

Differential System in algebra SBH

Authors Names	ABSTRACT
Ahmed Ali Hadhum Al Shajan^a Abbas A. AL-asdi^b Mortda Taeh Shadhan^c Publication date: 19 / 6 /2026 Keywords: SBH-alge., ideal, Differential System, fuzzy ideal of an alge.- SBH, Differential System of a alge.- SBH, fuzzy Differential System of an alge.- SBH.	In this research , we will be study some types of an alge.- SBH ,we prove some properties and solve some examples to relationship by the definitions.

1. Introduction

The notion In this paper.. of alge -BCK. introduced in 1966 by K. Isiki [6]., introduced In 1983 by Q.P.Hu with X. Lie the notion of in alge.- BCH [5], introduced In 1998 byY. Jon, and E. Rof and H. Kiem the notion of alge. some type of BH [5].

2. Some Basic About alge.-BH (BH-algebra)

We will take some definitions In this section for alge.-BCI, alge.-BCK, BCH in alge.and alge.- BH, with some of basics of, theorems and some examples.

Definition (2.1)[1]: A BCI in alge. If $(C, *, \text{CONS } 0)$ of a type $(2, 0)$, when C is a non - empty set with $\text{CONS } 0$ is a constant , hold, for all $z, y, a \in C$ and for all " $*$ " is a binary operation

- i. $((a * z) * (a * y)) * (y * z) = \text{CONS } 0, ,$
- ii. $(y * (y * a)) * a = \text{CONS } 0,$
- iii. $y * y = \text{CONS } 0, ,$
- iv. $y * a = \text{CONS } 0$ and $a * y = \text{CONS } 0 \rightarrow y = a,.$

Definition(2.2)[3]:A BCK in alge.is a BCI in alge. hold the one axiom: $\text{CONS } 0 * y = \text{CONS } 0$, for all $y \in C$.

Definition(2.3)[5]: A BCH in alge.is an alge. $(C, *, \text{CONS } 0)$, when C is set with $\text{CONS } 0$, holding the following, $\forall z, y, a \in C$ and, for all " $*$ " is a binary operation

- i. $y * y = \text{CONS } 0, \forall y \in C.$
- ii. $y * a = \text{CONS } 0$ and $a * y = \text{CONS } 0$ imply $y = a,.$
- iii. $(z * y) * a = (z * a) * y,.$

Definition(2.4)[7]: An alge.- BH is a set C , $\text{CONS } 0$ with " $*$ " an operation and hold a binary, $\forall y, a \in C$. holding the following:

- i. $y * y = \text{CONS } 0,$
- ii. $y * \text{CONS } 0 = y,$
- iii. $. y * a = \text{CONS } 0$ and $a * y = \text{CONS } 0$ imply $y = a,$

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THE MAIN RESULTS

In this paper study, the types of alge. and a Differential System In SBH-Alge. and. So that we prove theorems and some examples about alge. the differential System for First order and defined the function $y = y(x)$ “ is written as

$$Y^{(1)} = \text{FUNTI}(x, y)$$

When FUNTI (x, y) be a function and x be independent variable with y be dependent variable

3. Solve Alge.-SBH (SBH)

we will introduce in this section, the some of types concepts Alge.-SBH, Also we state some examples and prove theorems

Definition(3.1): An alge.- SBH is a set C , non - empty, a constant $\text{CONS } 0$ with “*” a binary operation, holding the following:

- i. $y * \text{CONS } 0 = y, \forall y \in C.$
 - ii. $y * a = \text{CONS } 0$ and $a * y = \text{CONS } 0$ iff $y = a, \forall y, a \in C.$
 - iii. $\forall a \neq \text{CONS } 0, y \neq \text{CONS } 0$ and $a \neq y$ then $[a * y] \neq \text{CONS } 0,$
- Then it is a. D. S. in SBH

Example(3.2):- let $C = \{\text{CONS } 0, c, v, p\}$ and operation $*$ is in the following table.

*	CONS 0	c	v	p
CONS 0	CONS 0	c	c	v
c	c	CONS 0	c	c
v	v	c	CONS 0	c
p	p	p	p	CONS 0

Then it is a. D. S. in SBH

Example(3.3):- let $C = \{\text{CONS } 0, c, v, p\}$ and operation $*$ is in the following table.

*	CONS	c	v	p
0	CONS	c	c	v
c	c	CONS	c	0
v	v	c	CONS	c
p	p	p	p	CONS

$\forall c \neq \text{CONS } 0, p \neq \text{CONS } 0$ and $c \neq p$, but $c * p = \text{CONS } 0$. Then it is not a D. S. in SBH

4. Solve Differential System by Alge.-SBH (D. S. in SBH)

we will explain In this section , the concepts a Differential System by Alge.-SBH, Also we state some examples and prove some theorems about the Differential System In SBH-Alge.or in alge.-SBH.

Definition (4.1):- a Differential System In SBH-Alge.or in alge.-SBH ,FUNTI (X,Y) Be A Function And (*) Be Any binary operation And Let (CONS 0) Denoted By (D. S. in SBH)

Let, $y=1 \rightarrow \text{FUNTI}(x,1) = \text{FUNTI}(x) \dots\dots\dots(1)$

Let, $x=1 \rightarrow \text{FUNTI}(1,y) = \text{FUNTI}(y) \dots\dots\dots(2)$

and satisfies

- i. $[\text{FUNTI}(x) * \text{CONS } 0] = \text{FUNTI}(x)$
- ii. $[\text{FUNTI}(x) * \text{FUNTI}(y)] = [\text{FUNTI}(y) * \text{FUNTI}(x)] = \text{CONS } 0$ iff $[\text{FUNTI}(x) = \text{FUNTI}(y)]$
- iii. $\forall \text{FUNTI}(x) \neq \text{CONS } 0, \text{FUNTI}(y) \neq \text{CONS } 0$ and $\text{FUNTI}(x) \neq \text{FUNTI}(y)$ then $[\text{FUNTI}(x) * \text{FUNTI}(y)] \neq \text{CONS } 0$,

Example (4.2) : let $(R, -, 0)$ be a field in a differential System and a function and called by FUNTI (x,y) =x.y is it (D. S. in SBH), $\text{FUNTI}(y) = y$ and $\text{FUNTI}(x) = x$, then

- i. $[\text{FUNTI}(x) * \text{CONS } 0]$

$\rightarrow [x-0]=x$ by def.

$\rightarrow \text{FUNTI}(x)$

- ii. $[\text{FUNTI}(x) * \text{FUNTI}(y)]$

$\rightarrow [x-y] = 0 = [y-x]$ by def.

$\rightarrow [FUNTI(y) * FUNTI(x)]$

then $[x=y]$

Conversely

$[x=y]$ then $[FUNTI(x) * FUNTI(y)]$

$\rightarrow [x-x] = 0 = [x-x]$ by def.

$\rightarrow [FUNTI(y) * FUNTI(x)]$

- iii. $\forall FUNTI(x) \neq CONS\ 0$, $FUNTI(y) \neq CONS\ 0$ and $FUNTI(x) \neq FUNTI(y)$ then $[FUNTI(x) * FUNTI(y)] \neq CONS\ 0$,
s.t $(R, -, 0)$ is (D. S. in SBH)

Example (4.3):- let $(Z/1, -, 0)$ be a field in a differential System and the function defined be $FUNTI(x, y) = xy + y$ Is it (D. S. in SBH)

Solve :- let $FUNTI(x) = x+1$, $FUNTI(y) = 2y$ then

i. $[FUNTI(x) * CONS\ 0]$

$\rightarrow [x+1-0] = [x+1]$ by def.

$\rightarrow FUNTI(x)$

ii. $[FUNTI(x) * FUNTI(y)] = [FUNTI(y) * FUNTI(x)] = 0$ by def.

$\rightarrow [x+1-2y] = [2y-x-1] = 0$ by def.

Conversely

$[FUNTI(x) * FUNTI(x)]$

$\rightarrow [x+1-x-1] = 0$ by def.

- iii. $\forall FUNTI(x) \neq CONS\ 0$, $FUNTI(y) \neq CONS\ 0$ and $FUNTI(x) \neq FUNTI(y)$ then $[FUNTI(x) * FUNTI(y)] \neq CONS\ 0$,

s.t $(Z/1, -, 0)$ is (D. S. in SBH)

since, it is hold if $y=x$.

Theorem (4.4) : Every an Alge.- SBH of X iff a D. S. in SBH of X

Proof.

Let C be a set and D. S. in SBH of X

T. p. is an Alge.- SBH

$\forall x, y \in C$, $FUNTI(x) = x$ and $FUNTI(y) = y$

i. $x * CONS\ 0 = [FUNTI(x) * CONS\ 0 = FUNTI(x)] = x$

ii. $x * y = [FUNTI(x) * FUNTI(y)] = CONS\ 0$

and $y*x = [FUNTI(y)*FUNTI(x)] = CONS\ 0$

imply $[FUNTI(x) = FUNTI(y)]$ by def.

$\rightarrow x=y$

and $x*x = [FUNTI(x)*FUNTI(x)] = CONS\ 0$

iii. $\forall FUNTI(x) \neq CONS\ 0, FUNTI(y) \neq CONS\ 0$ and $FUNTI(x) \neq FUNTI(y)$ then $[FUNTI(x)*FUNTI(y)] \neq CONS\ 0$,

Then, it is an Alge.- SBH

Conversely

Let, C be an Alge.- SBH of X,

To prove is a D. S. in SBH

Since, $FUNTI(x) = x$ and $FUNTI(y) = y$

i. $[FUNTI(x)*CONS\ 0]$

$= x*CONS\ 0$

$= x$ by def.

$= FUNTI(x)$

ii. $[FUNTI(x)*FUNTI(y)]$

$= x*y$ by def.

$= CONS\ 0$

and $[FUNTI(y)*FUNTI(x)]$

$= y*x$ by def.

$= CONS\ 0$

imply $x=y$ by def.

$\rightarrow [FUNTI(x) = FUNTI(y)]$

And $[FUNTI(x)*FUNTI(x)]$

$= x*x$ by def.

$= CONS\ 0$

iii. $\forall FUNTI(x) \neq CONS\ 0, FUNTI(y) \neq CONS\ 0$ and $FUNTI(x) \neq FUNTI(y)$ then $[FUNTI(x)*FUNTI(y)] \neq CONS\ 0$,

Then, it is a D. S. in SBH

Proposition (4.5): Let C be a set, SBH-Alge. and $\{FUNTI_i(x), x \in C, i \in \lambda\}$ be a D. S. SBH. Then

$\bigcap_{i \in \lambda} FUNTI_i(x)$ is a D. S. in SBH.

Proof: To prove $\bigcap_{i \in \lambda} FUNTI_i(x)$ is a D. S. in SBH.

Since $FUNTI_i(x)=x$ and $FUNTI_i(y)=y$

- i. $\bigcap_{i \in \lambda} FUNTI_i(x) * CONS 0 = \bigcap_{i \in \lambda} (FUNTI_i(x) * CONS 0) = \bigcap_{i \in \lambda} FUNTI_i(x)$
- ii. $\bigcap_{i \in \lambda} FUNTI_i(x) * \bigcap_{i \in \lambda} FUNTI_i(y) = \bigcap_{i \in \lambda} (FUNTI_i(x) * FUNTI_i(y)) = CONS 0$

And $FUNTI \bigcap_{i \in \lambda} FUNTI_i(y) * \bigcap_{i \in \lambda} FUNTI_i(x) = \bigcap_{i \in \lambda} (FUNTI_i(y) * FUNTI_i(x)) = CONS 0$

$\rightarrow \bigcap_{i \in \lambda} FUNTI_i(x) = \bigcap_{i \in \lambda} FUNTI_i(y)$ imply $x=y$

And $\bigcap_{i \in \lambda} FUNTI_i(x) * \bigcap_{i \in \lambda} FUNTI_i(x) = \bigcap_{i \in \lambda} (FUNTI_i(x) * FUNTI_i(x)) = CONS 0$

- iii. $\forall FUNTI_i(x) \neq CONS 0, FUNTI_i(y) \neq CONS 0$ and $FUNTI_i(x) \neq FUNTI_i(y)$ then $[FUNTI_i(x) * FUNTI_i(y)] \neq CONS 0$, it is hold

Then it is hold a D. S. in SBH

Example (4.6): Let $C = \mathbb{R}$ a real numbers, with operation $*$ and defined by:

$$FUNTI(x) * FUNTI(y) = \begin{cases} FUNTI(x), & \text{when, } x \neq y \\ 0, & \text{when, } x = y \end{cases}$$

For all, $x, y \in \mathbb{R}$

Then, C is a D. S. in SBH of C

Proposition (4.7): Let $\{FUNTI_i(x) \mid i \in \lambda\}$ it is a chain of a D. S. SBH. Then $\bigcup_{i \in \lambda} FUNTI_i(x)$ is a D.

S. in SBH of C .

Proof: to prove $\bigcup_{i \in \lambda} FUNTI_i(x)$ is a D. S. in SBH.

Since $FUNTI_i(x)=x$ and $FUNTI_i(y)=y$

- i. $\bigcup_{i \in \lambda} FUNTI_i(x) * CONS 0 = \bigcup_{i \in \lambda} (FUNTI_i(x) * CONS 0) = \bigcup_{i \in \lambda} FUNTI_i(x)$
- ii. $FUNTI \bigcup_{i \in \lambda} FUNTI_i(x) * \bigcup_{i \in \lambda} FUNTI_i(y) = \bigcup_{i \in \lambda} (FUNTI_i(x) * FUNTI_i(y)) = CONS 0$

$$\text{And } \bigcup_{i \in \lambda} FUNTI_i(y) * \bigcup_{i \in \lambda} FUNTI_i(x) = \bigcup_{i \in \lambda} (FUNTI_i(y) * FUNTI_i(x)) = \text{CONS } 0$$

$$\rightarrow \bigcup_{i \in \lambda} FUNTI_i(x) = \bigcup_{i \in \lambda} FUNTI_i(y) \text{ imply } x=y$$

$$\text{And } \bigcup_{i \in \lambda} FUNTI_i(x) * \bigcup_{i \in \lambda} FUNTI_i(x) = \bigcup_{i \in \lambda} (FUNTI_i(x) * FUNTI_i(x)) = \text{CONS } 0$$

iii. $\forall FUNTI(x) \neq \text{CONS } 0$, $FUNTI(y) \neq \text{CONS } 0$ and $FUNTI(x) \neq FUNTI(y)$ then $[FUNTI(x) * FUNTI(y)] \neq \text{CONS } 0$, it is hold

Then, it is a D. S. in SBH

Definition (4.8): Let C be a SBH-alge. and $G(X) = \{ x \in C \setminus \text{CONS } 0 * x = x \}$ is denoted by a .G-part of C.

Proposition (4.9): Let $X=G(X)$. Then every D. S. SBH is a commutation D. S. SBH

Proof :Let X be a D.S. in SBH

i. $FUNTI(x) * \text{CONS } 0$ by def.

$$= x * \text{CONS } 0 \text{ (by a comm).}$$

$\text{CONS } 0 * x$ by def. $G(x)$

$$= x = FUNTI(x)$$

ii $FUNTI(x) * FUNTI(y) = x * y$ (by a comm). $y * x$ (by def. SBH)

$$= \text{CONS } 0$$

and $FUNTI(y) * FUNTI(x) = y * x$ (by a comm). $x * y$ (by def. SBH)

$$= \text{CONS } 0$$

then $x=y \rightarrow FUNTI(x)=FUNTI(y)$ and $FUNTI(x) * FUNTI(x)$

$$= x * x \text{ (by a comm.)}$$

$$= x * x = \text{CONS } 0$$

iii. $\forall FUNTI(x) \neq \text{CONS } 0$, $FUNTI(y) \neq \text{CONS } 0$ and $FUNTI(x) \neq FUNTI(y)$ then $[FUNTI(x) * FUNTI(y)] = [FUNTI(y) * FUNTI(x)] \neq \text{CONS } 0$, it is hold

Then it is a comm. D. S. in SBH

Proposition (4.10): Let $C=G(X)$. Then every D. S. in SBH is hold:-

i. $FUNTI(x) * (FUNTI(x) * FUNTI(x)) = (FUNTI(x) * FUNTI(x)) * FUNTI(x) = FUNTI(x)$

ii. $FUNTI(x) * (FUNTI(y) * FUNTI(x)) * FUNTI(x) = FUNTI(x) * FUNTI(y)$

- iii. $(\text{FUNTI}(x) * \text{FUNTI}(y)) * (\text{FUNTI}(y) * \text{FUNTI}(x)) = \text{CONS } 0$
- iv. $(\text{FUNTI}(x) * \text{FUNTI}(y) * \text{FUNTI}(z)) * (\text{FUNTI}(z) * \text{FUNTI}(y) * \text{FUNTI}(x)) = \text{CONS } 0$
- v. $(\text{FUNTI}(x) * \text{FUNTI}(y) * \text{FUNTI}(x)) * (\text{FUNTI}(x) * \text{FUNTI}(y) * \text{FUNTI}(x)) = \text{CONS } 0$

Proof:-

- i. $\text{FUNTI}(x) * (\text{FUNTI}(x) * \text{FUNTI}(x))$ (by def. D.S in SBH)
 $= \text{FUNTI}(x) * \text{CONS } 0 = \text{FUNTI}(x)$
 and $(\text{FUNTI}(x) * \text{FUNTI}(x)) * \text{FUNTI}(x)$ (by def. D.S in SBH)
 $= \text{CONS } 0 * \text{FUNTI}(x) = \text{FUNTI}(x) * \text{CONS } 0$ by def. G(x)
 $= \text{FUNTI}(x)$
- ii. $\text{FUNTI}(x) * (\text{FUNTI}(y) * \text{FUNTI}(x)) * \text{FUNTI}(x) = \text{FUNTI}(x) * \text{FUNTI}(x) * (\text{FUNTI}(y) * \text{FUNTI}(x))$ by def. G(x)
 $= \text{CONS } 0 * (\text{FUNTI}(y) * \text{FUNTI}(x))$ (by def. D.S in SBH)
 $= \text{CONS } 0 * (\text{FUNTI}(y) * \text{FUNTI}(x)) = \text{FUNTI}(y) * \text{FUNTI}(x)$ by def. G(x)
 $= \text{FUNTI}(x) * \text{FUNTI}(y)$ by a comm.
- iii. $(\text{FUNTI}(x) * \text{FUNTI}(y)) * (\text{FUNTI}(y) * \text{FUNTI}(x)) = (\text{FUNTI}(x) * \text{FUNTI}(y)) * (\text{FUNTI}(x) * \text{FUNTI}(y))$ (by a comm).
 $= \text{CONS } 0$ by def. D.S in SBH
- iv. $(\text{FUNTI}(x) * \text{FUNTI}(y) * \text{FUNTI}(z)) * (\text{FUNTI}(z) * \text{FUNTI}(y) * \text{FUNTI}(x))$
 $= (\text{FUNTI}(x) * \text{FUNTI}(y) * \text{FUNTI}(z)) * (\text{FUNTI}(y) * \text{FUNTI}(z) * \text{FUNTI}(x))$
 $= (\text{FUNTI}(x) * \text{FUNTI}(y) * \text{FUNTI}(z)) * (\text{FUNTI}(y) * \text{FUNTI}(x) * \text{FUNTI}(z))$
 $= (\text{FUNTI}(x) * \text{FUNTI}(y) * \text{FUNTI}(z)) * (\text{FUNTI}(x) * \text{FUNTI}(y) * \text{FUNTI}(x)) = \text{CONS } 0$
- v. $(\text{FUNTI}(y) * \text{FUNTI}(x) * \text{FUNTI}(x)) * (\text{FUNTI}(y) * \text{FUNTI}(x) * \text{FUNTI}(x)) = \text{CONS } 0$
 $\setminus (\text{FUNTI}(y) * \text{CONS } 0) * (\text{FUNTI}(y) * \text{CONS } 0)$
 $\text{FUNTI}(y) * \text{FUNTI}(y) = \text{CONS } 0$

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