in which the decision variables are negative (i.e., a solution is infeasible). Thus, the final number of attempts values of nondominated solutions and objective

Doi: www.doi.org/10.25130/tjaes.22.74.1.42

functions will be 60, as shown in Table 2 in the appendix paragraph. It is noted from the above table that the proposed method calculated all possible nondominated solutions to the numerical problem. Accordingly, this gives the decision maker complete flexibility to choose the optimal solution, noting that the traditional methods of MOP that were applied in the previous paragraphs did not achieve such flexibility.

Step 4: From Table (2), the highest value of the three objective functions will be chosen to be:

$$\{f_1(\underline{x}) = 18541.50; f_2(\underline{x}) = 30666.36; and f_3(\underline{x}) = 20156.25\}.$$

The results obtained from the proposed method assume that the highest values of the three objective functions are taken individually, by substituting the resulting variables into the nondominated set of solutions that give each objective function its maximum possible maximization. After comparing all the vectors in Table 2 and taking the resulting decision variables into the optimal nondominated set of solutions that give each objective function its maximum available maximization, regardless of the compatibility of the solutions in the decision space, these resulting values in the objective functions using the proposed method may sometimes lead to an explicit violation of the constraints advantage in operations research. Therefore, in this case, traditional multi-objective optimization methods can be used, and the proposed method abandoned. However, if an alternative vector from Table 2 is taken, in which the values of the objective functions are relatively high together compared to the values of the other objective functions, the conflict problem will still exist, i.e., some objectives will be maximized and others minimized. Therefore, the proposed method was adopted as an alternative solution that attempts to minimize the conflict as much as possible and is acceptable to the decision-maker.

Step 5: It is now possible to equate the original objective functions mentioned in the numerical problem with the values of the maximum objective functions shown in step 4, as follows:

$$\text{Max}\begin{Bmatrix} f_1(\underline{x}) = 50x_1 + 100x_2 + 17.5x_3 = f_1^* = 18541.50 & \dots(1) \\ f_2(\underline{x}) = 92x_1 + 75x_2 + 50x_3 = f_2^* = 30666.36 & \dots(2) \\ f_3(\underline{x}) = 25x_1 + 100x_2 + 75x_3 = f_3^* = 20156.25 & \dots(3) \end{Bmatrix}$$

After solving the above equations using the (Gauss-Gordan) method, the greatest values of the objective functions were obtained together, and a set of nondominated solutions ($\underline{x}^*$) was also obtained. Therefore, it can be

Doi: www.doi.org/10.25130/tjaes.22.74.1.42

said that the optimal values satisfactory to the decision maker and in which all the objective functions are in the situation of maximization are:
$x_1^* = 222.942;\ x_2^* = 52.067;\ x_3^* = 125.014;$ and $f_1^*(\underline{x}) = 18{,}541.545;\ f_2^*(\underline{x}) = 30{,}666.389;\ f_3^*(\underline{x}) = 20{,}156.300.$

The optimal values for the set of nondominated solutions obtained after solving using the (Gauss-Gordan) method yielded the highest combined optimal values for the objective functions. However, the aforementioned method can sometimes explicitly violate the constraints advantage of operations research; therefore, in such cases, traditional multi-objective optimization methods can be employed. The final optimal values of the objective functions obtained using the (Gauss-Gordan) method with the proposed approach are substituted into Table 3 for comparison with the results obtained using traditional optimization methods, as shown below.
Table (3) can now summarize the difference between the results of the newly proposed method and the traditional MOP, as these results will be explained in the conclusion paragraph.

Table (3): A comparison of the optimal results of different methods

Objective functions	Proposed method	Compromise programming method	Relative weighting method	Constraint programming method	The difference between the proposed method and the compromise programming method	The difference between The proposed method and the relative weighting method	The difference between The proposed method and the constraint programming method
$f_1^*(\underline{x})$	18541.545	8041.825	7983.600	5487.625	10499.720	10557.945	13053.920
$f_2^*(\underline{x})$	30666.389	10950.350	10056.990	10950.740	19716.039	20609.399	19715.649
$f_3^*(\underline{x})$	20156.300	9355.250	9355.500	5920.500	10801.050	10800.800	14235.800
The total summation	69364.234	28347.43	27396.09	22358.87	41016.809	41968.144	47005.369

This situation (i.e., the comparison between the results of the newly proposed method and the common MOP) is also shown graphically outlined in Fig. 1.

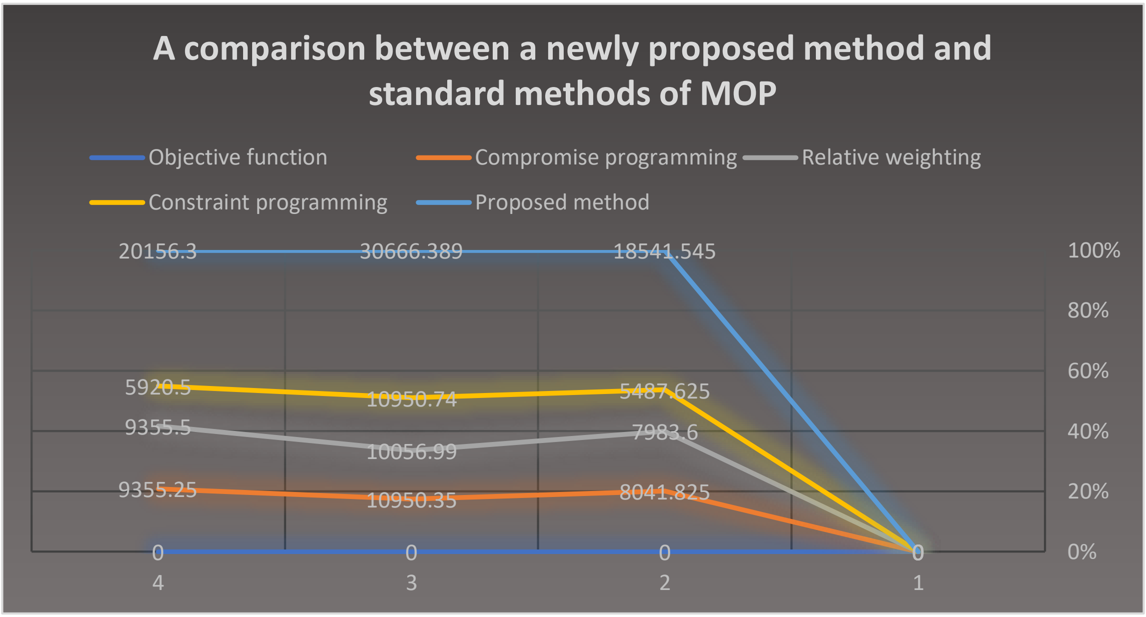

Fig. 1. A comparison between the newly proposed method and common MOP

6. Conclusion and Perspectives: The traditional approach used for MOP is mainly through the transformation of multiple objectives into one objective (i,e., into an LP problem), which is naturally the farthest from any experimental conditions contributed to solving real problems. While the proposed method in this paper showed some acceptable flexibility in finding the set of nondominated solutions, which allowed the decision maker to maximize the three objective functions (profit, quality, and time of achievement), respectively, in addition, the problem was not converted into LP. Instead, the optimum values were extracted with the multiplicity of the objective functions, noting that they are contradictory. The proposed method can allow the decision maker to make the optimal decision realistically and efficiently according to modern logic. This proposed method can also be applied to any real problem with multiple objectives, large or small; due to the significant development in computer programming and artificial intelligence programs, which are likely to develop even further in the future. In other words, the proposed method can be extended to a wide range of real-world multi-objective optimization problems, regardless of problem size or complexity. Its applicability is further supported by recent advances in computer programming and artificial intelligence techniques, which enhance computational efficiency and solution quality. Given the rapid pace of technological progress, these capabilities are expected to improve further,

Tikrit Journal of Administrative and Economic Sciences, Vol. 22, No. 74, Part (1): 860-882
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thereby increasing the practical relevance and scalability of the proposed approach for future operations research applications. As well as in Table No. (3), the difference was summarized according to the double classification between the results of the objective functions obtained using the multi-objective optimization traditional methods and among the application of the proposed new method through which all the set of non-dominated solutions was found. Three standard methods of multi-objective optimization (compromise programming, relative weighting, and constraint programming) and a new suggested method; were applied to the numerical problem and compared the results, showing the proposed method's efficiency. Furthermore, it is possible, through table (3), that Interpretating what has been accomplished using a newly proposed method as follows:

1. The objective function f_1 was increased by 10499.720 compared to the compromise programming solution; it was increased by 10557.945 compared to the solution using the relative weighting method, and finally, it was increased by 13053.920 compared to the solution using the constraint programming.
2. An increase in the objective function f_2 was achieved by 19716.039 compared to the solution of compromise programming, an increase of 20609.399 compared to the solution using the relative weighting. And finally, an increase of 19715.649 compared to the solution using constraint programming.
3. There was an increase of 10801.050 in the objective function f_3 compared to the solution using compromise programming and an increase of 10800.800 compared to the solution according to the relative weighting method. And finally, there was an increase of 14235.800 compared to the solution using constraint programming.
4. Suppose the decision maker is looking to maximize the three objective functions immediately. In that case, he can choose a set of nondominated solutions $(x_1^* = 222.942, x_2^* = 52.067, \text{and } x_3^* = 125.014)$ where the optimal values of the objective functions are: $\{f_1^*(\underline{x}) = 18{,}541.545, f_2^*(\underline{x}) = 30{,}666.389, \text{and } f_3^*(\underline{x}) = 20{,}156.300\}$. However, if the decision maker is searching a balanced solution, he can choose one of the solutions of the traditional MOP shown in the previous paragraphs.

As a conclusion, we observe that the considered newly proposed method does not only expand the application domain of multiple objective methods but also assumes a new approach to the solution of MOP most effectively.

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APPENDIX

Table (2): The values of nondominated solutions and objective functions

Extreme point	Solve for	Fix for	A set of nondominated solutions	$f_1(x)$	$f_2(x)$	$f_3(x)$
1	$y_{\{1,6\}}$	$x_{\{1,2,3\}}$	$x_{\{1,2,3\}} = 0$	.000	.000	.000
2	$x_{\{3\}}$; $y_{\{1,3,4,5,6\}}$	$x_{\{1,2\}}$; $y_{\{2\}}$	$x_{\{1,2\}} = 0$; $x_{\{3\}} = 125$	2187.50	6250	9375
3	$x_{\{3\}}$; $y_{\{1,2,4,5,6\}}$	$x_{\{1,2\}}$; $y_{\{3\}}$	$x_{\{1,2\}} = 0$; $x_{\{3\}} = 116.67$	2041.67	5833.33	8750
4	$x_{\{3\}}$; $y_{\{1,2,3,5,6\}}$	$x_{\{1,2\}}$; $y_{\{4\}}$	$x_{\{1,2\}} = 0$; $x_{\{3\}} = 82.81$	1449.22	4140.62	6210.94
5	$x_{\{3\}}$; $y_{\{1,2,3,4,6\}}$	$x_{\{1,2\}}$; $y_{\{5\}}$	$x_{\{1,2\}} = 0$; $x_{\{3\}} = 128.57$	2250	6428.57	9642.86
6	$x_{\{3\}}$; $y_{\{1,2,3,4,5\}}$	$x_{\{1,2\}}$; $y_{\{6\}}$	$x_{\{1,2\}} = 0$; $x_{\{3\}} = 268.75$	4703.12	13437.50	20156.25
7	$x_{\{2\}}$; $y_{\{2,3,4,5,6\}}$	$x_{\{1,3\}}$; $y_{\{1\}}$	$x_{\{1,3\}} = 0$; $x_{\{2\}} = 82.35$	8235.29	6176.47	8235.29
8	$x_{\{2\}}$; $y_{\{1,3,4,5,6\}}$	$x_{\{1,3\}}$; $y_{\{2\}}$	$x_{\{1,3\}} = 0$; $x_{\{2\}} = 111.11$	11111.11	8333.33	11111.11
9	$x_{\{2\}}$; $y_{\{1,2,4,5,6\}}$	$x_{\{1,3\}}$; $y_{\{3\}}$	$x_{\{1,3\}} = 0$; $x_{\{2\}} = 134.62$	13461.54	10096.15	13461.54
10	$x_{\{2\}}$; $y_{\{1,2,3,4,6\}}$	$x_{\{1,3\}}$; $y_{\{5\}}$	$x_{\{1,3\}} = 0$; $x_{\{2\}} = 75$	7500	5625	7500
11	$x_{\{2\}}$; $y_{\{1,2,3,4,5\}}$	$x_{\{1,3\}}$; $y_{\{6\}}$	$x_{\{1,3\}} = 0$; $x_{\{2\}} = 113.16$	11315.79	8486.84	11315.79
12	$x_{\{2,3\}}$; $y_{\{2,4,5,6\}}$	$x_{\{1\}}$; $y_{\{1,3\}}$	$x_{\{1\}} = 0$; $x_{\{2\}} = 82.35$; $x_{\{3\}} = 45.29$	10500	8441.18	11632.35
13	$x_{\{2,3\}}$; $y_{\{2,3,5,6\}}$	$x_{\{1\}}$; $y_{\{1,4\}}$	$x_{\{1\}} = 0$; $x_{\{2\}} = 82.35$; $x_{\{3\}} = 82.81$	12099.88	10317.09	14446.23
14	$x_{\{2,3\}}$; $y_{\{2,3,4,6\}}$	$x_{\{1\}}$; $y_{\{1,5\}}$	$x_{\{1\}} = 0$; $x_{\{2\}} = 82.35$; $x_{\{3\}} = 73.16$	9515.62	9834.56	13722.43
15	$x_{\{2,3\}}$; $y_{\{2,3,4,5\}}$	$x_{\{1\}}$; $y_{\{1,6\}}$	$x_{\{1\}} = 0$; $x_{\{2\}} = 82.35$; $x_{\{3\}} = 73.16$	9515.62	9834.56	13722.43
16	$x_{\{2,3\}}$; $y_{\{1,4,5,6\}}$	$x_{\{1\}}$; $y_{\{2,3\}}$	$x_{\{1\}} = 0$; $x_{\{2\}} = 32.26$; $x_{\{3\}} = 88.71$	4778.42	6855	9879.25
17	$x_{\{2,3\}}$; $y_{\{1,3,5,6\}}$	$x_{\{1\}}$; $y_{\{2,4\}}$	$x_{\{1\}} = 0$; $x_{\{2\}} = 37.50$; $x_{\{3\}} = 82.81$	5199.17	6953	9960.75
18	$x_{\{2,3\}}$; $y_{\{1,3,4,5\}}$	$x_{\{1\}}$; $y_{\{2,5\}}$	$x_{\{1\}} = 0$; $x_{\{2\}} = 6.06$; $x_{\{3\}} = 118.18$	2674.15	6363.50	9469.50
19	$x_{\{2,3\}}$; $y_{\{1,2,5,6\}}$	$x_{\{1\}}$; $y_{\{3,4\}}$	$x_{\{1\}} = 0$; $x_{\{2\}} = 39.06$; $x_{\{3\}} = 82.81$	5355.17	7070	10116.75
20	$x_{\{2,3\}}$; $y_{\{1,2,4,6\}}$	$x_{\{1\}}$; $y_{\{3,5\}}$	$x_{\{1\}} = 0$; $x_{\{2\}} = 14.04$; $x_{\{3\}} = 104.49$	3232.57	6277.50	9240.75
21	$x_{\{2,3\}}$; $y_{\{1,2,4,5\}}$	$x_{\{1\}}$; $y_{\{3,6\}}$	$x_{\{1\}} = 0$; $x_{\{2\}} = 100.83$; $x_{\{3\}} = 29.28$	10595.40	9026.25	12279
22	$x_{\{2,3\}}$; $y_{\{1,2,3,6\}}$	$x_{\{1\}}$; $y_{\{4,5\}}$	$x_{\{1\}} = 0$; $x_{\{2\}} = 26.69$; $x_{\{3\}} = 82.81$	4118.17	6142.25	8879.75
23	$x_{\{2,3\}}$; $y_{\{1,2,3,5\}}$	$x_{\{1\}}$; $y_{\{4,6\}}$	$x_{\{1\}} = 0$; $x_{\{2\}} = 78.29$; $x_{\{3\}} = 82.81$	9278.17	10012.25	14039.75
24	$x_{\{1\}}$; $y_{\{1,3,4,5,6\}}$	$x_{\{2,3\}}$; $y_{\{2\}}$	$x_{\{1\}} = 333.33$; $x_{\{2,3\}} = 0$	16666.50	30666.36	8333.25
25	$x_{\{1\}}$; $y_{\{1,2,4,5,6\}}$	$x_{\{2,3\}}$; $y_{\{3\}}$	$x_{\{1\}} = 175$; $x_{\{2,3\}} = 0$	8750	16100	4375
26	$x_{\{1\}}$; $y_{\{1,2,3,5,6\}}$	$x_{\{2,3\}}$; $y_{\{4\}}$	$x_{\{1\}} = 220.83$; $x_{\{2,3\}} = 0$	11041.50	20316.36	5520.75
27	$x_{\{1\}}$; $y_{\{1,2,3,4,5\}}$	$x_{\{2,3\}}$; $y_{\{6\}}$	$x_{\{1\}} = 113.16$; $x_{\{2,3\}} = 0$	5658	10410.72	2829
28	$x_{\{1,3\}}$; $y_{\{3,4,5,6\}}$	$x_{\{2\}}$; $y_{\{1,2\}}$	$x_{\{1\}} = 116.67$; $x_{\{2\}} = 0$; $x_{\{3\}} = 81.25$	7255.37	14796.14	9010.50
29	$x_{\{1,3\}}$; $y_{\{2,4,5,6\}}$	$x_{\{2\}}$; $y_{\{1,3\}}$	$x_{\{1\}} = 116.67$; $x_{\{2\}} = 0$; $x_{\{3\}} = 38.89$	6514.07	12678.14	5833.50
30	$x_{\{1,3\}}$; $y_{\{2,3,5,6\}}$	$x_{\{2\}}$; $y_{\{1,4\}}$	$x_{\{1\}} = 116.67$; $x_{\{2\}} = 0$; $x_{\{3\}} = 39.06$	6517.05	12686.64	5846.25

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Extreme point	Solve for	Fix for	A set of nondominated solutions	$f_1(x)$	$f_2(x)$	$f_3(x)$
31	$x_{\{1,3\}}$; $y_{\{2,3,4,6\}}$	$x_{\{2\}}$; $y_{\{1,5\}}$	$x_{\{1\}} = 116.67$; $x_{\{2\}} = 0$; $x_{\{3\}} = 128.57$	8083.47	17162.14	12559.50
32	$x_{\{1,3\}}$; $y_{\{1,3,4,5\}}$	$x_{\{2\}}$; $y_{\{2,6\}}$	$x_{\{1\}} = 71.87$; $x_{\{2\}} = 0$; $x_{\{3\}} = 98.05$	5309.37	11514.54	9150.50
33	$x_{\{1,3\}}$; $y_{\{1,2,5,6\}}$	$x_{\{2\}}$; $y_{\{3,4\}}$	$x_{\{1\}} = 116.07$; $x_{\{2\}} = 0$; $x_{\{3\}} = 39.29$	6491.07	12642.94	5848.50
34	$x_{\{1,3\}}$; $y_{\{1,2,4,5\}}$	$x_{\{2\}}$; $y_{\{3,6\}}$	$x_{\{1\}} = 89.02$; $x_{\{2\}} = 0$; $x_{\{3\}} = 57.32$	5454.10	11055.84	6524.50
35	$x_{\{1,3\}}$; $y_{\{1,2,3,5\}}$	$x_{\{2\}}$; $y_{\{4,6\}}$	$x_{\{1\}} = 92.97$; $x_{\{2\}} = 0$; $x_{\{3\}} = 47.95$	5487.62	10950.74	5920.50
36	$x_{\{1,3\}}$; $y_{\{1,2,3,4\}}$	$x_{\{2\}}$; $y_{\{5,6\}}$	$x_{\{1\}} = 59.02$; $x_{\{2\}} = 0$; $x_{\{3\}} = 128.57$	5200.97	11858.34	11118.25
37	$x_{\{1,2\}}$; $y_{\{2,3,4,6\}}$	$x_{\{3\}}$; $y_{\{1,5\}}$	$x_{\{1\}} = 10.42$; $x_{\{2\}} = 75$; $x_{\{3\}} = 0$	8021	6583.64	7760.50
38	$x_{\{1,2\}}$; $y_{\{2,3,4,5\}}$	$x_{\{3\}}$; $y_{\{1,6\}}$	$x_{\{1\}} = 104.74$; $x_{\{2\}} = 8.42$; $x_{\{3\}} = 0$	6079	10267.58	3460.50
39	$x_{\{1,2\}}$; $y_{\{1,4,5,6\}}$	$x_{\{3\}}$; $y_{\{2,3\}}$	$x_{\{1\}} = 53.92$; $x_{\{2\}} = 93.14$; $x_{\{3\}} = 0$	12010	11946.14	10662
40	$x_{\{1,2\}}$; $y_{\{1,3,5,6\}}$	$x_{\{3\}}$; $y_{\{2,4\}}$	$x_{\{1\}} = 220.83$; $x_{\{2\}} = 37.50$; $x_{\{3\}} = 0$	14791.50	23128.86	9270.75
41	$x_{\{1,2\}}$; $y_{\{1,3,4,6\}}$	$x_{\{3\}}$; $y_{\{2,5\}}$	$x_{\{1\}} = 108.33$; $x_{\{2\}} = 75$; $x_{\{3\}} = 0$	12916.50	15591.36	10208.25
42	$x_{\{1,2\}}$; $y_{\{1,3,4,5\}}$	$x_{\{3\}}$; $y_{\{2,6\}}$	$x_{\{1\}} = 3.07$; $x_{\{2\}} = 110.09$; $x_{\{3\}} = 0$	11162.50	8539.19	11085.75
43	$x_{\{1,2\}}$; $y_{\{1,2,4,6\}}$	$x_{\{3\}}$; $y_{\{3,5\}}$	$x_{\{1\}} = 77.50$; $x_{\{2\}} = 75$; $x_{\{3\}} = 0$	11375	12755	9437.50
44	$x_{\{1,2\}}$; $y_{\{1,2,3,6\}}$	$x_{\{3\}}$; $y_{\{4,5\}}$	$x_{\{1\}} = 220.83$; $x_{\{2\}} = 75$; $x_{\{3\}} = 0$	18541.50	25941.36	13020.75
45	$x_{\{1,2\}}$; $y_{\{1,2,3,4\}}$	$x_{\{3\}}$; $y_{\{5,6\}}$	$x_{\{1\}} = 38.16$; $x_{\{2\}} = 75$; $x_{\{3\}} = 0$	9408	9135.72	8454
46	$x_{\{1,2,3\}}$; $y_{\{4,5,6\}}$	$y_{\{1,2,3\}}$	$x_{\{1\}} = 104.74$; $x_{\{2\}} = 8.42$; $x_{\{3\}} = 114.13$	6079	10478.08	3460.50
47	$x_{\{1,2,3\}}$; $y_{\{3,5,6\}}$	$y_{\{1,2,4\}}$	$x_{\{1\}} = 63.54$; $x_{\{2\}} = 37.5$; $x_{\{3\}} = 58.98$	7959.15	11607.18	9762
48	$x_{\{1,2,3\}}$; $y_{\{3,4,6\}}$	$y_{\{1,2,5\}}$	$x_{\{1\}} = 56.84$; $x_{\{2\}} = 42.23$; $x_{\{3\}} = 56.18$	8048.15	11205.53	9857.50
49	$x_{\{1,2,3\}}$; $y_{\{3,4,5\}}$	$y_{\{1,2,6\}}$	$x_{\{1\}} = 36.51$; $x_{\{2\}} = 56.58$; $x_{\{3\}} = 47.66$	8317.55	9985.42	10145.25
50	$x_{\{1,2,3\}}$; $y_{\{2,4,6\}}$	$y_{\{1,3,5\}}$	$x_{\{1\}} = 45.77$; $x_{\{2\}} = 50.04$; $x_{\{3\}} = 42.78$	8041.15	10102.84	9356.75
51	$x_{\{1,2,3\}}$; $y_{\{2,4,5\}}$	$y_{\{1,3,6\}}$	$x_{\{1\}} = 43.30$; $x_{\{2\}} = 51.79$; $x_{\{3\}} = 42.92$	8095.10	10013.85	9480.50
52	$x_{\{1,2,3\}}$; $y_{\{2,3,6\}}$	$y_{\{1,4,5\}}$	$x_{\{1\}} = 60.20$; $x_{\{2\}} = 39.86$; $x_{\{3\}} = 60.24$	8050.20	11539.90	10009
53	$x_{\{1,2,3\}}$; $y_{\{2,3,4\}}$	$y_{\{1,5,6\}}$	$x_{\{1\}} = 44.94$; $x_{\{2\}} = 50.63$; $x_{\{3\}} = 41.77$	8040.97	10020.23	9319.25
54	$x_{\{1,2,3\}}$; $y_{\{1,5,6\}}$	$y_{\{2,3,4\}}$	$x_{\{1\}} = 4.64$; $x_{\{2\}} = 37.50$; $x_{\{3\}} = 81.07$	5400.72	7292.88	9946.25
55	$x_{\{1,2,3\}}$; $y_{\{1,4,5\}}$	$y_{\{2,3,6\}}$	$x_{\{1\}} = 30.32$; $x_{\{2\}} = 66.49$; $x_{\{3\}} = 38.83$	8844.52	9717.69	10319.25
56	$x_{\{1,2,3\}}$; $y_{\{1,3,4\}}$	$y_{\{2,5,6\}}$	$x_{\{1\}} = 48.71$; $x_{\{2\}} = 37.06$; $x_{\{3\}} = 65.04$	7279.70	10512.82	9801.75
57	$x_{\{1,2,3\}}$; $y_{\{1,2,6\}}$	$y_{\{3,4,5\}}$	$x_{\{1\}} = 22.28$; $x_{\{2\}} = 31.57$; $x_{\{3\}} = 74.46$	5574.05	8140.51	9298.50
58	$x_{\{1,2,3\}}$; $y_{\{1,2,5\}}$	$y_{\{3,4,6\}}$	$x_{\{1\}} = 77.59$; $x_{\{2\}} = 12.95$; $x_{\{3\}} = 53.72$	6114.60	10807.03	7263.75
59	$x_{\{1,2,3\}}$; $y_{\{1,2,4\}}$	$y_{\{3,5,6\}}$	$x_{\{1\}} = 45.22$; $x_{\{2\}} = 49.61$; $x_{\{3\}} = 43.52$	7983.60	10056.99	9355.50
60	$x_{\{1,2,3\}}$; $y_{\{1,2,3\}}$	$y_{\{4,5,6\}}$	$x_{\{1\}} = 48.64$; $x_{\{2\}} = 37.33$; $x_{\{3\}} = 64.57$	7294.97	10503.13	9791.75