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Bootstrap-Based Simulation for Estimating Confidence Intervals of Taguchi Loss Function Means

محاكاة قائمة على Bootstrap لتقدير فترات الثقة لمتوسطات دالة خسارة تاجوتشي

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Abstract : *Continuous improvement in healthcare organizations and services is essential to ensuring patient satisfaction. Despite the challenges and work pressures, these organizations strive to achieve stable and efficient performance. One of the most prominent indicators used to measure performance quality is the Taguchi loss function. The Taguchi loss function is used to calculate the financial loss resulting from deviations from the target value and ideal performance. This indicator is important in supporting quality improvement in healthcare organizations and services. To assess the efficiency and reliability of these estimates, confidence intervals must be estimated. However, traditional methods are often inappropriate, especially when sample sizes are small and the probability distribution of the data is unknown. Therefore, the bootstrap method was used due to its flexibility and ability to handle such data. The simulation results showed that equilibrium was achieved at a sample size of 150, with confidence intervals continuing to narrow significantly (around 1.17) and achieving high coverage rates for all methods, with the highest coverage rate reaching 95.3% for the Percentile and BCa methods. This indicates a good balance between precision and coverage and reflects the effectiveness of these methods when using a relatively large sample size.*

Keywords: ***Bootstrap, loss function of Taguchi , Estimation confidence interval , Simulation,healthcare organizations and services ,Bootstrap***

الملخص:تهدف ان التحسين المستمر في مؤسسات وخدمات الرعاية الصحية أمرًا أساسيًا لضمان رضا المرضى. ورغم التحديات وضغوط العمل، تسعى هذه المؤسسات جاهدةً لتحقيق أداء مستقر وفعال. ومن أبرز المؤشرات المستخدمة لقياس جودة الأداء دالة خسارة تاجوتشي . تُستخدم دالة خسارة تاجوتشي لحساب الخسارة المالية الناتجة عن الانحرافات عن القيمة المستهدفة والأداء المثالي. وتُبرز أهمية هذه المؤشر في دعم تحسين الجودة في مؤسسات الرعاية الصحية والخدمات. ولتقييم كفاءة وموثوقية هذه التقديرات، يلزم تقدير فترات الثقة. ومع ذلك، غالبًا ما تكون الطرق التقليدية غير مناسبة، خاصةً في حالات صغر حجم العينات وعدم معرفة توزيع الاحتمالات للبيانات. لذلك، استُخدمت طريقة إعادة المعاينة (bootstrapنظرًا لمرونتها وقدرتها على التعامل مع هكذا من البيانات. واظهرت النتائج محاكاة حيث حققت التوازن عند حجم العينة 150، مع استمرت فترات الثقة بالتضيق بشكل ملحوظ (حوالي 1,17) و تحقيق نسب تغطية مرتفعة لجميع الطرق، حيث بلغت أعلى نسب تغطية95,3% لطريقتي Percentile و .BCaوهذا يشير إلى توازن جيد بين الدقة ونسبة التغطية، ويعكس كفاءة هذه الطرق عند استخدام حجم عينة كبير نسبياً.

الكلمات المفتاحية: دالة الخسارة لتاجوتشي، تقدير فترات الثقة ، المحاكاة، مؤسسات وخدمات الرعاية الصحية ،طريقة إعادة المعاينة.

Introduction:

Taguchi's average loss is an important indicator for evaluating the quality of services and products, as it measures the degree of deviation from the optimal or target value. To obtain and improve the accuracy of estimates, the bootstrap method is used as a powerful statistical tool based on drawing from the original data. This method enables the analysis of estimates without the need for traditional assumptions and provides confidence intervals that reflect the accuracy and reliability of the results.

Some of what was studied in previous years on the subject of the study will be covered according to the researcher’s knowledge of it:

- The study (Manal& Alaa, 2019) used the Taguchi loss function for the costs of unutilized energy faced by Iraqi economic units due to unsuitable products. This function was used to measure and reduce these costs. The study found that these costs are hidden costs of external failure, and may constitute 90% of the total costs. The study recommended using the Taguchi function to calculate these costs, because it is mathematically independent and free of bias.
- The study (Sameer& Mahmoud, 2021) focused on the effectiveness of Taguchi's quality loss function to measure service quality, using a sample of (100) customers. The study discussed the importance of applying the quality loss function in restaurants to improve service quality and highlighted the importance of considering external costs when analyzing the costs of system component failure and the cost of system

components. The study concluded that developing Taguchi's quality loss function is essential for improving the quality of products and services.

- ❖ The study of (Bodhisattva, & et al., 2010) dealt with the smoothed bootstrap to improve the accuracy of the probability density function estimates for the Grenander estimator. The study also suggested applying the bootstrap method of (m") of ("n") as an alternative to creating samples that work to enhance the accuracy of the estimates. These proposals aim to provide consistent and effective estimates when processing data using bootstrap methods.

Taguchi loss function

The Taguchi Loss Function is one of the most common loss functions. It is based on the definition of Genichi Taguchi a Japanese scientist who contributed significantly to the development of quality engineering concepts through his innovative ideas. (Demetrescu et al., 2000) Taguchi created the Quality Loss Function (QLF) to measure the economic loss resulting from deviations in product characteristics from the target (ideal) value. He modeled the cost factor in a quadratic form (k), also known as the rework cost or waste, which expresses the financial impact resulting from deviations from the target. (Masoud et al.,2024:15),Taguchi also explained that losses do not only occur when specification limits are exceeded, but begin immediately after any deviation from the target value. The loss function is based on a quadratic model, which reflects the quadratic increase in loss as the deviation from the target increases. (Krishnaiah&Shahabudeen, 2012:188)

According to the following formula (Barker, (1990):

$$L(Y)=K(Y-m)^2 \qquad (1)$$

Where:

L(y): Represents the amount of loss resulting.

k: The cost coefficient or proportionality factor.

y: The actual value of the measured characteristic.

m: The target value (ideal target)

The Taguchi loss function is divided into three types (Chin&Hsing,2010:573)

1. Nominal-is-best:

$$\underline{L}(Y) = K[\sigma^2 + (\underline{Y} - m)^2 \qquad (2)$$

2. Smaller-is-better:

$$\underline{L}(Y) = K(\sigma^2 + \underline{Y}^2) \qquad (3)$$

3. Larger-is-better

$$\underline{L}(Y) = \frac{K}{\underline{Y}^2}[1 + \frac{3\sigma^2}{\underline{Y}^2}] \qquad (4)$$

Bootstrap

Bootstrap emerged as an alternative to the Jackknife method, which relied on deleting a single observation without replication, leading to problems related to data symmetry and small sample sizes. Efron (1979) introduced the bootstrap method as a more accurate and flexible method (Marcos&Anselmo,2015), based on resampling using the empirical distribution based on the original data, without the need for prior parametric assumptions about the population distribution. The bootstrap method is a modern non-parametric method, widely used in estimating statistics, checking bias, and confidence intervals. It is also efficiently applied in cases where it is difficult to know the probability distribution of the data (Chernick& Robert,2014)The bootstrap method is divided into two types: the non-parametric bootstrap, which relies on resampling from the empirical distribution, and the parametric bootstrap, which assumes a priori known distribution of the data. Basic steps to use Bootstrap (Marcos&Anselmo,2015)

1. A random sample of size (n) is drawn from the population [Z=(x1,x2,x3,…….xn)] with a return.
2. Using [Z*=(x1*,x2*,x3*,….,xn*)] instead of the original sample data is called a bootstrap sample.
3. The parameters of the sample drawn with a return are estimated.
4. Steps (1) and (2) are repeated B times, where ($B \geq 1000$)

Statistical analysis:

Monte Carlo simulation is one of the most important and popular simulation methods used in statistical analysis. In this study, the ready-made statistical software R (version 4.5.0) was used, as this software is distinguished by its ability to implement statistical applications on the one hand and develop statistical programs on the other. The simulation method was used to estimate the average Taguchi loss function, relying on the R program as the primary tool for statistical calculations. In the experiments, fixed loss coefficients (K = 500, 1000) were determined, along with the use of different bootstrap sample sizes (B = 10,000, 20,000, 30,000) to study the stability and accuracy of the estimators.

Data Generation:

Random variables subject to a finite uniform distribution were generated according to the following steps:

- The first stage:

This is the stage of determining the default values, as it is one of the most important stages upon which the remaining stages depend. The default values were chosen as follows:

1. Determine the sample size (n): Five sample sizes were chosen, which are: n=(40,60,150,300)
2. Define the default parameters for the experiment.
   - Taguchi loss coefficient k=[500,1000,1750,2500]
   - samples were selected with a B=10000
   - The significance level was set at α=0·05 for confidence intervals.

- The second stage:

This is the stage of generating the random variable y according to the regular distribution in the interval [1,5], i.e. U[1,5].

- The third stage:

This stage involves calculating the Taguchi loss function and the signal-to-noise ratio, which were calculated using the following equations:

$$\underline{\widehat{L^*}}_i(y) = K[\sigma_i^2 + (\underline{y}_i - m)^2] \qquad ,i=1,2,3,\ldots\ldots\ldots\ldots\ldots B$$

$$\frac{S}{N_{NTB-I}} = 10\ log\ log\ (\frac{\underline{y}^2}{S^2})$$

$$S^2 = \frac{\sum_{i=1}^{n} \quad (y_i - \underline{y})^2}{n-1}$$

The number of times the experiment was repeated was R=500.

Simulation results:

Table (1) : represents the simulation results when B=10000, K=500 with different sample sizes for the average loss function of Taguchi.

	Quality Loss Coefficient	Bootstrap Samples			Method		
	K=500	B=10000	Standard	t	percentile	Basic	BCa
	40	LOWER	805.7779	739.1541	759.5289	776.73	690.0052
		UPPER	1294.222	1235.846	1290.471	1323.27	690.0052
		Mean Cl Width	510.7573	509.2497	509.2497	509.2497	509.2318
		Coverage	91.5	91.3	92.1	91.3	92.8
	60	LOWER	765.7627	887.017	664.9564	725.7502	745.0532
		UPPER	1184.237	1329.65	1051.71	1157.583	1171.613
		Mean Cl Width	419.1013	418.4021	418.4021	418.4021	418.4332
		Coverage	91.9	90.7	92.5	90.7	92.8
	150	LOWER	896.401	875.043	881.8787	886.9142	819.7078
		UPPER	1163.599	1144.957	1151.455	1146.419	1093.626
		Mean Cl Width	266.7127	266.4153	266.4153	266.4153	266.374
		Coverage	94.6	94.9	95	94.9	95.3
	300	LOWER	922.3313	934.6502	942.5251	955.4703	876.6337
		UPPER	1114.335	1125.35	1134.142	1147.863	1066.7
		Mean Cl Width	189.166	189.0746	189.0746	189.0746	189.0172
		Coverage	65	65	65.3	65	66.4

A simulation was conducted to estimate the average quality loss using the bootstrap method with a very large sample size (B = 10,000) and a fixed loss coefficient (K = 500). The results of five different methods for calculating confidence intervals were compared: the standard method, the t-test method, the percentage method, the basic method, and the bias-corrected and accelerated method (BCa). The simulations were conducted on different sample sizes: 40, 60, 150, and 300. The interpretation of the sample results is as follows. At a sample size of 40, the confidence intervals were very wide, which is statistically unusual and may be an indication of sample heterogeneity or the presence of significant variance in the data. However, coverage remained relatively stable, and the BCa method remained the best in terms of coverage. At a sample size of 60, the resulting confidence intervals were relatively wide for all methods, indicating poor estimation accuracy at this size. Confidence interval coverage ranged between 90% and 92%, with the highest coverage being from The BCa method's share indicates its high efficiency compared to other methods, even in small sample sizes. However, we note an illogical variance in the upper values of some methods, especially Basic and Percentile, which may indicate an anomaly in the estimates or the influence of small data on the bootstrap distribution. Upon reaching a larger sample size (150), the results began to show a clear improvement in performance. The confidence intervals narrowed significantly, reflecting the increased accuracy of the statistical estimate. The coverage rate also increased to levels close to 95% for all methods, indicating that the sample size was sufficient to obtain more reliable estimates. Furthermore, the BCa method continued to achieve the highest coverage rate, enhancing its reliability as the best method used. Finally, when using a very large sample size (300), the confidence intervals continued to narrow, which is a good indicator of increased estimation accuracy. Surprisingly, however, the coverage rate dropped to only about 65%, a sharp and unexpected decline. This means that the confidence intervals had become so narrow that they no longer covered the true value of the mean in many iterations, which explains the possibility of

There is a bias or inefficiency at this large sample size, perhaps due to over-reducing the confidence intervals. In general, it can be said that the performance of bootstrap methods for estimating average quality loss depends greatly on sample size. Methods vary in their efficiency, but the BCa method has shown the best performance across various coverage sizes. Furthermore, there is a required balance between estimation accuracy (narrow confidence intervals) and reliability (high coverage), and this balance appears to be more achieved at medium sample sizes, such as 150, than at very small or large sample sizes.

Table (2): represents the simulation results when B=20000, K=500 with different sample sizes for the average loss of the Taguchi function.

Quality Loss coefficient	**Bootstrap Samples**	**Methods**				
K=500	B=20000	Standard	t	prudential	Basic	BCa
40	LOWER	859.2645	726.5908	841.3845	858.5481	1158.5561
	UPPER	1390.736	1223.409	1358.616	1366.452	1641.444
	Mean CI Width	511.3797	510.2687	510.2687	510.2687	510.0739
	Coverage	91.4	91.2	92.6	91.2	93.3
60	LOWER	859.2645	726.5908	841.3845	858.5481	1158.5561
	UPPER	1390.736	1223.409	1358.616	1366.45	1641.444
	Mean CI Width	511.3797	510.2687	510.2687	510.269	510.0739
	Coverage	91.4	91.2	92.6	91.2	93.3
150	LOWER	765.1300	869.5444	648.6481	871.6457	930.4508
	UPPER	1184.870	1297.122	1084.685	1311.688	1369.549
	Mean CI Width	419.7471	419.0981	419.0981	419.0981	418.8120
	Coverage	91.7	91.5	93.1	91.5	93.4
300	LOWER	903.9518	927.7056	899.3284	920.1895	893.7226
	UPPER	1092.715	1115.628	1084.005	1113.144	1079.611

		Mean CI Width	189.0750	188.9834	188.9834	188.9834	188.9609
		Coverage	67.6	66.9	67.8	66.9	68.6

Simulation results for estimating the average quality loss using different bootstrap methods. The loss coefficient K = 500 and the number of bootstrap iterations B = 20,000, with varying sample sizes.

When the sample size was 20, the results showed that the confidence intervals were relatively wide for all methods, with an average width of approximately 713 using the Standard method. The relatively narrowest method was Basic, with an average width of approximately 707.6, indicating that the estimates were not highly accurate at this size. However, the coverage rates were fairly acceptable, ranging from 89.9% to 94.6%. The BCa method recorded the highest coverage rate, indicating its relative superiority in performance even with a small sample size. The other methods provided coverage close to 90%, which is within the statistically acceptable limit, but reflects a relative disparity between methods in their ability to capture the true value of the average quality loss. When the sample size was 40, the results tended to be more balanced and accurate. Almost all methods showed narrower confidence intervals, with the interval width decreasing to approximately 510 for all methods, reflecting the accuracy of the estimate. Coverage ratios also improved, reaching 93.3% for the BCa method, followed by the Percentile method at 92.6%, confirming the stability of the statistical performance of the methods at this sample size. Results did not differ significantly at sample size 60; rather, the same values were repeated, which could indicate data entry errors or actual stability in the estimate at this stage. However, the main result remains that the methods continued to provide comparable levels of accuracy and coverage. At sample size 150, results improved further, with the average width of the confidence intervals decreasing to approximately 419, indicating a marginal narrowing compared to the previous two sample sizes. At this sample size, coverage ratios were relatively high, ranging from 91.5% to 93.4%, with BCa again being the most effective method, reflecting its ability to produce more reliable and stable estimates. This sample size reflects a transitional stage in statistical stability where the performance of bootstrap methods begins to approach ideal behavior. When the sample size reached 300, the confidence intervals became the narrowest of all sizes, averaging only about 189. This indicates high accuracy and low variance around the mean. However, the paradox here is that coverage rates decreased significantly despite the narrow intervals, ranging from only 66.9% to 68.6%, which are low rates that do not match expectations for larger sizes. This decrease is likely due to unseen bias in the estimates of some methods or to their reliance on assumptions that are not well-founded in the data used. It is noteworthy that the BCa method, despite its excellent performance at small and medium sizes, did not outperform significantly at this large size, which may reflect its limitations in dealing with data bias when there are a large number of observations. Overall, the results indicate that the performance of bootstrap methods gradually improves with increasing sample size until it peaks at medium sizes, such as 150, where accuracy and coverage are well balanced. At very large sizes, however, the narrow confidence intervals are not offset by improved coverage, suggesting that increasing size alone is not sufficient to ensure optimal estimates, and that data characteristics and distribution must be taken into account. The BCa method performed well at most sizes, particularly small and medium ones, making it a relatively reliable option for estimating average quality loss using bootstraps.

Table (3): represents the simulation results when B=30000, K=500 with different sample sizes for the average loss of the Taguchi function.

Quality Loss Cofficient	**Bootstrap Samples**	**Methods**				
K=500	B=30000	Standard	t	percentile	Basic	BCa
40	LOWER	1061.1352	796.2804	578.0201	579.2495	1059.5596
	UPPER	1563.865	1353.720	1046.980	1095.751	1565.440
	Mean CI Width	512.2956	511.1022	511.1022	511.1022	511.1497
	Coverage	93.6	93.6	94.6	93.6	95.1
60	LOWER	1072.6100	723.4573	644.6062	879.3061	845.6240
	UPPER	1494.057	1143.209	1038.727	1304.027	1271.043
	Mean CI Width	420.0324	419.2692	419.2692	419.2692	419.1768
	Coverage	91.8	90.6	92.6	90.6	92.9
150	LOWER	888.9961	925.7352	850.8243	933.6321	865.9415
	UPPER	1157.671	1200.931	1115.842	1199.701	1140.725
	Mean CI Width	267.0494	266.8722	266.8722	266.8722	266.8341
	Coverage	94.2	94.3	94.6	94.3	95.1
300	LOWER	936.3149	862.4060	931.5736	921.3433	932.8433
	UPPER	932.8433	1054.261	1118.426	1108.657	1127.157
	Mean CI Width	189.0739	188.9632	188.9632	188.9632	188.9523
	Coverage	68.1	67.4	68.3	67.4	69.5

The simulation results are shown at a fixed loss coefficient K = 500 and a number of bootstrap iterations B = 30,000. We note that the results were clearly affected by the sample size. At a sample size of 40, the confidence intervals were more consistent, with the upper and lower bounds falling within a less dispersed range compared to the previous sample. The width of the confidence interval was similar across the methods, reaching approximately 511 for almost all of them. This indicates a slight improvement in the accuracy of the estimate, and the coverage ratios maintained their good level, reaching 95.1% for the BCa method. and 93.6% for the Standard, t, and Basic methods, indicating that the methods remain effective at this size. At a sample size of 60, clearer signs of improved statistical stability emerged. Confidence intervals began to narrow compared to previous sizes, with the average confidence interval width remaining stable around 419. Coverage ratios also increased to good levels, reaching 92.9% for the BCa method and 91.8% for the Standard method, confirming that increasing sample size enhances estimation accuracy without negatively affecting confidence. At a sample size of 150, confidence intervals became more precise and clear. The bounds ranged between approximately 850 and 1200 for most methods, with a clear decrease in the average confidence interval width to only about 266. Coverage remained high, with the highest value recorded for the BCa method at 95.1%, followed by the Percentile at 94.6%, reflecting significant stability in the bootstrap results at this medium size. Finally, at a sample size of 300, confidence intervals reached The lowest level of width, where the average dropped to about 189, indicates high estimation accuracy. Surprisingly, however, coverage ratios decreased compared to previous sizes, not exceeding 69.5% in the best case (BCa), and reaching 67.4% in some methods, such as t and Basic. This decrease in coverage, despite the improved interval width, may be due to increased bias or data heterogeneity at this size level, indicating that formal accuracy does not always mean better reliability in capturing the true value. In general, it can be said that bootstrap methods were relatively effective in estimation, especially at small and medium sizes. The BCa method was more distinguished in terms of coverage ratios, while performance declined slightly at large sizes despite the improved interval width, which calls for a more precise statistical interpretation of the nature of the data or the variables studied.

Table (4): represents the simulation results when B=10000, K=1000 with different sample sizes for the average loss of the Taguchi function.

Quality Loss Cofficient	**Bootstrap Samples**	**Methods**				
K=1000	B=10000	Standard	t	Percentile	Basic	BCa
40	LOWER	1611.556	1478.308	1519.058	1553.460	1380.010
	UPPER	2588.444	2471.692	2580.942	2646.540	2369.990
	Mean CI Width	1021.514	1018.499	1018.499	1018.499	1018.463
	Coverage	91.5	91.3	92.1	91.3	92.8
60	LOWER	1531.525	1774.034	1329.913	1451.500	1490.106
	UPPER	2368.475	2659.299	2103.420	2315.166	2343.227
	Mean CI Width	838.2025	836.8042	836.8042	836.8042	836.8665
	Coverage	91.9	90.7	92.5	90.7	92.8
150	LOWER	1792.802	1750.086	1763.757	1773.828	1639.416
	UPPER	2327.198	2289.914	2302.909	2292.838	2187.251
	Mean CI Width	533.4255	532.8307	532.8307	532.8307	532.7480
	Coverage	94.6	94.9	95.0	94.9	95.3
300	LOWER	1844.663	1869.300	1885.050	1910.941	1753.267
	UPPER	2228.671	2250.700	2268.283	2295.726	2133.399
	Mean CI Width	378.3319	378.1492	378.1492	378.1492	378.0344
	Coverage	65.0	65.0	65.3	65.0	66.4

Simulation results for estimating the average quality loss at K = 1000 and the number of bootstrap iterations (B = 10,000), with statistical performance testing of several different confidence interval estimation methods under varying sample sizes. At a sample size of 40, the confidence intervals appeared narrower, and their average width decreased to approximately 1018, indicating estimation accuracy. The coverage rate for all methods was at acceptable levels, reaching 92.8% for the BCa method.

With an increase in the sample size to 60, the width of the confidence intervals decreased to approximately 836, reflecting continued improvement in estimation accuracy. The coverage rates continued to rise, reaching 92.8% for the BCa method and 92.5% for the Percentile method, reflecting the stability of the performance of these methods. It is noteworthy that some methods, such as the t-test method, began to show slight variations in coverage rates, decreasing to 90.7%, which may indicate that they are affected by the data distribution or their underlying assumptions. At a sample size of 150, the most stable features of the results became evident. The average width of the confidence intervals decreased significantly to approximately 533, and coverage ratios reached their peak, exceeding 95% in some methods. The BCa method achieved the highest percentage, 95.3%, demonstrating high accuracy and clear statistical balance. This size represents a stage of stability and maturity in estimation, as the data are sufficient to represent the true behavior of the mean and minimize variance and random effects. With a sample size of 300, the confidence intervals narrowed further, reaching a width of only about 378, demonstrating very high estimation accuracy. However, the paradox was the coverage ratios, which clearly declined to less than 66% for all methods. This sharp decline in coverage, despite the narrowness of the confidence intervals, indicates that these methods have become biased in their estimates or are unable to capture the true value, perhaps due to data characteristics or a misalignment in the assumptions of some methods with large sample sizes. This may be due to narrowing the intervals at the expense of capturing the true value, meaning that the methods are more confident in their estimates but less flexible in capturing the truth. In general, this table shows that the performance of bootstrap methods for estimating the mean quality loss gradually improves with increasing sample size until it reaches its peak at a size of 150, where it achieves an optimal balance between narrow confidence intervals and high coverage ratios. However, this performance begins to decline at the larger sample size of 300, where coverage ratios decrease disproportionately with the narrow intervals. This reflects the risk of overconfidence in estimates when relying solely on the data without taking into account biases or the behavior of the sample distribution. The BCa method performed better at most sample sizes, reflecting its robustness in estimation and bias correction compared to other methods, especially at medium sample sizes.

Table (5): represents the simulation results when B=20000, K=1000 with different sample sizes for the average loss of the Taguchi function.

	Quality Loss Cofficient	**Bootstrap Samples**	**Methods**				
	K=1000	B=20000	Standard	t	Percentile	Basic	BCa
	40	LOWER	1718.529	1453.182	1682.769	1717.096	2317.112
		UPPER	2781.471	2446.818	2717.231	2732.904	3282.888
		Mean CI Width	1022.759	1020.538	1020.538	1020.538	1020.148
		Coverage	91.4	91.2	92.6	91.2	93.3
	60	LOWER	1530.260	1739.089	1297.296	1743.291	1860.902
		UPPER	2369.740	2594.244	2169.371	2623.375	2739.098
		Mean CI Width	839.4942	838.1962	838.1962	838.1962	837.6240
		Coverage	91.7	91.5	93.1	91.5	93.4
	150	LOWER	1675.516	1754.579	1740.162	1936.875	1830.316
		UPPER	2204.484	2285.421	2273.171	2476.458	2369.684
		Mean CI Width	533.0919	532.7255	532.7255	532.7255	532.7165

		Coverage	93.3	93.5	93.6	93.5	93.8
	300	LOWER	1807.904	1855.411	1798.657	1840.379	1787.445
		UPPER	2185.430	2231.255	2168.010	2226.288	2159.221
		Mean CI Width	378.1501	377.9668	377.9668	377.9668	377.9219
		Coverage	67.6	66.9	67.8	66.9	68.6

Simulation results for estimating the average Taguchi loss function using the bootstrap method, with the number of iterations B = 20,000 and a fixed loss coefficient K = 1,000. This was used to test the performance of five types of bootstrap confidence intervals (Standard, t, Percentile, Basic, and BCa) under the influence of varying sample sizes. At a sample size of 40, the average width of the confidence intervals was approximately 1,020, an indicator of the accuracy of the estimate. Coverage ratios for all methods ranged between 91.2% and 93.3%, enhancing the reliability of the estimates at this size.At sample size 60, confidence intervals continued to shrink, reaching an average width of approximately 838, with high coverage ratios between 91.5% and 93.4%, reflecting stable performance for all methods, especially BCa and Percentile. At sample size 150, the average width decreased significantly to approximately 532 for all methods, and coverage ratios increased very closely, ranging between 93.3% and 93.4%, indicating that the five methods became more effective at this size, with similar performance across them. At sample size 300, the average width decreased to approximately 378, the smallest in all experiments, indicating high estimation accuracy. However, a significant decrease in coverage ratios was observed for all methods, ranging between 66.9% and 68.6%, indicating that significantly increasing sample size can reduce sample variance to a degree that negatively impacts coverage, especially when a fixed number of replications is used. In general, the performance of bootstrap confidence intervals improves as the sample size increases up to a certain point, but a balance between the number of iterations and the sample size is needed to maintain coverage levels close to the nominal percentage. The BCa method showed a slight advantage in most cases in terms of accuracy and coverage.

Table (6): represents the simulation results when B=30000, K=1000 with different sample sizes for the average loss of the Taguchi function.

Quality Loss Cofficient	**Bootstrap Samples**	**Methods**				
K=1000	B=30000	Standard	t	precential	Basic	BCa
40	LOWER	2122.270	1592.561	1156.040	1158.499	2119.119
	UPPER	3127.730	2707.439	2093.960	2191.501	3130.881
	Mean CI Width	1024.591	1022.204	1022.204	1022.204	1022.299
	Coverage	93.6	93.6	94.6	93.6	95.1
60	LOWER	2145.220	1446.915	1289.212	1758.612	1691.248
	UPPER	2988.113	2286.419	2077.454	2608.054	2542.085
	Mean CI Width	840.0647	838.5383	838.5383	838.5383	838.3535
	Coverage	91.8	90.6	92.6	90.6	92.9
150	LOWER	1777.992	1851.470	1701.649	1867.264	1731.883
	UPPER	2315.341	2401.863	2231.685	2399.402	2281.450
	Mean CI Width	534.0988	533.7445	533.7445	533.7445	533.6682
	Coverage	94.2	94.3	94.6	94.3	95.1
300	LOWER	1872.630	1724.812	1863.147	1842.687	1865.687
	UPPER	2254.037	2108.521	2236.853	2217.313	2254.313
	Mean CI Width	378.1478	377.9265	377.9265	377.9265	377.9047
	Coverage	68.1	67.4	68.3	67.4	69.5

Simulation results using a repeated bootstrap sample (B=30,000) with a quality loss factor (K=1000) to study the performance of five methods for estimating confidence intervals (Standard, t, Percentile, Basic, and BCa), across different sample sizes. At a sample size of 20, we note that the confidence intervals for all methods remain relatively wide, with the average width of the estimated interval being approximately 1405 to 1418, indicating that this small sample size does not provide high accuracy for the estimates. However, the BCa method recorded the highest coverage rate of 93.8%, while the coverage rates for the other methods ranged between 89.5% and 91.8%. This indicates that increasing the number of bootstrap samples did not necessarily lead to a significant improvement in width or coverage at this small sample size. With a sample size of 40, we note that the average interval width reached approximately 1,022 across all methods, with a clear increase in coverage rates, especially for the BCa method, which reached 95.1%, indicating reliable estimation. With a sample size of 60, the interval width decreased to approximately 838, and coverage rates stabilized at a high level, between 90.6% and 92.9%, reflecting consistent performance across the various methods, particularly since the Percentile and BCa methods demonstrated relatively better performance in terms of coverage. With a sample size of 150, the estimated interval width decreased to approximately 533, with coverage rates continuing to increase between 94.2% and 95.1%, indicating improved efficiency of the methods with a relatively large sample size. Clearly, all methods became more stable, and the BCa method maintained its slight advantage in coverage. For the larger sample size of 300, the confidence intervals reached their narrowest range (around 377), indicating high estimation accuracy. However, the coverage ratios declined, ranging between 67.4% and 69.5%, which is below the required nominal level (95%). This decline may be attributed to potential bias or an overly narrow interval, meaning that these methods, despite their high estimation accuracy, do not guarantee that the true value is captured in most iterations. In general, the results show that increasing the number of bootstrap iterations to 30,000 was not sufficient to compensate for the small initial sample size. Performance gradually improved as the sample size increased to a certain level (150), then coverage declined at the larger sample size (300), reflecting other complexities related to the shape of the distribution or the nature of the data that affect the efficiency of the estimation methods. The BCa method remains the most stable in terms of coverage across most sizes.

Conclusions:

The results showed that the performance of bootstrap methods in estimating confidence intervals is clearly affected by the sample size and the type of method used. Increasing sample size narrows confidence intervals and increases precision, but this may come at the expense of coverage in some cases, especially at very large sample sizes. In contrast, the corrected and accelerated (BCa) method proved highly efficient at almost all sample sizes, achieving an optimal balance between precision and coverage, thanks to its ability to correct for bias and flexibly handle the asymmetry of the empirical distribution. The t and Basic methods performed well at medium sample sizes, while the Percentile method was more sensitive to changes in sample size. Its coverage declined at small and medium sample sizes, but improved relatively at large sample sizes. Accordingly, BCa is recommended for use in cases where sample sizes are small or data are limited, as it provides accurate estimates and more stable coverage compared to other methods.

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